AN EXPLORATION OF HIGH SCHOOL FRENCH IMMERSION STUDENTS' COMMUNICATION DURING COLLABORATIVE MATHEMATICS PROBLEM-SOLVING TASKS

by

Karla Culligan

Bachelor of Arts, Mount Allison University, 2000

Bachelor of Education, University of New Brunswick, 2002

Master of Education, University of New Brunswick, 2008

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Supervisor:	Joseph Dicks, PhD, Faculty of Education
Examining Board:	Paula Lee Kristmanson, PhD, Faculty of Education
	Roger Saul, PhD, Faculty of Education
	David Wagner, PhD, Faculty of Education
External Examiner:	Miles Turnbull, PhD, Vice-Principal Academic Administration, Bishop's University

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ABSTRACT

The relationship between language and mathematics is complex, and arguably more so when students are learning mathematics through the medium of a second language. This study aims to describe, interpret, and understand how secondary French immersion mathematics students communicate, that is, how they use and attend to language and mathematics as they work collaboratively on problem-solving tasks in their second language. This study in grounded in sociocultural theory (e.g., Lantolf, 2000; Swain, 2000, 2008; Vygotsky, 1962, 1978) and the concept of the mathematics education register (e.g., Halliday, 1978, Pimm, 1987, 2007; Moschkovich, 2007) as the theoretical frameworks in order to highlight the social nature of learning and the key role of language in learning. With the theoretical frameworks guiding the remainder of the study, literature was reviewed that related to French immersion student achievement in mathematics, tensions inherent in bilingual mathematics classrooms, codeswitching or the use of the first language in second language mathematics, and what it means to "do" mathematics in a second language. This classroom-based study involved multiple site visits and working with 22 Grade 9 French immersion mathematics students in two different classes, along with their classroom teacher. Materials included a mathematics problem-solving task that required students to engage in reading, writing, oral interaction, hands-on modelling, and graphic representations. Data were collected via classroom-based audio recordings that were then transcribed verbatim; these data were triangulated with researcher fieldnotes, students' written texts, and post-hoc stimulated recall interviews. Data were analyzed using coding frameworks for language-related episodes (LREs), mathematics-related episodes (MREs), and instances of first language use using

a priori codes as well as emergent codes (Barwell, 2009a, 2009c; Halliday, 1985; Moschkovich, 2002, 2007; Swain & Lapkin, 1998, 2000, 2013). The discourse analysis was extended using Gee's (2014) theory and method, as well as the associated tools of inquiry (Gee, 2011). Several task-related findings suggested that student talk (rather than teacher talk) dominated the activity, that students mainly talked about the mathematics at hand, and that they used most of the anticipated problem-solving strategies to work through the task, although to varying degrees of thoroughness and success. Results showed that students engaged in various kinds of LREs and MREs, especially related to lexis and lexicogrammar, and also, although to a much lesser extent, other language forms. The LREs involved non-mathematical items, non-academic-mathematical items, and academic-mathematical items. The MREs mainly involved students' describing mathematical situations and expanding in order to provide explanations. Instances of first language use emerged with the LREs (especially lexical) and the MREs (especially expanding with repeat/restate). The first language was also used to move the task along and for interpersonal interactions (especially vernacular and to express feelings). Theoretical and practical implications for educators and policymakers are given based on the salient findings of the study. Suggestions for future research are also explored.

DEDICATION

For Shayna and Ryan.

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LIST OF ABBREVIATIONS

AAAL = American Association for Applied Linguistics

ACTFL = American Council on the Teaching of Foreign Languages

CLIL = Content and language integrated learning

CPF = Canadian Parents for French

CUP = Common underlying proficiency

EFI = Early French immersion

ELL = English language learner

ELPA = English Language Proficiency Assessment

EQAO = Education Quality and Accountability Office

ESL = English as a second language

FILA = French Immersion Language Arts

L1 = First language

L2 = Second language

LFI = Late French immersion

LRE = Language-related episode

FI = French immersion

FSL = French as a second language

MRE = Mathematics-related episode

NB EECD = New Brunswick Department of Education and Early Childhood Development

NCTM = National Council of Teachers of Mathematics (NCTM)

REB = Research Ethics Board

SCO = Specific curriculum outcome

SCT = Sociocultural theory

SLA = Second language acquisition

SUP = Separate underlying proficiency

UNB = University of New Brunswick

ZPD = Zone of proximal development

1.0 INTRODUCTION

"Language is about more than words; mathematics is about more than numbers" (Barwell, Leung, Morgan, & Street, 2005, p. 146). *Language* and *mathematics* are complex, multifaceted terms that can be defined differently depending on one's particular areas of interest, focus, and study. My work explores oral language, with a special focus on spoken interaction. It also investigates mathematics, or, more specifically, how students communicate mathematically when working collaboratively. This work responds to current needs identified in the L2 and mathematics education literature for increased understanding of the nature of student talk in the classroom. Recent work in L2 classrooms has explored how language use and language learning can co-occur during collaborative interactions (e.g., Swain, 2010). In mathematics education, there is recent increased emphasis on the importance of students' mathematical communication skills (e.g., National Council of Teachers of Mathematics [NCTM], 2000) and on the complex and important role of language in mathematics (e.g., Morgan, Craig, Schüte, & Wagner, 2014). Scholars in both fields have called for more research that investigates these phenomena, since they remain under-explored and have significance related to various aspects of teaching and learning (e.g., pedagogy, policy, programming). Moreover, mathematics education researchers have identified these research needs as being particularly acute in bilingual and multilingual contexts (e.g., Morgan et al., 2014). Thus, in this study, I examine the ways in which students use and attend to language and mathematics while working collaboratively on a mathematics problem-solving task in their second language (L2). The context is French immersion (FI) mathematics at the high school level, and the participants include Grade 9 students as well as their classroom

teacher. Reflecting the opening citation of this paragraph, the study views language and mathematics as complex entities and explores each in a complex and multifaceted way—going beyond the "words" and "numbers." This approach is underpinned by sociocultural theory (SCT; e.g., Vygotsky, 1962, 1978) and the *mathematics register* (Halliday, 1978), which has its roots in functional linguistics (e.g., Halliday, 1978, 2009). The main data, audio recordings of students' classroom interactions, are analyzed following Gee's (2011, 2014) discourse analysis theory and method. These data are supplemented by other data sources, including observations, artefacts, and interviews. Using a combination of coding frameworks and in-depth qualitative analysis, this study contributes to a deeper understanding of these participants' oral interactions in the classroom. In turn, the findings have significance for researchers and educators in other bilingual mathematics contexts who aim to better understand the complex ways in which language and mathematics interact.

In this introductory chapter, I preface my doctoral study by providing a glimpse of my own academic and professional experiences and how these relate to my current research interests. I also contextualize my research by presenting a selection of background literature related to the FI program broadly and more specifically in New Brunswick. I explain the research problem I aim to address in my research study.
In Chapter 2, I outline the theoretical framework guiding and grounding my research approach. The chapter also provides an overview of existing research related to language and mathematics and helps situate my own study. I discuss how my own work aims to contribute to and expand upon this important literature by addressing research gaps and responding to calls from scholars in the field, and pose my research questions. Next, in

Chapter 3, there is a description of the study itself, which includes information about the ethics procedures, research site, participants, and materials. Also in Chapter 3, I explain the methodology, and the data collection and analysis methods I employed during my work. These are closely linked to my theoretical framework, which continually guided my approach to data collection and analysis. Chapter 4 presents the results of the study. The presentation of the results is organized with respect to the research questions posed. Descriptive quantitative analysis through coding frameworks provides an overview of results related to each question. These analyses are then enhanced with qualitative analyses, in which I write about the most salient findings in an in-depth, descriptive way in order to build a rich analysis of mathematical communication contextualized in the L2 classroom. I conclude my dissertation in the final chapter, Chapter 5, with an interpretation of key findings and their implications for FI mathematics classrooms, which in turn may inform other additional-language mathematics contexts, and school language policies. I close the chapter and the dissertation with an exploration of potential avenues for future research, and concluding remarks.

1.1 Personal Academic and Professional Experience

During my Bachelor of Arts degree, I, like all students, had to choose an area of study in which to major and minor. At the time, I was unsure of my future career path (or any part of my future path, for that matter); however, I was becoming increasingly interested in teaching as a career. My two passions were French and mathematics, which were also high demand "teachable" subjects, and so pursuing these as my major and minor seemed logical. Upon graduation from my B.A. at Mount Allison University, I pursued teaching as a career and I continued to focus on French as a second language

(FSL) and mathematics in the secondary stream during my Bachelor of Education degree at the University of New Brunswick. This served me well as I moved into the teaching world, and I became a high school FI mathematics teacher. My academic and professional background led me to develop a keen interest in not only language, but also mathematics and how the two play a role in, and interact during, the mathematics learning process. I became increasingly fascinated by how students were able to learn both language and content (mathematics) through the medium of their L2.

As a graduate student enrolled in the Master of Education degree program, I was able to explore some of the questions that I had regarding FI students, their program choices, and their experiences in secondary FI mathematics. While this experience shed light on a number of issues, it also led to more questions. For this reason and many others, I was grateful to have the opportunity to continue my exploration of secondary FI mathematics and develop more expertise in the fields of L2 and mathematics education. My doctoral work represents a continuation of my learning, grounded in my academic and professional experiences and interests. In order to broadly contextualize the study at hand, I will explain the key tenets of the immersion program in which my experience has been based and describe the research problem under investigation.

1.2 Background

When it comes to school-based language learning, immersion programs are a global phenomenon; however, immersion is a Canadian invention. The program began in St. Lambert, Quebec, in the 1960s as a response to parents' desire for an alternative to traditional FSL programming. In these traditional FSL programs, students studied an L2 through short periods of direct language teaching, which, in St. Lambert during this time

period, were largely based on traditional drill-and-practice, grammar-focused approaches to language learning. Many, including the Anglophone parents in St. Lambert, found that this type of approach led to unsatisfactory performance in French, with graduating students being unable to communicate proficiently in their L2. In response, a novel way of learning French was introduced in St. Lambert and became known as "immersion" (Lambert & Tucker, 1972). There are several key characteristics of an immersion program, however it can generally be described as a school-based language-learning program in which students whose first language (L1) is not French learn this target language not only in French language classes but also through content classes such as science, history, and mathematics (Day & Shapson, 1996; Rebuffot, 1993; Swain & Johnson, 1997).

According to Swain and Johnson (1997), research conducted in the early days of the FI program centred on three key questions: (a) Would students learn the L2 (French) through its use as the medium of instruction? (b) Would content outcomes be learned through a language that students had not yet fully acquired? and (c) Would students' L1 (English) be maintained and developed through such a program? As many in the L2 education field have noted (e.g., Cummins & Swain, 1986; Day & Shapson, 1996, Johnson & Swain, 1997; Lapkin & Swain, 1984; Rebuffot, 1993; Swain & Lapkin, 1982), early research found that not only were graduates of the FI program achieving higher French proficiency than graduates of traditional French programs, FI students were also achieving as well in content courses and English language arts as their counterparts in the regular program.

After its beginnings in St. Lambert, FI spread rapidly throughout Quebec and Canada in the 1960s and 1970s (see Rebuffot, 1993, for a discussion). The success of the initial St. Lambert program was a key reason for FI's growing popularity. Moreover, the dissemination of positive research and program evaluation results led increasing numbers of parents to choose FI for their children. At the same time, Canada's *Official Languages Act* of 1969, which established English and French as national official languages, further solidified the desire on the part of many parents and students to learn French as an L2 in Canada for social, political, and economic reasons (Rebuffot, 1993; Swain & Johnson, 1997). Presently, FI remains a popular program choice in all 10 Canada's provinces and two of its three territories. The latest report from Canadian Parents for French (CPF, 2017) indicated that the latest enrolment percentages (for year 2015-2016) range from 6.8% of eligible enrolment (Alberta) to 32.0% (Quebec); in New Brunswick, Canada's only officially bilingual province, FI enrolment is at 28.5%.

Since the 1960s and 70s, immersion programs based on the Canadian model have spread to many parts of the world, with several different languages represented and a number of program variations with regard to grade point of entry, percentage of instruction in the L2, and so on. As Kristmanson and Dicks (2014) and others (e.g., Baker, 2011; Swain & Johnson, 1997) have suggested, immersion education can take on a variety of forms, especially when looking at the program on an international scale. For example, Swain and Johnson (1997) noted four main varieties of immersion programs, many examples of which are discussed in more recent explorations of the state of immersion in North America and globally (Kristmanson & Dicks, 2014): (a) immersion in a foreign language (e.g., FI programs in Australia); (b) immersion for majority-

language students in a minority language (e.g., FI programs in Canada); (c) immersion for language support and language revival (e.g., Catalan immersion programs in Catalonia, Maori immersion programs in New Zealand, FI programs in Louisiana in the United States); and (d) immersion in a language of power (e.g., English immersion programs in Hong Kong).

Further to this broad categorisation of four main types of immersion programs, Swain and Johnson (1997) have established a number of core features common to all immersion programs. At the same time, the authors have suggested that these features exist on a continuum within each program. The "prototypical immersion program" (p. 6) would exhibit each of the core features working at its fullest potential, however, in reality, most programs operate within a variable degree of fidelity to these features. Moreover, there are several different features that almost always vary and that differentiate one immersion program from another. Table 1 shows both the prototypical and variable features of immersion programs.

Like other school-based language learning programs (e.g., mainstream programs with short class periods of L2 or foreign language teaching, such as core French in Canada) the target L2 is taught as a school subject in language arts classes (Baker, 2011). However, as shown in Figure 1, one fundamental and unique characteristic of immersion programs is their reliance on language learning through the use of the L2 as a medium of instruction in content courses. Consequently, it is important for researchers and educators to closely examine how language and content are addressed simultaneously in the immersion classroom. With regard to mathematics, large-scale and smaller-scale quantitative work has shown that FI students achieve results that are comparable to or

sometimes exceed those of their counterparts in English Core programs (Bournot-Trites & Reeder, 2001; Cummins & Swain, 1986; Turnbull, Hart, & Lapkin, 2003; Turnbull, Lapkin, & Hart, 2001). While these quantitative studies provide valuable insights into big trends, there is a paucity of qualitative research examining FI students' experiences. This type of qualitative work is important since it has potential to enhance our understanding of quantitative results stemming from large-scale studies.

Table 1

Prototypical and Variable Features of Immersion Programs

Prototypical Features	Variable Features
1. The immersion language is the medium of instruction. 2. The immersion curriculum parallels the local L1 curriculum. 3. Overt support exists for children's L1 and all home languages. 4. The program aims for additive bilingualism. 5. Exposure to the immersion language is largely confined to the classroom. 6. Students enter with similar levels of proficiency in the immersion language. 7. All the teachers are bilingual. 8. The classroom culture recognises the cultures of the diverse language communities to which its students belong, including immigrant communities.	1. The grade level at which immersion is introduced (e.g., early, middle, or late). 2. The extent of immersion (e.g., full or partial). 3. The ratio of L1 to the immersion language at different stages within the program. 4. Continuity across grade levels. 5. Bridging support (when moving from L1 to immersion language instruction). 6. Resources. 7. Commitment. 8. Attitudes toward the immersion language. 9. Status of the immersion language. 10. What counts as success.

Note. From Swain and Johnson (1997); Swain and Lapkin (2005).

Several reports and studies have aimed to explain the reasons for FI students' overall high achievement in mathematics (and in other content areas, English, and French) but have also acknowledged that the situation is complicated and multifaceted and, therefore, difficult to explain. For instance, one explanation for FI students' high academic achievement overall when compared to their English program counterparts could be that FI students tend to come from higher socioeconomic backgrounds and are more likely to have parents with a postsecondary education, both of which correspond to higher academic achievement in general (Hart & Lapkin, 1998; Statistics Canada, 2008). Again, explanations for this are not straightforward. Factors such as self-selection and program attrition may play an important role. Late FI programs in particular (students enter the program in Grade 6 or 7, around age 11 or 12) might tend to attract students who are academically strong, as these programs require students to deal with more complex content in a new language (Dicks, 2008). Also, many FI students who struggle academically or otherwise choose to leave the program (or are "counselled out") by the time they get to high school (Statistics Canada, 2008; Turnbull, Hart, & Lapkin, 2003). Nonetheless, even when these factors are taken into account FI students still tend to outperform students in regular English programs (Statistics Canada, 2008). Issues of FI students' performance, particularly as it relates to mathematics, are discussed in further detail in Chapter 2.

1.3 Research Problem

Students in FI at the secondary level are expected to learn complex and abstract mathematical concepts through a developing L2. In the field of second language acquisition (SLA), *comprehensible input* (Krashen, 1985), *comprehensible output* (Swain,

1985), *negotiation of meaning* (Long, 1985; see also Ellis, 1997; Swain, 2000), and the importance of oral *interaction* through dialogue (Ellis, 1997; Long, 1985, 1996, 2007; Swain, 2000, 2008) are key concepts related to how learners acquire an L2 and thus have influenced the field of L2 education. All of these concepts relate to ways in which oral language, particularly through interaction, can be used to promote language learning in the L2 classroom. This represents a move away from the more traditional, grammar-based approaches to language teaching in favour of communicative-based approaches that began in the 1970s (Stern, 1983). Recently, oral language has begun to be considered an important part of mathematics education as well (Pimm, 1987; see also NCTM, 2000). This reflects current trends in mathematics education that emphasize the need for students to gain deep, conceptual understanding of mathematics while engaging in mathematical practices such as decision making, reflection, reasoning, and problem solving. An increasing importance is now placed on students' mathematical communication (which includes the production of oral as well as written communication) during the learning of mathematics as a way of sharing ideas, clarifying understanding, and making connections (NCTM, 2000).

More research is needed in order to understand more fully how language and, in particular, oral interaction operate in the mathematics classroom. For example, some mathematics education scholars have identified a need for more research on "what is actually happening in classroom interactions, on the nature of communication among students . . . and on the effects of particular language choices" (Morgan et al., 2014, p. 846). Moreover, this need for increased study is especially great with regard to bilingual classrooms. As Morgan et al. (2014) have suggested, "multilingual classrooms, especially

ones in which learners are studying mathematics using a language different from their mother tongue/first language, represent contexts where some of the above considerations of teaching and learning are significantly more obvious and acute" (p. 848). Barwell et al. (2005) have argued that results stemming from research in this area will have implications at the levels of policy (e.g., curriculum, classroom language policies, what counts as mathematical language) and practice (e.g., how language really plays out in the classroom, changing deficit discourses about "failure" to use mathematical language, meaning making).

In this vein, more research is needed to explore how bilingual FI students work together through mathematical problem-solving tasks. In conducting this research, I endeavoured to fully explore both language and mathematics, and their relationship with one another. Rather than simply conducting L2 research that happens to be situated in a mathematics classroom, or, conversely, studying mathematics while paying some attention to language issues, one of my key research goals was to keep both language and mathematics at the forefront of the study and explore their relationship in a complex, contextualized, and meaningful way. As Barwell (2004) has suggested, "the challenge is to work with both language and mathematics and keep them both in view" (p. 244). In effect, conducting research that lies at "the intersection between the two academic communities" (Barwell, 2005c, p. 97) has been identified as not only one of the most significant challenges but also one of the greatest needs in both fields:

> In general, research conducted within mathematics education has been concerned with the teaching and learning of mathematics, drawing on ideas from linguistics as theoretical and methodological tools in this endeavour. Research in linguistics,

> on the other hand, where it has focused on mathematics classrooms, has been interested in the linguistic aspects of the teaching and learning of mathematics, such as the nature and acquisition of the mathematics register. There has been little attempt to bring research from the two communities together, in order to relate, for example, issues of the acquisition of the mathematical register with the acquisition of mathematical concepts. (Barwell, 2005c, p. 97)

Moreover, while FI research is extensive and dates back to the program's inception in the 1960s, it has often focused on issues related to L2 and L1 proficiency, acquisition, and achievement, particularly in early entry (i.e., Kindergarten or Grade 1) programs. While these are clearly extremely important studies, more work is needed with regard to FI mathematics, later entry (i.e., Grade 6 or 7) programs, as well as studies based in FI in high school (especially in mathematics). My work therefore aims to respond to these identified gaps in the literature from both the L2 and mathematics education fields, a task that, although challenging, has the potential to contribute rich, meaningful data and analysis to two major fields in education. Thus, based on the problem as described, the overarching purpose of this research is as follows: This research intends to describe, interpret, and understand the nature of high school FI students' communication as they work collaboratively on mathematical problem-solving tasks in their L2.

As a first and crucial step in this process, I adopted a particular theoretical framework that aligns with my own sense of bilingualism, L2 education, and mathematics education, and that I felt would serve me throughout my research journey. In the next chapter, I lay the theoretical foundation that guides my literature review as well as my overall approach to this study.

2.0 THEORETICAL FRAMEWORK AND LITERATURE REVIEW

In the words of Moschkovich (2010):

> The most significant challenge is that research examining language and mathematics learning must be grounded not only in current theoretical perspectives of mathematics cognition and learning, but also in current views of language, classroom discourse, bilingualism, and second language acquisition. Becoming familiar with two or more sets of research literature can be a daunting task. (p. 2)

Indeed, familiarizing myself with the theoretical research based in both the second language (L2) and mathematics education fields, along with the seminal and cutting edge empirical research from both, has been challenging. At the same time, the exercise has proved (and continues to prove) important, useful, and enlightening and has been essential to gaining a solid foundation in both fields. This is a key component of one of my main research goals, which is to keep both language and mathematics in focus and at the forefront of the work.

As with all literature reviews, when I embarked on this one I found that, despite the previously identified research gaps, there are a large number of theoretical works and empirical studies upon which to draw. Although I have tried to be as thorough as possible, I have nonetheless had to make choices about what to select and focus on in the literature review. My choices have been guided by my past experiences (personal, professional, and academic), my research interests, the identified research problem, and my theoretical stance; and I believe that the works I have chosen to feature here represent those that are most pertinent to my own. Despite having made these careful and thoughtful selections,

this chapter remains substantial in large part due to its attention to both the L2 and mathematics education fields.

The theoretical framework for my research is grounded in two key concepts—one of which stems mainly from the L2 education field, while the other, from the mathematics education field. However, the two theoretical constructs are extremely complementary and actually very intertwined, as will become apparent in this chapter. Both derive from many of the same scholars' works based in the linguistics field, and, more specifically, those that view language as a social construct used for meaning making. Thus, my guiding theoretical framework is built upon (a) *sociocultural theory* and (b) *the mathematics education register*. This theoretical foundation in turn guided the remainder of my literature review.

The literature reviewed in preparation for my research was drawn from several main areas of study. Using my research problem and theoretical frameworks as guides for inquiry, I explored seminal and current works related to content (especially mathematics) teaching and learning in FI, and issues of language in other (non-FI) bilingual mathematics school contexts. Research related to questions of L1 use is interwoven throughout these main areas of exploration, since the issue of L1 use or codeswitching appears time and time again as an emergent issue in several studies, and I also highlighted codeswitching in a separate section in the literature review. In keeping with my theoretical approach and due to its pertinence to my work, much of the literature reviewed stems from studies using SCT-based research approaches.

Thus I begin Chapter 2 with a description of my theoretical framework. In these sections, I lay the foundation for the literature review and indeed the entire study,

presenting the lens through which I looked in order to select literature for review, design the study, and collect and analyze data. Following these theoretical sections, I move on to discuss literature more empirical in nature, but which fit with my theoretical focus by way of their own research designs.

2.1 Sociocultural Theory

One way in which a number of researchers interested in L2 education aim to understand language use, and thus language teaching and learning, is by approaching their work from a sociocultural theory (SCT) perspective. This is particularly so when researchers are interested in how language is used during oral interactions, especially of a collaborative nature, due to SCT's emphasis on the social, interactive nature of learning and its primary interest in language as the key cognitive tool used during these interactions. Sociocultural theory is primarily informed by the work of Vygotsky (e.g., 1962, 1978), a Russian scholar whose ideas have gained popularity and have had increasing influence on the field of education in general, and L2 education in particular, since his writings were translated and became more widely accessible in the 1960s and 1970s.

In this section, I present two of Vygotsky's overarching theories, involving the social nature of higher mental functions and the mediation of higher mental functions by cultural tools. Within these two overarching theories I discuss three specific subthemes that emerge: the *zone of proximal development*, *concepts development*, and *egocentric speech*. While it is important to explore all three of these three key subthemes along with the two overarching theories in Vygotsky's work because they form the foundation of much of the SCT research conducted in the L2 education field, I go on to focus mainly on

one subtheme as an SCT approach to L2 research in particular: the zone of proximal development (e.g., Cole, 1985; Donato, 1994; Lantolf, 2000; Lantolf & Appel, 1994; Ohta, 2000). I do this because it is this subtheme that mainly informs my own research study. More specifically, I discuss two concepts taken from L2 education literature that are used to explore interactions in the zone of proximal development: *collaborative dialogue* and *languaging* (Swain, 2000, 2008, 2010; Swain, Kinnear, & Steinman, 2011; Swain & Lapkin, 1998). This section concludes with final thoughts on Vygotsky's SCT, its potential for L2 research in general, and its pertinence to my own research in an FI mathematics context.

2.1.1 The Social Nature of Higher Mental Functions and the ZPD

Guiding my exploration of Vygotsky's (1962, 1978) writings is work by neo-Vygotskian scholars such as Wertsch (1985, 1993) and, in particular, Wertsch and Tulviste's (1994) discussion of Vygotsky's main contributions to developmental psychology, which they frame within two key overarching theories: "his claims about the social origins and social nature of higher (i.e., uniquely human) mental functioning and his uses of culture" (p. 334). Wertsch, in earlier publications (e.g., Wertsch, 1985, 1993), has described three overarching theories within Vygotsky's work: a reliance on genetic, or developmental, analysis; the social origins of individual higher mental functioning; and the mediation of human action by tools and signs (role of culture). However, Wertsch (1985) underscored the interrelatedness of the overarching theories, and in particular, how the "very notion of origins in the second [overarching theory] points toward a genetic analysis" (p. 15). Consequently, because of the interrelated nature of the overarching theories that emerge in Vygotsky's work and because the key aim of this

section is to focus on language as it is used in the zone of proximal development, I proceed following the two-pronged approach found in Wertsch and Tulviste (1994). Thus, this overview of Vygotsky's work begins by examining his overarching theories on the social origins of mental functioning and follows with a discussion of his use of culture.

Before elaborating on Vygotsky's overarching theory regarding the social origins of *higher mental functions*, it is useful to define the term. Vygotsky defined *higher mental functions* as those that are unique to humans, and that are mediated by tools and sign systems such as language. Examples of higher mental functions include thinking, voluntary attention, and logical memory, in both social as well as individual forms of activity (Wertsch, 1993).

Vygotsky's beliefs about this are perhaps best summed up in his general genetic law of cultural development, which Wertsch (1993) has referred to as Vygotsky's "most general statement about the social origins of individual mental functioning" (p. 26). This law stated that:

> any function in the child's cultural development appears twice, or on two planes. First it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category. This is equally true with regard to voluntary attention, logical memory, the formation of concepts, and the development of volition . . . [I]t goes without saying that internalization transforms the process itself and changes its structure and functions. *Social relations or relations among people genetically underlie all higher functions and their relationships* [emphasis added]. (Vygotsky, cited in Wertsch, 1993, p. 26)

In other words, in SCT emphasis is placed upon the processes occurring between people, on the intermental plane (interpsychological), and the resulting processes occurring within the individual, on the intramental plane (intrapsychological), are seen as derivative and "emerging through the mastery of internalization of social processes" (Wertsch & Tulviste, 1994, p. 335). According to Wertsch and Tulviste (1994), Vygotsky believed that the mind "extend[s] beyond the skin" and, consequently, "mind, cognition, memory, and so forth are understood not as attributes or properties of the individual, but as functions that may be carried out intermentally or intramentally" (p. 336).

Vygotsky's focus on the intermental and intramental planes, and the interaction between these as a way to construct knowledge, formed the basis of his general genetic law of cultural development and was a key tenet of his overarching theory of the social nature of cognitive development. However, despite Vygotsky's emphasis on the social construction of knowledge, he also described a genetic, or developmental, component at play. This is clear from even the title of his "general genetic law of cultural development." As Wertsch (1993) explained, for Vygotsky "a 'natural' and a 'cultural,' or 'social,' line of development interact to create the dynamics of change" (p. 22). Vygotsky's belief in an interplay between a certain "natural" course of development and the equally, or perhaps more important way in which this natural development is constructed socially, provides the basis for virtually all his work and is also closely intertwined with his views on the role of culture within the developmental framework. From this line of thinking, at least one important idea for L2 research in particular, namely the notion of the "zone of proximal development" (Vygotsky, 1978, p. 86), can be viewed as a subtheme of

Vygotsky's overarching theory of the sociocultural origins of individual mental functioning (Wertsch & Tulviste, 1994).

Vygotsky (1978) defined the zone of proximal development (ZPD) as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). Within the framework of the intermental and intramental planes, the ZPD, or a child's potential level of development, corresponds to the former while the child's actual level of development corresponds to the latter (Meece, 1997; Wertsch & Tulviste, 1994). The ZPD takes into account the dynamic nature of development in that it considers not only learners' present achievements but also their potential. The ZPD can be a reliable predictor of where children are headed in their developmental process. In effect, according to Vygotsky (1978), "what is the zone of proximal development today will be the actual developmental level of tomorrow—that is, what a child can do with assistance today she will be able to do by herself tomorrow" (p. 87).

The idea of the existence of a ZPD has had a direct influence on educational practices in general and in L2 settings more specifically. The ZPD is often put into play, for example, in a teaching technique referred to as "scaffolding" (Lake, 2012, p. 53; Wood, Bruner, & Ross, 1976; see also Auger & Rich, 2007; Meece, 1997). Interestingly, scaffolding is a "teacher-friendly concept that is associated with Vygotsky although not named by him" (Swain, Kinnear, & Steinman, 2011, p. 26). Scaffolding refers to a learning activity in which a more capable leader or leaders, who could be the teacher or peers, guide(s) the process with questions and prompts. Meanwhile, learners gradually

take increasing responsibility for the situation until eventually they are able to manage the task on their own (Auger & Rich, 2007; Cole, 1985; Meece, 1997). In this way, it is clear how the scaffolding technique relates directly to a Vygotskian ZPD. Thus, the theory of a ZPD combined with the pedagogical technique known as scaffolding has "powerful implications for how one can *change* intermental, and hence intramental, functioning" (Wertsch & Tulviste, 1994, p. 337). In the context of L2 teaching and learning, scaffolding has become a widespread and accepted technique used to support students' language use and learning. Key ideas that drive L2 education, such as the input hypothesis (Krashen, 1985), the output hypothesis (Swain, 1985) and the negotiation of meaning (Long, 1985, 2007; Swain, 2000) can be supported through scaffolding in the L2 classroom. (These concepts will be explored in further detail in upcoming sections.)

