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David Wagner

Joseph Dicks

Paula Kristmanson

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Students' Language Repertoires for Prediction

David Wagner
University of New Brunswick, Canada
Joseph Dicks
University of New Brunswick, Canada
Paula Kristmanson
University of New Brunswick, Canada

Abstract: Communication about prediction is complex in a number of ways. First, language is by nature recursive — language is an indicator of meaning as well as a force that shapes meaning. Second, the same language used to communicate prediction in uncertain environments is used for other purposes. In this article, we describe how the recursive nature of language impacted the choices we made in a cross-sectional longitudinal study aimed at gaining insight into children's language repertoires relating to conjecture. We then explore some Grade 6 students' communication about prediction to develop insight into their meaning and meaning-making with prediction language. From this we raise questions about interpreting data from such contexts. Finally, we discuss implications for educators.

Keywords: mathematics education, language, degrees of certainty, conjecture, authority.

The understanding of possibility, risk, and certainty, like the understanding of any mathematical idea, is mediated by language. Certain language repertoires are necessary to convey the ideas. At the same time, the language used to describe these ideas shapes the way people conceptualize them. This recursive nature of language compelled us to develop a research project to investigate children's language repertoires in relation to conjecture. Having noted similarities in the language of conjecture and of prediction, we structured the classroom activities and interviews in the project to prompt students to make predictions. In this paper, we focus on our research choices in relation to this endeavour. First, we describe choices we made to gain insight into children's language repertoires. Second, we use some of the data from the project to identify issues relating to interpreting data in the characteristically mathematical contexts of conjecture and prediction.

Moving beyond our academic interest in mathematics education, we will argue that the issues we identify may be significant for understanding everyday experience. In particular, we will raise questions about the impact of mathematics class experiences that involve uncertainty on experience outside the classroom. We will also raise questions about the impact of intertextuality between uniquely mathematical ways of communicating about conjecture and everyday ways of interacting about authority.

The investigation of conjectures (hypotheses) is one of the most important mathematical processes. Much mathematics teaching focuses on enabling students to perform particular mathematical procedures, such as adding fractions, factoring polynomials, and calculating probability. These skills appear as standards in curriculum documents and frameworks (e.g., CCSSO, 2010) that are used by curriculum planners and teachers. Research and professional literature, including curricula (e.g. New

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Brunswick Department of Education, 2010) and curriculum frameworks, point to the necessity of students learning these intended outcomes through the exploration of mathematical problems.

When people explore a mathematical problem together, as with mathematical investigations in classrooms, it is necessary to have a way of suggesting an idea before knowing it is true. Rowland (2000) noted the centrality of such conjecture to mathematics, and coined this "space between what we believe and what we are willing to assert" (p. 142) as the Zone of Conjectural Neutrality (ZCN). Because of the recursive relationship between language and experience, the language resources available affect the possibilities for making conjectures.

As our research exemplifies, the language of conjecture shares language that describes probability. Rowland's work refers often to the necessity of expressing uncertainty for conjecture, and he draws heavily on linguistics literature that describes the way people express uncertainty. Our research illustrates the complex relationship between probability, itself an important mathematical concept, and conjecture, which is at the heart of teaching for understanding.

Our interest in language is not aimed to identify correct language. Rather it focuses on the language students use and asks what their language choices might tell us about the way they think about uncertainty. There is a range of English words that relate to uncertainty. Mathematics educators are likely to have particular ideas of what the words mean, which would differ from ideas of others. For example, in addition to everyday use of the word 'risk,' the concept has been studied in the fields of mathematics, psychology, business, and engineering. We find a general consensus that it references probability and uncertainty, especially as they relate to (perceived) consequences (e.g., Slovic, 2000). Our focus in this article is on the meaning and meaning-making we observe while students are confronted with uncertainty.

Communicating About Uncertainty

Our theoretical perspective for this research draws on the work of Vygotsky (e.g., 1962, 1978) and Wertsch (1991) related to the connections between thought and language, and, in particular, the central role that language as social interaction plays in the process of learning.

Nevertheless, we have found it a challenge to avoid deficit framing because of the shaping force of one's language repertoire. Indeed, we suggest that it is not possible to completely avoid deficit framing when analyzing language use. Deficit framing suggests that one's own way of speaking or thinking is superior by evaluating whether or not others have acquired the same skills. In the study of linguistic variation for numbers, which is the only area of mathematics register variation that has been documented significantly, Swetz (2009) pointed out how cultures have been rated on the scope of their number systems. In our research we are more interested in the potential for linguistic variation to open up opportunities to understand mathematics differently. For example, in the context of language repertoires for number, numbers are verbs in Mi'kmaq (Lunney Borden, 2010). Our conversations among ethnomathematicians suggest that this is not uncommon, though in English, numbers are adjectives or nouns. A question warranting attention is how this distinction affects one's conception of counting and arithmetic operations.