In effect, Vygotsky described the relationship between development and instruction as "two processes that exist in very complex interrelationships" (Vygotsky, cited in Wertsch, 1985, p. 70). That said, according to Vygotsky, although instruction "creates" the ZPD, a child could only progress "within certain limits that are strictly fixed by the state of the child's development and intellectual possibilities" (Vygotsky, cited in Wertsch, 1985, p. 70). Again, in this explanation, one can detect the interplay between Vygotsky's ideas of the social origins of higher mental functions and of the developmental model. Nonetheless, the notion of "learning leading development" (Lake, 2012, p. 56), rather than the other way around, is the overarching feature of the ZPD and the ultimate tenet of Vygotsky's thinking. When Vygotsky's work became more widely available, this idea represented quite a novel departure from influential, although increasingly challenged (see, e.g., Egan, 2002, for an in-depth critique), more traditional

models of development based on Piagetian stages (see Beilin, 1994, for an overview of Piaget's work). In the vein of a Vygotskian ZPD, as Lantolf and Pavlenko (1995) explained, "development does not proceed as the unfolding of inborn capacities, but as the transformation of innate capacities once they intertwine with socioculturally constructed *mediational means*" (p. 109). This notion of mediational means is the second main overarching theory found in Vygotsky's work, as described in the two-pronged approach taken in Wertsch and Tulviste (1994) and which I use to organize my discussion here. In this second key overarching theory, the idea that language is of utmost importance in the social learning process emerges. Thus, it also provides an important foundation upon which to build the current study.

2.1.2 Cultural Tools (Language) as Mediational Means

As previously noted, Wertsch and Tulviste (1994) have explained that, in addition to the first general theme regarding the social nature of individual cognitive functions, a second general theme can also be seen throughout Vygotsky's work: the role of culture, or more precisely "cultural tools," in the development of this higher mental functioning. Recall that Vygotsky believed that the mind extends beyond the skin. In an example of this, he has suggested that "human mental functioning . . . involves cultural tools, or mediational means," and furthermore, that these cultural tools are "socially evolved and socially organized" (Wertsch & Tulviste, 1994, p. 341). Examples of these cultural tools, which Vygotsky referred to as "psychological tools" (e.g., Vygotsky, cited in Wertsch & Tulviste, 1994, p. 342), include "*language* [emphasis added]; various systems for counting; mnemonic techniques; algebraic symbol systems; works of art; writing; schemes, diagrams, maps, and mechanical drawings; all sorts of conventional signs; and

so on" (Vygotsky, cited in Wertsch & Tulviste, 1994, p. 342). The use of tools does not simply make a so-called predetermined process of any given human mental functioning easier. Rather, the use of these cultural tools, and in particular language, transforms the functioning itself (Wertsch & Tulviste, 1994). Clearly, embedded in Vygotsky's views on the role of social interactions in the development of individual learning is an important link with language, as this is the cultural tool that is of primary concern to Vygotsky and the main mediational means through which social interactions occur.

For Vygotsky, the idea of mediation underscores "the claim that human action, on both the individual and social planes, is *mediated* [emphasis added] by tools and signs" (Wertsch, 1993, p. 19). According to Wertsch (1993), in Vygotsky's *Thinking and Speech* (or *Thought and Language*, 1962), Vygotsky placed "primary emphasis . . . on how different forms of speaking are related to different forms of thinking" (Wertsch, 1993, p. 30). Wertsch (1985) has pointed out that there are a number of important points to consider with regard to cultural tools and higher mental functions. First, Vygotsky believed that the introduction of a cultural tool, namely language, into a mental function fundamentally changes that function. And second, he argued that these cultural tools are social in nature, both on the broader, sociocultural domain and on the smaller, intermental domain; this intermental domain was Vygotsky's primary focus (Wertsch, 1993). In this vein, Vygotsky (cited in Wertsch, 1985) stated that, "the primary function of speech, both for the adult and for the child, is the function of communication" (p. 81). It follows that if the primary aim of language is to communicate, then language will be formed to meet this aim. Furthermore, if language also plays a key role in the development of individuals'

mental functions, then such functions will also be indirectly shaped by what happens during communication (Wertsch, 1985).

Wertsch (1985) described Vygotsky's account of the functions of speech by discussing four oppositional pairs of functions: signaling versus significative, social versus individual, communicative versus intellectual, and, finally, indicative versus symbolic. I focus here on the fourth distinction because the notion of indicative function versus symbolic function lays the groundwork for concept development and egocentric speech, which are informative for the ways in which language and discourse are discussed subsequently throughout this dissertation (rooted in functionalist perspectives, and as being contextualized and studied in use). The indicative-symbolic distinction is also arguably the most directly linked to the ZPD (which, as already explained, is a key theoretical foundation for this study). Moreover, while Vygotsky himself recognized many potential uses of mediational means, he, too, focused in his writing mainly on these two uses in particular (concepts development and egocentric speech, Wertsch, 1993), which seems to indicate the importance he placed on them. I discuss these here as subthemes of the overarching theme of cultural tools as mediational means.

The indicative function of speech serves its purpose in contextualized settings. Here, "the structure and interpretation of linguistic signs depend on their relationships with the context in which they appear" (Wertsch, 1985, p. 95). Meanwhile, language can also be used in abstract, decontextualized reflection, in which case it is serving the symbolic function. This symbolic function is further connected to the notions of generalizability and social interaction. As Vygotsky (1962) explained, "human communication presupposes a generalizing attitude, which is an advanced stage in the

development of word meanings. The higher forms of human intercourse are possible only because man's [*sic*] thought reflects conceptualized actuality" (p. 7). Vygotsky (1962) claimed that early levels of generalization and social interaction are based on the indicative function of speech, while more advanced levels are due to the symbolic function of speech. To explain how the transition from speech that uses only the indicative function to that which also uses the symbolic function occurs, Vygotsky focused on the issue of word meaning. And, it is in his exploration of word meaning that the notion of concept development is addressed.

2.1.2.1 Concept development.

Vygotsky believed that when it comes to word meaning, children progress from "unorganized heaps," where their understanding is very subjective, to "complexes," where their understanding is more objective but connected with concrete tasks, to "scientific concepts," where they are able to understand decontextualized relationships, (Vygotsky, cited in Wertsch, 1985, pp. 100-102), and through their development, participate in abstract forms of reasoning (Vygotsky, 1962; Wertsch 1985, 1993). It is important to note that, "what is decontextualized are the mediational means, which have come to be treated as abstract objects of reflection rather than as embedded in the context of other forms of intermental or intramental action" (Wertsch, 1993, p. 39). For Vygotsky, all of this is apparent through children's use of language. For example, when scientific concepts develop, a child could not only use the words *furniture* and *table* when appropriate in connection with these objects, but could also use these words to make statements of logical equivalence, for example, "all tables are furniture" (Wertsch, 1985, p. 103).

The development of "scientific concepts" is "based on one of the organizing principles of human language, the capacity of words to enter into decontextualized relationships with other words," or sign-sign relationships (Wertsch, 1985, p. 108). This development is the final step in the decontextualization of mediational means, which are the essence of what makes mental activity (thinking) possible in humans (Wertsch, 1985). Vygotsky saw instruction, particularly the type that children would receive in formal schooling, as playing an important role in the development of these so-called scientific concepts. Furthermore, he differentiated these concepts from "spontaneous concepts" such as those that were often the focus of Piagetian studies, arising from children's spontaneous interaction with everyday objects in the everyday environment (Wertsch, 1985, p. 102; see also Shayer, 2003).

In Vygotsky's examination of the development of scientific concepts, he focused on how language as a mediational means comes to be used in decontextualized situations. In contrast, in his analysis of "social," "egocentric," and "inner" speech (Vygotsky, 1962, pp. 14 & 149), the second subtheme I explore under the cultural tool theme, Vygotsky focused on the potential for human speech to increase contextualization, or on "ways in which speech comes to serve as its own context" (Wertsch, 1993, p. 39). At first, this may seem contradictory, since Vygotsky had also proposed that development is based on the potential of language to function in decontextualized, or abstract, settings, in sign-sign relationships. However, the "context" to which Vygotsky referred for egocentric and inner speech is a linguistic context and, "unlike the sign-sign relations involved in reflective conceptual reasoning, the linguistic context of an utterance does not remain constant across contexts" (Wertsch, 1985, p. 109). For example, if a speaker says, "I saw

a man on the street" and follows that sentence with, "He was very tall," the listener's interpretation of the word *he* depends on the linguistic context provided by the first sentence. The interpretation of *he* will change according to the linguistic context provided (Wertsch, 1985, pp. 109-110). The decontextualized relationships of scientific concepts and the relationships found in linguistic contexts are similar, however, in that neither directly depends on extralinguistic context (i.e., a context composed of non-linguistic, concrete objects). Furthermore, both play a role in mediation and therefore are key to higher mental functioning, and both involve a form of sign-sign relationships. However, while the sign-sign relationships are relatively constant in scientific concepts utterances, they vary and depend on unique utterances in egocentric and social speech (Wertsch, 1985).

2.1.2.2 Egocentric speech.

In the very nature of Vygotsky's chosen terminology, for example the use of the word *speech* rather than some of the more fashionable terms of the time such as *thinking* or *mental processes*, one can see the importance he placed on the social nature of knowledge building, especially through oral interaction. As Wertsch and Tulviste (1994) have pointed out, "Vygotsky's use of the term *speech* here reflects the fact that he viewed individual mental functioning as deriving essentially from the mastery and internalization of social processes" (p. 338). True to the interrelatedness of Vygotsky's theories, inner and egocentric speech are language-specific ideas that are linked strongly to Vygotsky's theory of a ZPD. According to Vygotsky, after children engage in verbal social interaction, using "social," or "external," speech, they begin to use "inner" speech to plan their action. This is a key step on the way to self-regulation and the appearance of

egocentric speech, which occurs during the transition from social speech to inner speech, marks the beginning of this shift toward self-regulation (Meece, 1997; Vygotsky, 1962; Wertsch & Tulviste, 1994). As Swain (2011) has explained, the changes that occur within the individual in the ZPD represent "the movement from other- to self-regulation" (n.p.).

According to Wertsch (1985, 1993), the notion of egocentric speech was a key focus for Vygotsky. However, to more fully understand what Vygotsky meant by egocentric speech it is useful to compare to Piaget since egocentric speech is perhaps more often associated with his work than with Vygotsky's, and, as previously noted, since Piaget has had an arguably profound influence on education (e.g., Beilin, 1994) despite some past and recent criticisms of his work (e.g., Egan, 2002). Moreover, according to Wertsch (1993), although Vygotsky's analysis of egocentric speech was influenced by various other theories, it was Piaget's that provided the "particular impetus" (p. 40) for Vygotsky's work. However, despite some similarities among the analyses of these two theorists, Vygotsky's account of egocentric speech differs remarkably from Piaget's. Because of the different emphasis each theorist placed on language, the fundamental difference can be summed up in the following statement: "Egocentric speech, from the Vygotskian perspective, is not an external indication of thinking [as in the Piaget perspective]—egocentric speech *is* thinking" (Duncan, 1995, p. 462). To elaborate, I will explain how Vygotsky's theory differs in two distinct ways from Piaget's: in his views on (a) the function and (b) the fate of egocentric speech.

Both Piaget and Vygotsky agreed that egocentric speech appears in the child's development at around 3 years of age. Aside from this rather small point of agreement, however, there are key differences to note. With regard to the function of egocentric

speech, in Piaget's view it is a type of monologue, whereby even when potential listeners are present the child speaks but has no intention of communicating, or being understood. Egocentric speech, according to Piaget, serves no real purpose and is symptomatic of the general egocentricity of the child at this age and stage. Speech, for Piaget, was thus separated into two functional categories: social and egocentric (Piaget, 1923/1959; Wertsch, 1985). In contrast, Vygotsky (cited in Wertsch, 1985) viewed the function of egocentric speech as essentially social, and claimed that, "the initial function of speech is the function of communication, social contact, influencing others" (p. 114). For Vygotsky, egocentric speech begins to emerge as children begin to develop the capacity to plan their action. In other words, egocentric speech marks the beginning of the emergence of self-regulation.

Because of their mainly opposing views on the function of egocentric speech, Vygotsky and Piaget also expressed different ideas regarding the fate of egocentric speech. Both scholars recognized that egocentric speech eventually disappears, usually around age 7. However, the key difference lies in each work's interpretation of what becomes of egocentric speech once it disappears. For Vygotsky (in Wertsch, 1985), it does not so much "disappear" as it is actually being internalized and forms inner speech. Unlike Piaget (in Beilin, 1994), for whom egocentric speech simply loses its egocentric quality and disappears with progressive socialization and development, Vygotsky (cited in Wertsch, 1985) viewed egocentric speech as

> a transitional form from external [social] to inner speech . . . It grows out of its social foundations by means of transferring social, collaborative forms of behaviour to the sphere of the individual's psychological functioning . . . Thus the

> overall scheme takes on the following form: social speech – egocentric speech – inner speech. (p. 117)

Vygotsky explained that out of an undifferentiated speech present in the child's early development, there emerges a differentiated speech with different structures and different functions, namely speech for oneself (inner speech) and speech for others (communication), and egocentric speech marks the transition between the two.

The Piagetian and Vygotskian explanations of egocentric speech reflect not only some key differences between their views on the relationship between language and thought, but also the difference in their fundamental orientations. In Vygotsky's view, language helps form thought whereas for Piaget, language is more a symptom or reflection of development, it is merely a "window" into cognitive development (Beilin, 1994, p. 261). Thus, while language never became a serious focus for Piaget, it was almost always at the forefront of Vygotsky's analyses (Beilin, 1994; Wertsch & Tulviste, 1994).

Ideas related to concept development and egocentric speech (the movement between social and inner speech) show how SCT privileges social interaction and the role of language as a cultural tool and mediational means within these interactions. Through these social interactions, the ZPD theorizes ways in which language can be used as a scaffold to facilitate movement from other- to self-regulation. The learning potential of individuals can be explored within this zone, when they interact with other individuals toward a common learning goal or activity. While discussed here in a somewhat separate manner, the theories are of course very interrelated, with one building upon and interweaving with the other. Together, they provide a theoretical foundation upon which

to pursue research like this, that aims to understand the complexities of students' communication as they interact in the classroom.

2.1.3 A Sociocultural Theory Approach to Research in Second Language Education

Sociocultural theory can provide a solid theoretical framework for researchers interested in classroom-based interactions involving L2 teaching and learning. According to Lantolf and Pavlenko (1995), this approach

> erases the boundary between language learning and language using; it also moves individuals out of the Chomskian world of the idealized speaker-hearer and the experimental laboratory, and redeploys them in the world of their everyday existence, including real classrooms (Lave and Wenger, 1991). In so doing, it situates the locus of learning in the dialogic interactions that arise between socially constituted individuals engaged in activities which are co-constructed with other individuals rather than in the heads of solipsistic beings. (p. 116)

This emphasis on the collaborative, co-constructed nature of language *learning* through language *using*, via dialogue, is key to the theoretical underpinnings of my own research. To this end, I pay special attention to the SCT notions of what Swain has termed "languaging" (Swain, 2008, 2010; Swain, Kinnear, et al., 2011, p. 34) and "collaborative dialogue" (Swain, 2000, p. 7), especially as they occur during a ZPD.[1]

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[1] Here, I use Swain, Kinnear, et al.'s (2011) and Swain and Lapkin's (2013) suggested approach of thinking about the ZPD as an activity rather than a space. Thus, activities occur "during" a ZPD, rather than "within" a ZPD.

Swain's (e.g., 2000, 2008; Swain, Kinnear, et al., 2011; Swain & Lapkin, 1998, 2013) notions of languaging and collaborative dialogue have emerged as novel ways to explore L2 use and learning in educational contexts. Swain (2000) has argued that L2 acquisition had previously been focused on Krashen's (1985) comprehensible input as the key to L2 learning. That is,

> that humans acquire language in only one way—by understanding messages, or by receiving "comprehensible input." We progress along the natural order . . . by understanding input that contains structures at our next "stage"—structures that are a bit beyond our current level of competence. (We move from *I*, our current level, to *I* + 1, the next level . . .). (p. 2)

Learners receive this comprehensible input when they interact with an interlocutor or interlocutors. However, Swain and others (e.g., van Lier, 2000) have viewed interaction as not only a source of comprehensible input but also as an opportunity for learners to produce comprehensible output (Swain, 1985) or use the target language, and thus learn said language. Furthermore, more recently Swain (2000) has argued for an expanded view of output that recognizes that "in 'saying,' the speaker is cognitively engaged in making meaning. . . . 'Saying', however, produces an utterance that can now be responded to—by others or by the self" (p. 102). In this view, more emphasis is placed on the social, interactional component of language use and learning.

This type of focus on interaction reflects a move in SLA theory and literature that explores the interactional structure of conversations between L2 learners and their interlocutors (whereas much of the initial research in the field focused on interactions between native and non-native speakers). The *interaction hypothesis* (Long, 1996) for

example recognized the importance of *negotiation of meaning* (e.g., Long, 1985; see also *negotiation* for *meaning*, Long, 2007; Swain, 2000) not only between native and non-native speakers outside of classrooms, but also between L2 learners within educational contexts. This kind of research explores the ways in which L2 learners modify their talk for each other (input and output, e.g., slowing speech, repetitions) so as to ensure that messages are understood, in other words, how they negotiate meaning within and related to particular contexts. Similarly, Donato (1994) has also highlighted the limitations of focusing solely on input and output and neglecting to examine these in their communicative contexts. He referred to the input/output construct of L2 learning as "the message model of communication" in which "the goal of conversational partners during a communicative event is the successful sending and receiving of linguistic tokens" (p. 34). Donato (1994) has suggested that research framed within this model "masks fundamentally important mechanisms of L2 development and . . . the social context is impoverished and undervalued as an arena for truly collaborative L2 acquisition" (p. 34).

In fact, Swain (2000) has described her expanded view of L2 learning as moving "beyond" the output hypothesis and toward a notion of *collaborative dialogue*, which was defined as

> knowledge-building dialogue. In the case of our interests in second language learning, it is dialogue that constructs linguistic knowledge. It is what allows performance to outstrip competence. It is where language use and language learning can co-occur. It is language use mediating language learning. It is cognitive activity and it is social activity. (p. 97)

Clearly rooted in SCT, the work of Swain (2000) and others (Donato, 1994; Swain & Lapkin, 1998) suggests that when language learners work together on a linguistic problem-solving task, they are able to "notice" (Swain, 2000, p. 99) or identify what they do not know (linguistically speaking). They are then able to construct linguistic knowledge and "their output, in the form of collaborative dialogue, is used to mediate their understanding and solutions" (Swain, 2000, p. 102). Swain and Lapkin (1998, 2000; see also Gutiérrez, 2007) have described what they termed "language-related episodes" (Swain & Lapkin, 1998, p. 326), or LREs, that occur during collaborative dialogue among language learners. An LRE has been defined as "any part of a dialogue where the students talk about the language they are producing, question their language use, or correct themselves or others (Swain & Lapkin, 1995)" (Swain & Lapkin, 1998, p. 326). These LREs may occur either in the learners' target language or in their L1. Furthermore, Vygotsky's work suggests that "the knowledge building [the learners] have collectively accomplished may become a tool for their further individual use of their second language" (Swain, 2000, p. 104). This has important implications for learners' engagement in collaborative dialogue and scaffolding in the ZPD and the significance of LREs within these activities.

Extending and building upon the notions of comprehensible input, comprehensible output, negotiation of meaning, collaborative dialogue, and LREs, Swain, in more recent work (e.g., Swain, 2008; Swain, Kinnear, et al., 2011), has used the term "languaging" (Swain, Kinnear, et al., 2011, p. 34) to describe more broadly what happens when learners engage in collaborative dialogue. Specifically, according to Swain, Kinnear, et al. (2011), languaging "is the process of making meaning and shaping

knowledge and experience through language. Languaging organizes and controls (mediates) mental processes during the performance of cognitively complex tasks" (p. 151). For Swain (2008), languaging does "refer to producing language" but, beyond a straightforward output hypothesis, it refers, "in particular, to producing language in an attempt to understand—to problem-solve—to make meaning" (p. 96). Moreover, languaging can happen between individuals or it can happen within an individual, in private speech. Thus, collaborative dialogue is one type of the broader meaning-making, problem-solving, organizational activity that is languaging. Specifically, "it is when language is used to mediate conceptualization and problem-solving, whether that conceptualization or problem-solving is about language-related issues or science issues or mathematical ones, that languaging takes place" (Swain & Lapkin, 2013, p. 107). When this mediation occurs as an oral interaction between individuals, for example students working together on mathematical problem solving, languaging is said to occur in the form of collaborative dialogue (Swain & Lapkin, 2013).

Both collaborative dialogue and languaging are clearly closely tied to Vygotsky's notion of a ZPD, in which an "expert" provides assistance so that a "novice" can achieve at a level beyond which they would have been capable alone. With regard to participants in collaborative dialogue in the L2 classroom context, a number of scholars (e.g., Donato, 1994, 2000; Lantolf, 2000; Lantolf & Appel, 1994; Ohta, 2000; Swain, 2000; Swain & Lapkin, 1998, 2000) have suggested that scaffolding can be provided by other learners or peers during a ZPD and not just by an "expert" (e.g., the teacher). Specifically, Donato (1994) showed that "collaborative work among language learners provides the same opportunity for scaffolded help as in expert-novice relationships in the everyday setting"

(p. 41). Ohta's (2000), Donato's (1994), and also Swain and Lapkin's work (1998, 2000) demonstrated how peers provided mutual or "collective scaffolding" (Donato, 1994, p. 46) during a language-based problem-solving task, resulting in collective performances that exceed the individuals' capabilities. This last point relates to Vygotsky's notions of internalization and regulation, as the learners who collaborate and construct a ZPD create a zone "in which each person contributes something to, and takes something away from, the interaction" (Lantolf & Pavlenko, 1995, p. 116). Furthermore, as Swain, Kinnear, et al. (2011) pointed out, true to the metaphor of the scaffold, the idea is that assistance provided by others during group work in the classroom setting is "gradually dismantled when the structure/individual can mediate (regulate) itself" (p. 26).

At first glance, the notion of the ZPD and its associated concept of scaffolding may resemble Krashen's (1985) original notion of comprehensible input and $I + 1$, discussed earlier. However, Vygotsky's ZPD takes into account the dynamic nature of learning as it occurs during interaction; indeed, as pointed out earlier, some scholars view the ZPD as an activity, rather than a static "space." Thus, the culture, context, and relationship between the participants are all considered important in the interaction. In contrast, in the $I + 1$ construct, the expert transmits input to the novice while focusing essentially on the language level or stage of the latter (Swain, Kinnear, et al., 2011). During a ZPD rather, and thus during collaborative dialogue and languaging, Swain, Brooks, and Tocalli-Beller (2002) explained that, "[language] acquisition occurs *in* interaction, not *as a result of* interaction" (p. 173, original emphasis). In L2 classrooms, it is clear that collaborative dialogue and languaging are very much a part of interactions that can occur during a ZPD, or may even serve to create it. The communication that

occurs during collaborative dialogue (within the framework of the ZPD) is one way in which L2 researchers can explore how learners use their L2 to make meaning.

Conducting research from an SCT perspective, however, particularly when the approach centres on participants engaging in collaborative dialogue while working on problem-solving tasks in their L2, requires some important considerations. For example, research conducted by Storch (2002) and Watanabe and Swain (2007) has examined interaction patterns and the effects of proficiency differences in collaborative dialogue occurring between adult learners of English as a second language (ESL). With regard to interaction patterns, Storch found that four distinct patterns emerged during learners' interactions: collaborative, dominant/dominant, dominant/passive, and expert/novice. Watanabe and Swain added a fifth category, that of expert/passive. Both studies found that dialogue is more effective (i.e., produces more LREs and more turns per LRE) when the participants adopt a collaborative pattern. When it came to posttest scores, which reflected the extent to which participants had learned a particular aspect of language that had been discussed during their problem-solving dialogue, participants whose role was either collaborative or expert (in the expert/novice pattern) achieved better results (Watanabe & Swain, 2007).

In both the Storch (2002) and Watanabe and Swain (2007) studies, patterns of interaction seemed to have the greatest effect on the frequency of LREs. However, the language proficiency levels of the individuals in each participant pair were found to sometimes affect these patterns of interaction. For example, the expert/novice pattern was more likely to be (although not always) adopted by the participant pair if one individual had a high level of language proficiency (expert) and the other a low level (novice).

Consequently, these findings have important implications for researchers and educators regarding the effectiveness of collaborative work in terms of language and, perhaps, content learning in an immersion setting. The research suggests that pair work is effective, including when learners of different proficiency levels are grouped together, but that close attention must be paid to the interactive patterns adopted by the pairs; thus, the findings have relevance for the current study, which was focused on student work undertaken in pairs and/or small groups.

Despite some challenges, SCT can provide a solid theoretical framework for L2 classroom-based research that allows for an exploration of social interaction during collaborative problem-solving tasks mediated by language. Moreover, approaching L2 research from an SCT point of view can be especially valuable for researchers who are interested in how students make meaning within complex, situated contexts like the L2 mathematics classroom.

2.1.4 Concluding Remarks on Vygotsky and Sociocultural Theory

Vygotsky's work, and thus SCT, is not without its criticisms. Perhaps more so than many other theories in education and L2 learning, the limitations inherent in Vygotsky's writings have been explored and debated. There are a number of possible reasons for this, not the least of which includes the fact that Vygotsky's life was relatively short and thus some scholars suggest that there simply was no chance for him to develop in more detail some of his lesser-explored ideas. It is beyond the scope of this dissertation to discuss these limitations at length; however, I will identify some of the primary concerns that have been expressed as well as some counter-claims.

With regard to Vygotsky's views on culture, Wertsch and Tulviste (1992, 1994) have remarked that his perspective is ethnocentric, resulting in a Eurocentrism that makes findings from cross-cultural studies difficult to interpret. This view assumed that modern European cultural tools and forms of mental functioning are superior to those of other cultures and peoples, without framing the work in an in-depth way within the particulars of the given research context (schooling, for example). However, there are indications in Vygotsky's work that he was moving away from this kind of thinking, toward a view that "particular forms of mental functioning are associated with particular institutionally situated activities" (Wertsch & Tulviste, 1992, p. 554) rather than with some sort of developmental hierarchy. In moving forward with SCT, Wertch and Tulviste (1992, 1994) suggest that visioning mental functioning in terms of a "cultural toolkit" could help avoid the ranking of individuals or groups as "genetically" inferior or superior to others.

In addition to the problematic nature of Vygotsky's arguably limited views of culture, it has also been suggested that, while greatly concerned with social forces, he was interested in these more on a small scale (e.g., the intermental plane) than on a larger scale and thus may have ignored wider historical, social, cultural, or economic forces. Arguably, Vygotsky could have paid more attention to these macro-level forces and how they might influence the politics and policies of schools and classrooms. However, scholars have pointed out that Vygotsky's primary focus was on the intermental and intramental forces at work in social interaction; thus it seems justified that he devoted most of his attention to that topic (Wertsch & Tulviste, 1992, 1994).

Finally, it has been noted that Vygotsky's claims about the existence of a "natural development" (the genetic role), particularly as this relates to the emergence of word

meaning, need to be strengthened. However, scholars have also pointed out that there was little relevant research available to Vygotsky for study at the time; furthermore, the genetic development component of language was not of primary concern for Vygotsky and consequently this topic remained underdeveloped when compared to the social aspects of language (Wertsch, 1985, 1993; Wertsch & Tulviste, 1992, 1994). Perhaps Vygotsky would have eventually delved deeper into these and other topics as his career progressed—I and others who read his work can only speculate about how his writings and ideas might have evolved, since his life was cut tragically short when he died of tuberculosis in 1934, at the age of 37.

Despite these criticisms, it is clear that Vygotsky has contributed substantial and important theories that have strongly influenced the field of education and, more recently, the field of L2 education. This is partly because, in Vygotsky's work, which involves the social origins of individual mental functioning and the role of cultural tools as mediators of individual mental functioning, language plays the key role. The common thread of language throughout the themes and subthemes explored here reinforces Wertsch's (e.g., 1985, 1993) assertion that any themes that emerge in relation to Vygotsky's work will be very interrelated and a separation, while useful for analysis and discussion, is largely artificial.

With this in mind, in this review of Vygotsky's work I focused on those theories that relate most strongly to language and learning and how these develop in social interactions in settings, such as, for example, the classroom. I focused on how these theories can apply in particular to L2 classrooms and how SCT can provide the theoretical underpinnings for a novel approach to research in this field. Adopting SCT as

one of two parts of my theoretical framework enabled me to approach my work in such a way as to gain a fuller understanding of the complex phenomenon that is students' oral interactions in the FI mathematics classroom. Furthermore, when I extended the SCT lens to mathematics, I was able to explore the ways in which language and mathematics are intertwined and socially situated. In effect, scholars in the field of mathematics education who are interested in language, particularly those based in bilingual classroom contexts, are beginning to apply an SCT lens more and more to study mathematical communication. In the next section, I discuss how this approach to mathematics education provides the second part of my theoretical framework for my own research study.

2.2 The Mathematics Education Register

This section begins with an exploration of some of the foundational work with regard to mathematics and language, and the special ways in which these concepts are at play in mathematics classrooms. In terms of what *language* means within the field of mathematics education, Morgan et al. (2014) recently noted that the literature focuses on a number of different aspects. For example, the term *language* might be used to refer strictly to words, or might also include forms of non-verbal communication (e.g., mathematical symbolism, diagrams, graphs, gestures). In other cases, the work might focus on *language* as a means of communication used by a particular group or

community, as might be the case in bi- or multilingual classroom settings.[2] Finally, some literature centres on the style, phraseology, and vocabulary particular to mathematics, in other words, the *mathematics register*.

I will not attempt to explore all of the potential definitions of language in mathematics in this literature review. With regard to establishing my own theoretical framework, I believe that the emergence of the notion of a mathematics register represents a pivotal departure point in the literature on language and mathematics, and a key concept for understanding subsequent and current developments in the field. The research that I have found to be most relevant to my own has drawn upon the idea of mathematics register (Halliday, 1978), and, by extension and of key importance, a *mathematics education register* (Pimm, 2014; see also Morgan et al., 2014). These theoretical constructs are rooted in the larger works of Halliday (e.g., 1978, 1985, 2009) in the area of functional linguistics (i.e., the idea of language as a social, semiotic activity; the study of contextualized language-in-use). Consequently, while the mathematics education register provides a focused theoretical framework for the current

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[2] From this point forward, I use the terms *bilingual* and *bilingualism* to include *multilingual* and *multilingualism*, respectively. Taking Baker's (2011) approach, "while bilingualism and multilingualism are different, where there is similarity multilingualism is (for the sake of brevity) combined under bilingualism" (p. 3). Furthermore, I recognize the complex and problematic nature of defining *bilingual* and *bilingualism* with respect to dimensions such as ability, use, language balance, age, development, culture, and contexts.

study, I provide more in-depth support for that theory by drawing on Halliday's broader work both in this section and, later, in Chapter 3 when explaining the methodology of the current study. In addition to this literature, work on language in mathematics education that explores the particularities of language use in bilingual settings is particularly relevant to my own research (e.g., Barwell, 2009b; Moschkovich, 2010). In my theoretical framework I include these foundational ideas regarding mathematics, language, the mathematics education register, and the mathematics education register as it manifests in interactions in bilingual classrooms. Thus, the tenets of SCT and the mathematics education register come together to form the foundation of this work.

2.2.1 Language Register

As scholars in the field of mathematics education have noted (e.g., Barwell et al., 2005; D. Wagner, personal communication, 2015; Pimm, 1987), in academic, professional, or other social discussions statements are made that represent a range of views of the relationship between mathematics and language. At one end is the view that "mathematics is language-free", and at the other end, the view that "mathematics is a language". In my reading of literature on the topic, I have discovered possible explanations for the belief in either one or the other of these views, or in one that falls somewhere along the continuum between the two.

As Barwell (2005a) has explained, mathematics, in the school setting at least, might be seen as language-free because it has traditionally been viewed as a subject within which there is very little or no room for "ambiguity," or, in other words, "any possibility of more than one interpretation for a mathematical expression (p. 118).

Building on this, Pimm (1987) has suggested that, according to this traditional view, there is no need to discuss mathematics in the classroom because

> there are right and wrong answers to everything, together with clear-cut methods to be taught and learnt for finding them. . . . While there might be open problems at the frontiers of mathematics, it is all sorted out and written down at the school level. (p. 47)

Any alternative interpretations are viewed as misinterpretations that apparently stem from a poor understanding and use of mathematical language (Barwell, 2005a).

At the opposite end lies the view that mathematics is itself a language. As Pimm (1987) explained, this type of perception may be due to mathematics and language sharing some similar traits, such as having a complex and rule-governed writing system, an abstract nature, and allowing for confusion between symbol and object. However, despite some of these similarities, mathematics is not a natural language like English, Russian, Vietnamese, or French. Rather, as Halliday (1978) argued in his seminal work, *Language as Social Semiotic*, mathematics should be viewed as a register within natural language.

In order to better understand Halliday's (1978) explanation of register and, therefore by extension, of mathematics register, it is useful to first consider briefly some of his general thoughts on the nature of language. The very title of his work (*Language as Social Semiotic*) points toward Halliday's (1978) conception of language as a social enterprise. In it, he stated that,

> at the most concrete level, this means that we take account of the elementary fact that people talk to each other. Language does not consist of sentences; it consists

> of text, or discourse—the exchange of meanings in interpersonal contexts of one kind or another. (p. 2)

The "exchange of meanings" is a key concept in Halliday's (1978) work and he goes on to say that "we all the time exchange meanings, and the exchange of meanings is a creative process in which language is one symbolic resource—perhaps the principal one we have, but still one among others" (Halliday, 1978, p. 4).

This emphasis on the exchange of meanings between people as a fundamental part of language, and on language as the principle resource we have, fits well with the SCT framework I described earlier, and underpins Halliday's (1978) general explanation of register:

> Types of linguistic situation differ from one another, broadly speaking, in three respects: first, what is actually taking place; secondly, who is taking part; and thirdly, what part the language is playing. These three variables, taken together, determine the range within which meanings are selected and the forms which are used for their expression. In other words, they determine the "register." (p. 31)

These "three respects" fall within Halliday's (1978, 2009) broader notion of the *metafunctions* of language:

> These are to be interpreted not as functions in the sense of "uses of language," but as functional components of the semantic system. . . . They are the modes of meaning that are present in every use of language in every social context. (Halliday, 1978, p. 112)

Halliday (1978, 2009) referred to three main metafunctions: ideational, interpersonal, and textual. The ideational function represents the component through which cultural

experience and the individuals' experience is expressed (speakers' meaning potential as observers). Through the interpersonal function, speakers express their own attitudes and judgments and try to influence those of others (speakers' meaning potential as intruders). Finally, the textual component represents language that is operating in a context of situation, rather than in some kind of vacuum (speakers' text-forming potential). This is the component that makes language relevant to each unique context, making it contextualized, and through it, the other two metafunctions are actualized.

From this broad idea of language register situated within a social context, Halliday (1978) and, later, Pimm (1987) proposed what they view as a register that is specific to communication in mathematics: a mathematics register.

2.2.1.1 The mathematics register.

As discussed, for Halliday (1978, 1985, 2009) the exchange of meanings, or meaning making, between people represents an important aspect of language, and this is no less the case than when communicating about mathematics. Building on his general ideas about linguistic registers, Halliday (1978) noted the following in terms of a register specific to mathematics:

> A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a "mathematics register," in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)

In other words, the mathematics register includes meanings (semantics), words and structures ("lexicogrammar"), and symbols. When it comes to the lexicon, the mathematics register may include the creation of new words, either out of native or non-native word stock; the invention of brand new words (calquing); the reinterpretation of existing words; the borrowing of words from another language; or the creation of locutions. However, vocabulary is just one part of the mathematics register. Grammar is another and it is closely related to the vocabulary, sometimes with no clear line dividing the two; thus Halliday's (1978) use of the term *lexicogrammar*. Crucially, however, rather than just vocabulary and grammar, "it is the meanings, including the styles of meanings and modes of argument, that constitute a register" (Halliday, 1978, p. 195).

Building on Halliday's (1978) foundation, researchers interested in mathematics education and language have discussed ways in which the mathematics register and the notion of meaning making can be applied to the particular context of mathematics classrooms. Like Halliday, these scholars (e.g., Barwell, 2007; Pimm, 1981, 1987, 2007) have also noted that emphasis should be placed on meaning making. In some of the earlier literature, researchers identified certain potential sources of difficulty that students might encounter with the mathematics register in the classroom setting. For example, with regard to the meaning making process, Pimm (1987) argued that in most mathematics classes, both the everyday and the mathematics registers are used. However, because everyday words can take on different meanings when used mathematically (e.g., *mean* can signify "not nice" in everyday talk or, more mathematically speaking, "the average"), failure on the part of students to understand these differences can lead to errors and communication breakdown.

Pimm (1981, 1987, 1988) also identified metaphor as a key factor in mathematical classroom communication. He has suggested that students' failure to understand mathematical metaphors, and their underlying analogies, can be another source of misunderstanding in the classroom. Pimm (1987) named two types of mathematical metaphor. First, there are *extra-mathematical metaphors*, which attempt to explain a concept in terms of the real world (e.g., an equation is a balance). Second, there are *structural metaphors*, which are "a metaphoric extension of ideas from within mathematics itself" (Pimm, 1987, p. 95). According to Pimm (1987), structural metaphors are of particular concern because they are often more difficult to identify (e.g., the extension of the notion of triangle to the spherical triangle). In either case though, problems may arise from the inappropriate or misunderstood extension of a metaphor, or when a metaphor is taken literally, and "confusion arising from seeing such (often unanalyzed) truths fail contaminates not only the extended system, but also the one from which it grew" (Pimm, 1987, p. 108).

Clearly, when the mathematics register meets the mathematics classroom, there are special concerns that need to be considered. The mathematics classroom has its own culture and its own social and linguistic codes that influence communication; bilingual classrooms have additional discourses to consider regarding language use and policies. Researchers in mathematics education have aimed to explore some particular classroom concerns related to the broader mathematics register. In so doing, a number of ideas have emerged regarding the existence of a kind of mathematics *education* register.

2.2.1.2 The mathematics register in the classroom.

Prior to the important works discussed in the previous section, language was not really the focus of theory or research in mathematics education. This was an interesting discovery for me, as a new researcher in the field, since my experience is based in our current research climate, which is more open to explorations of mathematics and language. Furthermore, before reading this seminal literature, my knowledge of research in the field had been drawn mainly from more recent studies. My professional experience as an immersion mathematics teacher had also led me to the impression that mathematics and language "go together" and that it had always been so. While studies exploring mathematics and language still represent a relatively small portion of mathematics education research, it is nonetheless now not uncommon to see studies in mathematics education that focus on verbal communication and draw upon transcribed classroom interactions as data sources. In the 1990s, calls for increased emphasis on communication in mathematics classrooms (e.g., *NCTM Principles and Standards for School Mathematics*, 2000, with first and earlier editions released in the late 1980s and throughout the 1990s) in order to provide opportunities for students to learn and be able to communicate deeper conceptual understandings of mathematics, have led researchers to pay more and more attention to language in the mathematics classroom (Pimm, 1987, 1991). Pimm (2010) has commented on this shift in research climate:

> What a difference forty years can make. To look at academic research journals in mathematics education now, one could be forgiven for thinking that it has always been the case that augmented verbatim transcripts of classroom or other interactions, that autograph student work in its myriad forms, that teacherly

> inscriptions in public spaces (such as boards or, latterly, screens) have always been both the focus and manner of reporting in academic study. (p. vii)

Thus, in the last four decades or so, the mathematics register and, perhaps more importantly for educators, how the mathematics register is used in the mathematics classroom have become an increasingly recognized area of study.

Mathematics education scholars interested in language have noted that the mathematics register of the classroom, in other words the mathematics education register, has its own unique characteristics. While it may be useful to consider some of the similarities between the ways in which students and "professional" mathematicians might communicate, it is also useful, and necessary, to consider the differences between their contexts and, thus, their communication and practices. Some differences may arise because students bring a range of experiences to the classroom through their participation in the outside world that may be part of their mathematical repertoire. Additionally, and of key importance, is the social nature of the classroom environment. As noted previously, the mathematics register is more than just vocabulary or grammar—it is also the meaning making process. As Pimm (1991) explained, "part of learning mathematics is gaining control over the mathematics register so as to be able to talk like, and more subtly to mean like, a mathematician" (p. 18). The practice of meaning making through social classroom interactions adds a layer of context and thus complexity to the mathematics register, to where it becomes the mathematics education register. For instance, using the term "vector" as an example, Barwell (2007) noted:

> The idea of a vector has no intrinsic meaning; the word "vector" has no intrinsic meaning; an arrow on a page has no intrinsic meaning. Meaning has to do with

> our engagement with these things, and for me, that engagement is a social process. (p. 32)

The mathematics education register involves the ways in which students make sense of what they encounter, as individuals and through their collaboration with their teachers and each other.

By focusing on meaning making and the ways in which students communicate in the mathematics classroom, researchers have been exploring the discourse of school mathematics. In my review, I highlight and draw upon work from those scholars who work in bilingual mathematics classrooms and whose work draws on SCT concepts (e.g., Barwell, 2004; Moschkovich, 2003). A key theoretical tenet in these approaches rests on a social, situated view of mathematics. Taken at the broad, big "D" discourse level (big "D" Discourses, Gee, 2014, i.e., ways of thinking, believing, valuing, enacting a particular sort of identity; explained in detail in Chapter 3, Section 3.2.1), "mathematical Discourse includes not only ways of talking, acting, interacting, thinking, believing, reading, writing but also mathematical values, beliefs, and points of view" (Moschkovich, 2003, p. 326). With regard to a mathematics education register or mathematics classroom discourse, they can share qualities that have traditionally been valued in mathematical communities of various kinds, including "being precise and explicit, searching for certainty, abstracting, and generalizing, . . . [and perhaps less often] imagining" (Moschkovich, 2003, p. 327). That being said, researchers have also been careful to point out that mathematics classroom discourse "is not a single set of homogeneous practices . . . it varies across individuals, communities, time, settings, and purposes" (Moschkovich, 2003, p. 327). Moreover, researchers like Moschkovich (e.g., 2002, 2003,

2009, 2010) and Barwell (e.g., 2005a, 2007, 2009b) have underscored the need to value so-called "everyday" discourses for their mathematical potential in the classroom, as well as the importance of recognizing other "non-textbook" definitions and discourses as also being potentially very mathematical when taken in context (this will be discussed in greater detail in Section 2.6).

Viewing the mathematics education register in this way represents a social, discursive approach to mathematics (e.g., Barwell, 2005a, 2007, 2009b; Moschkovich, 2003, 2007; Pimm, 2007). Rejecting one of the more traditional views of mathematics as being language-free, "this approach involves a shift in theoretical orientation to language, literacy and mathematics, seeing them as less essentialist, less decontextualized, more fluid, 'fuzzy' and shifting with context" (Barwell et al., 2005, p. 143). When applied to the mathematics classroom, what follows is a rejection of deficiency models and, "from this perspective, many 'problems' . . . come to be seen instead as a resource" (Barwell et al., 2005, p. 143). Rather than focusing on students' problems or failures with regard to their ability to communicate mathematically, this approach allows researchers, including myself, to refocus on what students bring to the mathematics classroom. This view of mathematical communication represents an inclusive interpretation of what constitutes the mathematics education register.

2.2.2 Concluding Remarks on the Mathematics Education Register

The mathematics education register is a complex system that involves not only the study of discourse related to the mathematics register but also its use within the multifarious classroom context. Through SCT and the mathematics education register, my theoretical framework views both language and mathematics as social, meaning-

making processes, rooted in interactions between people. This requires a shift in traditional thinking, to viewing language "problems" as resources in the mathematics classroom.

Despite some common ground, there is still not one coherent, all-encompassing definition of the "mathematics education register" or what clearly constitutes "mathematical communication." It is, as Barwell et al. (2005) noted, "fuzzy," and highly dependent on context. Indeed, Moschkovich (2007) has argued that there are "multiple mathematical discourse practices" (p. 27) and Barwell (2014) has cautioned against what Bakhtin (1981, cited in Barwell, 2014) likened to a centripetal force, which drives us towards an unattainable unitary language. To quote Pimm (2014): "Does it make sense to talk about a coherent mathematics education register? And as for talking about *the* mathematics register, I am confident Barwell would see the use of the definite article as reflective of a unitary force at work" (p. 974, original emphasis).

Despite the fuzziness, however, understanding the evolution of the mathematics register and its applications in mathematics education provides an important context and underpinning for my own research, and for embracing the multiple meaning-making potentials of mathematical talk. With regard to language and mathematics, it is through this theoretical lens that I approach my work. Taken together, SCT and the mathematics education register represent an overarching theoretical framework for my particular research in FI mathematics classrooms. Each area of work is distinct in some ways, however the ideas stemming from each one inform the other and, I argue, blend together cohesively. This framework allows me to approach my research viewing the relationship between language and mathematics as complex, intertwined, social, and situated in

context. To explore this relationship further, I turn now to my review of selected literature of a more empirical nature from both the L2 and mathematics education fields.

2.3 French Immersion and Student Achievement

As Day and Shapson (1996) noted, "the distinguishing feature of immersion is that students learn language primarily through subject matter rather than by formal language teaching" (p. 1). Consequently, at the FI program's inception in Canada in the 1960s, one of the main concerns for researchers, educators, and parents was how well students would fare learning not only the target L2 but also the various subject content outcomes through this approach:

> It was uncertain how well students would learn French under conditions where it was used as a medium of communication to teach curriculum content areas, but also it was uncertain whether the curriculum content would be adequately learned and the first language adequately maintained and developed. (Cummins & Swain, 1986, p. 37)

Many studies were conducted in the first years of the FI program and have continued to be undertaken that measure FI students' achievement in both French as an L2 and in the content courses. I do not undertake a full review of literature related to French proficiency achievement or L1 achievement here, because my interests and research problem relate more to process than to product. Nonetheless, I felt it was important to have an understanding of what this important research has shown over the years and thus I devote some time here to providing an overview.

2.3.1 Achievement in French and English

In what follows, I present the highlights and key findings from decades of FI research. However, for more detailed accounts and summaries of these important early studies, see Cummins and Swain, 1986; Day and Shapson, 1996; Lapkin & Swain, 1984; Lyster, 1992; Marrie and Netten, 1992; Netten and Spain, 1992; Rebuffot, 1993; Rehorick and Edwards, 1992; Swain and Lapkin, 1982; Turnbull, Lapkin, Hart, and Swain, 1998; Turnbull et al., 2001; Turnbull et al., 2003. In brief, these and other studies have found that, in general, FI students achieve a high level of proficiency in the L2, French, described as near "native-like," particularly in terms of receptive skills (i.e., listening and reading). Students also acquire a relatively high level of proficiency in productive skills (i.e., speaking and writing), although not as high as the receptive skills and rarely achieving "native-like" levels (e.g., Cummins & Swain, 1986; Day & Shapson, 1996). Nonetheless, immersion students' French proficiency levels have been found to be consistently higher that those of students who follow regular core programs (see, e.g., Cummins & Swain, 1986). The appropriateness of the "native" or L1 French speaker as a target for L2 language speakers is an area of considerable debate that, although I will not enter into it here, deserves thoughtful consideration (see, e.g., Baker, 2011; Day & Shapson, 1996). The debate does, however, relate (among a number of other issues) to the notion of L1 use or *codeswitching* while using the L2, a topic that I will return to in a later section of this literature review.

With regard to FI students' L1 (English) proficiency, empirical research has shown that although students learning literacy solely in the L2 are usually initially behind in L1 skills compared to students instructed solely in the L1, FI students perform equally

as well as non-FI students on standardized tests within one year of the introduction of English language arts. This phenomenon has been described as an immersion "lag" (e.g., Cummins & Swain, 1986) and will be revisited later. That FI students "catch up" in English language arts and achieve as well as their English program counterparts can be explained by the power of the L1 majority language, the support provided for it (home, media), and Cummins's (1981, 2000) theories of language interdependence and transfer.

Overall, large-scale studies of student achievement in French and English suggest that FI students are not disadvantaged by being in an immersion program and in many cases seem to achieve results that exceed those of their English-program counterparts (although explanations for this phenomenon are not straightforward, as discussed in the introductory chapter of this dissertation). While significantly less researched, the finding that FI students are not disadvantaged also tends to hold true for students considered "at-risk" for academic difficulties (see, e.g., Genesee, 2007 for caveats and a discussion). With regard to FI students' achievement in mathematics, results are also generally positive. The next section highlights some of the key work done in this area.

2.3.2 Achievement in (French) Immersion Mathematics

A number of large-scale studies have been conducted over the years in order to investigate Canadian FI students' performance on standardized mathematics assessments. Such studies conducted in the early days of the program, in the 1970s and 1980s for example, determined that, in general and in the long term, FI students are not disadvantaged compared to their non-immersion counterparts in terms of mathematics achievement. However, when considering these types of studies, both those conducted early on in the FI program and more recently, it is important to bear in mind two

important variables: (a) the intensity and point of entry of the program (early total immersion, early partial immersion, late immersion, etc.); and (b) the language of the mathematics assessment (i.e., French or English). Throughout most of the earlier studies, FI students were tested in English in mathematics. This was due in large part to parents' concerns regarding students' ability to transfer mathematical skills and knowledge acquired in French to an eventual English setting (e.g., high school, university, job market). These studies determined that students in the early total immersion program (beginning in Kindergarten or Grade 1, 90-100% instruction in French), who had received mathematics instruction in French but wrote assessments in English, achieved mathematics scores that paralleled or exceeded those of students in the English program. With regard to students enrolled in early partial (beginning in Kindergarten or Grade 1, approximately 50% instruction in French) or late immersion programs (beginning in Grade 6 or 7, approximately 80% instruction in French), some studies, although not all, showed that these students' results were inferior to those of students in the English program. However, these studies also found that for early partial or late immersion students, this initial gap, likely due to insufficient L2 comprehension at the outset of the program, was resolved in the long term, with students eventually achieving mathematics scores that were comparable to students in the English program (Cummins & Swain, 1986; Day & Shapson, 1996; Swain & Lapkin, 1982).

More recent large-scale studies regarding mathematics achievement in immersion have largely corroborated earlier research conducted in the Canadian FI context. Some of these recent studies have also been based in Canadian FI classrooms, whereas others have been situated in international immersion contexts. In Canadian FI, two large-scale studies

in particular confirmed earlier findings. Turnbull et al. (2001) analyzed Grade 3 students' assessment scores on standardized provincial mathematics tests (Education Quality and Accountability Office [EQAO] testing) in Ontario, in which students were rated on a level scale of *1* to *4* (*1* being the lowest and *4* the highest). All FI programs in the study were early immersion programs, but which had introduced English instruction at different points and with differing levels of intensity (thus resulting in programs ranging from early total to early partial). Of further consideration, some school districts chose to administer the mathematics tests in English, while others chose to use French translations. Therefore, in considering the results reported in the study, it is important to bear in mind the language of testing, the point of introduction of English instruction (in this study, either Grade 1/2, Grade 3, or Grade 4), and the intensity of English instruction as measured by accumulated hours (which were, in this study, 0 in the early total immersion program; ranging from ≤ 420, to ≤ 500, to ≤ 750, in other early immersion programs; and 1400 in an early partial [50/50] immersion program).

Turnbull et al. (2001) found that in mathematics, "the levels profile of immersion students was, overall, broadly similar to that of non-immersion students" (p. 16). More specifically, there was no "systematic relationship" (p. 20) between the points of introduction and intensity of English instruction and performance on the Grade 3 mathematics assessment, however there was some variability. For example, the weakest performance in mathematics was among students in two programs in which English instruction was introduced in Grade 1 or 2 but resulted in less than 500 or less than 750 hours accumulated instruction as of Grade 3. These students, fewer in number due to this

being an uncommon type of immersion program, underperformed when compared to other immersion students, and also when compared to students in the English program.

With regard to language of testing, school districts had the choice of whether to test immersion students using the English mathematics assessment or a French translation, even though students in all immersion programs had received French mathematics instruction. All districts with early total immersion programs chose translated (French) tests, while districts that had introduced some level of English instruction in Grade 1/2 chose both versions with a tendency toward the translated (French) tests. The exception was districts with early partial immersion (50/50), all of which chose English tests. All students in early total immersion, who, as noted, wrote French translation versions of the tests, performed similarly to their English program peers. Students of other immersion programs, some of whom had written the tests in English and some of whom had written them in French, also achieved results that were comparable to students in the English program (save the exception noted previously; Turnbull et al., 2001).

However, when comparing achievement scores between immersion students who wrote English tests and those who wrote French tests, the results were mixed. The results for students in immersion programs receiving 420 hours or less of accumulated English instruction in Grade 3 are perhaps the most noteworthy, because the districts in this category were divided almost equally in their choice of testing in English or French. In this category, students who wrote the test in English did markedly better than students who wrote the test in French translation. As the authors noted, these results seem counterintuitive, as mathematics instruction had occurred in French, and thus worthy of further investigation in future research. However, in immersion programs receiving 500

hours or less of accumulated English instruction, the French test group outperformed the English test group. In spite of that, in programs receiving 750 hours or less of accumulated English instruction, the situation was reversed. However, the authors noted that in both of these categories (≤ 500 and ≤ 750), only one district had chosen the English version of the test, thus making it difficult to make sound comparisons. Based on these mixed outcomes, the authors concluded that the results, at least, suggest that overall FI students are not disadvantaged when writing mathematics tests in English (Turnbull et al., 2001).

Turnbull et al. (2003) also examined Grade 6 students' results on EQAO provincial tests in Ontario. The study was not longitudinal, meaning the Grade 6 students were not the same cohort as the Grade 3 students described in the authors' 2001 article. With regard to the Grade 6 students, again a wide variety of FI programs were represented, each with different points of introduction to English instruction and varying amounts of accumulated English instructional hours. Whereas in Grade 3, FI students generally only matched (although they sometimes slightly exceeded) non-immersion students' scores, in this study, the authors noted that the Grade 6 FI students markedly outperformed students in the regular English program on mathematics assessments (as well as on reading and writing assessments). This was true when FI students' results were compared both to students in the "regular" English stream and to students in an identified English "enrichment" stream. In this study, all mathematics tests were written in English and there was no systematic relationship found between mathematics results and the grade at which English was introduced or accumulated hours of English instruction. The authors proposed and explored two possible explanations for the immersion students'

outperformance of the English cohorts: "the 'extended lag' explanation" (p. 13) and "the selection explanation" (p. 16). Touched upon briefly in the Introduction, it is worth expanding these ideas here.

Turnbull et al.'s (2003) extended lag explanation focuses mainly on the English reading and writing scores reported in the article. It builds on the lag hypothesis put forth by earlier research in FI (e.g., Cummins & Swain, 1986) as mentioned previously, that suggested that immersion students experience a delay in academic achievement, particularly in English reading and writing, upon entry to the immersion program. The lag is described as temporary, and researchers have found that within 1 to 2 years of the introduction of English instruction, FI students catch up to their peers in the English program in terms of English reading and writing and general academic performance in other subjects (Swain & Lapkin, 1982, cited in Turnbull et al., 2003; see also Cummins & Swain, 1986). As Turnbull et al. (2003) explained, there appears to be a "threshold amount of English language instruction needed to set in motion gains in English language performance [and it is] . . . quite low and easily met by virtually all early immersion program formats" (p. 15). The extended lag explanation proposes that there is "a second, higher threshold of English-language instruction that once crossed results in added gains in immersion students' performance in English" (p. 15), which could explain the FI students' higher achievement in this area when compared to their English program counterparts.

The second of Turnbull et al.'s (2003) possible explanations for the FI students' outperformance of the English cohort is related to a common criticism of immersion programs, which is that of elitism. As the authors noted, attrition from the FI program has

been largely linked to academic difficulty. This is not a straightforward relationship, as issues of student and parent choice, lack of resources, the possibility of students being counselled out of the program, and more, all come into play. Nonetheless, the authors explained that it is the students with weaker academic performance who tend to leave the FI program over the years. Feasibly then, FI students' outperformance of English regular program students on the mathematics and other assessments could be a result of having mainly the strongest performing students remaining in immersion by the time they all reach Grade 6. The authors tested and explored both the extended lag and the selection hypotheses, but found it difficult to provide empirical proof for either.

A final empirical study, this time on a smaller scale, conducted in the Canadian FI context is also noteworthy with regard to insights on student achievement. Bournot-Trites and Reeder (2001) conducted evaluative research in British Columbia that addressed a unique situation in FI mathematics. The school in the study had proposed to increase instruction in French from 50% to 80% of instructional time in their FI program, which included a switch from mathematics being taught in English to mathematics being taught in French in Grades 4 through 7. Parents were concerned about whether the students being taught mathematics in French would be disadvantaged in comparison to the English mathematics group. The last cohort of students to receive English mathematics instruction was tested, along with the subsequent year's French mathematics group. Both groups wrote the mathematics test in English. Overall, results showed that the students who had studied mathematics in French outperformed the English mathematics students on these standardized tests (Stanford Diagnostic Mathematics Test). The results alleviated parents' concerns and also supported the notion of positive transfer (Cummins,

1981, 2000) in that students were able to apply mathematical concepts learned in French to problems in English. The authors discussed plausible explanations for the FI students' performance, such as the possibility that students' expectations of increased difficulty in the French mathematics cohort could have led them to work harder and focus more on the lessons. The authors suggested that this increased work ethic and concentration may have led to a deeper understanding of the mathematical concepts.

Along with these noteworthy studies based in the Canadian FI context, a few international empirical studies have also examined student performance in immersion mathematics. As previously noted, immersion programs share a number of common characteristics but implement these along a continuum. Furthermore, immersion programs, particularly when compared across international borders, vary in regard to several other factors. It is useful to consider results from international studies in immersion contexts even though these programs may differ slightly or in some cases quite substantially from the Canadian FI context since a number of parallels could be drawn with regard to learning mathematics in an L2 regardless of context. It is also imperative, however, to keep these differences in context in mind while reflecting the research findings, since the context always influences research and its interpretations.

Overall, empirical studies of student performance based in international immersion programs corroborate the findings from research conducted in the earlier days and the more recent years of the Canadian FI program. One of these international studies includes research based in a content and language integrated learning (CLIL) context in Finland. Jäppinen (2005) explained that CLIL approaches to content teaching through a foreign language are based on Canadian FI programs but are adapted to reflect European

cultural, social or linguistic needs. In this large-scale study the CLIL was modeled after Canadian early total immersion; the immersion languages included in the study were English (60% of students), French (30%), and Swedish (10%), and in each of the immersion programs the students' L1 was Finnish. Three different age groups of immersion students, aged 7-9, aged 10-12, and aged 13-15, were tested in mathematics, as were three control groups of non-immersion students. Students were tested four times, from the spring of 2002 through the fall of 2003. The immersion students had received mathematics instruction in the L2 and the non-immersion students in their L1, but all students were tested in the L1, Finnish. The tests aimed to measure students' *cognitional development*, which, for Jäppinen, represented "thinking and content learning processes" (pp. 150-151). Cognitional development was "assumed to manifest itself in understanding, using and applying concepts and conceptual structures of the contents taught through a foreign language in mathematics" (p. 151).

Jäppinen (2005) reported that when the results for the first age group (7-9) and third age group (13-15) of immersion students were compared to those of the corresponding age groups of non-immersion students, there were no significant differences between any of the four test measures. With regard to the second age group, immersion and non-immersion students' scores were equal for the first test. For the second, third, and fourth measurements however, the immersion students outperformed the non-immersion students. The author suggested that these results support content learning via CLIL and also support the notion of positive transfer. With regard to the immersion students' outperformance of non-immersion students in the second age group, Jäppinen acknowledged that, while efforts were made to control for class composition by

choosing similar schools located in similar settings, it is possible that the results could be influenced by factors such as socioeconomic status, home environment, or the nature of the learners (echoing similar statements made in Canadian FI research).

Also on the international level, de Courcy and Burston (2000) conducted a large-scale study in Australia in an early partial immersion program. In this program, immersion students were instructed in the L2, French, for approximately 45% of the time and in English, the L1, for 55% of the time from Grades Preparatory to Year 6. The researchers were interested in whether mathematics skills learned through the L2 could be accessed in the L1. Specifically, the study examined whether students who had been taught mathematics in French could be successful on standardized tests (Progressive Achievement Test-maths [PAT Maths]) covering the same mathematics concepts but administered in English. The authors were also interested in comparing the immersion students' results to the national averages of non-immersion students. The immersion students in Years 3, 4 and 5, all of whom had been taught mathematics in French, were divided into two groups per Year. Stratified random sampling was applied to the class lists, whereby students of high, average, and low mathematics "ability" (p. 79), as identified by their teachers, were distributed approximately equally amongst the groups. All students wrote the mathematics tests in both French (L2) and English (L1).

In general, the immersion students in all three Years, whether they wrote the test in French or English, performed higher than the national average. With regard to language of testing, the results from the Year 4 and 5 students were examined. For Year 4 students, there was no statistically significant difference in results between the group that wrote the test in English and the group that wrote the test in French. For Year 5 students,

however, the students tested in English did significantly better than those tested in French. The authors suggested that this was likely due to a lack of French vocabulary that would enable students to be capable of dealing with the higher cognitive demands associated with reading the French word problems. In the article, the authors also provided an item analysis to further investigate this point (de Courcy & Burston, 2000).