We want to bring the same kind of question to the study of different language repertoires for expressing uncertainty. However, at this point, we focus on the range of language strategies within English and French. In this article, we focus on English only. Thus our research addresses the larger question: How does linguistic variation express itself in relation to understanding probability? Because

linguistic variation in mathematics (besides the area of number) has not been researched significantly, the discussion requires careful research to move forward.

Reports on research of probability learning have included many examples of language spoken and written by children and their teachers. The same can be said about most subfields of inquiry in mathematics education, in which reporting generally focuses on understanding without significant consideration of the language that mediates this understanding. Morgan et al. (2014) have said that this approach is naïve: "Naïve conceptions of language as a transparent means of transmission of ideas from speaker to listener have been seriously challenged by current thinking about communication" (p. 847). A further naiveté identified by Morgan et al. (2014) is evident in research that views "language as a barrier to learning that must be overcome" (p. 846).

Our review of the subfield of inquiry into probability learning identifies the first kind of naiveté (language as a transparent window) and thankfully not the second (language as a barrier). We position our article as a small first step away from the naïve use of language as a transparent window into probability understanding. It is a small step because we merely problematize apparent meaning-making. The concern we have in our reporting is that it may be taken as a caution to speak and write more clearly, which would relate to the second kind of naiveté. Rather, we will claim that ambiguity is inevitable. However, it is important to understand the nature of this ambiguity.

We see some development of attention to language and related developments in its conceptualization within the subfield of research on probability teaching and learning. Ben-Zvi and associates moved from a naïve conception of language as transparent medium to language in interaction with activity. Ben-Zvi and Arcavi (2001) conducted a study that is similar to ours in many ways. They explored the conceptions of data among children of a similar age to children in our study. In their analysis, they attended carefully to the linguistic distinctions made by their participants. They identified the distinctions among the things said by their participants as representations of different conceptualizations: "The verbal abilities of these students allowed us to follow, at a very fine level of detail, the ways in which they begin to make sense of data, data representations, and the 'culture' of data handling and analysis" (p. 35). This is an example of research that conceptualizes language as a transparent window into understanding. Later, Ben-Zvi et al. (2012) presented a more complex conceptualization of language, as they trace its development in individuals as they interact with increasingly complex probability tasks: "prediction tasks, helped in promoting the students' probabilistic language" (p. 913).

The research we describe in this article is inspired by an unexpected result from an earlier collaboration that was focused on authority in mathematics classrooms (e.g., Herbel-Eisenmann, Kristmanson & Wagner, 2011; Herbel-Eisenmann & Wagner, 2010), for which we drew on Rowland (2000) for interpretation of students' use of modal verbs. The ambiguity of meaning among modal verbs highlighted our attention to the two mathematical phenomena of probability and reasoning. Our current attention to modal verbs was bolstered by the strong attention to modal verbs in literature on additional language teaching and on teaching in multilingual contexts. Thus our interest in uncertainty focuses us on participants' repertoires for expressing modality, especially on the use of modal verbs.

Modality

Modality refers to linguistic tools for expressing degrees of certainty, for example the use of modal verbs like *must* and *could*. "It must be six" is stronger, and thus has higher modality than "It could be six." Rowland (2000) identified assertions that appear without expressions of modality as *root modality*. These assertions, which may be called bald assertions, are stronger than even the highest

modal expressions because saying 'it is six' does not even recognize the question of other possibilities. Figure 1 illustrates a range of meanings of modal verbs (from Herbel-Eisenmann, Kristmanson & Wagner, 2011, p. 2).

<table>
<tr><td>it is six</td><td colspan="2">high polarity (root modality)</td></tr>
<tr><td>it must be six</td><td rowspan="4">modulated polarity</td><td>high modality</td></tr>
<tr><td>it might be six</td><td rowspan="2">low modality</td></tr>
<tr><td>it could be six</td></tr>
<tr><td>it cannot be six</td><td>high modality</td></tr>
<tr><td>it is not six</td><td colspan="2">low polarity (root modality)</td></tr>
</table>

Figure 1. Range of meanings of modal verbs

Some modal verbs—e.g., 'can'—are ambiguous. "You can be excused from the table" indicates a degree of obligation; "You can finish the race" indicates ability; "I can help you" indicates inclination; and "It can be a six (because one of the remaining cards in the deck is a six)" indicates probability. When students (or others) hear the word *can*, we wonder what it means to them? Linguists, Martin and Rose (2005), codified these forms of meaning of ambiguous modality language, including categories that describe degrees of certainty relating to usuality, probability, obligation, inclination, and ability (p. 50). Their distinction between usuality and probability is not very clear to us. Rowland (2000) distinguished among the various meanings using the terms *alethic* (or logical), *deontic*, and *epistemic*. Alethic modality is most relevant to communication about probability. For example, in such a context, "the next card *could be* a six" is a statement of certainty if one knows that there is a six in the pile from which cards are being drawn. However, *could* often expresses doubt. Deontic modality references obligation or authority. Epistemic modality references levels of belief and thus is most important in conjecture.