In general, de Courcy and Burston's (2000) research study showed that immersion students' scores matched or exceeded the national averages for non-immersion students. Furthermore, the results showed that immersion students could transfer mathematics skills learned in French and apply them in English. "Specifically, children taught maths in French do not need to be re-taught in English in order to succeed in tests in English" (p. 93). However, the authors noted that the Year 5 students in particular did suffer a disadvantage when tested in the L2. This was especially true in the case of word problems. The researchers believed that given additional time and more direct instruction in the L2, these word problems could be completed successfully by students in French. The authors also recommended that "students' ability in reading in their native language (for most, English) be harnessed to facilitate reading in French, rather than being seen only as a source of interference" (p. 94).

In contrast to the mainly positive results reported in the Canadian FI studies and the Australian and Finnish studies, empirical research based in Hong Kong immersion classes has questioned the effectiveness of content teaching through the L2 in terms of student achievement. Marsh, Hau, and Kong (2000) conducted a large-scale longitudinal study to examine the mathematics results of students in a late English immersion program at a Hong Kong high school. Students wrote standardized assessments in all of their

subjects, in the language of instruction for that particular subject. In the case of mathematics, the language of instruction, and therefore the language of testing, was English (L2). In this study, immersion in the L2 showed positive student results for English (L2) and Chinese (L1) performance, but there was a remarkable negative effect on their performance in science, geography and history. There was also a small negative effect detected on mathematics performance. Thus, this study suggested somewhat mixed results with regard to student success when content is taught in the L2. However, these results must be interpreted within their specific context, which in this case differs in a number of ways from many other immersion contexts.

While both the Finnish and Australian immersion contexts, having early total or early partial immersion programs, respectively, more closely resemble the research contexts of the studies conducted in Canadian FI, Marsh et al. (2000; see also Kristmanson & Dicks, 2014) have recognized that the Hong Kong immersion context within which their study was situated differs in a few key areas when compared to many other international immersion programs. On a large scale, unlike Canadian FI programs, the Hong Kong immersion program would be considered immersion in a language of power rather than immersion in a second or foreign language, which carries with it special considerations regarding power and political issues (there are issues related to language power and politics in Canadian FI as well, but the context, of course, is different). Several smaller-scale factors also differed. The point of entry into immersion was later in the Hong Kong program than in most Canadian programs (in Hong Kong, immersion only begins at the secondary level, a delayed late entry). The authors argued that plausibly, the Hong Kong students' knowledge of the L2 did not have time to

develop enough to handle the linguistic demands of the complex and often abstract content presented at the high school level. Also, there was often a lack of qualified teachers who were fully bilingual in both the L1 and L2 and also had an expertise in the content areas. Finally, the authors acknowledged that the program provided little bridging support and that there may have been an overall lack of support for the L2.

The large-scale and smaller-scale, quantitative, empirical studies conducted in national and international immersion contexts are important for knowing about students' academic achievement in content subjects, in this case mathematics, when taught via the medium of the L2. Notwithstanding the importance and value of such studies, it is also imperative to note their respective limitations. All of the studies discussed here were large scale (mostly involving standardized testing), with the exception of Bournot-Trites and Reeder (2001), and all were quantitative in nature, thus the richness and different perspectives provided by small-scale, qualitative research is absent. Furthermore, these studies employed quasi-experimental designs, using control groups to draw comparisons between immersion and non-immersion students, and/or immersion students writing English tests and immersion students writing tests in translation. The usual limitations of control groups in quasi-experimental design would apply. While researchers in these studies did their utmost to control for factors such as socioeconomic status, academic ability, gender, motivation, home environment, and so on, none of these influences can truly be accounted for when conducting social science research in education. Finally, some immersion programs in the various studies differed from one another significantly in their designs (e.g., early total immersion, early partial immersion, late immersion). Having explored literature related to student achievement in FI in Canada and other

immersion programs around the globe, I turn now to a more qualitative look at what it is like for students doing mathematics in an L2 in Canadian FI, other immersion programs, and other non-immersion bilingual classroom settings.

2.4 Bilingual Mathematics Classrooms

The existence of research of a qualitative nature that explores what it is like for students to do mathematics in an L2 is extremely limited in the Canadian FI context. In other international contexts, researchers in the field of mathematics education having an interest in language and, in particular, in bilingual mathematics classrooms (though not necessarily immersion programs), have provided important work to consider. Although this kind of research is emerging in the mathematics education field, it nonetheless still represents a relatively small portion of the entire research body. There are three broad considerations that I have gleaned from reading this international research on bilingual mathematics education that I would like to address prior to discussing some of the finer details and pertinent findings of these studies. These three key considerations—tensions, context, and politics and policy—are important for moving forward with my own research study in a bilingual mathematics classroom. They are briefly discussed here, prior to proceeding with the remainder of the literature review, because they need to be reflected upon when reading each study.

2.4.1 Tensions

One of the first things that must be considered when researching bilingual mathematics classrooms is "tensions" and these tensions must then continually be reflected upon when interpreting any piece of literature or conducting any classroom-based research. Barwell (2009b, 2009c, 2010) has described tensions that are inherent

and continually at work in virtually all bilingual mathematics classrooms. These tensions are between:

1. language and mathematics,
2. formal and informal language used to discuss mathematics,
3. students' home languages and the official language of schooling,
4. mathematical understanding and the social value of an L2, and
5. policy goals and classroom practice.

When reviewing literature that explores language issues in the bilingual mathematics classroom in various contexts, the tensions are evident. However, the ways in which the tensions are enacted and experienced are highly dependent on the particular context and the policies and practices at play in the classroom. These five tensions are something to consider and are woven throughout any discussion of literature regarding bilingual mathematics classrooms. These tensions infiltrate the next two broad considerations for researching bilingual classrooms as well: context, and politics and policy.

2.4.2 Context

One of the first things that need to be considered when researching bilingual mathematics classrooms is context. What is a bilingual mathematics classroom? There is no one description that fits what such a classroom might look like, however Barwell (2009b) provided a general definition that is useful:

> Mathematics classrooms are considered to be multilingual [or bilingual] if two or more languages are used overtly in the conduct of classroom business. And mathematics classrooms are *also* considered to be multilingual if students *could*

use two or more languages to do mathematics, even if this does not actually occur. (p. 2, original emphasis)

As with any classroom, and perhaps more so when it comes to bilingual mathematics classrooms, each one has its own unique context, composition, and culture.

The acknowledgment of the wide variety of contexts for bilingual mathematics education research does not imply that there is nothing to be gained from consulting research conducted in a setting different from one's own. Research based in one context may be quite relevant to that based in another. For example, Barwell (2009b) suggested in the introductory chapter of *Multilingualism in Mathematics Classrooms: Global Perspectives*, that his edited book has two related aims: first, to "give a sense of the diversity of what multilingual mathematics classrooms can be like" (p. 1); and second, to "explore issues arising in these particular contexts in such a way that this exploration informs practice across contexts" (p. 2).

It was essential to gather research from other particular contexts to inform my own. However, recognizing and reflecting on the differences between contexts was also important because the tensions that have been identified by Barwell (2009b, 2009c, 2010, 2014; see here Section 2.4.1) as relevant to and continually at play in most bilingual mathematics classrooms may manifest differently or to different degrees depending on context. In additive immersion contexts, majority-language L1 speakers choose to learn a minority, but nationally official L2; the situation is additive since the idea is to add the L2 to the L1, the latter of which continues to be valued and supported. An example of additive immersion is FI in Canada, which is considered a strong form of bilingual education that promotes bilingualism and biliteracy. This differs (and in many cases quite

substantially so) from submersion contexts such as mainstream (often English) classrooms in which speakers of a minority L1 are expected to "sink or swim" in English; in submersion contexts the idea is usually to replace the students' L1 with the L2 (English). Mainstream English programs, particularly those with no supports for English language learners, in the United States or the United Kingdom are examples of submersion environments. Both of these contexts differ, in turn, from those in which the immersion is in a foreign language of power (i.e., English). In many of these cases, such as English immersion in Hong Kong or South Africa, for example, the L1(s) and English have different official statuses and power relations (see Baker, 2011, for an in-depth look at various forms of bilingual education). Thus, the context of the particular bilingual mathematics classroom relates very much to the next topic of consideration: politics and policy.

2.4.3 Politics and Policy

The policies, particularly the language policies, of bilingual mathematics classrooms are inextricably linked to, and largely driven by, politics (micro and macro). For example, Moschkovich's work (e.g., 1999, 2002, 2003, 2005, 2007, 2009a, 2009b, 2010) is based in mainstream English classrooms in the United States where the target language of teaching and learning mathematics is English. However, Moschkovich has focused many of her studies on how languages are used in the classroom among the students for whom Spanish, an unofficial minority language, is the L1. Although Spanish is considered a minority language in the United States broadly speaking, in these classrooms (like in many across the country) the Spanish L1 students make up the majority of the classroom composition and, in some cases, all of it. In spite of this, the

idea of using all available languages (i.e., Spanish and English) as resources for mathematical communication has remained unexplored or unaccepted in schooling. This relates to the political language climate in the United States, where speaking English, the language of power and de facto national language, is very important politically, educationally, and economically speaking and is tied to views on national unity. Recent political events in the United States, such as the presidential election of 2016, has brought to the forefront the degree to which language/culture-related Discourses (big "D" Discourses, Gee, 2014, i.e., ways of thinking, believing, valuing, enacting a particular sort of identity; explained in detail in Chapter 3, Section 3.2.1) run deep, affecting almost every aspect of public and private activity (including education).

Situated in a very complex political and language climate in South Africa, where there are no fewer than 11 recognized official languages, Adler and Setati (e.g., Adler, 1998, 1999; Setati, 2002; Setati & Adler, 2000) have highlighted the dilemmas teachers face when it comes to classroom language policy and student learning. Decisions about whether or not to school children in English or in their home language, for example, Tswana, can be politically motivated. In many cases, English is the chosen target language of teaching and learning. There is demand by parents for access to English, a global, power language, and learning mathematics in English is seen to be more advantageous to students than doing so in another language. Consequently, even though in these classrooms English is just one of many languages known to students and teachers, English-only classroom language policies are put in place so that students are exposed not only to mathematical content, but also to the English language. In the South African

context, the situation is further complicated by colonialist histories that carry with them issues of power and marginalization.

With some similarities, but a different context in Pakistan (Halai, 2004, 2009), students from varied L1 backgrounds (e.g., Gujrati, Katchi, Sindhi, Urdu) but usually sharing Urdu as a common language, study English mathematics for many of the same reasons motivating English schooling in South Africa. In Pakistan, language educational policies are politically driven and affect the ways in which home languages other than English are viewed, valued (or not), and used (or not).

In other contexts, particularly those in which mathematics education takes place within a bilingual education program, such as FI in Canada, students' L1 (English) is the shared majority language and they learn mathematics in an L2. In these contexts, students all have similar proficiency, that is, very limited to no proficiency, in the L2 at the outset of their schooling (e.g., Swain & Johnson, 1997; Swain & Lapkin, 2005). However, this is markedly less so in urban areas of Canada in recent years (see Swain & Lapkin, 2005). In these bilingual mathematics classrooms, where students have chosen to learn an L2, most often a minority language (but at times a politically and/or economically powerful one, that may even have status as a national official language, such as French in Canada), classroom policy usually requires that the L2 be the only language of teaching and learning. Arguments could be made that this type of L2-only policy is more reasonable considering these kinds of contexts, when compared to other kinds. In another example, in Welsh-medium schools and classrooms in Wales, Welsh is the main language of communication in the mathematics classroom, however students' L1 may be Welsh or English and there appears to be a more relaxed policy when it comes to language use

within the classroom (e.g., Jones, 2009). Nonetheless, in both of these contexts students' choice to learn an official national language influences the classroom language policy.

While the Canadian FI and Welsh contexts are different from each other in some ways, taken together they are different from the contexts described by Adler and Setati, Moschkovich, Barwell (in most of his work), or Halai. In the case of English and Welsh in Wales, for example, Jones (2009) claimed that the two languages have "comparable currency within the education system" (p. 127); and, with regard to English and French in the Canadian context, despite some historical (and current) bumps in the language road, a similar argument could be made for the languages in question. In the other contexts mentioned, the political structure is arguably different with issues of power, marginalization, colonialism, and mainstreaming at play. In this way, while politics and policy are an important part of any bilingual mathematics classroom, these factors manifest differently depending on context.

With regard to politics and classroom language policy, I have highlighted research studies from different contexts that I have found most relevant to my own work to illuminate two key points. The first is that each political context brings with it its own language in education policy. Second, policy affects if and how different languages are valued (or not) and/or used (or not) in bilingual mathematics classrooms. The ways in which language policy is put into practice is complex and, again, manifests differently in different contexts. Moreover, the five tensions put forth by Barwell (2009b, 2009c, 2010) weave through the contexts, politics, and policies of bilingual mathematics programs and classrooms.

Despite the particulars of these various research contexts, a common thread throughout the literature that resurfaces over and over again in almost every bilingual classroom context, mathematics or otherwise, is that monolingual language policies are often at play and that these policies are challenged in practice with regard to the use of multiple languages in the classroom. The use of two or more languages within a single speech utterance, or codeswitching, is evidently a key topic of concern for researchers and educators in the fields of L2 education and those interested in bilingual contexts in mathematics education. This phenomenon is also a key focus of my own study. Thus it bears important consideration in the literature review next.

2.5 Codeswitching

As Planas and Setati-Phakeng (2014) have pointed out, language policy is fixed within bilingual mathematics classrooms, but there is room, in practice, for teachers and learners to make language choices. Despite policies in a variety of contexts that dictate that only the target language be used in bilingual classrooms, including mathematics, research has shown that bilingual students sometimes draw upon and use their L1 when it is different from the target language of learning and instruction. In general, use of the L1 while speaking the L2 can be referred to as codeswitching, where the L1 and the L2 are linguistic codes (Levine, 2011). Some scholars nuance a switch made at the word level as "code mixing" rather than codeswitching (Baker, 2011). However, Baker (2011) has explained that "codeswitching has generally been used to describe any [language] switch within the course of a single conversation, whether at word or sentence level or at the level of blocks of speech" (p. 107). I operationalize and define codeswitching for the

context of my work as the use "of linguistic material from two or more languages in the same sentence or conversation" (Levine, 2011, p. 50).

Since approximately the 1960s, codeswitching has been systematically studied among bilingual populations from sociolinguistic standpoints. Researchers now recognize it as a complex and varied activity, serving different purposes in any given conversation. Historically (and to some extent even today), however, some researchers, educators, L2 learners, and other members of the general public have viewed codeswitching in a negative way. It can be seen as a "communication deficit" (Baker, 2011, p. 106) and as reflecting an inferior grasp of one or both of the languages at play. Pejorative terms such as "Spanglish" (Spanish-English), "Wenglish" (Welsh-English), or "Franglais" (French-English), are "used in a derogatory fashion to describe what may have become accepted language borrowing within a particular community" (Baker, 2011, p. 106). "Language interference" was a term commonly used to refer to the mixing of languages, a term now regarded as pejorative by many bilinguals, who more recently prefer less deficit-oriented terms such as "transfer" or "cross-linguistic influence" (Baker, 2011, p. 107). Such issues relate strongly to language assessment, as, historically, monolingual "native speaker" proficiency has been the idealized linguistic target for L2 learners and the measure against which their language proficiency has been assessed. Social, political, and cultural factors may also cause codeswitching to be viewed negatively. For example, where conflict exists between ethnic groups, codeswitching may be perceived as disloyal (Baker, 2011).

Many of the traditionally negative views of codeswitching likely stemmed from the larger issue of negative views of bilingualism in general. As scholars have recently

explained (e.g., Baker, 2011; Cummins, 2000), in the past, much of the discourse and research about bilinguals had been unfavourable. Baker (2011) has suggested that these negative discourses were particularly acute in English-speaking regions. This may be due to colonial mentalities and, especially more recently, the adoption of English as a lingua franca. Such research normalized the monolingual, and went further by suggesting that the bilingual brain was deficient. However, present research has led to new, more positive understandings of bilingualism and the bilingual brain. Cummins's (1981, 2000) common underlying proficiency (CUP) model and theories of language interdependence mark a major turning point. Recent advances in brain-based research by leading scholars like Bialystok (e.g., 2001, 2007, 2009) have further served to better researchers' understandings of bilingualism and cognition; results of these studies have also shown a number of positive effects associated with bilingualism.

As Baker (2011) has explained, prior to Cummins's theories, many viewed the two languages in the bilingual brain as existing in a sort of balance (weighing scales). In the balance view of bilingualism, the L1 occupies one side of the balance, while the L2 resides on the other. The similar balloon analogy views the L1 and the L2 existing as language balloons inside the bilingual's head. Much like the ideas behind the balance view, as the L2 balloon is pumped up, so the L1 balloon must deflate in order to "make room." Moreover, the monolingual is seen to have one well-filled balloon, while the bilingual is pictured as having two inadequately filled balloons. These analogies imply at least two key assumptions: First, that as the L2 "increases," the L1 must "decrease" in order to maintain balance and make room in the brain. Second, that the L1 and the L2 are very much separate and distinct from one another. (Baker, 2011).

Cummins (in Baker, 2011) has referred to these outdated and naïve views of bilingualism and cognition as the separate underlying proficiency (SUP) model of bilingualism. Perhaps not surprisingly, as Baker (2011) has explained, ideas based on the SUP model led to negative and deficiency-oriented views of bilingualism and of codeswitching in particular. Based on SUP conceptions, bilingualism was thought of as a "condition" which should be avoided at all costs and, should one be "afflicted," it was determined to be the cause of a variety of problems including confusion, low intelligence, learning disorders, identity conflicts, schizophrenia, and more. However, new views of bilingualism based in large part on Cummins's (1981) work, have offered a different and much more positive view of the bilingual brain.

In contrast to the SUP, Cummins's (1981, 2000) seminal and widely regarded work on the CUP model of bilingualism and theory of language interdependence requires an integrated view of language and a positive outlook on bilinguals' cognition and their linguistic practices—even when these practices include codeswitching. Cummins' theories are often explained using an iceberg analogy, in which two peaks of an iceberg appear separate above the surface of the water. However, underneath the surface, the peaks are joined as one large iceberg. Thus with regard to language, while any bilingual individual's L1 and L2 may appear different based on their surface features, the two are joined at the deeper, cognitive level. The model argues that there is a common underlying proficiency and both languages function as one through a central operating system. The thoughts that accompany speaking, listening, reading, and writing come from the same central engine, irrespective of the language of operation. When languages are viewed as an integrated system, codeswitching need not be seen as a deficit marker.

Because of this CUP (Cummins, 1981, 2000), skills learned in one language need not be relearned in the other. This point has implications for bilingual schooling in particular, as this is what purportedly enables the positive transfer of ideas, thoughts, and skills from one language to the other. In addition, the CUP model argues that the brain has capacity to store not just two but any number of languages. Finally, according to Cummins's model, educational attainment and skills can be developed just as well through two, or more, languages as through one (see Baker, 2011, for a discussion). Studies such as those reported here regarding students' mathematics achievement when having learned concepts in one language but having been tested in another, as well as those studies that report FI students' high achievement even in English language arts, support the notion of language interdependence and positive transfer. Outside the context of schooling, research by leading scholars in the field of brain-based language research, such as Bialystok (e.g., 2001, 2007, 2009), which suggests that knowing and using more than one language can offset dementia, increase flexible thinking, and enhance problem solving capability and divergent thinking, has also served to begin to change what have in the past been some negative misguided notions of bilingualism and alleviate some if its associated anxieties (see also Baker, 2011, for a discussion).

Due in large part to these cognitive theories, such as the CUP model and language interdependence hypothesis, and cutting-edge brain based research that now view languages as integrated within the human mind, codeswitching has been able to be studied increasingly from non-deficit viewpoints in sociolinguistics research. Rather than continually and, as Levine (2011) suggested, futilely "striving for monolingualism in a multilingual world" (p. 19), an integrated conceptualization of languages and bilingual

identity accepts codeswitching as a naturally occurring phenomenon among bilinguals that plays an important role in communication. Cognitive-sociolinguistic theoretical models such as the markedness model (Myers-Scotton, 1993, 1998; see also Levine, 2011) have argued that, "in their language own use, individuals exploit the relationships that become established in a community between a linguistic variety [e.g., choices of one language rather than another] and who uses the variety, and where and how it is used" (Myers-Scotton, 1998, p. 18). These models have underscored the idea that codeswitching involves choices and is not a haphazard practice based primarily on filling lexical gaps. In the markedness model, limitations on choices should not be equated with limitations on repertoire—speakers are viewed as rational decision makers and may choose to use an unmarked (expected) or marked (unexpected) code for a variety of social purposes (Myers-Scotton, 1998). Indeed, from a sociolinguistic perspective, Baker (2011) identified 13 overlapping purposes of codeswitching. It may be used to: (a) emphasize a point; (b) overcome a lexical gap; (c) express a concept that has no equivalent in translation; (d) reinforce a point or request; (e) clarify a point; (f) express identity; (g) relate a conversation held in a different language; (h) interject; (i) ease tension and inject humour; (j) signify a change of attitude or relationship; (k) exclude people from a conversation; (l) discuss certain topics; and (m) copy peers or, in the case of children, adults. Recent research has shown that children as young as 2 years of age are developing communicative competence and can be sensitive to context when it comes to choosing the language of communication.

Moving even further beyond these more recent (and more positive) cognitive and social ways of thinking about the simultaneous use of multiple languages, emerging work

on *translanguaging* considers how bilingual learners in various contexts use not only their different languages but also other resources in order to communicate and engage in meaning making (e.g., Creese & Blackledge, 2010; Garcia, 2009; Garcia & Wei, 2014). In an emergent field such as this, particularly one that involves complex subjects like language and cognition, singular definitions are difficult to find. Nonetheless, regardless of particular scholarly orientation (e.g., sociolinguistic, psycholinguistic), researchers interested in translanguaging generally seek to move beyond the perceived limitations of terms such as *bilingual*, *multilingual*, or even *plurilingual*, all of which are considered to "refer to a plurality of autonomous languages . . . whether at the individual (bilingual/plurilingual) or societal level (multilingual)" (Garcia & Wei, 2014, p. 15). Thus, translanguaging "goes beyond what has been termed code-switching . . . , although it includes it, as well as other kinds of bilingual language use and bilingual contact" (Garcia, 2009, p. 37). Translanguaging is a practice, although not a homogeneous one; indeed, "translanguagings are *multiple discursive practices* in which bilinguals engage in order to *make sense of their bilingual worlds* [original emphases]" (Garcia, 2009, p. 37). Exploring codeswitching as one piece of a translanguaging practice allows educators and researchers to investigate bilingual students' use of multiple languages alongside or within their other communicative resources.

This research has prompted these and other scholars to raise questions, then, about the place of codeswitching in bilingual classrooms. As previously discussed, in many bilingual classrooms (mathematics and otherwise), monolingual language policies exist in which classroom communications are required to occur only through the target language of teaching and learning; this includes Canadian FI classrooms. However, as seen here,

codeswitching is viewed as a natural and even beneficial activity in which bilinguals engage outside of the classroom. In the next section, I review literature taken from Canadian FI and other bilingual mathematics classrooms that explores whether and how codeswitching occurs in these contexts.

2.5.1 Codeswitching in the Second Language Classroom: Language Arts and Other Content Areas

As Baker (2011) has explained, the negative views of bilingualism and codeswitching at a cognitive level (as already discussed) have carried over into the field of language education in general. Due to deficiency-oriented views stemming from the outdated SUP model, ideas that value a monolingual norm, and because codeswitching has been traditionally viewed as a deficiency, for example, as indicating a need to fill a gap in the L2 lexicon or a lower language proficiency level, classroom policies have often banned codeswitching as a practice. In these views, the L1 is a source of interference (rather than a source of positive transfer) for the L2. In school-based language learning programs, where the main goal after all is to teach and learn the target language, the situation becomes further complicated and there are additional, more pedagogically-based reasons for why codeswitching may be unaccepted or banned.

There is very little research investigating codeswitching in Canadian FI programs. In these programs in particular, L1 use has not been and is still not considered a sound pedagogical strategy and the topic is quite controversial. As Turnbull and Dailey-O'Cain (2009) have explained, in most L2 classrooms and certainly in immersion programs (in Canada and elsewhere), "first language use is generally expected to be rare or nonexistent" (p. 1). For example, based on this longstanding tradition, the *Foundation for French*

Language Arts in French Immersion in Atlantic Canada (New Brunswick Department of Education, Educational Programs & Services Branch, 2001) described the underlying principles of the FI program under the heading "*All French All The Time*" (p. 35). The document stated that "it is . . . essential that French be the only language of communication" (p. 35) in the FI classroom. Similarly, the American Council on the Teaching of Foreign Languages (ACTFL) has proposed "that language educators and their students use the target language as exclusively as possible (90% plus) at all levels of instruction during instructional time" (ACTFL, 2012, n.p.). That L1 use in immersion classrooms should be forbidden has become taken for granted by many over the years. This stems from a number of longstanding cognitive and pedagogical arguments or beliefs. For example, some L2 educators believe that L2 learning can truly only occur in the L2 and many have simply internalized long-standing district or school policies on bans of L1 use. Others may view monolingual children's language acquisition as the most successful, which points toward learning through a single language as being the most effective approach. Some may continue to regard the educated monolingual speaker as the standard to which all L2 speakers should be held. By extension, codeswitching practices are seen as indicative of lower language proficiency. Many educators also see the L1 as a source of negative interference with regard to L2 development. Furthermore, the original notions of comprehensible input (Krashen, 1985) and output (Swain, 1985; Swain has extended this hypothesis considerably in subsequent works, see, e.g., 2000, 2008, 2010) and the "time-on-task" or "maximum exposure" hypotheses, as Cummins (2007) has explained, argue against an educational approach that includes both L1 and L2 use (p. 174). Finally, L1 use may, for some educators, connote grammar-translation

methods that have fallen by the wayside in favour of more communicative-based approaches. Teaching and learning through two languages feel too reminiscent of the

> dreaded grammar-translation methods that communicative language proponents loathe—after all, unless it is compensated by further target-language talk, codeswitching reduces exposure to that all-important comprehensible input in the target language (Krashen, 1982). As this argument goes, codeswitching also detracts from opportunities for negotiating meaning while interacting with other learners or native speakers in the target language. (Turnbull & Dailey-O'Cain, 2009, p. 3)

Moreover, immersion programs, particularly the Canadian FI model, have been touted as the most successful in terms of producing functional bilinguals; and a key tenet of Canadian FI has always been exclusive L2 use in the classroom (Turnbull & Dailey-O'Cain, 2009).

Despite the longstanding nature of these arguments, they have been explored and challenged by researchers in L2 education (a small number of these are based in immersion contexts) in recent years who have taken a new look at existing theories and proposed new ideas and approaches (e.g., Cook, 2001; Cummins, 2000, 2001, 2007; Dailey-O'Cain & Liebscher, 2009; Macaro, 2009; Swain, 2000, 2012; Swain & Lapkin, 2000, 2013; Turnbull & Dailey-O'Cain, 2009). There are a number of reasons for this. First, in spite of the reasons for what has been long-held policy banning the L1 from L2 classrooms, research has shown that teachers and students in these environments use both the L2 and the L1 to different degrees (e.g., Antón & DiCamilla, 1999; Behan, Turnbull, & Spek, 1997; Cummins, 2007; Dailey-O'Cain & Liebscher, 2009; Gutiérrez, 2007;

Macaro, 2009; McMillan & Turnbull, 2009; Swain & Lapkin, 2000, 2013; Turnbull, 2001; Turnbull, Cormier, & Bourque, 2011). As Cook (2001) has suggested, "the L1 creeps back in, however many times you throw it out with a pitchfork" (p. 405). Second, as previously explained, Cummins' (1981, 2000) CUP model and theories of language interdependence argue against a separation of languages for bilinguals, and this would include students in L2 programs and classrooms. Third, there are now increasing calls to view L2 students as developing bilinguals rather than poor imitators of an unattainable monolingual speaker-ideal (e.g., Cummins, 2007; Dailey-O'Cain & Liebscher, 2009; Macaro, 2009; Turnbull & Dailey-O'Cain, 2009). Cummins (2007), for example, has recently challenged what he referred to as one of three "monolingual instructional assumptions," namely the "direct method assumption," which posits that "instruction should be carried out exclusively in the target language without recourse to the students' L1" (p. 222). Cummins (2007) explained that alternatives to this direct method assumption have been explored more successfully in contexts where L2 instruction is directed at minority language speakers (rather than in FI environments). However, Cummins (2007) has asserted that immersion educators should also challenge this assumption. The direct method assumption is based on large-scale studies from the early days of FI (1960s and 1970s) that suggested that students' performance in the L2 correlated positively with increased L2 use on the part of the teacher and also relates to Krashen's (1985) notion of comprehensible input. Cummins (2007) has not denied the empirical evidence demonstrated by these seminal research studies, but he has also called for consideration of other studies that suggest that L1 use serves purposes both

sociolinguistically (e.g., see Baker, 2011) and pedagogically (e.g., see Swain & Lapkin, 2000, 2013; Turnbull, 2001; Turnbull et al., 2011).

Further arguments for a re-visioning of the place of L1 use in L2 and immersion language classrooms are largely based on SCT views of language as a cognitive tool. Researchers have approached this work from Vygotskian (1962, 1978) and neo-Vygotskian (e.g., Donato, 1994; Lantolf, 2000; Lantolf & Appel, 1994; Swain, Kinnear, et al., 2011; Swain & Lapkin, 2013; Wertsch, 1985, 1993) viewpoints, which, as explored in the theoretical sections of this chapter, underscore the social roots of all individual higher mental functions. Researchers' adoption of SCT-informed paradigms have led to work which has explored the potential benefits of students tapping into their L1 as a resource for L2 learning, particularly during oral interactions. In these studies, several of the key theories relevant to theoretical framework the current study (e.g., collaborative dialogue, scaffolding, languaging) resurface and are used to study codeswitching in these various L2 educational contexts.

Researchers in non-immersion L2 classroom contexts have found that L1 use can be a learning strategy that serves both cognitive and social functions in L2 classrooms. Regarding the cognitive functions of student L1 use, Antón and DiCamilla (1999) noted that in an adult beginner Spanish as a foreign language class, use of the L1 (English) during collaborative dialogue allowed students to construct "collective scaffolding" (Donato, 1994), through which they mutually helped each other through a problem-solving task. Antón and DiCamilla also found that the L1 allowed students to work through cognitively difficult tasks and served metalinguistic functions. With regard to the more social functions of student L1 use, Antón and DiCamilla found that the L1 enabled

the beginner level Spanish L2 students in their study to establish "intersubjectivity," that is, "a shared perspective on the task" (p. 240).

In a university context in the United Kingdom, Gutiérrez (2007) used *microgenesis*, an SCT-informed approach to studying individuals' learning as it unfolds during interaction, to explore Spanish L2 learners' collaboration. The author found, similarly to Antón and DiCamilla (1999), that learners of Spanish as a foreign language working in triads engaged in collective scaffolding and were able to achieve together what had been previously unattainable by the individuals. Gutiérrez also noted that the L1 (English) was used in three ways to request assistance from the designated peer "expert" during problem-solving, collaborative tasks. Assistance consisted of either a straightforward reply (translation), a paraphrase followed by an L1 reply, or co-constructed assistance that followed an L1 request. An L1/L2 balance was struck as a result of students', or especially, the expert's desire for L2 learning and his or her insistence on its use being balanced by an awareness of the partner's needs.

Work by Dailey-O'Cain and Liebscher (2009) examined how codeswitching practices observed in naturalistic bilingual settings also emerge in the L2 classroom. Based in two German language classrooms at a Canadian university, the authors found that students used the L1 as self-scaffolding and that teachers also used the L1 as a scaffold for learners. The authors underscored the importance of distinguishing between student and teacher codeswitching because students may adopt and enact their own distinct practices, independent of teacher policy or even modelling. Furthermore, and studied from an SCT perspective, the authors suggested that "some of the codeswitches take on different meanings depending on whether the students or the teacher perform

them" (p. 143). For example, the authors noted that students sometimes codeswitched in order to facilitate and/or advance their L2 and content learning, a practice that might be accepted by the teacher (e.g., through the choice to codeswitch in response). However, at other times student codeswitching represented a move to a "comfort" zone. Teachers, in the way they responded to student codeswitches (e.g., by not using the L1), were able to communicate the importance of target language use. Finally, similar to Antón and DiCamilla (1999), Dailey-O'Cain and Liebscher (2009) also concluded that students used their L1 in a social function to establish intersubjectivity.

In addition to these studies that are based in L2, but not immersion, classrooms, a small number of important studies on teacher and student codeswitching have been conducted in the Canadian core French (FSL) or FI contexts. The majority of these studies have focused on language arts classrooms and not content, or more specifically mathematics, classrooms, but they are nonetheless important for understanding L1 use in FI. (I address those few that are based in mathematics contexts in the next section.)

Turnbull (2001) has referred to his own 2000 study of teacher L1 use in Canadian FSL classrooms and results from other studies in order to acknowledge the usefulness of the L1 while cautioning against an over-reliance on it. In his own study, Turnbull (2001) noted that teacher use of the L2 varied greatly, and ranged from 21% to 72%. He also cited another study (Duff & Polio, 1990, cited in Turnbull, 2001) in which teachers used the L2 approximately 10% of the time. Referring to the lower percentages of L2 use, Turnbull (2001) suggested that "teacher educators must help teacher candidates and practicing teachers make principled decisions about the judicious use of the L1, while maximizing their TL [target language] use" (p. 537).

A small number of studies examining student (rather than teacher) L1 use have also been based in the Canadian core French and FI contexts. Swain and Lapkin (2000) conducted research in early total FI language classrooms examining student L1 use during collaborative, task-based learning. These Grade 8 students were from two classes that were comparable in academic performance and socioeconomic background. Students in each class worked in pairs and were presented with the same story, but one class completed a dictogloss activity (in which students took notes while listening to an audio recording of the story and then wrote a story based on their notes) and the other a jigsaw activity (in which students pieced together pictures they were given in order to retell the story orally and then produce a written version). The researchers recorded students' conversations and analyzed them using the turn as the unit of analysis. For both activities, the authors found that students used the L1 for three main purposes: (a) moving the task along, (b) focusing attention, and (c) interpersonal interaction. Each of these three main categories was further divided into more specific subcategories.

Swain and Lapkin (2000) found that the amount of L1 use was similar for each class completing the two different activities. The dictogloss class had 21% of turns in English, the L1, and the jigsaw class had 29% of turns in the L1. For both classes, the L1 was used most frequently for task management purposes (35% for the dictogloss and 43% for the jigsaw). The researchers found that, in general, the pairs of students who received higher ratings for content and language on their tasks also used less L1 overall. The researchers did note, however, that the dictogloss task was more constraining when compared to the jigsaw task. They also noted that many other factors, for example, learner perceptions (e.g., of the tasks, the teacher, the researchers, learning French, the

recording equipment) or the compatibility of the pairs, likely also influenced both L1 use and student performance. Overall, Swain and Lapkin (2000) suggested that "use of the L1 should not be prohibited in immersion classrooms, but neither should it be actively encouraged as it may substitute for, rather than support, second language learning" (p. 268). However, reflective of Vygotsky's and Cummins's theories and their own SCT lens, the authors concluded that "to insist that no use be made of the L1 in carrying out tasks that are both linguistically and cognitively complex is to deny the use of an important cognitive tool" (p. 269).

Two notable studies on student L1 use have also been carried out in Canadian FI content classrooms, although not mathematics. First, in a study based in students' first year of a Grade 7 extended French program in social studies class, Behan et al. (1997), like Swain and Lapkin (2000), suggested that L1 use "can both support and enhance L2 development" (p. 41). Moreover, because Behan et al.'s study was based in a social studies class, they added that, the L1 was also "functioning simultaneously as an effective tool for dealing with cognitively demanding content" (p. 41). The authors designed an experiment in which four groups of four students each were monitored while working together to prepare an oral presentation. All four groups were encouraged to use as much French as possible during their collaborative work. Two of the groups were closely monitored and reminded to speak French by the teacher, whereas the other two groups, although addressed in French, were not reminded to speak French during the task. Through microgenetic analysis informed by SCT, the authors found that the groups who were left to converse in the language of their choice spoke more often, and exhibited more evidence of learning. When students used the L1, it was mainly for reasons related

to vocabulary and task organization. The authors were clear about not advocating unlimited L1 use in the FI classroom, however they went on to say that, in the content (social studies) class in particular, "limited L1 use may benefit both L2 development and content mastery" (p. 42).

Also informed by key tenets of SCT, but with a focus on quantitative analysis from a quasi-experimental design in this particular study, Turnbull et al. (2011) explored late FI students' L1 use in science classes. Two groups of Grade 7 students, enrolled in their second year of a late FI program, were taught using either the experimental literacy-based approach designed by the researchers or the standard approach as prescribed by the provincial curriculum document (for a detailed description of each approach, see Cormier & Turnbull, 2012; Turnbull et al., 2011). Students participated in oral interviews, both before and after the teaching interventions, and a written task. A series of complex statistical analyses showed that, in general, the experimental group showed increased complexity in their final interview responses, but that these students required the L1 (English) in order to express the complex ideas. Furthermore, the English or codeswitched turns correlated positively to increased complexity and better results in both written French and science content knowledge. Consequently, the authors suggested that "language acts as an important cognitive tool to help make sense of complex science content" (Turnbull et al., 2011, p. 194).

Whereas Turnbull et al.'s (2011) study was based in science classes and Behan et al.'s (1997) work was based in a social studies class, research exploring L1 use in FI mathematics classes is lacking, particularly when it comes to student L1 use. That said,

there are a small number of studies of note based in the Canadian FI mathematics context and a more substantial number related to other L2 mathematics contexts.

2.5.2 Codeswitching in French Immersion and Other Second Language Mathematics Classrooms

Regarding this specific context, that is, FI mathematics, I have my own personal experiences. In my past role as a teacher in FI mathematics, my goal was to simultaneously ensure students' comprehension of the mathematical content and to promote L2 comprehension and use. Nonetheless, it seemed logical at times, particularly at the high school level where I was teaching, when mathematical concepts are often abstract and intangible, that use of the L1 to reinforce or re-explain concepts could be a positive strategy. Likewise, when students were discussing the concepts amongst themselves, it seemed natural and I thought perhaps beneficial for them to call upon their L1 as a resource. However, since, as discussed earlier, the practice of teachers or students using the L1 as a resource in the Canadian FI classroom is strongly discouraged or even banned, I always felt guilty when either my students or I used the L1 (English) in the classroom. This complex dual role of language and content teacher has long been recognized in the FI literature (e.g., Swain, 1996). The codeswitching dilemma I faced, myself, as an FI mathematics teacher also seems pervasive and a common thread throughout studies based both in Canadian FI and other non-immersion L2 mathematics contexts.

With respect to teacher L1 use in Canadian FI mathematics, McMillan and Turnbull (2009) conducted individual interviews with, and observations of, two late FI teachers in order to explore their perceptions and beliefs surrounding their use of the L1

(English) and how these beliefs were enacted in their classrooms. Both teachers taught a variety of content courses (mathematics, science, social studies, health) in Grade 7 late FI, the students' first year of the program. One teacher, Frank (pseudonym), used little to no L1 with his students, while the other teacher, Pierre (pseudonym), used the L1 much more frequently but in a judicious manner, particularly at the beginning of the school year. Even though Frank used very little L1 and saw its use as practically unavoidable at times (especially in the early months of the program) feelings of guilt were associated with both teachers' L1 use. Both Pierre and Frank felt that, due to the many French-English cognates and its use of numbers and symbols, mathematics was one subject that could be more easily taught exclusively in the L2 from the outset. However, Frank noted that the need for some L1 use actually seemed to increase with time, as the mathematical concepts and word problems became more challenging and complex. This links to other findings based in immersion content classrooms exploring student L1 use, and previously discussed here (e.g., Behan et al., 1997; de Courcy & Burston, 2000; Turnbull et al., 2011), which underscored that students' L1 was used to deal with cognitively complex content in the L2, especially in the early phases of their program when proficiency was limited.

Offering an FI student perspective on their own and their teachers' L1 (English) use, in my master's research (Culligan, 2008, 2010, 2015) I explored secondary students' experiences in FI mathematics. The study used a phenomenological approach and open-ended interviews were the main source of data collection. Several main themes emerged from the data, and one of those themes was L1 use. Many students indicated that, for both themselves and their teachers, it was important to use the L2 as much as possible in FI

mathematics. However, they also felt that L1 use provided vocabulary support and helped link to prior knowledge in the L1. Students also perceived that they used the L1 more often when they needed to express themselves, particularly when it came to socializing with peers or when trying to formulate a question related to mathematics. In general, while they were not specifically asked about L1 use and codeswitching as part of their experiences in FI mathematics, the number of related comments and the extent of those comments suggest that the issue preoccupied students' thoughts.

In addition to these few studies conducted in FI mathematics in Canada, there are a number of studies, referenced in the upcoming paragraphs, exploring teacher and student L1 use in L2 mathematics classrooms in non-immersion contexts around the globe. This body of literature stems from researchers in mathematics education who are interested in language and mathematics and who work in bilingual settings. For example, a body of work has investigated teacher codeswitching in English-medium mathematics classrooms in South Africa, while another study explored teacher perceptions of translanguaging as a classroom practice in mainstream English mathematics in the United States in which there were high numbers of Spanish L1 students. There is also research on bilingual students' use of their L1 when doing mathematics in their L2 in monolingual educational settings (e.g., in the United States, United Kingdom, Pakistan; and in Germany, from a translanguaging perspective). Other work has focused on bilingual programs that are more immersion-like, in which students study through two official national languages (e.g., in Wales). Although these contexts differ from Canadian FI, they also offer key understandings related to L1 use in L2 across bilingual mathematics classroom settings.

The studies I chose to feature in more detail here informed my own work with regard to their focus on codeswitching practices as well as their theoretical orientation. What Setati and Adler (2000) remarked about their own work holds for all of the studies included in this section of the literature review: "All of these studies have been framed by a conception of mediated learning, and of the communicative and cognitive functions of speech" (p. 246). Moreover, the authors featured herein have also referred to the mathematics education register in their work, and view language and mathematics as intertwined and socially situated. They also adopt a broader view of mathematical communication and avoid using deficit discourses. These studies offer a different perspective on codeswitching practices too since, being from the mathematics education field rather than the L2 education field, they focus mainly on the role of language as it relates specifically to mathematical communication and learning. While relevant to my work in the Canadian FI context, these studies simultaneously broaden my understanding of the complexities that must be considered within different contexts.

In South Africa, the target language of many classrooms is English and the students' (and teacher's) L1 can be any one of several official languages. The South African context could be described as a case of "immersion" in a language of power, and is distinct from the Canadian FI context due to at least three main factors. First, in South African classrooms the situation is highly multilingual. Students in the same classroom often have different L1s which may, in turn, be different from the teacher's L1 (although it could be argued that this is the case more and more in Canada, see, e.g., Cummins, 2007; Swain & Lapkin, 2005 for a discussion). Second, education in the various L1s is sometimes seen as inferior, thus the L1 and L2 do not enjoy the same status. And finally,

resources and vocabulary are sometimes nonexistent in the various L1s, which may create a gap in the bridging support deemed an asset for an immersion program (Setati, 1998).

Despite these differences, this research has contributed a more qualitative look at bilingual mathematics classrooms compared to the important but quantitative and mainly large-scale studies examining academic achievement conducted in Canadian FI and other international immersion programs (e.g., Finland, Australia). In particular, as part of their qualitative explorations, Adler, Setati, and others have extensively discussed the concept of codeswitching in the mathematics classroom (Adler, 1998, 1999; Setati, 1998; Setati & Adler, 2000; Setati, Adler, Reed, & Bapoo, 2002). These researchers focused primarily on teacher codeswitching practices but also on how teachers and students work together to navigate between languages. Data for these studies were collected by observing and recording classroom teaching and via individual interviews with teachers. This work takes a social, discursive approach to language and mathematics that supports the use of students' main language as a resource while reflecting on the politics and power structures at play in the context. Throughout these studies, mathematics teachers described codeswitching as a "dilemma," and both a controversial and necessary practice. For example, Adler (1998) explained that the teacher participants

> believed on the one hand that use of learners' main language was necessary at times for understanding. On the other, they held strong views that it was their responsibility to work only in English in class . . . and facility with English is best acquired in use and when the use of other language is restricted. (p. 25)

Setati et al. (2002) noted that codeswitching was most prevalent at the secondary level, and that mathematics and science teachers switched more often than language arts teachers. This may be due to teachers' perception that the heavier content load at the secondary level, and in these subjects in particular, required an increased support of L1 (or main language) use.

On a different continent, Mejía Colindres (2015) noted similar strategies used among middle school teachers in mainstream English mathematics classrooms where 99% of the students were L1 Spanish speakers. Three teachers (two of whom were bilingual Spanish/English from birth, one of whom had learned English later in life) were interviewed and, although to some differing degrees, they commented that they generally used English for academic mathematical terms and taught mostly in English. In spite of this, the teachers also reported engaging in translanguaging practices in the classroom (in this case, the systematic use of and alternation between English and Spanish, particularly with regard to using different modes for input vs. output, based on Baker, 2011). Translanguaging, which included codeswitching, was "used to make sense of the content and to elicit students' thinking, but not necessarily to support students' first language" (Mejía Colindres, 2015, p. 154). The teachers also used translanguaging strategies for sociolinguistic purposes, such as establishing rapport and in affirming common identity.

Also in the United States, but with a focus on students' classroom communication, Moschkovich (2005) analyzed codeswitching in a conversation between two Grade 9 students solving a mathematics problem. The students were both L1 Spanish speakers but had been enrolled in mainstream English mathematics classes in the United States for a number of years. Drawing on her findings, Moschkovich (2005) challenged the

historically negative view of codeswitching as a signifier of language deficiency, and, in this case, also a deficiency in mathematical knowledge. Taking a sociolinguistic approach, she noted that the bilingual students' codeswitching served a variety of purposes. The codeswitches reflected a level of communicative competence and also reflected community norms; they provided stylistic switches in the conversation (to add colour, emphasis, etc.); they related to memory and to routines; and they were a resource for elaborating ideas. Moschkovich (2005) asserted that, in the bilingual mathematics classroom in particular, students' L1 might be used to "explain a concept, justify an answer, describe mathematical situations or elaborate, expand and provide additional information" (p. 138).

Duarte (2016), in a study based in Germany, explored similar issues related to minority-language students' communication in mainstream classrooms, but from a translanguaging (i.e., use of multiple languages together) perspective. Drawing on sociocultural theory in order to understand the nature of adolescent peer-peer interactions during their work on tasks in various content areas, including mathematics, this study found that students' talk was predominantly on-task during translanguaging practices. The majority of this on-task talk occurred in German, even in the translanguaged exchanges (other languages used reflected the various minority-language L1 backgrounds of the students, e.g., Russian, Bosnian, Turkish). These translanguaged interactions were mostly of a cognitively demanding nature, that is, they occurred when students were trying to make sense of the task at hand or when they were co-constructing answers. In a few instances, the students translanguaged for reasons related to task management (e.g., sorting out materials needed).

A recent study by Barwell (2014) set in a Canadian context explored instances of student L1 use during their interactions while solving word problems and producing oral and written explanations. These Grades 5 and 6 students were Cree L1 speakers and were in an ESL classroom in an Anglophone elementary school in Quebec. Using fieldnotes, audio recordings of whole-class and small-group interactions, samples of student work, visit reports and extended interviews with the teacher and some students, this classroom ethnography draws on Bakhtin (1981, cited in Barwell, 2014) to discuss what Barwell (2014) argued were the centripetal and centrifugal forces acting on language in this bilingual mathematics classroom: "Centripetal forces represent the drive for standardization; centrifugal forces arise from the inherent diversity of language practices" (p. 911). Monolingual language policies, such as the English-only policy of this classroom, represent an example of a centripetal force. Specifically, Barwell (2014) noted that there are tensions at work in the classroom with regard to the use of students' L1, language policy and classroom practice, and formal versus informal language. Moreover, these tensions were noted to be particularly strong during three instances: (a) the use of Cree during mathematical discussion, (b) interaction around mathematical word problems, and (c) the production of oral and written explanations of mathematical thinking. Though set in Canada, this study's context is also different from Canadian FI in notable ways, including, for instance, the perceived status of the students' L1 (Cree).

In yet another different region of the world, Halai's (2009) examination of codeswitching is based in English-medium mathematics classrooms in Pakistan. This context is quite different from Canadian FI in a number of respects. In this study, the language of instruction was English. Students came from Gujrati-, Katchi-, Sindhi-, and

Urdu-speaking backgrounds, but all were fluent in Urdu and used it as a common language (when codeswitching). Despite the fact that English was the target language, neither teachers nor students were fluent in the language. The author noted that Pakistan had recently moved from a more traditional, rote learning and drill-based mathematics curriculum to one that reflects the current focus in North American curriculum on communication and problem solving in mathematics (such as found in NCTM, 2000). Thus, there is now more talk in the mathematics classroom than ever before and this talk is supposed to be occurring in students' L2 (English). Halai (2009) noted that the politics of language use were present in the classroom, and that although students reverted to Urdu when working together on problems, English was positioned as more prestigious and "official." Echoing statements coming from L2 education, the author argued that "there is a need for teachers, teacher educators and policy makers to look into ways of maximizing the potential of codeswitching through appropriate policies, teaching practice or curriculum materials" (p. 61). Halai (2009) was also careful to note that more research is needed to adequately explore how codeswitching facilitates or hinders mathematics learning.

A final international study that explores codeswitching in mathematics is based in a context that shares a number of similar traits with Canadian FI. Jones (2009) described the Welsh educational context as similar to Canadian FI and especially so at the elementary level. There are some notable differences too, for example that in Welsh-medium schools in Wales, students, especially at the secondary level, may have Welsh as their L1 or English as their L1, and may in turn be studying mathematics in either Welsh (as an L1 or an L2) or English (as an L1 or L2). Furthermore, at the secondary level

students largely have access to academic materials in both languages simultaneously. In Welsh-medium schools, codeswitching happens and is by-and-large less controversial than in Canadian FI. Jones (2009) was mainly interested in whether bilingual resources and student and teacher codeswitching help or hinder mathematical learning. Audio recordings of whole-class interactions, fieldnotes, and teacher and student interviews were collected as data. The author used a social, discursive approach to analysis based on Gumperz's (1982, cited in Jones, 2009) notion of "contextual cues." Jones (2009) noted that Welsh and English have comparable currency in the mathematics classroom and that Welsh was used more for classroom management (because this exposure to "incidental" language was viewed as way to help students learn more Welsh), teacher explanations were repeated in both languages (which seemed to facilitate understanding), and teachers switched from Welsh to English when referring to calculations or numbers (most teachers had themselves been taught mathematics in English).

Thus, although views on codeswitching are overall much more positive than they once were, the situation remains complicated. On the one hand, many researchers have argued that educators should embrace codeswitching in the L2 language classroom (immersion and otherwise) and the L2 mathematics classroom as part of the bilingual norm (e.g., Cummins, 2007; Moschkovich, 2005; Swain, Kirkpatrick, & Cummins, 2011). As Moschkovich (2005) has argued, both within and outside the classroom, "codeswitching will continue to seem 'odd' only if it is compared to a monolingual norm, to some imagined set of 'pure' or 'normal' language practices, or to an ideal monolingual speaker-hearer who functions in a homogeneous monolingual speech community" (p. 140). Some have even gone so far as to advocate for reframing classroom codeswitching

to a view of "code choice" (Levine, 2011) in order to underscore the notion that for bilinguals, codeswitching is a complex and nuanced activity that services sociolinguistic and oftentimes cognitive purposes. Levine (2011) has argued for principled incorporation of codeswitching and code choice activities in L2 classrooms, so that learners might harness and exploit the potential benefits of such activities. Swain and Lapkin recently (2013) presented a thorough discussion of many of the immersion-based studies cited here viewed through a "Vygotskian sociocultural perspective." In that article, the authors suggest three key implications with regard to L1 use in immersion classrooms, one of which relates directly to student L1 use during oral interactions (the other two related to relationships between teacher and students, and to teacher L1/L2 use):

> Students should be permitted to use their L1 during collaborative dialogue . . . in order to mediate their understanding and generation of complex ideas (languaging) as they prepare to produce an end product (oral or written) in the target language. However, as student proficiency in the L2 increases, students should increasingly be encouraged to language using the L2 as a mediating tool. Further, when new and complex material is introduced within and across grades, students should again be allowed to make use initially of their L1 to language, that is, to mediate their thinking. (pp. 122-123)

Thus, on the other hand, while calls are being made for increased acceptance (or even encouragement) of L1 use for specific purposes, there is simultaneously an argument against over-reliance on the L1 and increasing use of the L2. Furthermore, researchers have recognized that codeswitching, and the degree to which it can and should be encouraged in the bilingual classroom, is highly contextualized to the particular

language-learning environment (e.g., immersion or not) and its goals; the languages, teachers, and students involved; and the language policies at play. Many researchers who have advocated for codeswitching/L1 use have also been often careful to use terms such as *judicious use*, *principled use*, or *optimal use*, to describe the extent to which codeswitching might be used to serve these sociolinguistic and/or cognitive purposes in the L2 classroom (see, e.g., Cummins, 2007; Levine, 2011; Swain & Lapkin, 2000; Turnbull & Dailey-O'Cain, 2009).

Having investigated the FI context by exploring student achievement; the related tensions, contextual considerations, politics, and policies at work; and theories and questions regarding L1 use; I turn next to focus more precisely on mathematics classrooms in particular. In order to do so, and with the complexities of the bilingual classroom in mind, I discuss what it means to "do mathematics," particularly in an L2.

2.6 Doing Mathematics in a Second Language

My conceptualization of "doing mathematics" is based in the theoretical framework developed earlier in this chapter. In other words, doing mathematics in the classroom context is a cognitive, social, situated, broad, and multifaceted enterprise. Furthermore, when it comes to students doing mathematics in an L2, there are a number of special key points of consideration as I have already discussed. For example, describing and analyzing the unique bilingual classroom context of my study is of utmost importance for my own interpretations and for those who might read my work and apply it to their own contexts. In addition, the classroom context and the larger languages forces at work ("politics") influence the classroom's language policies. Exploring how these policies are enacted in practice in high school FI mathematics is also a key area of

interest for this study and is part of "doing mathematics" in this context. Finally, as previously discussed, in all bilingual classrooms, both language classrooms and content classrooms, the phenomenon of codeswitching is important to explore. Thus, codeswitching is also viewed in this study as part of the "doing" of mathematics.

Along these lines, other mathematics researchers and educators who are interested in language in bilingual mathematics classrooms have studied ways in which mathematics and language are used during particular instances of "doing mathematics." The results reported in these studies serve to build my own understandings of "doing mathematics" in this study. I turn now to these studies, which approach their analyses with a social, situated view of mathematics and language, as the final component of the literature review.

In my theoretical framework, I discussed how viewing mathematics and language through a sociocultural, discursive lens is a key way to reject deficiency models of learners. I suggest that moving away from deficit-oriented views of mathematics students may be of particular importance for those who are bilingual. Scholars such as Barwell (2005b) and Moschkovich (1999, 2009a, 2010), who focus on mathematics and language in a general sense but who conduct much of their research in mainstream English classrooms in the United Kingdom (Barwell) and the United States (Moschkovich) with English language learners (ELLs), have argued that research has too long focused on the obstacles facing bilingual students rather than highlighting the resources they bring to the mathematics classroom. Researchers Adler and Setati (e.g., Adler, 1998, 1999; Setati, 2002; Setati & Adler, 2000), based in highly multilingual classrooms in South Africa, where English is often the target language of teaching and learning but not the students'

L1, also call for a move away from deficit views of bilingual mathematics learners and underscore the teaching challenges associated with this particular multilingual context. Thus, adopting a resource-oriented view of students' communication in their doing of mathematics has been an important step in my own study. In order to learn more about how to move away from deficit discourses, notions of multiple meanings and the valuing of so-called "everyday" talk in the mathematics classroom are important.

2.6.1 Multiple Meanings

One way in which researchers are moving away from deficit-oriented views of bilingual mathematics learners is through exploring the potential of multiple meanings in the mathematics classroom. For example, recall what Barwell (2005a) described as ambiguity in the mathematics classroom. Traditional views of mathematics might only value standard, canonical textbook definitions of mathematical concepts or terms as an acceptable part of the mathematics register or a mathematics education register. Any alternative interpretation of a given mathematical concept is seen as a "problem" to be overcome and one that is due to incorrect, inattentive use of language rather than ambiguity that might be inherent in the mathematical idea. However, Barwell (2005a) found that when ambiguity, that is, the possibility of multiple interpretations, was introduced by the teacher in a discussion with elementary students regarding 2-dimensional shapes, what followed was a sophisticated mathematical discussion that generated opportunities for meaning making, and opportunities for constructing both linguistic and mathematical understanding. The tension here is one of the valuing of informal talk (contrasted with the more common practice of devaluing) versus formal mathematical language. Barwell (2005a) concluded:

> Informal language can be used to explore and develop sophisticated mathematical ideas and to participate in mathematical practices. I do not wish to suggest that students should not learn to use formal aspects of mathematical discourse. This learning, however, is not identical with learning mathematical vocabulary. Rather, students' development of the use of mathematical discourse is intertwined with their development of mathematical thinking. (p. 125)

This may be especially the case for bilingual students, for whom the target language of teaching and learning is not their L1. For these students, the opportunity to explore ambiguity, or multiple meanings (Moschkovich, 2003, 2009a), in mathematics is an opportunity to generate discussion that supports student learning. Moschkovich (2009a) explored bilingual students' interactions with their teacher in a middle school mathematics class. This was an English-medium mathematics class, however the students were all L1 Spanish speakers and were former dual language immersion students (a type of immersion program in which L1 speakers of two different languages, in this case English and Spanish, are combined in approximately a 50-50 ratio in the same class and in which instruction occurs in both languages to an approximately equal degree; see Baker, 2011, for a discussion). The teachers were bilingual and both they and the students spoke a combination of English and Spanish in their classes, although the students only spoke English during the excerpt in this article. In order to encourage students' participation in an exploration of the multiple meanings of "I went by" (i.e., students were saying things like "I went by twos" or "I went by fives" when explaining how they chose to number their axes on a graph), the teacher did not give "right" or "wrong" answers but rather used students' different interpretations to extend the discussions. In

addition, the teacher used other strategies to support participation such as using student-generated products, gestures, and objects; accepting and building on student responses; and using a central mathematical concept as a foundation. Moschkovich (2009a) explained:

> Multiple interpretations can serve as resources for instruction in bilingual classrooms. A positive perspective on multiple interpretations is particularly important for bilingual classrooms . . . multiple interpretations need not be seen as obstacles but can be used as resources for explaining and using important mathematical concepts. (p. 95)

Moreover, Moschkovich (1999, 2010) has argued for less focus on mathematical vocabulary, and especially less on criticizing students' language errors, and more focus on "the mathematical meanings learners construct . . . in, through, and with language" (Moschkovich, 2010, p. 12). Indeed, in a longitudinal study in which I have been involved as a graduate student (project leader D. Wagner), we have explored how FI students at Grades 3, 6, and 9 make meaning of probability and certainty by using different kinds of language—formal and informal (e.g., Culligan, Dicks, Kristmanson, & Wagner, 2014; Wagner, Kristmanson, & Herbel-Eisenmann, 2011). For example, through this work, we found that the students were able to express ideas related to probability by using modals from "everyday" language (e.g., *pourrait*, *je suis certain que*) in a mathematical way, and older students in particular (e.g., Grade 9) also used some more formal mathematical terminology (e.g., *la probabilité*).

Accepting and even encouraging meaning making and multiple interpretations through open discussions between teacher and students, and among students themselves,

in various languages about various mathematical concepts, can be one way to explore the resources (linguistic and mathematical) they bring to the classroom. In order to do this effectively, it is important that researchers and educators also recognize the mathematical potential in students' so-called everyday talk.

2.6.2 Hearing the Mathematical in Everyday Talk

Another example of shifting a deficit model of learners to a resource-oriented view is the blurring of the rigid distinction between so-called "everyday" and mathematical discourses, which is also one of the five tensions present in bilingual mathematics classrooms and described earlier (e.g., Barwell, 2005a, 2009b, 2009c, 2010; Moschkovich, 2003, 2007). Recent studies have suggested that while this type of distinction can be useful "for describing mathematics learning as moving from everyday to more mathematical ways of talking" (Moschkovich, 2003, p. 325), it can also be limiting in the mathematics classroom. For example, if researchers and educators focus solely on academic, mathematical discourse, they could miss "hearing the mathematical in student talk":

> Learning the mathematical meanings of words describes one important aspect of learning mathematics . . . However, the relationship between the everyday and the mathematics registers and communication in the classroom is more complex. First, mathematical discourse involves more than word meanings. Second, everyday meanings are not only obstacles but also resources for developing mathematical competence. And lastly, As Forman (1996) points out, in the classroom the everyday and the mathematical discourses are not separate but interwoven in discussions. (Moschkovich, 2003, p. 326)

Thus, a social, discursive view of the mathematics education register views everyday talk and the mathematical register as intertwined (Moschkovich, 2007; Setati, 2002; Setati & Adler, 2000) and views everyday talk as a resource rather than a problem.