Modality can be expressed using other language strategies, in addition to modal verbs. For example, the adverb 'probably' expresses strong confidence short of certainty, and the adverb 'possibly' represents the opposite end of the scale, with approximately the same modality (the same level of certainty), but strong confidence that an event is nearly impossible. However, there are no standardized quantifications of these expressions in everyday use. Nevertheless, in quantitative research methodologies certain expressions of modality are taken to have numeric thresholds relating to correlation coefficients, for example. Similarly, in textbooks and other resources for learners of additional languages, modal verbs have appeared with numeric ranges given as percents.

In addition to the alethic and epistemic modality, which relate most closely to probability and reasoning respectively, our analysis will identify modal expressions as referencing emotions — for example, desires and fears. Pratt et al. (2012) demonstrated the influence of emotion in their study of the priority heuristic. The participants' worries influenced their assessment of probabilistic situations due to empathy for the subject of the context story, which was related to the risk in a medical procedure. One of the conclusions the researchers draw from their study is that "Teaching about risk carries with it certain obligations. We see one pedagogic challenge as sensitising people to their own decision-making, including their emotionally-charged heuristic thinking" (p. 940).

Our analysis will focus on the range of meaning in participants' language choices, not on the conceptualization of probability. Nevertheless, our data could provide examples of various heuristics and misconceptions identified in the stochastic reasoning literature. Most prominently, we see our participants straddling the boundary between what Fischbein (1975) referred to as 'primary intuitions'

and 'secondary intuitions'. The primary intuitions are "cognitive acquisitions which are derived from the experience of the individual, without the need for any systematic instruction", and the 'secondary intuitions' are "acquisitions that have all the characteristics of intuitions but . . . are formed by scientific education, mainly in school" (p. 117). The finer-grained distinction developed by Jones et al. (1999) — distinguishing among prestructural, unistructural, multistructural, and relational thinking — could be used to identify student thinking on Fischbein's fuzzy boundary, but we are more interested in distinctions in language than on rating or evaluating children's understanding.

As noted above, we are especially interested in the way children use language to express modality in mathematics contexts (and beyond) because modality is important in conjecture, as noted by Rowland, and to describe uncertainty, and it is also important to understand other points of view, as noted by Shaffer (2006). In our research, we did not aim to look for holes in children's language repertoires. Rather, we focused on attending to the ways they talked about their understanding, to help us see a range of ways to talk about and understand conjecture and uncertainty.

Methodological Choices

The data for our cross-sectional longitudinal study comprise audio- and video-recordings from English-medium and French Immersion instructional contexts in an Anglophone region in Canada. Students worked in groups in class and were subsequently interviewed, extending the group work. At the end of the interviews we asked the students about the meaning of the words they used to describe degrees of certainty: How do the participants distinguish between obligation and probability, as noted above?

For each mathematical context we tried to avoid using specialized mathematical language ourselves. We know from second language acquisition literature that good language learners are generally good at noticing and, subsequently, using the language used in interactions with more able speakers (Long, 1996). Furthermore, people tend to follow the grammatical patterns of their interlocutors, for example in research interviews (Wagner, 2003). We wanted to hear what language skills the children in our research used to communicate their ideas without setting them up with the specialist language to build on. As we struggled to construct problems without use of specialist language, we found that larger narrative contexts made this possible. Other strategies we considered became grammatically awkward.

Our narratives also made the problems accessible to very young children, perhaps partially because of the lack of specialty language, but mostly, we think, because they connect to children's experience. If we were interested in assessing the level of probability knowledge among our participants, we would have to be wary of the effect our narrative contexts could have on supporting their developing conceptualizations. Pratt & Noss (2002) engaged children in micro-world exploration and noted how their understanding developed through their experience within the research. The same caution would apply if we were interested in rating or assessing our participants' language repertoires. As noted by Ben-Zvi et al. (2012), whom we cited above, their participants seemed to develop language for describing probability when engaging in well-constructed tasks involving increasingly challenging questions about uncertainty. However, we are not as interested in rating our participants' understanding and language, as we are in documenting the range of language repertoires used to interact with uncertain contexts.

In addition to embedding our questions in a narrative context, we attempted to avoid specialized uncertainty language when we interviewed participants about their predictions in contexts based on

uncertainty. At first, our team agreed it would be acceptable to use a language strategy only after the participant did, not before. This proved extremely difficult; indeed, in the interviews we often used words we intended to avoid, and sometimes used incorrect or awkward structures in attempts to avoid this. After completing most of the first year's classroom and interview interactions, we agreed amongst our team that we should be less paranoid about avoiding specialty language, but thought that this issue might impact the interpretation of the data.