Like the need to recognize ambiguity or multiple meanings as a resource, the need to see the mathematical in everyday talk might also be more acute in bilingual classrooms. For example, Barwell (2009a), in his study of 9- and 10-year-old English language learner (ELL) mathematics students, found that as they worked through the mathematics task (writing their own word problems) three main things occurred: (a) students drew on personal experience, (b) students paid attention to the word problem genre, and (c) there was interaction between language learning and mathematical thinking. Students used everyday talk to make sense of and contextualize the mathematical concepts and word problems. Even when the talk was seemingly unrelated to mathematics in that it was, for example, language focused, Barwell (2009a) argued that "these language-focused exchanges are not separate from the students' mathematical thinking. The significance of these exchanges is that they contribute to the development of the mathematical dimension of the students' word problems" (p. 74). For example, in a discussion focused on the pronunciation of the number *15* (one student questioned whether the pronunciation is *fifty* or *fifteen*), students simultaneously negotiated the meaning of both the language and the mathematics involved in the word problem in question. Similarly, in an earlier analysis of the same data, Barwell (2005b) noted that the ELL students' attention to written form allowed opportunities for language-related episodes (LREs) to emerge (Swain & Lapkin, 1998); this established an important link to seminal research based in L2 education. In a reflexive relationship between language and mathematics, this attention to written form

"contextualizes discussion of mathematical structure . . . and vice versa" (Barwell, 2005b, p. 215).

In this study, doing mathematics involves students' communication practices during collaborative mathematical problem solving. Consequently, doing mathematics involves more than textbook mathematical vocabulary and syntax (although these things are also part of doing mathematics)—it also involves the ways in which students use language to make meaning of the mathematical problems and content, it involves multiple meanings, and an attention to the mathematical in students' "everyday" talk. This view of doing mathematics stems from sociocultural ideas of learning and the key tenets included in the notion of a mathematics education register.

2.7 Conclusion

I have devoted considerable space in this chapter to developing my theoretical framework for my research for two reasons that I consider of key importance. First, the theoretical framework represents the foundation upon which the entire remaining structure rests. It guided all the facets of my doctoral work including the literature I chose to read and feature, the approach I took to working with my participants, the methodology I chose, the data I collected, the lens through which I analyzed and interpreted these data, the way in which I wrote about the data, and the way in which I presented and interpreted my final results. In this qualitative work, my theoretical framework, along with my own background (personal, professional, academic), is a continual filter through which I experience my research study and which guides my interpretations. As Halliday (1978) once suggested: "Our experience of reality is never neutral. Observing means interpreting; experience is interpreted through the patterns of

knowledge and the value systems that are embodied in cultures and in languages" (p. 203).

The second key reason for a fully developed and solid theoretical framework is the fact that my research aims to link two fields of work: L2 education and mathematics education. My study highlights both the L2- and the mathematics-related aspects of FI students' interactions and my research questions reflect this aim. In seeking to develop expertise in both areas of interest, I have put substantial time into developing a theoretical framework that considers both fields and that reflects a research approach appropriate to both. Similarly, with regard to my review of literature, I presented studies that reflect work that is based in both L2 and mathematics education. There is a lot to consider in terms of language, mathematics, and meaning making through oral interaction, and how these elements interact with each other in context. Moreover, bilingual classroom settings often present additional layers of complexity and considerations when it comes to language and mathematics. A theoretical framework based in SCT and a mathematics education register enables me to explore these complexities in a multifaceted and deep way.

Whereas the idea of analyzing interactional discourse has been more common in the field of L2 education, it was not until the development of the notion of the mathematics register that researchers in the mathematics education field really began examining classroom communication by way of detailed transcripts of audio and video recordings, reflecting a change in thinking that began to shift towards the social nature of mathematics and the role language plays within it. Over the years, the idea of the mathematics register has been extended in terms of what might constitute a mathematics

education register, in other words, the specialized mathematical language of classrooms. From there, a number of scholars have taken an interest in mathematics and language as viewed from a social, discursive theoretical perspective particularly in bilingual classroom contexts.

Having built the theoretical framework, this chapter moved on to discuss the results found in studies of language and content classrooms based in L2 and mathematics education contexts, and based specifically in immersion when possible. I provided a brief overview of studies of FI students' achievement in the program overall, and especially with regard to English L1 and French L2. Decades of research in Canadian FI contexts has shown that, overall, FI students' performance parallels or exceeds that of their English program counterparts in English L1 (once the initial lag period has passed), French L2, and mathematics, particularly for students enrolled in early total immersion.

Regardless of whether the context is Canadian FI or another bilingual context in which students are doing mathematics in their L2, there are important tensions to consider that relate to language and mathematics, and these considerations weave through research conducted in these environments. When considering these "universal" tensions, however, it is also imperative to acknowledge how they manifest and enact differently depending on the context, politics, and policy of the particular classroom environment.

Another overarching factor to consider when researching L2 mathematics classroom discourse is the practice of codeswitching. This issue has emerged in almost every qualitative study that has explored bilingual students doing mathematics; however, many questions remain about what the role should be, if any, for codeswitching or L1 use in an L2 classrooms (mathematics or otherwise). Many are beginning to view

codeswitching as a resource for both mathematical and language learning, rather than a "problem." to be overcome. However, these arguments are not straightforward. When it comes to FI mathematics classrooms, teachers have a complex dual role, which is to teach both mathematics and language.

This study focuses on FI students' oral interactions while "doing" mathematics in their L2. "Doing mathematics" can be interpreted in different ways, and what counts as mathematical communication is open for debate. In this literature review, I discussed work that views mathematics as a social activity that is situated in a particular context. In these studies, which were based in bilingual (though non-immersion) mathematics classrooms, the authors pointed out that mathematical communication is more inclusive than just standard textbook vocabulary and ways of speaking. Thus, doing mathematics involves all the ways in which students talk and make meaning through a problem-solving activity. Ambiguity in mathematics is recognized as a possibility; consequently, multiple meanings are also possible. Students' meaning-making potential, which includes the mathematics they bring to the table through their so-called everyday talk, is valued as part of mathematical communication. Such a view of doing mathematics allows the focus to remain on students' resources for communication and doing mathematics, rather than on their deficits.

There was a substantial amount to be learned about SCT, the mathematics education register, students' achievement in FI, the nature of bilingual mathematics classrooms, and what it is like when students do mathematics in their L2. Despite all the knowledge I gained during the writing of this theoretical framework and literature review

chapter, I found I was left with a number of questions at the end. These questions became the focus of my doctoral research.

2.8 Research Questions

Having reflected on my experiences, articulated a research problem, established my theoretical framework, and explored existing literature pertinent to my study, I posed three research questions that guided my inquiry:

1. How do FI students at the secondary level attend to language, that is, what language-related episodes emerge, while working collaboratively during mathematical problem solving in their L2 (French)?
2. How do these students communicate mathematically while working collaboratively during mathematical problem solving in their L2 (French)?
3. When and how do these students use their L1 (English) while working collaboratively during mathematical problem solving in their L2 (French)?

I believe that exploring answers to these research questions will contribute important understandings to current discussions about language and mathematics in immersion education and, more specifically, to how bilingual students make meaning through language and mathematical communication in the FI classroom context. Through a resource-oriented approach (i.e., one that aims to understand what students do [rather than what they do not do] and why and how they do it, one that adopts a broad and inclusive view of students' linguistic and mathematical skills, one that values students' collaborative and varied approaches to linguistic and mathematical problem solving) these research questions aimed to help me describe, interpret, and understand FI students' mathematical communication during their interactions while working on a mathematical

problem-solving task in their L2. I sought to examine students' attention to both language and mathematical concepts during these interactions and, relatedly, aimed to explore whether and how students' L1 was used.

3.0 METHODOLOGY

When I began thinking about which methodology to choose for my study, my key step was to return to my research questions. The questions, which are qualitative in nature, led me to select a qualitative methodology. Nonetheless, I found it was also useful to think about some numerical analysis of my data, for example, counts of certain linguistic or mathematical terms, or counts of instances of L1 use. Looking at the data in this quantitative way helped me to identify key trends and clarify some of my questions and interpretations. However, the numerical analysis is strictly descriptive and therefore the research, overall, remains highly qualitative in nature.

According to Locke, Spirduso, and Silverman (2007), "one of the most common purposes of qualitative research is served when investigators pose the basic question, 'What's going on here?'" (p. 96). Denzin and Lincoln (2000) have offered a definition of qualitative research that I have found useful when conceptualizing my work:

> Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible . . . qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or to interpret, phenomena in terms of the meanings people bring to them. (p. 3)

Moreover, as Locke et al. suggested: "It is the participant's experience in that context that the researcher seeks to capture and understand" (p. 96). Thus, the context of a phenomenon is a key consideration in qualitative research designs and is inseparable from the participants and their experiences. This idea of qualitative research as "situated"

and as tied inextricably with its participants' experiences fits very well with my theoretical framework and with the approaches to research I explored in the qualitative studies included in my literature review in Chapter 2.

Qualitative research is further defined by its inductive nature. Specifically, "qualitative researchers do not formulate hypotheses and gather data to prove or disprove them (deduction). Rather, the data are gathered first and then synthesized inductively to generate generalizations, models, or frameworks" (McMillan, 2008, p. 274). This statement also rang true for me throughout my data collection process. As I collected data and began preliminary analysis, two events that I often felt occurred somewhat simultaneously, I found my methodology and methods evolving as themes emerged.

Qualitative research as an overarching paradigm, with its emphasis on description, interpretation, participant perceptions, context, situatedness, and inductive data analysis, was the most appropriate means by which to investigate and explore my research questions. Within the overarching qualitative paradigm, however, I needed a more precise methodology of investigation to guide my data analysis. The research questions, in broad terms, aimed to describe, interpret, and understand the nature of students' oral communication while working collaboratively on a mathematical problem-solving task. Thus, discourse analysis was the choice of methodology that would best qualitatively investigate the phenomenon in question. Moreover, in my review of pertinent literature I took careful notice of the methodologies used by other scholars to explore similar phenomena; discourse analysis was by far the most widely used methodology for exploring students' language use in both L2 and mathematics classroom contexts.

In this chapter, I begin by describing the methods used to collect data for this study. In the Methods section I discuss the ethics review procedures, and then explain the processes of site and participant selection. Details about the study's participants are given. I then explain the process through which the materials used in the study were selected and developed, and also how these materials aim to elicit problem-solving activity and link to the mathematics curriculum. The data collection is explained in detail, in terms of both the various methods used as well as the unfolding of the numerous site visits. I move on next to the Data Analysis section of this chapter. This section includes an exploration of "discourse" (what it is and how one might define it), which provides a foundation for the discourse analysis methodology used in this study. I also explain the more specific a priori coding schemes that were used in the data analysis, which also allowed for the emergence of novel themes. Issues of validity are addressed, and the triangulation strategies used in the data collection and analysis are discussed. I conclude the methodology chapter with summative thoughts on the entire data collection and analysis processes.

3.1 Methods

In order to explore my research questions, and in line with my discourse analysis methodology (described in detail in the upcoming Data Analysis section), the primary source of data for this study is audio recordings collected during student interactions (collaborative dialogue) in the classroom. These interactions occurred as students worked together on mathematics problems in their FI mathematics classroom; the recordings were subsequently transcribed. Additional data sources for deepening the analysis and for triangulation purposes include fieldnotes/research diary, interviews, and students' written

work. This study is classroom-based, in that it took place in a physical classroom in a particular high school, with a teacher and her students. In this Methods section, I describe and explain in more detail the research ethics process, the data collection site and participants, the materials (including the mathematics task assigned to the students), and the data collection procedures.

3.1.1 Ethics

As this study involves research with human subjects (participants), the research proposal was reviewed by the University of New Brunswick Research Ethics Board (REB). Prior to submitting my proposal for official review by the REB, I made informal contact via email in March 2015 with a teacher with whom I had worked for a number of years in the same school. The teacher expressed interest in working together with me on my research project, which at that point I had explained to her in broad terms.

I submitted my ethics application to the Faculty of Education at the beginning of September 2015. Upon receipt of the feedback from the faculty reviewers, I made some edits to the application and sent the revised version to the UNB REB in mid-September. On September 21, 2015 I received notice that the UNB REB required edits to be made to five areas of the application, the most substantial being some modifications to the informed consent procedures. (In the end, it was decided that both student participants and their parents would need to give active consent in order for students to be able to participate in the study.) I revised the application, addressing all five queries, resubmitted it, and the application was formally approved on September 22, 2015 (REB file #2015-095, see Appendix A for a copy of the ethics approval letter).

With regard to informed consent, following the official approval of the research project by the REB information and approval letters were sent to the school's district superintendent and principal via email. The teacher who intended to participate in the study, as well as all students in her target classes and their parents, received hard copies of an information letter and informed consent form (see Appendix B for all information letters and informed consent forms), which I distributed during visits to the school.

At this point in the project (November 2015), the teacher and I had decided to work with her two Grade 10 classes (at the time she was teaching two classes of Grade 10 and two classes of Grade 9 FI mathematics). I visited the Grade 10 classes during their regular mathematics class time, where I spent approximately 20 minutes introducing myself, distributing the student and parent letters and forms, and responding to questions regarding the project. Students brought the letters and forms home and those who wished to participate returned them, signed, to the teacher. Later in the school year (February 2016), when we had finished working with the Grade 10 students and began working with the teacher's two Grade 9 classes, the same process of visiting the classroom to distribute the letters and forms was repeated for those new Grade 9 classes. During the entire preparation for and undertaking of the data collection for the research, I worked closely with the teacher to ensure that the relationship continued to be mutually beneficial. The teacher was interested in having a partnership with a colleague from the university (me) so that she could offer to her students opportunities to participate in some innovative mathematics activities that she might not otherwise think of nor have the time to prepare on her own (Participating Teacher, personal communication/fieldnotes, November 13, 2015). I worked my school visits around the teacher's and students' school schedules so

as to minimize disruption to their regular routine. The teacher was involved in the planning, preparation, and approval of all data collection materials (e.g., the mathematics activities) that were used in the study. Activities were chosen to not only aim to elicit data related to the research, but also to provide students with valuable mathematics and language learning experiences in the classroom.

As will be described in further detail in an upcoming section of this chapter, students' classroom interactions, and student and teacher interviews were audio recorded using a digital USB audio recorder. Following each data collection session, audio files were transferred to my password-protected computer and erased from the digital recorders. All audio data were also transcribed, and these transcriptions were stored on my computer as well. Hard copies of items such as fieldnotes, the signed informed consent forms, and students' written work were stored in a secure, locked office on the UNB campus. These hard copies were also digitally scanned as a precaution, and the digital files were stored on the same computer. As an additional precaution against data loss, the audio files and transcriptions, as well as the digitally scanned copies of the fieldnotes, signed informed consent forms, and students' written work, were also backed up on two external backup data storage devices, which were also stored in the same office. Every effort was made during the final reporting of the data in this dissertation to ensure anonymity of the participants. All participants (teacher and students) were assigned pseudonyms, which are used throughout the dissertation.

3.1.2 Site and Participants

This study took place in a New Brunswick public high school (Grades 9 to 12) with a student population of just under 1,700. The school is one of two high schools in

the Anglophone school district in the city; there are two Francophone schools in the city as well (Grades K to 12). The city itself is majority Anglophone: On the 2011 Census, 86.7% of the population reported English only as mother tongue, 7.0% reported French only, and 5.1% reported only a non-official language (21.4% reported bilingualism in official languages, i.e., knowledge of English and French). The high school in this study offers FI programming for Grades 9 through 12, and students in the FI program represent both the early (EFI) and late (LFI) entry programs, depending on which one was offered at the "feeder" middle school from which they came. In Grades 9 and 10 at the school, EFI and LFI students are integrated in the same content classes (i.e., mathematics, science, social studies) but are separated according to program in their French immersion language arts (FILA) classes. Mathematics is offered in French to FI students from Grade 9 through to Grade 11, and, in fact, students are required to take mathematics in French in Grades 9 and 10 in order to continue to be enrolled in the FI program. In Grade 11, students must take mathematics but can choose to take it in either French or English. Most Grade 11 FI students, however, tend to take mathematics in French in order to satisfy graduation requirements for the FI program completion certificate (Culligan, 2008). With regard to numbers of FI mathematics classes, during the school year in which this study took place there were seven Grade 9 FI mathematics classes and four Grade 10 FI classes. This compares to nine Grade 9 and 13 Grade 10 English mathematics classes.

This school was chosen as my research site because I had previously taught there (but not since 2007) and therefore it was relatively easy to gain access through my connections to teachers whom I had known and were still teaching at the school

(including the participating teacher as well as the mathematics department head teacher). The ability to gain access can be a common stumbling block for conducting classroom-based research (Bogdan & Knopp Biklen, 2007). Furthermore, the school had a sizeable number of FI mathematics teachers and classes, so I knew my chances of gaining access to work with one or more classes would likely be good. Thus the site and participants were chosen following the practice of purposeful sampling, a common one in qualitative research. As McMillan (2000) described, "in purposeful sampling . . . the researcher selects particular individuals . . . because they will be particularly informative about the topic" (p. 119). That being said, the approach could probably be best described as random purposeful sampling (Creswell, 2007) since, once the particular target classes were decided upon, I welcomed the entire class to participate by providing all students in those classes with information letters and the opportunity to give informed consent; no specific students were targeted as participants within the classes and all students who wished to participate were included in the study.

As touched upon previously in the Ethics section, this study's participants were one teacher and some of her Grades 9 and 10 students (i.e., those who consented to participate in the study). I had known the teacher quite well when I had taught at the school. At that time, we had mostly a "professional friend" relationship (i.e., we saw each other mostly while at work in the school or when attending social functions that were outside of school but school-related, such as volunteer or extracurricular activities). However, we were friendly and close, especially relative to other coworkers. We had always worked well together in the past on various school-related projects. At the time of this study, she was still teaching at the same school whereas I had moved on to studying

and working at UNB approximately 9 years earlier. We had crossed paths in the city from time to time during that span and were still very friendly when we saw each other on those occasions, however we had not stayed in touch to any great degree. She and I both expressed our delight at the chance to work together again and we began this journey excited and motivated.

The participating teacher, Mme Nathalie, had been teaching for 15 years at the time of this study. Although her teacher training was primarily in the area of social studies and FI, she had been teaching FI mathematics in Grades 9 and 10 for the past 11 years. She is fluently bilingual (English and French) and identifies her mother tongue as English.

The main group of student participants came from a pool of two Grade 9 classes with whom we worked during the second half of the 2015-2016 school year (from February to May). The teacher and I also worked with participants from two Grade 10 classes during the first semester, however, this round of data collection resulted in more of a piloting and field-testing round rather than a "true" data collection episode. Interestingly, initially I had anticipated collecting data only from the Grade 10 classes. However, due to the low turnout of consenting participants (only five from the two Grade 10 classes), I had to adjust my data collection plan and eventually sampled the Grade 9 classes in the second semester, with much greater success (22 participants). Therefore, the main focus in this dissertation is on the Grade 9 data (although I will describe the Grade 10 piloting experience in the Data Collection section). Table 2 shows demographic information for the Grade 9 student participants, who are the focus of the study. The table includes not only information related to students' academic achievement in French

Immersion Language Arts (FILA) 9 and Mathematics 9, but also their English Language Proficiency Assessment (ELPA) result. The ELPA is a provincial standardized reading test administered to all students in Grade 9 (Com = Complete; Inc = Incomplete; AA = Appropriate Achievement; BAA = Below Appropriate Achievement; SA = Strong Achievement). Provincial results for 2015-2016 school year were BAA = 19.7%, AA = 72.3%, and SA = 8.1%.

Table 2

Grade 9 Student Participant Demographic Information

Pseudonym	Group (Letter, Period)	Program (Early or Late)	FILA 9 (Mark in %)	Math 9A, 9B (Marks in %)	ELPA (Result)	Other language(s)
Mac	E1	EFI	81	95, 99	Com (AA)	
Mike	E1	EFI	78	81, 97	Inc (BAA)	
Scot	L1	EFI	85	91, 92	Com (AA)	
Ava	L1	LFI	71	76, 70	Com (AA)	
Jane	M1	EFI	76	90, 94	Com (AA)	
Beth	M1	EFI	92	87, 93	Com (AA)	
Sara	H1	EFI	81	82, 88	Com (AA)	
Brit	H1	LFI	65	60, 72	Com (AA)	
Bea	H1	EFI	72	84, 85	Com (AA)	
Kim	H4	LFI	84	82, 87	Com (AA)	
Eve	H4	LFI	90	94, 94	Com (SA)	
Amy	E4	EFI	87	86, 91	Com (AA)	
Hali	E4	EFI	89	93, 93	Com (AA)	

Pseudonym	Group (Letter, Period)	Program (Early or Late)	FILA 9 (Mark in %)	Math 9A, 9B (Marks in %)	ELPA (Result)	Other language(s)
Dan	D4	EFI	72	85, 90	Com (AA)	
Zac	D4	EFI	82	92, 92	Com (AA)	
Ella	M4/E4[a]	LFI	100	98, 100	Com (AA)	1
Hana	M4[a]	LFI	94	96, 96	Com (SA)	
Sue	L4	EFI	96	97, 97	Com (SA)	
Liz	L4	LFI	93	94, 94	Com (SA)	
Mae	K4	LFI	85	72, 77	Com (AA)	
Max	K4	EFI	70	87, 87	Com (AA)	
Mya	K4	LFI	83	86, 91	Com (AA)	

Note. EFI = early French immersion; LFI = late French immersion; ELPA = English Language Proficiency Assessment.

[a]Hana was absent for 2 out of 3 days (day #2 & #3) of the classroom-based recordings, therefore Ella worked with the E4 group on those days.

As shown in Table 2, 22 Grade 9 students (from two different classes, period 1 and period 4) chose to participate in the study. All students participated in all parts of the study, with the exception of Max, who chose to not participate in the interview portion. None of the participants withdrew from the study.

The teacher participant was not offered any incentives to participate in the study, but did perceive benefits to the working relationship with me as the researcher (e.g., getting help planning and executing the mathematics activities, having the chance to talk about mathematics teaching and learning). In addition, the participating teacher was able

to have a supply teacher on three occasions (at the beginning, middle, and the end of the project) arranged through the university, so that she could have release time to meet with me to attend to matters related to the research project (e.g., planning the course of action, preparing the activities, discussing the students' finished products). The student participants were not offered any incentives to participate, however I did supply a pizza party for them at lunchtime at the school (with permission from the school principal) as a thank-you and celebratory gesture at the end of the project.

In qualitative research, it can be difficult to know how many participants are "enough." However, it is useful to keep in mind that the goals of qualitative research centre on conducting quality, rich, in-depth, complex analysis using smaller-sized samples, rather than attempts at generalizations and the achievement of statistical significance requiring large samples. This can be especially so when conducting discourse analysis—in one of the key guiding studies for my research, Swain and Lapkin (1998), for example, found that they were able to conduct in-depth discourse analysis using a very small number of samples, each just 5 minutes in length, of transcribed audio recordings. Scholars who are experts in the field of qualitative research (e.g., Bogdan & Knopp Biklen, 2007; Creswell, 2003, 2007; Merriam, 2009) have concurred that despite the overall tendency toward smaller numbers, the number of participants still varies significantly from study to study. Creswell (2007), for example, noted qualitative studies of different types with one, three, 10, 20, 30, and up to 325 participants. Each individual researcher must consider when he or she has reached sufficiency and saturation in the data, as well as more practical aspects such as the availability of time, money, and resources (Seidman, 2006). With 22 Grade 9 participants in total arranged into 8 pairs

and two groups of three, and three data recording sessions for each pair/group ranging from 15 to 55 minutes each, plus supplemental data collected from each participant pair/group (e.g., interviews, written work), I felt confident that I had enough (probably more than enough) data with which to conduct a thorough analysis. The fact that I was a participant observer in the data collection sessions (for the in-class activity recordings and the interviews) and was able to observe the data collection as well as listen back to a number of the recordings along the way, also helped to build my confidence in the number of participants as well as the quality of the data being collected. Nonetheless, as a safeguard I conducted a second round of data collection with the Grade 9 participants to ensure that I would have enough data with which to work. In the end, the second round of data was not needed (saturation was achieved based on the first round of data collection), and the results presented in Chapter 4 focus on the first round of data collection from the two Grade 9 classes. Next, I describe the materials used to undertake this data collection.

3.1.3 Materials

When I first began thinking about recording students as they worked together on mathematics problems, I was not sure what types of problems these would be. From my own classroom teaching experience, I knew that some students tended to talk to each other more than others for what seemed like a variety of different reasons (e.g., preferred learning strategy, motivated by topic, motivated by grades, talkative personality). In order to elicit student communication from as many students as possible, I knew the teacher and I would have to carefully choose the mathematics problems. Indeed, the choice of good problems would prove to be much more important than I had originally anticipated.

3.1.3.1 Grade 10 "pilot" materials.

At our first meeting together, in October 2015, Mme Nathalie and I decided to work with her two Grade 10 classes for the research project. We could have just as equally decided to work with the Grade 9 classes instead of or as well as the Grade 10s, but at the time I thought that working with two Grade 10 classes of 20+ students each would provide enough data. Also, I had thought that I would have been able to know students' results on the Grade 10 French Immersion Literacy Assessment, which was a provincial assessment that had been conducted in previous years, and that these results would provide an interesting contextualization of students' proficiency in their L2. However, it turned out that the province ceased offering this assessment the year this study was conducted. Nonetheless, we pursued the Grade 10 option since that was our original plan.

In November 2015 the teacher and I met for a full day (one of her three supply release days taken over the course of the research project) in order to review the project in detail and plan the data collection materials. The approach we tried, in this Grade 10 data collection round, was to give students an in-class written assignment that Mme Nathalie identified as "typical" of her classroom. It was a written assignment that she would have intended to assign to the students as in-class work, whether this research project was happening or not. The assignment was a standard high school mathematics written assignment on linear functions and relations, with all the questions taken from the student textbook. The assignment aimed to mainly address the following curriculum outcome on which the class was currently working:

> *RAS RF3 : Démontrer une compréhension de la pente en ce qui a rapport à l'élévation et la course, des segments de droite et de droites, le taux de variation,*

> *des droites parallèles et des droites perpendiculaires. [RP, R, V]* (New Brunswick Department of Education and Early Childhood Development [NB EECD], 2014, p. 49)
>
> [Translation: SCO RF3: Demonstrate an understanding of slope with respect to: rise and run; line segments and lines; rate of change; parallel lines; and perpendicular lines. [PS, R, V] (NB EECD, 2012, p. 47)]

All students, including the five Grade 10 participants in the study, were allowed to use their notebooks and textbooks, and were encouraged to consult with their partner(s) while working on this written assignment. The assignment began with easier questions (i.e., procedural, straightforward answers required) and increased in difficulty level (i.e., involving multiple procedures and making connections) toward the end; the problems all came from the assigned course textbook. While some of the questions asked students to "justify" their answers, and thus aimed to elicit written (or perhaps oral, if students chose to discuss with a partner) communication, overall the assignment was not expressly communicative in nature. The problems did contain mathematics that related, importantly, to the curriculum outcome and the assignment was a typical example of one that the classroom teacher had used in the past, so we decided to move forward and use it in this first round of data collection. However, these materials did not generate much in the way of student discussion and interactions. The sparse discourse combined with the low number of student participants meant that I finished the Grade 10 data collection round with minimal data with which to work. Although initially discouraged, I used this as a learning opportunity and, thus, the experience provided me with a valuable piloting round on which I based my subsequent materials development and data collection sessions. I

describe the precise data collection procedures in the upcoming Data Collection section. First, however, I elaborate on the materials used as a stimulus for the Grade 9 data collection.

3.1.3.2 Grade 9 materials.

After having few students to work with from the Grade 10 classes in the first semester, and after the materials did not generate as much student discourse as I had hoped, Mme Nathalie and I agreed that we would do additional data collection in the second semester of the school year—this time with her two Grade 9 classes. Mme Nathalie felt that she had a positive rapport with these classes, whereas she reflected that she felt she had struggled in this regard with the Grade 10 classes and thought that this might have contributed to the low turnout of signed informed consent forms (personal communication/fieldnotes, February 18, 2016). Moreover, the material we used with the Grade 10 students was a rather procedural worksheet that, in retrospect, might explain the lack of interest and enthusiasm, which we both also felt may have contributed to the low number of participating students and the minimal amount of discourse during data collection. Whether due precisely to those reasons or others (e.g., different age group, different student personalities), the participant numbers were much higher second semester with the Grade 9 students. Whereas we only received five signed consent forms in Grade 10, to my delight (and relief) we received 22 from the Grade 9 students. In order to address our concerns about the Grade 10 material, the materials that Mme Nathalie and I chose for the second round of data collection, with the Grade 9 students, were of a different nature. This time, we chose a problem-solving activity from the NCTM Illuminations website: a lengthy, hands-on activity called “Planning a Playground.”

3.1.3.2.1 Choosing a problem-solving task.

The importance of the choice of research materials was something that emerged during the Grade 10 piloting phase of data collection. Prior to that experience, I would describe my approach to choosing the mathematics tasks as somewhat naïve in that I mainly believed that, given the opportunity, students who were asked to collaborate while working on a mathematics task, any task, would do so. Thus, while the worksheet we used as data collection material for the Grade 10 participants represented a solid, useful exercise of problems to practice procedural skills, it was likely not the best choice for promoting student discourse. It became clear to me that, in this research context at least (and likely for any classroom), the choice of mathematical task would be of utmost importance if the goal were to elicit communication.

Mme Nathalie and I chose the Planning a Playground problem-solving activity (I will describe the activity in more detail in the following sections) for a number of reasons. First, we were concerned about providing a "good" mathematics problem for the students that was a "true" problem-solving activity and, as such, would be more likely to generate interest and communication among the students. Defining problem solving for research purposes is not straightforward—as Wilson, Fernandez, and Hadaway (1993) remarked, "every exercise of problem-solving research has gone through some agony of defining mathematics problem solving" (p. 58). Researchers' and educators' definitions of problem solving can be influenced by many things, for example, our ideas about mathematics, curriculum, teaching, and learning; or our goals for the classroom or for our research. I draw primarily from seminal work on problem solving by Polya (e.g., 1957) and resources produced by the NCTM (e.g., 2000, which also draws on Polya's work) to

enact a working definition of problem solving for this study. Polya's work on problem solving is widely recognized in mathematics education, and the New Brunswick K-12 Mathematics Curriculum is heavily influenced by the NCTM's *Principles and Standards for School Mathematics* (2000). For its part, the NCTM (2010) has offered the following description of problem solving:

> "Problem solving" refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development. . . . Such tasks—that is, problems—can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interests and curiosity. (p. 1)

Thus problem solving has implications for students' mathematical (and language) activity. The NCTM (2010) definition helps guide my own interpretation of the language of problem solving throughout my data analysis.