The first year's participants were in Grades 3, 6, and 9. We had them play a modified version of *skunk*, a game often used in the teaching of probability (e.g., Brutlag, 1994; Neller & Presser, 2004). We had them play in pairs so that they would be more likely to talk with each other about their ideas and strategies. We introduced the game with a narrative like this, varying slightly between contexts because we did not script the narrative:

> One day, I was picking strawberries in the forest. After a while, when my basket was quite full, a skunk wandered into the berry patch. I ran away so the skunk would not spray me. I lost the berries in my basket when I ran off.

This narrative also gave a reason for calling the game *skunk*. Participants had a pile of beans (representing the berry patch), a cup (the basket), and a bowl (home). When the researcher rolled the die and called out the number, participants put that number of berries in their basket. A six represented the skunk. When it was rolled, everyone would lose the berries in their baskets. On the other hand, if they "went home" (dumping their beans into their bowl) before the appearance of the skunk, their berries were safe. We played seven rounds — one berry-picking expedition for each day of the week.

We played the game with participants in their classrooms first. The following day we interviewed groups of students and played again but with six cards bearing the numbers one to six instead of the die. The interviewer would not replace the cards into the deck until the deck was completely played out, at which time it would be reshuffled. Thus the participants experienced the difference between independent and mutually exclusive events in probabilistic situations.

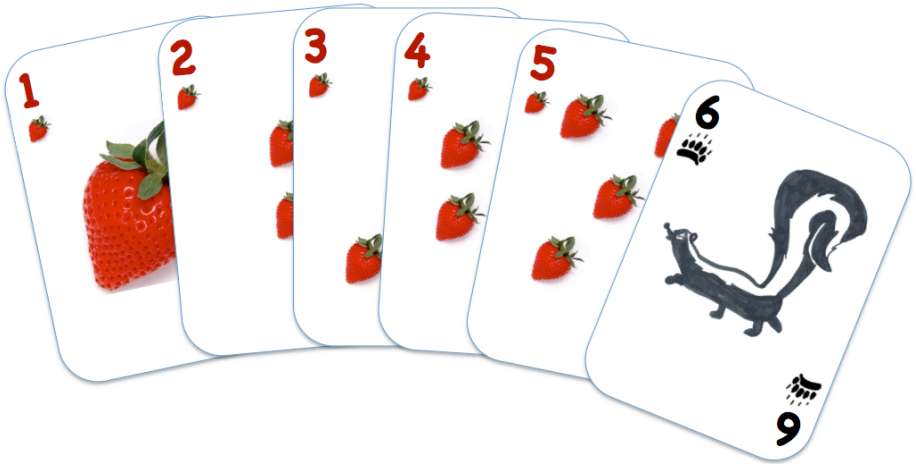

Figure 2: Skunk cards

During the game, the interviewer asked the participants to say why they made their choices about when to "go home." After the game, the interviewer asked participants about specific things they said, asking for clarification on meaning. The camera operator was helpful in this regard, acting as a second

interviewer. She or he could make notes on what participants said, which was relatively difficult for the primary interviewer who was busy with the cards and interaction.

For this article, we focus on one interview with four Grade 6 students playing the game of *skunk*. This group of students was not identified by their teacher as exceptional in any way. The school is in an area that has relatively low socio-economic indicators. As noted above, these four students played *skunk* in class the day before, and subsequently one of our research team interviewed them — first playing *skunk* with cards instead of a die, and then asking them about some language meanings. We asked them to play *skunk* in pairs, and they somehow came to an implied agreement that the pairs were competing against each other.

Though we focus on the interview with the four students described above, we make some references in our discussion to other data within the project to illuminate certain findings through comparison. For this reason we describe the second year research prompts as well. Instead of playing *skunk*, participants from Grades 4, 7, and 10 (catching some of the same students as the previous year, one grade earlier) predicted the 50th car on different trains based on the first seven cars. The narrative context of this situation had the researcher tell a story about waiting with a friend for a train at a level crossing, and deciding to predict what kind of car the fiftieth car would be. Trains were then shown using presentation software, with an engine and the first six or seven cars, each labelled with their number, as shown in Figure 3. After students made their predictions about the 50th car, we had the train accelerate and then decelerate to settle on the 50th car. As with the game of *skunk*, we had students work in groups to draw out communication.