In addition to a working definition of the term, having an understanding of the activity of problem solving is also important. Seminal works in the mathematics education field (e.g., Polya, 1957; Polya & Conway, 1985) have noted four rather distinct phases of students' problem-solving activity:

> First, we have to *understand* the problem; we have to see clearly what is required. Second, we have to see how the various items are connected, how the unknown is linked to the data, in order to obtain the idea of the solution, to make a *plan*. Third, we *carry out* our plan. Forth, we *look back* at the completed solution, we review and discuss it. (Polya & Conway, 1985, pp. 5-6, original emphasis)

These phases, although listed as if distinct, can be viewed as a cyclical, dynamic framework that can help mathematics classroom researchers think about what is happening during students' problem-solving activities as shown in Figure 1:

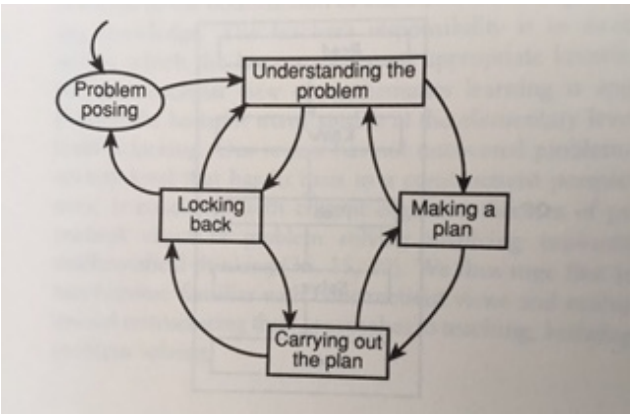

Figure 1. Framework emphasizing the dynamic and cyclic nature of problem-solving activity (Wilson, Fernandez, & Hadaway, 1993).

Throughout these phases of problem solving, several classic strategies are often used in order to find solutions; those cited in the NCTM's *Principles and Standards* (2000) include: "using diagrams, looking for patterns, listing all possibilities, trying special values or cases, working backward, guessing and checking, creating an equivalent problem, and creating a simpler problem" (p. 54). These strategies are inspired by Polya's (1957) seminal work in the field, in which he identified many of the same strategies (sometimes worded slightly differently) and others, for example: guess and check, look for a pattern, make an orderly list, draw a picture, eliminate possibilities, solve a simpler problem, use symmetry, use a model, consider special cases, work backwards, use direct reasoning, use a formula, solve an equation, and be ingenious. Therefore, the teacher and I anticipated that we might see this type of four-phase, cyclical process during the

mathematics activity and that we might expect students to use some of these classic problem-solving strategies.

With a working definition of problem solving and the goal of finding a good problem-solving task in mind, we reflected on how we might best identify some appropriate choices for Grade 9 materials. The NCTM (2010) has identified at least 10 criteria for teachers to consider in the selection of good quality, worthwhile mathematics problems, which are:

1. The problem has important, useful mathematics embedded in it.
2. The problem requires higher-level thinking and problem solving.
3. The problem contributes to the conceptual development of students.
4. The problem creates an opportunity for the teacher to assess what his or her students are learning and where they are experiencing difficulty.
5. The problem can be approached by students in multiple ways using different solution strategies.
6. The problem has various solutions or allows different decisions or positions to be taken and defended.
7. The problem encourages student engagement and discourse.
8. The problem connects to other important mathematical ideas.
9. The problem promotes the skillful use of mathematics.
10. The problem provides an opportunity to practice important skills. (p. 1)

The NCTM (2010) acknowledged that not every mathematics problem a teacher selects would be able to target each and every one of the 10 criteria. However, the NCTM (2010) suggested that the first four criteria should, ideally, be a focus of every good quality

mathematics problem; from there, the other criteria can be addressed according to the teacher's particular instructional needs. In our case, for instance, criterion #7, "encourages student engagement and discourse", was also of utmost importance.

A second key reason for using a different type of mathematics task (a problem-solving task rather than a more traditional written assignment comprising relatively straightforward textbook problems) in this Grade 9 round of data collection, was that Mme Nathalie was keenly interested in providing her students with an opportunity to participate in these kinds of problem-solving activities, particularly the kinds that might have a hands-on component and that were communication-based, since she felt that she, herself, did not often incorporate these kinds of activities into her mathematics classroom (personal communication/fieldnotes, February 18, 2016). The main reason she cited for not doing so was lack of time (to find good problems, prepare materials, and so on). Thus, my research project presented the ideal occasion for Mme Nathalie to be able to offer these kinds of tasks to her students, since I was available to track down and sift through the potential activities and then prepare any necessary materials.

Finally, after my experience with the pilot round of Grade 10 data collection, I found that the typical classroom worksheet had not seemed to generate much interaction among the students. While the task may not have been the only reason for this (e.g., student personalities, students' age, the research setting, and a myriad of other factors could have contributed to this), I suspected that it played some role, as did Mme Nathalie. When I revisited the NCTM (2000) "Communication" standard, what I found further confirmed my suspicions:

> Students need to work with mathematical tasks that are worthwhile topics of discussion. Procedural tasks for which students are expected to have well-developed algorithmic approaches are usually not good candidates for such discourse. Interesting problems that "go somewhere" mathematically can often be catalysts for rich conversations. (p. 60)

This was yet another reason that we both agreed to a choose richer, problem-solving oriented task, ideally having a hands-on component, versus the more procedural, traditional word problem-type task from the pilot round of data collection (with the Grade 10s).

With all of this in mind I set about finding some potential mathematics problem-solving tasks that would address some of these key criteria, with a particular focus on criterion #7—"the problem encourages student engagement and discourse"—since analyzing student discourse was the main goal of this study. When Mme Nathalie and I met at our next full planning day, at the beginning of March 2016, I had a few options prepared for us to look at together. We ultimately chose one task that was quite elaborate, which we anticipated would take a minimum of two full class periods to complete (in practice, it actually required three periods). Next, I give details about the task, including its preparation, materials, and potential solutions.

3.1.3.2.2 Planning a playground.

The problem-solving activity Mme Nathalie and I chose was from the NCTM Illuminations website (n.d.), which has described itself in the "About Illuminations, What We Do" section as follows: "Illuminations works to serve you by increasing access to quality standards-based resources for teaching and learning mathematics, including

interactive tools for students and instructional support for teachers." The activity, which was called "Planning a Playground," seemed like it would fulfil many of the criteria for choosing good problems, as discussed in the previous section. Once we had decided on this particular activity, I piloted it with my mathematics education students. This particular mathematics education course was focused on the role of language in the teaching and learning of mathematics and in it, we often engaged in various mathematics activities and then reflected on our language use. Thus, I felt that doing the activity with these preservice teachers would be valuable learning for them, while, at the same time, provide me with a sense of whether or not the activity might work well for my data collection purposes. The piloting provided valuable information: The education students liked the activity, thought it was engaging and interesting, and felt that it would encourage the Grade 9 students to communicate. I was particularly interested in knowing whether the hands-on component of the activity, which included the use of large-scale models, was something they felt was of value, since I thought that the activity could be done without these models, but they suggested that the models added a valuable visual, hands-on element to the activity they felt that many students would really enjoy; moreover, the hands-on component was one that Mme Nathalie had expressed specific interest in including. Consequently, I decided to retain that part of the activity. Once I had incorporated some of the suggestions from this piloting phase and had produced what I felt would be a good final version of the activity, I went through the entire task myself from beginning to end in order to test it out.

The basic premise of the Planning a Playground activity was as follows: A "real-life" scenario was set up in which there was a narrative about a daycare that had received

funding and a donation of new equipment in order to build a new playground. The daycare was entertaining different design possibilities for this new playground, however, there were a number of constraints involved. The new playground would be close to a busy highway therefore it had to be fenced for the children's safety. The builder had 200 ft. of fencing to work with, which came in 1-ft. panels. There were eight pieces of playground equipment in total (slide, dome, carrousel, two teeter-totters, swing set, and two picnic tables), each having its real dimensions and its required dimensions. The real dimensions were the actual size of the given piece of equipment; the required dimensions were the actual space needed around each piece when building the playground, in order to conform to safety standards (e.g., the swings themselves only measured 8 ft. by 5 ft., but they required a building space of 32 ft. by 36 ft. so that the children could swing safely without bumping into other children walking by). The students worked in pairs or groups of three in order to prepare a playground design for the daycare. They were aiming to construct a playground with the maximum area possible, given the 200 ft. of fencing in 1 ft. panels, that would also accommodate all of the pieces of equipment taking into account their required areas. (See Appendix C for a copy of the entire Planning a Playground/*Garderie Bout de chou* activity.)

There were several steps to solving the problem. Students were first given large-scale, hands-on models in order to get a physical sense of planning the playground and be able to easily manoeuvre the fencing and equipment as needed (see Figure 2). The fencing was represented by 200 paper clips (each paper clip representing a 1-ft. fence panel) that could be linked together to enclose the fence and could be easily readjusted to different dimensions. Large pieces of paper were used to represent each piece of

equipment; each piece of paper was cut out to scale, in paper clips (again, 1 paper clip = 1 ft.), and could be rearranged within the fencing as needed in order to find a good fit. Thus, students worked initially with a scale model of the playground where 1 paper clip = 1 ft. Once the students had found a configuration they felt worked, they responded to some written questions regarding their process. Next, students were asked to create a precise scale drawing of their playground design using graph paper, rulers, and compasses, since some of the equipment, like the dome, for instance, was circular (see Figure 3). Students were asked to answer written questions regarding maximum area and their scale diagram, and about why they chose their particular design and why their design was the best choice for the daycare (see Figure 4). A key area of mathematical interest in the problem is that although a square fence with sides measuring 50 ft. each gives the maximum interior area (50 ft. x 50 ft. = 2500 $ft.^2$), the playground equipment will not fit into a square (i.e., the pieces of equipment overlap each other and/or the fence). As NCTM Illuminations (n.d.) noted, "this is an interesting demonstration of how a real-world context can change a purely mathematical result" (Lessons section, "Planning a Playground"). By manipulating their large-scale models or using their graph paper drawings, students should then work out that the next maximum area possible, that will accommodate all of the pieces of playground equipment, is a rectangle with sides measuring 48 ft. x 52 ft. There is not much difference between these dimensions and the square measuring 50 ft. x 50 ft. Consequently, students would need to take care and be precise when using the models and/or drawing their scale diagrams. Otherwise, they may not notice that the equipment does not fit within the square shape. Once they determine their final dimensions, students must then decide how to best arrange the equipment

within the space. While nothing in the written instructions of the activity prohibited using rounded or circular shapes for the fencing, the shapes of the various pieces of playground equipment along with the rigid fencing supplied in 1-foot panels made working with perfectly rounded shapes difficult.

Figure 2. Large-scale models for the Planning a Playground activity.

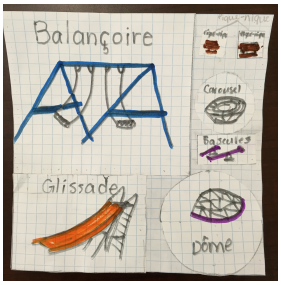

Figure 3. Example of student scale drawing for the Planning a Playground activity.

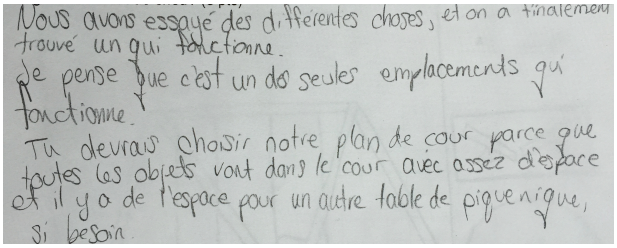
Nous avons essayé des différentes choses, et on a finalement trouvé un qui fonctionne.
Je pense que c'est un des seules emplacements qui fonctionne.
Tu devrais choisir notre plan de cour parce que toutes les objets vont dans le cour avec assez d'espace et il y a de l'espace pour un autre table de pique nique, si besoin.

Figure 4. Example of student written work for the Planning a Playground Activity.

In order to do this, students could be expected to use one or more of the common problem-solving strategies discussed earlier in this section (NCTM, 2000; Polya, 1957; Polya & Conway, 1985):

- Using symmetry: Perfect squares are mathematically (e.g., $200 \div 4 = 50$, $50 \times 4 = 200$, $50 \times 50 = 2500$, $\sqrt{2500} = 50$) and visually (e.g., shape, equal

symmetry, centre) appealing. Thus, students' "mathematical and/or aesthetic instincts" could lead them to immediately gravitate toward a square-shaped fence (50 ft. x 50 ft.). However, even if they choose the square immediately, students should also then check that 2500 (50 x 50) is the maximum area possible (e.g., by finding the results of different multiplications). They should then check to see if their 50 ft. x 50 ft. fence would accommodate the equipment by using their models. When or if they discover that the square will not accommodate the equipment, students might then try simply adjusting the dimensions by 1 (e.g., 49 x 51), then by 2 (e.g., 48 x 52), and so on, until they find a solution that fits the equipment. At this point, a second problem-solving strategy has become involved in the process: guessing and checking. Making small adjustments like these would be considered reasonable guesses, given that the 50 x 50 layout is very close to being able to fit all the equipment.

- Guessing and checking:
    - Students might try sides of 50 right away, using their number sense and realizing that 50 x 4 = 200 (or 200 ÷ 4 = 50). Creating a fence with four equal sides (i.e., a square fence) simply makes the numbers easy to work with, with most students likely able to handle the mental math involved (e.g., $200 \div 4 = 50$, $50 \times 4 = 200$, $50 \times 50 = 2500$, $\sqrt{2500} = 50$), so it is a reasonable guess to begin with. Students would then need to follow the same steps described above, for checking the calculations for maximum area as well as the fit of the equipment.

    - Alternately, students might begin their guessing in a more "random" fashion using any numbers that could make up the four sides of a rectangle (e.g., 30 x 70) and continue guessing and checking until they find the factors that give the largest possible result (2500). Students would then need to follow the same steps described above, for checking the calculations for maximum area as well as the fit of the equipment.

- Using diagrams/Using a model: Using the physical models (paper clips and large paper cut-outs) or by drawing scale diagrams on graph paper, students might visually try different arrangements (of the fencing and the playground equipment) until they find what works. The large-scale models were pre-scaled (in paper clip units), whereas if using graph paper students would need to ensure they have scaled the diagram properly (e.g., 1 ft. = 1 unit, or 1ft. = 2 units).
- Listing all possibilities: Students might begin a list of all possibilities of perimeters and calculate their corresponding areas in order to be certain they have found the maximum area (students might organize this in a table). Students would then need to follow the same steps described above (in "Using symmetry" and "Guessing and checking"), for checking the fit of the equipment.

There are a number of possibilities for arranging the equipment within the fence, and Figure 5 shows one possible solution.

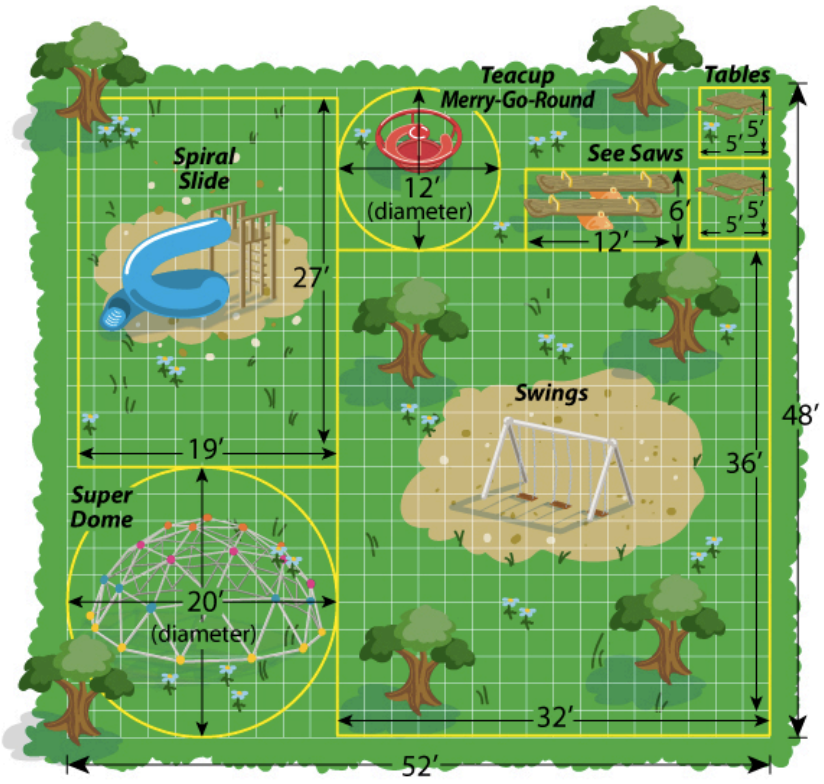

Figure 5. One solution to the Planning a Playground activity, NCTM Illuminations website.

The Planning a Playground activity involved key mathematical ideas for Grade 9 such as measurement, perimeter, area, and scale. Specifically, it related to the following Grade 9 mathematics curriculum outcomes (given here in French, with their English translations following):

> *RAS : N5 : Déterminer la racine carrée des nombres rationnels positifs qui sont des carrés parfaits. [C, L, R, RP, T]* (NB EECD, 2013, p. 32)
>
> SCO: N5: Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T] (NB EECD, 2011, p. 29)
>
> *RAS : FE4 : Dessiner et interpréter des diagrammes à l'échelle de figures en deux dimensions. [L, R, T, V]* (NB EECD, 2013, p. 81)

SCO: SS4: Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V] (NB EECD, 2011, p. 79)

In addition, throughout the *Mathematics 9 Curriculum Document* teachers are encouraged to use perimeter within applications of other SCOs (e.g., NB EECD, 2013, p. 51, 63, 64).

In looking closely at the curriculum outcomes, it is useful to take the time to explain the different letter codes and abbreviations, and what kinds of information these provide. An SCO indicates a Specific Curriculum Outcome for the grade level in question or, in French, *Résultat d'apprentissage spécifique* (*RAS*). Table 3 shows the abbreviations and meanings, in English and French, for the four key curricular strands that run throughout the K-12 New Brunswick mathematics curriculum.

Table 3

Four Key Curriculum Strands in the New Brunswick K-12 Mathematics Curriculum, in English and French

English		French	
Strand	Abb.	*Domaine*	*Abb.*
Number	N	*Le nombre*	*N*
Patterns and Relations	PR	*Les régularités et les relations*	*RR*
Shape and Space	SS	*La forme et l'espace*	*FE*
Statistics and Probability	SP	*La statistique et la probabilité*	*SP*

The letter-number code following each SCO, for instance, in our above example, SCO: "N5", simply indicates that this SCO is the fifth outcome in the Number strand.

The letters appearing in square brackets following each written outcome represent mathematical processes that underpin the entire K-12 mathematics curriculum, and that should be or have the opportunity to be used in the teaching and learning of the particular outcome: "The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning" (NB EECD, 2011, p. 6). These codes also appear differently in English and French, therefore I have used Table 4 to show the codes and the processes they represent, in both languages.

Table 4

Seven Mathematical Processes in the New Brunswick K-12 Mathematics Curriculum, in English and French

English		French	
Process	Code	*Processus*	*Code*
Communication	[C]	*Communication*	[*C*]
Problem solving	[PS]	*Résolution de problèmes*	[*RP*]
Connections	[CN]	*Liens*	[*L*]
Mental Math	[ME]	*Calcul mental et estimation*	[*CE*]
Technology	[T]	*Technologie*	[*T*]
Visualization	[V]	*Visualisation*	[*V*]
Reasoning and Estimation	[R]	*Raisonnement*	[*R*]

Specifically, the Planning a Playground activity offered students the opportunity to engage in Communication, Problem Solving, Mental Math, Technology (low-tech), Visualization, and Reasoning and Estimation.

In addition to addressing the SCOs N5 and SS4, and providing students with mathematics that was conducive to at least six out of seven key mathematical processes, the activity also fit at least one of the main broader goals of the New Brunswick K-12 Mathematics Curriculum, which was the goal of developing "mathematically literate students" (NB EEDC, 2011, p. 2). In particular, the Planning a Playground activity would require students to "use mathematics confidently to solve problems," "communicate and reason mathematically," and "make connections between mathematics and its applications" (NB EECD, 2011, p. 2). Thus, with regard to the Grade 9 mathematics curriculum and the curricular goals for the province in general, the Planning a Playground activity seemed like a good match.

The Planning a Playground activity also satisfied a number of the criteria for choosing a worthwhile problem as discussed earlier (NCTM, 2010). The playground problem had important, useful mathematics embedded in it (it addressed at least two SCOs in the Grade 9 curriculum and had potential to provide opportunities for several key mathematical processes and general goals for mathematical literacy); it required higher-level thinking and problem solving (re-thinking the square, finding a solution to the fit problem, understanding and explaining why the square did not work); it contributed to the conceptual development of students (linking perimeter to area, maximizing area); it created an opportunity for the teacher to assess what her students are learning and where they are experiencing difficulty (students were monitored during their discussions and also submitted written work); it could be approached by students in multiple ways using different solution strategies (symmetry, guess and check, large-scale physical models, graph paper scale diagrams, calculations, listing all possibilities); it had

various solutions or allowed different decisions or positions to be taken and defended (different arrangements of the equipment were possible, students were asked to explain why their design was the best); it encouraged student engagement and discourse (students could get hands-on with the models, students could use art skills, students were motivated and engaged in conversations about how to solve the problem and how to explain their answers); it connected to other important mathematical ideas (perfect square is always the largest area, how "real life" applications can require adjustments to purely mathematical results); it promoted the skilful use of mathematics (estimating, visualizing, and calculating perimeter, area, square roots, perfect squares; creating and interpreting scale; using low-tech mathematical tools such as graph paper, rulers, compass); and it provided an opportunity to practice important skills (especially communicating mathematically by describing and explaining).

The Planning a Playground activity was entirely in English on the NCTM Illuminations website. I translated the activity myself, and then had the translation checked by a university colleague, a native speaker of French, who helpfully suggested a few minor edits. I then asked Mme Nathalie to also review the activity and she reported that she was satisfied with it. Because the activity was to be completed as part of the regular mathematics class (all students would be doing the activity, although not all would be participating in the research portion), I prepared enough paper copies for the entire two classes, a total of 49 (plus a few extra copies). Over the next few weeks, I constructed and prepared the rest of the required materials for the activity: 12 small re-sealable bags with 200 paper clips each, 12 sets of large paper model cut-outs, graph paper, rulers, and compasses. I was able to get everything ready by the beginning of

March 2016 and Mme Nathalie and I set the dates for two consecutive days in mid-March to do the activity. Once we actually did the activity, we found that we needed to add a third day in order for some student pairs/groups to finish and I give more details about this next, in the Data Collection section.

As previously mentioned, Mme Nathalie and I also engaged in a second problem-solving task (an interesting although much shorter, less elaborate problem that took approximately 20 minutes) with the Grade 9 students, and with it, I also collected students' written work and conducted follow-up interviews (as I had done with the playground activity, described in detail in the next section). This second round of Grade 9 data collection was done as a precaution, in case I did not gather sufficient and appropriate data from the Planning a Playground activity. In the end, however, the playground activity proved effective as a stimulus and thus data stemming from that task (i.e., classroom-based recordings, interviews, written work) are the focus of this dissertation.

3.1.4 Data Collection

As discussed in the previous section, the development of the materials (i.e., the mathematics activities and problems) was a process that took a lot of time from beginning to end. At times during the process of developing the materials, data collection (and data analysis, for that matter) was also occurring. In total, I visited the research site, the high school, 17 times from November 2015 to June 2016 for meetings with participants, classroom observations, and data collection (recordings, interviews, information gathering). During that time span, the teacher and I also met at the university on three occasions: twice in November 2015 at the beginning of the project (one of these meetings

was for a full day), and once in March 2016 prior to beginning the data collection with the two Grade 9 classes (for a full day).

As a classroom-based research study, this project required fieldwork. Consequently, there were a number of issues on which I needed to reflect and various challenges that arose with regard to data collection. As Bogdan and Knopp Biklen (2007) have noted, qualitative researchers in education, once they have overcome the first challenge of gaining access to the research site, must then navigate the waters of the particular research environment in which they are entering. I feel that having been a high school FI mathematics teacher myself, and at the very same school, I felt quite comfortable transitioning into the school research site. On my first visit to the school, which was for a meeting with the participant teacher only, I not only stayed for our formal meeting time but also accompanied her on her noon hour supervision duty to continue talking. On that first day, I remarked to her that it felt like I had never left (fieldnotes, November 26, 2015). Once I entered the classroom, I was again overcome by a feeling of familiarity—students, teaching, bells, announcements, and so on. However, I quickly realized that these were not "my classes": Students were welcoming or, more commonly, indifferent to my presence initially and did not speak to me as much as to their regular teacher, Mme Nathalie. During the class activities as the school year progressed, the role I played could best be described as "participant observer" (e.g., Bogdan & Knopp Biklen, 2007; Creswell, 2007) in which I spent a lot of time observing the student participants but I also participated in the ongoing activities, which mainly involved me responding to their questions and requests for help as appropriate. Initially, these questions and requests were always related to the mathematics task at hand,

however, as time went on and I visited the classes more, some students began to ask questions reflective of the more "everyday" nature of the classroom (e.g., "can I go to the washroom?", "do you have an extra pencil?"), which I interpreted as indicative of their increasing comfort with me and their viewing me as having some kind of belonging (or authority) in their class. Just like teaching in my own classroom, some students tended to reach out to me more than others (perhaps they enjoyed socializing more in general, or perhaps they felt a good rapport with me specifically). The more intimate environment of the interviews with the students helped me to get to know them better since we had time to sit down together and talk in a two-on-one (in a few cases, three-on-one) setting. Overall, by the end of the data collection I had gotten to know all the students to some extent, some more than others. Certainly, I knew all their names and, at minimum, knew how they tended to operate in mathematics class, and became familiar with their overall demeanour.

In the next sections, I describe in detail the different types and phases of data collection that were happening at various points during the 2015-2016 school year. I begin with the collection of the primary source of data for this study: audio recordings of student interactions in the classroom. I then go on to describe the collection of other data used to triangulate those from the audio recordings, namely, interviews, documents, and fieldnotes.

3.1.4.1 Audio recordings of students' interactions.

At the heart of this study is the exploration of FI students' oral interactions as they work together, communicating during mathematical problem solving in their L2. Consequently, my main source of data for analysis is the audio recordings of these interactions. Mme

Nathalie and I had developed the materials as described previously and these mathematics tasks were used to generate student discussion. The collection of all audio files was done using small USB recording devices. Each device was labelled with a letter (A, B, C, and so on) and each pair of students (or group of three) working together was assigned a letter, which corresponded to their particular recorder (e.g., Group A used recorder A). The same recorder was used always with the same group for all data collection episodes to facilitate organization of the files. I took careful note of students' names, their group/recorder letter, the dates, and the data collection activities taking place, and organized the information into a spreadsheet in Excel. After each data collection episode, I transferred the audio files from the USB recording device onto my computer, backed up the computer files using both my external storage drives, then deleted the audio files from the recorders. I was able to use iTunes on my computer to listen to the audio files.

At the beginning of each round of data collection, Mme Nathalie and I spoke to the classes briefly as a group. We reminded students that their written work would be collected at the end of the activity in question and reviewed by both of us. We explained that the original copy of the work would then be returned to the students, and that I would keep a copy for research purposes. Mme Nathalie and I also asked the students to speak in French as they worked together, just as they (the students) normally would in class. I reiterated that there was no "right" or "wrong" way of either speaking French or discussing the mathematics, and that I was interested in just listening to what they had to say during their problem solving. All of this information had been explained in the information letter sent out with the informed consent forms (for both students and

parents), however I felt it was important to underscore these points with the student participants at the beginning of each data collection round.

In the reporting of the results in Chapter 4, the focus is on data collected with the Grade 9 classes (the Grade 10 data collection having resulted primarily in an information-gathering, pilot round). Therefore, I describe in detail here the data collection process for the two Grade 9 classes. In these two classes (a Period 1 class and a Period 4 class), there were nine participants in one and 13 in the other. For the Planning a Playground activity, Mme Nathalie and I relocated the entire two classes (participants and non-participants alike) to the school library each during their regular class period, where they would have floor and table space with which to work in order to use the large-scale models and the other materials. Recorders were given to the student participant groups, and students were advised that they could and should move the recorders if needed (e.g., onto the floor if they were working there). During the task, Mme Nathalie and I were both in the class (library) at all times, and there was a lot of activity (talking and moving). The site felt like a "typical mathematics classroom" (based on my personal teaching experience) in which students were talking and working on a major project and the data recording process appeared to cause minimum disruption. The teacher and I allowed and encouraged the students to work independently as much as possible (so as to allow the focus to be on students' interactions with each other), but of course we circulated from time to time and responded to the students when they requested our attention. I interacted mainly with the student participant groups, however at times I did attend to other students who were not being recorded. For each class, we spent a total of two full class periods (of 65 minutes each) on two consecutive days working in the library. Audio recordings

gathered on these two days were approximately 55 minutes each. After that, some students needed a bit more time to finish the activity and so we allotted a third class period, which took place in the regular classroom (participating and non-participating students were all together in the room). The recordings from the third class period ranged from 3 to 29 minutes (median of 17 minutes, mean of 16.4 minutes), and two of the groups did not have a third recording because they had finished their work by the end of the previous class. To simplify, throughout the remainder of this dissertation I refer to the classroom task as a "3-day" activity and when referring to a recording collected during a period on a particular day, I refer to "day 1," "day 2," or "day 3." The classroom recordings were of good quality, however it was at times difficult to hear students particularly during the height of the large-scale modelling phase of the playground activity.

Taking into account all the rounds of data collection (the Grade 10 pilot and the two Grade 9 activities), I had an estimated 2,600+ minutes of audio recordings of students' classroom interactions. I decided that I would transcribe all of the recordings, and I will discuss the transcription in detail in an upcoming section. Although all classroom-based recordings were transcribed, the final reporting in this dissertation focuses on those from the Grade 9 Planning a Playground activity, which constituted approximately 1200+ minutes of the total and provided high quality data to analyze. Despite the large amount of data I had collected from these classroom-based audio recordings alone, I decided it was important to seek out additional sources of data in order to delve deeper into the analysis of the students' interactions.