Figure 3: First train

The sequences presented to students varied considerably to defy expectations of certain kinds of patterns. The cars were distinguishable by colour and shape — Yellow (Y) cars were rectangular boxcars, green (G) cars were tankers, and blue (B) cars were flatbeds carrying big triangles. Train 1 showed Y,G,B,Y,G,B,Y and continued with a pattern of threes (YGB). Train 2 showed Y,G,Y,Y,G,Y and continued with a pattern that increased the number of Ys before each G — i.e., Y,G,Y,Y,G,Y,Y,Y,G, etc. Of course, the initial seven cars could have suggested a pattern of threes (YGY) similar to the previous train — i.e., Y,G,Y,Y,G,Y,Y,G,Y, etc. For this train, we stopped the train at around the 25th car to let students reconsider their predictions. We invited students to tell us their reasoning whenever possible. Train 3 showed B,G,B,B,G,G, etc. and continued with B, B, B, G, G, G, etc. with increasing groups of B and G. The interviews on the following day started with Train 4 showing Y,B,G,Y,Y,B,G. It continued with groups of four (YBGY) — i.e., Y,B,G,Y,Y,B,G,Y etc. Train 5 started with Y,B,G,B,P,B,Y and continued with a random collection of cars, in which the colours started to misalign with the shapes and new kinds of cars appeared, including ones carrying animals. As with Train 2, we stopped train 5 at around the 25th car so we could talk with the students as they reconsidered their predictions. In addition to the confounding randomness of the fifth, (perhaps "avant-garde") train, there was no 50th car — it had only 42 cars. As with *skunk*, we ended these interviews with questions about distinctions among various language choices we heard the students use.

Language Used to Express Uncertainty

The four 11- and 12-year olds in the group whose interview we focus on for this article show considerable language repertoires, which we found to be the case for even the most mathematically and linguistically novice students in this project, even the Grade 3 French Immersion students who were in their first year of French-medium learning. As noted above, we were the most careful about and attentive to modal verbs in our analysis because of our earlier research and teaching work, but we identify other ways to describe levels of certainty as well. We present here a narrative of the game played in the interview, followed with more detailed analysis of the discussion about the meaning of some key modal verbs.

Playing skunk

We began the interview with a question about the dice version of the skunk game played in class the day before: "You know how the skunk came on the six, what if the rule was that the skunk came on the three instead of the six? Would you get more berries or less?" (turn 30). The students demonstrated either a misconception (possibly due to the prevalence of sixes and the paucity of threes on the day before) or based their answers on primary intuition. Nevertheless, they gave a few expressions that they seemed to feel had similar meaning. Chris discounted the "difference" between the two sets of rules by saying "It wouldn't really make much of a difference" (turn 31). Dale needed no special language to agree, simply saying, "No" (turn 32). Chris added the expression "an even chance" with an indicator of reasoning — 'because': "Because they are all an even chance" (turn 33). Finally, Terry said, "it's just the roll of the dice" (turn 38), apparently using the word 'just' to suggest that nothing different happens with either set of rules. Thus there seemed to be four different ways of expressing the idea that the probability remained unchanged.

We then moved to playing *skunk* with the cards. After shuffling the deck of six cards, the first cards played were the 2, the 1, and then the skunk. Each pair of students lost the three berries collected before the skunk had arrived. The three cards (2, 1, and skunk) were laying face up on the table. The interviewer, holding the remaining three cards, gestured that the students should decide to stay or go home. Dale pointed at the skunk card and said, "Well, the skunk is right there" (turn 74). Dale needed no special language to indicate certainty — the gesture along with the bald assertion sufficed.

The interviewer then drew attention to the next unrevealed card: "Okay, so what about this one?" (turn 84). Chris responded with, "That would be like around five." This expression employs a mix of uncertainty language. "Would be" expresses prediction. "Like" lowers the modality even more, as does "around" (though it is more normally used to describe estimation).

The 4, and the 3 were played next so five cards now laid face up on the table (2, 1, skunk, 4, 3). Chris asked, "Do you have a second skunk in there?" (turn 95). The interviewer was surprised by this question and responded, "I showed you the cards before. What cards were they?" (turn 96). Chris again expressed doubt, "Trickster" (turn 97). Chris seemed to think the interviewer might have secretly switched one of the cards.

The interviewer played out the deck, picked up the cards and shuffled them. This time the skunk came out first. One of the pairs of students went home before the skunk was played and thus averted losing their berries. The interviewer asked, "Did you know? Did you know that this was the skunk?" (turn 124). Chris said, "No, I just, kind of had a feeling" (turn 126). Dale said, "had a feeling" (turn 127) simultaneously with Chris. This reference to having a feeling did not seem to be associated with emotion. Rather, it seemed to be a strategy for expressing the act of prediction. Interestingly, both Chris and Dale introduced the expression simultaneously.

By contrast, close to the end of the game, emotion became palpable and the students talked about risk. Chris said, "Don't want to take our chances" (turn 160). Leslie then reacted with emotion to the emergence of the skunk as the first card of the newly shuffled deck (this was the second time the skunk was the first card in the deck): "Oh, come on!" (turn 164) laughing with what appeared to be a mix of delight and frustration. After the five remaining cards were played out, for which the students wisely elected to stay in the berry patch, the cards were reshuffled and the students discussed their strategy in earnest. Leslie said, "I'm scared" (p. 191), expressing emotion but perhaps also expressing awareness of the possibility of the skunk appearing early in the deck again. Terry assessed the risk of loss in relation to the possibility of catching up to the other pair: "I mean, if you guys still stay, then we really don't have anything to lose" (turn 198). Later Terry added, "One of us won't lose everything" (turn 204), and Leslie responded, "It is probably going to be us" (line 205). This instance of 'probably' seemed to be based both on calculation and fear.

As noted in the above narrative, for each idea there seemed to be more than one way of expressing it, including very simple statements that indicated agreement with another utterance. Also noted above, the prediction, conjecture, and emotion seem to be intertwined in the students' utterances.

Discussing meanings of modal verbs

We turn now to analysis of the modal verbs. The modal verb *have* expresses high modality because it refers to events that must occur. The interviewer used it first (though trying to avoid doing so) in turn 111, but it did not get used again until turn 229 when Chris talked about the difference between playing skunk with cards and with the die: "It's easier this way because when the skunk first came you just don't have to worry." It wasn't used again until the interviewer asked questions about its meaning. Here is an abbreviated version of that discussion (omitting diversions).

319 *Interviewer*: [Yesterday] I heard Terry say when you're working in your groups, "Do we both have to write this down?" So what's the difference between "it has to be the skunk" and "she has to write this down"? Is the "has to" the same? "This has to be the skunk." "She has to write it down." Do you notice a difference between them?

...

346 *Terry:* Do we both need to, like, do we both need to write it down?

347 *Interviewer:* No, but it's a proper use of the word. But is it the same as "this has to be the skunk"?

348 *Terry:* No.

349 Interviewer: No? Why not?

350 *Terry:* Because you know it has to be.

351 *Dale:* It absolutely has to be.

352 *Interviewer:* It absolutely has to be.

353 *Terry:* Yeah.

354 *Interviewer:* But when asking "do you have to" it's not absolutely.

355 *Terry:* No, yeah.

356 *Interviewer:* Okay

357 *Dale:* Because the fire bell or something could ring or something and you all go outside and you don't have to write it down.

358 *Interviewer:* Don't have to write it down but if the fire bell rung this would still be the skunk.

359 *Dale:* It would still be the skunk.

We note that, to clarify meaning, the students introduce new vocabulary that was not part of the interview up to this point. Terry used the modal verb structure "need to" to emphasize the necessity of "have to." Dale introduced the adverb *absolutely* to further emphasize this sense. The students distinguished between instances of 'have to' depending on context.

We had a similar conversation about the modal verb *can* which had been used in its various forms, including *can't*, by the students in the interview. We started this part of the conversation by referencing Dale's writing in class earlier. When asked what is the greatest number of berries they could get in a day, Dale had written, "You can get any number because it could just keep going." (This was with playing skunk with a die.) The researcher also referred to Dale saying in the interview that it is different with the cards because "we can't keep going." Again, this is an abbreviated version of the conversation that ensued.

391 *Interviewer:* That *can't* – If you're wanting to go visit your friend, and your mother or father says that you can't go over to your friend's house, is it the same kind of *can't*

392 *Terry:* No, that means you're not allowed.

393 *Interviewer:* Not allowed. So how do you know, if you teacher says that you can't do something, whether she is teaching your something?

394 *Terry:* It means no, you're not allowed to.

395 *Interviewer:* You're not allowed to.

396: *Terry:* Yeah.

397 *Interviewer:* Of how do you know it's not the kind of *can't* that Dale said? Where it just can't possibly happen? How can you tell the difference?

398 *Terry:* By the way she says it.

399 *Chris:* Yeah.

...

418 *Camera operator:* When you said earlier "you can't win," which one is that closest to? Remember, when you looked at your basket and you said, "Oh, we can't win." Is that like the "you're not allowed" or is it.

419 *Terry:* It would be you can't.

420 *Leslie:* You don't.

421 *Terry:* Like you, it's impossible, like.

422 *Leslie:* Yeah, it's impossible.

423 *Terry:* Well, it was because if you added it all up, the skunk...

424 *Dale:* You'd only get, like, fifteen.

425 *Terry:* The skunk would have come.

426 *Chris:* Yeah, you'd only get fifteen so if the skunk is that

427 *Leslie:* Or seventeen.

428 *Dale:* No, because we had thirty-five and then I counted them all up form the five and we still wouldn't have enough.

429 *Interviewer:* Okay.

430 *Terry:* Because the skunk was gone.

431 *Interviewer:* It would have been impossible.

432 *Terry:* Yeah, yeah.

433 *Interviewer:* So if someone says can't, … if I told you that you can't divide by zero in a lesson on dividing would you think that that means that you're not allowed to or that it is impossible to do?

434 *Chris:* That it is impossible.

435 *Interviewer:* Why would you think that?

436 *Terry:* Because you can't divide by zero.

437 *Interviewer:* Why can't you?

438 *Terry:* Because it is impossible.

439 *Interviewer:* How do you know?

440 *Chris:* Because you can't.

441 *Terry:* Because you can't.

442 *Chris:* If it is zero, you can't put it in any groups.

In this case, Terry introduced the adjective *impossible*, to clarify the meaning of *can't*. No one had used the word before this in the interview. As with "have to", the students distinguished among instances of *can* and *can't* based on context. During and after this interview, we wondered how students could make this distinction for instances in which they do not know a convincing logical argument for the assertion. With the example of division by zero, the students now knew that it is impossible, but how might they have thought about it the first time they heard their teacher say, "you can't divide by zero"?

Overview of linguistic strategies

The students introduced three adverbs/adjectives that indicate degrees of probability into the interview. The word *probably* was first used by Leslie and not used again by others. When Leslie and Terry were considering whether or not to make the same choice about going home or not as the other group, Terry remarked, "One of [our groups] won't lose everything and the other would" (turn 204), and Leslie replied, "It is probably going to be us" (turn 205). The adverbs *absolutely* and *impossible* (sometimes an adjective) came up in the conversations about language choices when the students were trying to explain what the modal expressions meant, as noted above.

Other modal verbs used included *would*, which was first used (accidently) by the interviewer and used liberally later by the students, and *may* as in "you may be able to win" (Dale, turn 263). Another specialized linguistic form used by a student was the if-then statement, first used by Chris: "If it was two

numbers, then it would make a difference" (turn 39). This was in the context of discussion about the playing *skunk* with a die.

In addition to the relatively specialist terminology for modality (the modal verbs and adverbs), students expressed degrees of certainty in other ways. Terry introduced the modal expression "I think" in a conversation about playing skunk with the die. The researcher had asked if the number of berries they gathered would be different if the skunk came on a one instead of a six, to which Terry replied, "I think it would because we roll the one a lot" (turn 45). Terry introduced another expression to describe the differences between playing skunk with cards and with the die. In turn 236 Terry said, "You never know what is going to happen (with the die)." Terry also said, "the odds are harder" (line 273) when the probability of success became lower. Dale was inventive too, and used the expression "I had a feeling" (turn 126) after "going home" to stay safe. This statement was in reply to the researcher asking, "Did you know that this was the skunk?"

Finally, the absence of any modal expressions is significant in the consideration of modality as well. The use of bald assertions can replace strong modal verbs or adverbs. Dale said, "the skunk is right there" (line 74) while pointing at the skunk card, as yet unrevealed but evidently the skunk by deduction. We might expect "the next one has to be the skunk" or "I am certain that the next one is the skunk" but the bald assertions serves the same function. Chris did the same on line 82 saying "it's there." In this interview (and others), there were many instances of this method for expressing certainty.

Discussion

The four students in the interview described above demonstrated a wide repertoire of language for expressing degrees of certainty. Each of them used a range of expressions, and each of them introduced expressions that no one else had used before. Terry was the most talkative in the discussions about language meaning, but we caution that it would be unwarranted to make conclusions in comparison to the others on this basis. Many of the expressions introduced by the students came late in the interview, which tells us that if the interview had been shorter, we would not have known whether or not the students had these expressions in their repertoires. This serves as an exemplary caution against deficit-based assessments. The development of increasing linguistic strategies may also raise questions about the work of Ben-Zvi et al. (2012), cited above — were the participants in that study increasing their linguistic strategies or simply waiting to employ their strategies when they are needed? Both are reasonable explanations.

Another phenomenon that challenges deficit-assessment is that when one student said something, there was no need for the others to say it again or even speak about it unless they disagreed. We cannot assume someone does not possess certain language simply because they do not use it. However, we can claim that a student has an expression in their language repertoire if they introduce it. This is why we went to the lengths that we did for structuring our prompts carefully.

Related to this, we note that if students use an expression that has just been used by the teacher or interviewer, perhaps in a recent class, or in the interview itself, the student use may not be fully independent. They may not be able to use the expression autonomously at a later time or in a different context.

In addition to using (and introducing) specialist language, the students in the interview at times demonstrated ability to convey their meaning using very limited technical language. In particular, they could make their ideas clear when talking about the extremes of certainty — when events were impossible or certain. The more specialised language seemed to be relied upon either for describing

events that were somewhere between impossible and certain (aligning with a result from Ben-Zvi et al. (2012)), and for clarifying meaning on the extremes when pressed to do so.

As noted above, Rowland (2000) introduced the idea of the zone of conjectural neutrality to describe language that specifies degrees of certainty, which is "in defiance of the cultural norm that the pupil is judged to be 'right' or 'wrong'" (p. 211). He claimed it to be helpful for a conjecturing atmosphere. We note that the same terminology is used to describe probability, and thus specialized modality language can defy situations in which predicted results may be between impossibility and certainty. We have only begun to consider the implications for pedagogy considering the phenomenon that language is shared for both conjecture and probability spaces.

This brings us to discussion of the second research context, which was set up to be similar to but distinct from the game of skunk — a twist on the context. In both contexts, students were making predictions. What is the difference between a train and a pile of cards, both of which are sequences of physical objects? One difference is that the cards are shuffled in front of the students and train cars are sequenced with some sort of intention in advance. Nevertheless, our experience of real trains is that the sequence of cars seems to be quite random, or in groups (e.g., the boxcars first, followed by a bunch of tankers, followed by a few flatbeds, and finally the rest of the tankers). We have never seen trains with patterns similar to the ones introduced in our research — patterns like yellow boxcar, green tanker, blue flatbed, yellow boxcar, green tanker, blue flatbed, etc. A Grade 4 student in the second year of research involving the trains became increasingly frustrated with the rest of the class identifying what the 50^{th} car would be. This student kept saying that it is impossible to know, while the class continued to ignore him. This student refused to make predictions. This reminds us of an observation noted by both Falk (1981) and Chernoff (2009) — that people often see randomness when it is not present, and see order when randomness is present.

This tension also points to the presence of some sort of pedagogical contract in which students generally expect intention from their teachers. We would suggest that this contract extends to researchers, whom, from our experience, are associated with teachers. Even in the game of *skunk*, when the interviewers showed all the cards to the students and shuffled the cards directly in front of them, the students sometimes expected some kind of lesson — the appearance of a second skunk card, for instance (for example, Chris expressed this fear in the example above). With the trains the phenomenon was more obvious; the students (with some exceptions, most notably the Grade 4 student noted above) assumed that the patterns would continue even though the researcher and teacher never said that these were patterns and the described context was one of a real-life train. The apparent frustration displayed by almost all the participants when they saw the fifth train (the random, avant-garde train) made clear to us the students' expectations for pattern. There is something about the transposition of a narrative into a mathematics classroom that changes it to a scenario in which everything should be predictable (and known by the teacher, or researcher). We suggest that this transposition may confound some claims in the literature based on classroom interactions or research interviews about probability.

In our research project, student predictions were based on both the probabilities inherent in the given scenarios and the students' second-guessing of teacher/research choices in constructing scenarios for pedagogic or other reasons. This raises questions about how students experience probability learning. Uncertainty in the mathematics classroom is experienced differently than it is outside the classroom. Furthermore, we note that the language of conjecture shares language with probability and risk-related emotion, and so we wonder whether this ought to confound similarly our understanding of the way students experience proof and reasoning.

Finally, we turn our attention to implications beyond the classroom. Increasingly significant social phenomena, such as climate change, involve both calculations of risk, which are based on assumptions, and conjectures (hypotheses). The fact that risk calculation and conjecture share terminology may complicate communication about such social phenomena. Furthermore, language used to express risk calculation and conjectural language of certainty are also used to express authority, as demonstrated in the above conversation about authority — notably the discussion about the modal verbs 'have to' and 'can't'. When people in the public sphere who appear to be scientific make claims that sound authoritative, how are listeners to know whether these claims are warranted expressions of calculation-based certainty? It is incumbent upon mathematics teachers to be aware of these shades of meaning and the risk of ambiguity on such important social issues.

This brings us to the question about what educators might do in the face of the ambiguity in this language. Pratt et al. (2012) conclude their study of emotion-laced contexts of risk assessment saying, "Teaching about risk carries with it certain obligations. We see one pedagogic challenge as sensitising people to their own decision making, including their emotionally-charged heuristic thinking" (p. 940). We suggest that this imperative is warranted not only for discussion of emotion but also for discussion of linguistic ambiguities. As educators we are obligated to help students and other educators become aware of the meaning associated with the language of prediction (uncertainty), emotion, authority, and reasoning. The ambiguity of this meaning is probably inevitable however. First, because uncertainty is by nature worrisome and thus triggers emotion. Second, people who want to establish authority will co-opt the language of logic to emphasize what they consider to be necessity. Third, reasoning requires the acknowledgment that one might be unsure of an idea. Developing a fuller repertoire of language to express ideas may help people negotiate meaning, but will probably not make meaning and meaning-making entirely clear because of the inherent connections among these concepts.

Perhaps an appropriate way to close this article is to employ one of the ambiguous expressions discussed here — good educators *have to* (or must) make their students aware of the ambiguity in prediction and reasoning language. We have to because it is a moral obligation. And we have to because there is no way around this awareness when we aim for clarity.

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