3.1.4.2 Interviews.

Although the classroom-based audio recordings of students' interactions were the main focus of the data analysis, I wanted to triangulate these data with other sources (I discuss triangulation and validity in further detail in an upcoming section). In L2 research, interviews are a common data collection tool used with a variety of qualitative (and sometimes in conjunction with quantitative) approaches, either as a main source of data or as a supplementary data source (Mackey & Gass, 2005). With regard to discourse analysis in particular, Gee (2014) suggested interviewing as an appropriate way to build validity and extend the data analysis from the main discourse source. Thus, one main purpose of conducting interviews in this study was related to the triangulation of the classroom audio data. However, the interviews also were used for many of the purposes identified by McKay (2006) as commonly associated with L2 classroom research, such as to find out more about: participants' background (e.g., students' knowledge of other languages); participants' reported behaviour (e.g., when and with whom do students use English); and/or about participants' opinions and attitudes (e.g., students' feelings about the use of particular classroom activities or the content of classroom materials). Moreover, in Swain and Lapkin's (1998) study, which was a key inspiration for my own work in terms of theoretical framework and data analysis, the authors had called for future work that combined an analysis of students' collaborative dialogue with follow-up interviews that would deepen understanding of the processes at work during students' interactions. These authors also believed that follow-up interviews could shed light on such matters as students' opinions regarding what they liked or disliked about different tasks, and the efficacy of certain tasks over others for language learning. Gass and Mackey (2000), in

reference to Swain and Lapkin's (1998) study, outlined a number of ways in which interviewing, particularly of a stimulated recall nature, could be used in order to do this. For all of these reasons, I decided to conduct follow-up interviews with each of the student pairs (or groups of three) during which I posed questions related to the mathematics activity they had worked on (using a stimulus to prompt), to doing mathematics in an L2 in general, and to any other point of interest that arose from my observations during the activities or during the interview itself.

These post hoc interviews took place 13 or 14 days following the final day (day 3) of the Planning a Playground mathematics activity. The interviews were semistructured in that I used a pre-developed "interview guide" (Creswell, 2003; Merriam, 2009; Seidman, 2006), with key questions and potential probes to guide my questioning; however, I attempted as much as possible to conduct the interviews in a conversation style and follow-up questions were often tailored on-the-spot to the specific responses and interests of the individual participants as they unfolded during the interviews themselves (Creswell, 2003; Merriam, 2009). All guiding questions were open-ended, as were follow-up probes that used techniques such as asking participants how they "felt" about something, or to "give an example." I was cognizant of avoiding leading questions as well as simple "yes" or "no" questions. (McKay, 2006; Merriam, 2009; Seidman, 2006). During my Master's research (2008), I had used phenomenological interviews as the main source of data; therefore, I had some previous experience with this style of open-ended interview. From that experience, I had also learned and attempted to put into practice another key technique of effective qualitative interviewing, which is to ensure

that participants, not the researcher, are doing most of the talking (Seidman, 2006). (See Appendix D for a copy of the interview guide.)

As mentioned previously, although the interview questions themselves were open-ended, some did draw upon a particular stimulus—previously elicited data—in order to generate participant responses (Mackey & Gass, 2005). I sometimes used the students' written work that they had submitted at the end of the mathematics activity, which I brought with me into the interviews, as a prompt to stimulate discussion. Other times, I would refer to my observation fieldnotes, using the data I had gathered during the students' interactions while working on the activity as an interview question starting point. On still other occasions, I referred to specific interactions as they occurred in the audio files and used these as a stimulus (I could not do this for every interview, since, due to time limitations, I was not able to listen to every recording prior to conducting the follow-up interviews). In this way, the interviews, while being semistructured, also had an element of stimulated recall methodology.

Stimulated recall is a well-developed methodology in and of itself, with a theoretical framework based in introspective methods and several ways in which to collect specific types of stimulated recall data aimed at addressing various research goals (see Gass & Mackey, 2000). While I do not enter into a full exploration of stimulated recall methodology and methods here, I do highlight the key ways in which it influenced parts of the interviews in this study. Interviews, including those using elements of stimulated recall, have the potential to augment analyses based in other methods. With regard to building on Swain and Lapkin's (1998) study in particular, in which they analyzed pairs of students' interactions while working on linguistic tasks, Gass and

Mackey (2000) suggested, "prompting the two learners to introspect about their thoughts during the interaction . . . using videotapes of the collaborative dialogues as the stimuli" (p. 135). This was the inspiration for the stimulated recall component of my own interviews: My goal was to better understand students' use of language and mathematics not only through my own interpretations of the recordings of their classroom interactions but also by asking them to reflect on their own data. Rather than using video (as Gass and Mackey, 2000, suggested), I used the students' written work, my observations, and excerpts of interactions taken directly from the audio recordings as the stimuli. After considering using video recordings and discussing the decision with my supervisor and members of my committee, I ultimately chose to only use audio recordings for a number of reasons. The first two and most important reasons are interrelated: I felt that the prospect of video recording might deter some students from participating and, since my data analysis did not rely expressly on visual analysis (e.g., gesture, positioning), I felt that the potential risk of deterring participants did not outweigh any perceived benefits of video. In addition, the data collection was to take place within the regular classroom and school environment and I felt that there would be a number of significant logistical challenges associated with video such as equipment setup and tear down, space limitations, recording only the participants (challenging because of the movement required for the playground activity as well as space), being only one person (I was not working with a research team and the teacher was occupied with the running of the classroom), and so on. Finally, I was engaged in multiple forms of data collection (e.g., taking fieldnotes, observing, conducting informal and formal interviews, collecting

written work) and I felt that these data sources would provide rich detail that would offset anything that might be collected via video.

With regard to the formal post-hoc interviews, the following examples (taken directly from this study) are illustrative of the kinds of questions I asked when I referred to, and thus used as a stimulus, something I had heard when listening to the audio recording of students' interactions, or to parts of the students' written work:

- "*Parfois j'ai remarqué qu'on commence à parler en anglais et je commence à m'intéresser à ça. Eve, tu as dis, « ça c'est trois quatre » et Kim a dit, « c'est trois quarts » et ensuite tu as dit, « **three quarters** ». Ensuite tu (Eve) as juste répété tout simplement en anglais, « **three quarters** ». Alors pourquoi tu as dit ça, « **three quarters** »?* ("Sometimes I noticed that you start speaking English and I'm starting to get interested in that. Eve, you said, 'that's three four" and Kim said, 'it's **three quarters**'. Then you (Eve) simply just repeated in English, '**three quarters**'. So why did you say that, '**three quarters**'?)
- "*Une question où j'ai remarqué qu'il y avait un peu de discussion, où vous n'étiez pas certaines comment répondre à la question, c'était la partie C : « Pourquoi votre plan était le meilleur choix? » Parle-moi de cette question-là.*" ("One question where I noticed that there was a bit of discussion, where you weren't sure how to respond to the question, was part C: 'Why was your plan the best choice?' Talk to me about that particular question.")
- "*Pouvez-vous regarder cette partie de votre travail? Le numéro 3a? Pouvez-vous m'expliquer comment vous avez travaillé ensemble pour répondre à ceci?*" ("Can

you look at this part of your work? Number 3a? Can you explain to me how you worked together to answer this?")

Thus I tried to capitalize on the key potential of using a stimulus during interviewing, which, at its root, is as "an attempt to explore learners' thought processes and strategies by asking learners to reflect on their thoughts after they have carried out a predetermined activity" (Gass & Mackey, 2000, pp. 37-38).

Overall, the interviews served to provide insights into students' backgrounds, experiences, and perceptions (examples of this will be shown in Chapter 4 with the presentation of the results of the study). Moreover, through the use of stimuli such as dialogue excerpts and written work, the interviews allowed student participants the opportunity to express opinions and discuss thoughts and strategies related to the specific tasks on which they had worked. The small-group setting also seemed to allow some students to have increased opportunities to express themselves in an environment that was perhaps more conducive to discussion. Finally, from my own standpoint as a classroom-based researcher, I felt the interviews allowed me to get to know the participants better, which enhanced my ability to generate insights during data analysis.

3.1.4.3 Student texts.

As mentioned in the previous sections of this chapter, all student participants produced written work as part of the playground mathematics activity. Their written work was collected so that Mme Nathalie and I could look at the students' final products. From a pedagogical standpoint, having the written work along with our own observations from the in-class activity deepened our understanding of how the student groups undertook the task. From a research standpoint, the written work provided the opportunity to trace

students' collaborative dialogues through to their final products. The written work was eventually returned to the students, however I made copies to keep for analysis purposes. Because the materials (activities and questions) were mathematical in nature and also asked students questions to which a written response would be appropriate, the student texts often comprised mathematical notation/symbols, non-mathematical words/sentences, and graphical text. For example, students wrote down numerical calculations, written responses to explain their mathematical choices and reasoning, and produced sketches and scale diagrams.

Written discourse can be the main focus of a discourse analysis (see, e.g., Gee, 2014; McKay, 2006), however, in this case, the written texts were used to supplement the audio interactions data and at times were used as a stimulus during the interviews. Because students' interactions were directly related to the mathematics task at hand, it was helpful to be able to follow the discussion topics to their "finalized" states: the written texts. For instance, when students discussed potential different solutions to a problem at length it was sometimes difficult to interpret, from the audio only, what they had decided upon as their final, "official" solution. Having the written work as a reference enabled me to track and connect their discussion through to the written product. Subsequently, I was able to use the written work to prompt students during our interviews together. The written texts allowed for a more in-depth analysis of the oral discourse, as will be elaborated in Chapter 4.

3.1.4.4 Research diary and fieldnotes.

Throughout my entire journey as a doctoral student, from the time I first enrolled in the program until present day as I write this dissertation, I have kept a "research diary" or

journal. As Holly and Altrichter (2011) explained, the research diary is an important way for qualitative researchers to reflect on the entire research process, from the initial phases of study design, through data collection, and finally to analysis. It is a place where researchers can reflect on their own research biases they may bring to the process, and capture insights and ideas related to any facet of the study as they arise. In my research diary, I have jotted down experiences, questions, reflections, and ideas at various points throughout my research journey. The diary included three physical/digital objects: a hardcover notebook, my smartphone, and an email filing system. I would bring the notebook with me to classes, conference presentations, meetings, and any event at which I thought I might need to write down something related to my doctoral work. At times, however, something would strike me at the mall, at home with my children, when travelling, or at some other time when I did not have my notebook. On these occasions, I entered my notes, fittingly, into the "notes" function on my smartphone. Finally, a substantial amount of communicating, idea sharing, questioning, and reflecting occurred over email—with my supervisor, members of my committee, other colleagues, and with Mme Nathalie, my participant teacher. As part of my research diary, I have kept all email correspondence in an organized email filing system. These three items comprising my research diary—the notebook, the smartphone notes, and the email system—have enabled me to access what I thought about and gathered along the way, as well as report accurately on timelines, dates, and statements made during personal communications. The research diary was part of my global research experience and contained notes about anything from lists of articles or books I intended to add to my literature review, to interesting potential possibilities for approaches to data coding, to questions I wanted to

remind myself to ask my supervisor. With regard to more focused note-taking, related more directly to the research project itself and especially to the data collection phase, I also used fieldnotes as an additional data source.

During the data collection, which included the school site visits, off-site meetings with the teacher participant, audio recording of students' interactions, and interviews, I also made observations by way of fieldnotes. In qualitative research, fieldnotes are "the written account of what the researcher hears, sees, experiences, and thinks in the course of collecting and reflecting on the data" (Bogden & Knopp Biklen, 2007, pp. 118-119). During and following the various data collection episodes, I wrote fieldnotes either by jotting them down on loose-leaf paper I had with me during my visits to the classroom or entering the notes into my smartphone. I then filed the handwritten notes in a three-ring binder or typed them, and the phone notes, into a Microsoft Word document, which I then printed, and inserted into the binder. As with interviewing and text analysis, Bogden and Knopp Biklen (2007) have pointed out that fieldnotes can be the main source of data collection in a qualitative study, particularly in those that are based on direct participant observation, or they can be used as an "important supplement to other data collecting methods" (p. 119). In this study, the fieldnotes, like the interviews and the written texts, are used as supplemental data sources (to the interaction audio recordings), in order to augment and triangulate the main discourse analysis. Fieldnotes can be both descriptive and reflective (Bogden & Knopp Biklen, 2007; Merriam, 2009), and mine contained elements of both genres. In terms of descriptive notes, mine included some of the commonly seen elements such as: reconstruction of dialogue, description of physical setting, accounts of particular events, depiction of activities, and description of my own

behaviour (Bogden & Knopp Biklen, 2007). My short reflections are also scattered throughout the descriptive fieldnotes, or I sometimes included more lengthy reflections, often referred to as "memos'" (Bogden & Knopp Biklen, 2007; Merriam, 2009). These more reflective pieces address a variety of typical issues, such as: reflections on analysis reflections on method, reflections on ethical dilemmas, and reflections on my own frame of mind (Bogden & Knopp Biklen, 2007).

More specifically, Bogden and Knopp Biklen (2007) have remarked that fieldnotes "[help] the researcher to keep track of the development of the project, to visualize how the research plan has been affected by the data collected, and to remain aware of how he or she has been influenced by the data" (p. 199). This aspect of fieldnote-taking became of utmost importance in this study, as these notes helped me reflect on the materials used during the various rounds of data collection and led to the eventual development of new materials (Planning a Playground activity). The timelines, meeting dates, and on-site data collection dates were all also clearly laid out in the fieldnotes. These detailed notes greatly facilitated the final reporting in this dissertation on the development of the materials and the unfolding of the data collection. Furthermore, the fieldnotes also provided me with data regarding the classroom environment (the physical layout; the participants' and my own behaviours; my impressions of the experiences; conversations between me and the teacher, me and the students, or the teacher and the students; etc.) and thus were an invaluable resource during the writing of the Data Collection section. Indeed, Merriam (2009) has described how qualitative fieldnotes become memos that become data, and these are often used when writing about the study's methodology, in particular.

The fieldnotes, written student texts, and interviews all provided data that supplemented the main source of data for this study, which was the audio recordings of students' communication while they worked on the mathematics problem-solving tasks in the classroom. These audio data, however, had to be put into a usable form in order to become part of the written data analysis. Consequently, in addition to the all-important data collection that occurred during the course of this study, the transcription of the audio data also represented a significant undertaking.

3.1.5 Transcription

As described in the previous sections on data collection, both the students' classroom interactions and their interviews were audio recorded. In order to present a written analysis of these audio data, I needed to transcribe them into written form. Faced with the extensive amount of data I had collected, I thought about the various possibilities for accomplishing what, at first, seemed like an impossible task. Two initial key decisions to be made were whether or not I would transcribe all of the data and whether or not I would do it myself. Because the recordings of students' classroom interactions were my main source of data and the focus of the research, I decided to transcribe these recordings myself and I transcribed all recordings in their entirety. Although it is recognized (e.g., Poland, 2002) that it can be challenging to do so I wanted to be sure to try to capture in the written transcripts some of the elements present in the audio, such as hesitations, pauses, back channelling, tone of voice, laughter, sighs, and so on, where such characteristics of the participants' discourse seemed meaningful. Despite making an effort to do this, it is important to note that scholars have argued that transcripts can never truly capture all that can be conveyed through speech and thus they have suggested that

the recordings themselves, not the transcripts, be considered the raw data (e.g., McKay, 2006). I tend to agree with this, at least to some extent, as one need only to look at the detailed transcripts produced in conversation analysis studies as proof of how difficult it is to thoroughly indicate, in a transcript, what is going on during spoken interaction (e.g., Peräkylä, 2005).

This leads to a related, important reason that I decided to perform the transcription of the interaction recordings myself: In many ways, the act of transcribing, perhaps especially in the case of a discourse analysis, actually marks the beginning of the data analysis itself. As Gee (2014) explained:

> A discourse analysis is not based on all the physical features present, not even all those that might, in some conceivable context, be meaningful, or might be meaningful in analyses with different purposes. Such judgments of relevance (what goes into a transcript and what does not) are ultimately theoretical judgments, that is, they are based on the analyst's theories of how language, contexts, and interactions work in general and the specific context being analyzed. In this sense, a transcript is a theoretical entity. It does not stand outside an analysis, but, rather, is part of it. (p. 136)

Depending on the methodology and the research focus then, transcripts range from being extremely detailed (narrow) to much less detailed (broad); any of the approaches along the spectrum is acceptable. Transcripts can show as much or as little detail necessary in order to achieve the goals of a particular study, and these decisions are made by researchers as part of their interpretations of the data (Gee, 2014; Mackey & Gass, 2005). Gee (2014) has recommended that for discourse analysis, researchers might choose to

transcribe with more detail, that is, more narrowly, than they feel is needed at least in the beginning, since it may be less clear at that point what will ultimately prove of interest in the analysis. Transcripts can always be broadened afterward and should be if the researcher sees fit, as too much unnecessary detail can prove distracting in an analysis that in the end, for example, focuses on larger themes rather than the micro-details of a particular interaction.

Finally, I found that it was important to transcribe the classroom-based interactions myself for the simple fact that they were at times quite challenging to decipher. As I will discuss in further detail in a subsequent Data Analysis section in this chapter, "real" classroom discourse, especially students' interactions with each other, can be quite chaotic. In my study for example, French, English, and mathematical terms were all being used; the students were engaged in a problem-solving mathematics task in their "real" classroom therefore their interactions were thoroughly unscripted and oftentimes quite disorderly (i.e., a lot of cutoffs and overlapping speech); the environment was noisy; there was a lot of physical movement during the modelling phase of the playground activity in particular; and finally, it would be difficult to know who was doing the talking unless one had known the students (and thus their voices) fairly well, as I had. Despite my familiarity with the students, the mathematics tasks and terminology, and the research context in general, I found the transcription of the classroom activities required significant effort. I was worried that passing these recordings along to another transcriber, no matter how skilled, would have posed quite a challenge.

In order to have some common understanding, transcription conventions have been developed that one can use to convey different features of spoken interaction. There

is some variability in the notions used, therefore it is important to provide an explanation for the transcription codes used in any given study. For my own work, I consulted primarily resources from the field of L2 research (e.g., Mackey & Gass, 2005; McKay, 2006) to develop a transcript scheme. To that end, in this study I use the following transcript conventions for the interaction data as shown in Table 5:

Table 5

Transcript Conventions

Symbol	Meaning
.	A period at the end of a word, phrase, or clause means falling intonation (statement).
?	A question mark at the end of a word, phrase, or clause means rising intonation (question or expressing uncertainty).
!	An exclamation mark at the end of a word, phrase, or clause means an exclamation or excitement, indicated by rising volume.
,	A comma indicates continuing intonation.
:	A colon indicates a lengthened sound or syllable. More colons indicate a more prolonged sound.
-	A hyphen after an initial sound means a cutoff or false start.
. . .	Three dots indicate a noticeable pause.
<u>Word</u>	Underlining indicates some form of stress on the underlined word(s).
Word	Bold indicates word(s) uttered in the L1 (English).
[ ]	Square brackets indicate overlapping speech.
"word"	Words in quotation marks indicate citing oneself or other(s), or reading.

Symbol	Meaning
(?)	A question mark in parentheses indicates incomprehensible word(s).
(Word)	Word(s) in parentheses indicate that the transcriber is not certain of hearing the word(s) correctly.
(( ))	Double parentheses indicate transcriber's comments.

I found these transcript conventions to be intuitive and able to communicate an appropriate level of detail for the purposes of my analysis of the students' classroom-based interactions. Most of the transcription was completed during a concentrated period of time over the fall of 2016 (some had been done prior to that).

With regard to the interview data, which were collected as a means to supplement the main interaction data, the transcribing was done differently. The two main differences were that someone other than me transcribed the interviews and the transcription was done more broadly than for the interaction data. The same transcription conventions hold for the interview data, however some phenomena occur markedly less often in the interviews (e.g., cutoffs and overlapping speech) and overall the focus was on the broader discourse for the analysis of the interview data. Thus, the interview transcripts are much broader, relative to the interaction transcripts. After having transcribed one interview myself, I decided to have someone else transcribe the remainder largely due to time constraints. I would have preferred to transcribe all the interviews myself for the same reasons as described earlier, however the time simply did not permit. I am grateful to have been able to work with a transcriber whom I knew and who did high quality work. Even though I was not transcribing the interviews myself, I was able to touch base often

with the transcriber to see how things were going and, in turn, the transcriber was able to ask me at any point along the way to clarify things being heard on the interview recordings (e.g., inaudible or incomprehensible speech, questions about mathematical terms, questions about who I thought was speaking in cases where that had gotten difficult to follow). Overall, the transcriber and I agreed that the interviews proved less challenging to transcribe as they proceeded in a relatively "orderly" fashion (i.e., generally only one person was speaking at a time and the environment was less chaotic and noisy).

Thus the transcription process, along with the ongoing fieldnotes taken during data collection, marked the very beginning of the data analysis process. In the next section, I describe in detail the theoretical and methodological approach to the complete data analysis.

3.2 Data Analysis

Lee and Petersen (2011) have noted that social analysis, in general, has become increasingly interested in discourse. This reflects an epistemological shift, one that questions the nature of knowledge and aims to explore the increasingly textualized world. These authors suggested that, partly due to this increased interest, approaches to discourse analysis are growing and varied. Nonetheless, they outlined three major theoretical traditions guiding the field that I find useful for an initial framing of my methodological approach: (a) critical theory tradition; (b) linguistics; and (c) socially-oriented frameworks emerging from within sociology, psychology, and English studies. In addition to the three main branches identified by Lee and Petersen, Peräkylä (2005) has identified a fourth theoretical tradition, (d) historical discourse analysis. With such

far-reaching roots, the focus of discourse analysis can look different from one tradition to the next. For example, critical discourse analysis tends to explore "ways in which texts of different kinds reproduce power and inequalities in society" (Peräkylä, 2005, p. 871). In linguistics, researchers' overall aims often involve "uncovering the features of text that maintain coherence in units larger than the sentence" (Peräkylä, 2005, p. 871). On the other hand, social psychology (also referred to as discursive psychology), an example from Lee and Petersen's third framework, "involves research in which the language use . . . underpinning mental realities, such as cognition and emotion, is investigated" (Peräkylä, 2005, p. 871). In historical discourse analysis, rather, "constitution of subjects and objects is explored in historical context—or, in Foucault's terms, through *archaeology* and *genealogy*" (Peräkylä, 2005, p. 871).

My own work falls mainly within the second theoretical tradition noted above: linguistics. However, the field of "linguistics" itself is broad and varied, as are the numerous approaches to the study of language within it. This study is situated firmly within applied linguistics (rather than "pure" linguistics), that is, linguistics as applied in the real-life context of education. In general, the American Association for Applied Linguistics (AAAL) defined applied linguistics on their website (n.d.) as:

> an interdisciplinary field of inquiry that addresses a broad range of language-related issues in order to understand their roles in the lives of individuals and conditions in society. It draws on a wide range of theoretical and methodological approaches from various disciplines—from the humanities to the social and natural sciences—as it develops its own knowledge-base about language, its users

and uses, and their underlying social and material conditions. ("About Applied Linguistics" section)

In this vein, my study, broadly speaking, explores applied linguistics in education through a focus on *language in use*, which is language anchored in the social, contextualized unfolding of classroom interactions. The study investigates the *functional* nature of linguistics through language in oral interaction that is, furthermore, grounded in an educational context. This could be contrasted with, for example, analyses of smaller linguistic units often characteristic of pure linguistic analyses of a more *formal* or *structural* nature (I discuss these italicized terms in more detail in the next sections).

Aiming to provide an organizing structure with which to work, Schiffrin (1994), described six different approaches to discourse analysis, all based in the linguistics field: speech act theory, interactional sociolinguistics, ethnography of communication, pragmatics, conversation analysis, and variation analysis. For the purposes of my research, I draw mainly on ideas taken from pragmatics. Even the term *pragmatics*, however, is broad, thus defining it can be challenging and elusive: pragmatics "deals with three concepts (meaning, context, communication) that are themselves extremely vast and unwieldy" (Schiffrin, 1994, p. 191). Moreover, Gee (2014), on whose work I draw heavily in order to frame my own discourse analysis methodology, simply uses the term "*discourse analysis*" to mean what is elsewhere often termed *pragmatics*. For Gee (2014), discourse analysis can refer to the more formal study of language, that is, the study "of the connections among and across sentences as they follow one after the other" (p. 20); it can also refer to the study "of language-in-use in specific contexts" (p. 20) or the more functional study of language, in contexts ranging from the micro to the macro. In this

dissertation, I follow Gee's (2011, 2014) lead and adopt the term *discourse analysis* to describe my particular methodological approach. In the next sections, I elaborate on how my operationalized definition of *discourse* weaves through my guiding discourse analysis methodology.

3.2.1 Approaches to Discourse Analysis

Prior to describing the discourse methodology for this study in more detail, it is useful to first reflect more in-depth on a definition of *discourse*. I begin with a comparison of the two broader paradigms that often frame views of discourse: *formal* (also referred to as *structural*) and *functional*, which represent the distinction between the *form* and *function* of language (McCarthy, 1991). These overarching linguistic paradigms represent different views of language and thus different orientations for defining discourse. Broadly speaking, there are four ways in which the two views differ:

1 Formalists (e.g., Chomsky) tend to regard language primarily as a mental phenomenon. Functionalists (e.g., Halliday) tend to regard it primarily as a societal (social) phenomenon.

2 Formalists tend to explain linguistic universals as deriving from a common genetic linguistic inheritance of the human species. Functionalists tend to explain them as deriving from the universality of the uses to which language is put in human society.

3 Formalists are inclined to explain children's acquisition of language in terms of a built-in human capacity to learn language. Functionalists are inclined to explain it in terms of the development of the child's communicative needs and abilities in society.

> 4 Above all, formalists study language as an autonomous system, whereas functionalists study it in relation to its social function. (Leech, 1983, cited in Schiffrin, 1994, pp. 21-22)

Functionalism, with its emphasis on the social aspects of language and its uses and functions in communication, fits more closely with my theoretical framework. Halliday (e.g., 1978, 1985, 2009), who is arguably the founder of functionalism and whose work played an integral role in informing my theoretical framework in Chapter 2 (e.g., Halliday, 1978, *Language as Social Semiotic*), resurfaces here, creating ties with the foundation of my discourse analysis methodology (described in finer detail in the next sections of this chapter). Moreover, the sociocultural view of language as a mediational, meaning making tool, that is used during communication such as collaborative dialogue in classroom interaction (e.g., Swain, 2000; Vygotsky, 1978), provides the underpinning for viewing language in a way that holds context and language-in-use to be important. The link between the functionalist-linguistic and sociocultural paradigms is clear.

So, definitions of discourse are influenced by the broader language paradigm to which a researcher ascribes. Formalists mainly work with the classic definition of discourse as "language above the sentence or above the clause" (Stubbs, 1983, cited in Schiffrin, 1994, p. 23). Working from this definition, analyses of discourse might entail a "focus on the way different units function in relation *to each other*" (Schiffrin, 1994, p. 23, original emphasis), or, in Gee's (2014) words, a study of "the connections among and across sentences as they follow one after the other" (p. 20).

On the other hand, definitions of discourse that are more compatible with functionalist views—discourse as language-in-use, in other words, "language actually

used in specific contexts" (Gee, 2014, p. 19)—require different approaches to discourse analysis. According to this view, when it comes to language-in-use, language cannot (or at least should not) be separated from its social, communicative functions and contexts. Thus, with regard to an analysis of discourse as language-in-use, Brown and Yule (cited in Schiffrin, 1994) offered the following remarks: "it [discourse analysis] cannot be restricted to the description of linguistic forms independent of the purposes or functions which these forms are designed to serve in human affairs" (p. 31).

Those informed mainly by the functionalist paradigm seem unanimous in their assertion that the study of language-in-use, that is, how language serves communicative (and other) functions in specific contexts, is key. However, even within this overarching framework there are a variety of approaches to conducting this type of discourse analysis. According to Schiffrin (1994), most fall within one of two schemes: etic or emic. Coming from the etic direction, researchers focus on matching particular units (e.g., utterances or actions) to a predetermined list of functions served by language. Alternatively, from an emic standpoint, researchers might begin with how particular units (e.g., utterances or actions) are used and then, based on analysis, interpret or infer the functions of these units (Schiffrin, 1994). Of course, any particular analysis might use a combination of both of these techniques. These kinds of discourse analyses, which often draw heavily from Halliday's (e.g., 1985, 2009) seminal works on functional grammar and *systemic functional linguistics*, contribute valuable information to the field in terms of understanding how grammar functions in communication and social interaction and thus, represent an important consideration and starting point for the present study.

Seeking to take my own discourse analysis beyond functional grammar and to delve deeper into a discourse that is contextualized within the communication of the classroom and, more specifically, the secondary FI mathematics classroom, I began to explore complimentary and more appropriate ways of extending my analysis. I sought out a theory and method of discourse analysis that would best help me answer my research questions. This search brought me to Gee (2011, 2014), who had the following to say about discourse analysis:

> I assume any discourse analysis is aware of (and honours) the general form-function correlations that exist in the language being analyzed. In some cases, form-function analysis is all we may do, and such analyses can be informative and important. However, most often the real action of discourse analysis, where it has its biggest bite, is at the level of analyzing situated (context specific) meanings. (Gee, 2014, p. 83)

Prior to closing this section on definitions of discourse and, by extension, discourse analysis, it is also useful to address the important distinction between *discourse* and *Discourse*, that is, little "d" discourse and big "D" Discourse. Here, I have already discussed ways in which one might define little "d" discourse. Big "D" Discourse, on the other hand, includes little "d" discourse, but also takes into account much broader ideas of what goes into peoples' use of language. Gee (2014) described Discourse as "ways of thinking, believing, valuing, and using various symbols, tools, and objects to enact a particular sort of socially recognizable identity" (p. 222). When Discourses are at play during interactions between people, as they always are, "it is as if socially significant forms of life (identities), formed in history via social work, talk to each other—continue a

long-running conversation they have been having" (Gee, 2014, p. 25). In my study, I am cognizant of the Discourses that may be at play during students' interactions in the classroom, such as the Discourses of being: a student; a "good" or "bad" mathematics student; a French immersion student; a friend, boy, girl, son, daughter, and so on; a hockey player, basketball player, non-athlete, rebel, and so on. There are many Discourses influencing a person's language-in-use and interacting and intersecting with each other at any given time. Gee (2011, 2014) presented "The Big 'D' Discourse Tool" as one way (among many) that researchers can approach their main discourse analysis (the other tools will be discussed in an upcoming section). In this study, however, my primary concern is with the analysis of students' discourse (that is, little "d" discourse), through the use of a series of other discourse analysis tools and coding more pertinent to my work. However, I keep in mind the Discourses potentially at play throughout my analysis, as these can help situate the discourses and provide context and background. From this point forward, my use of the term *discourse* should be taken to mean little "d" discourse, unless otherwise indicated.

Moving forward with these definitions of discourse and some foundational ideas about discourse analysis in mind, in the next sections, I outline in finer detail the theory and method for discourse analysis developed by Gee (2011, 2014) that allowed me to analyze my data in light of their particular "situated meaning" in the secondary FI mathematics classroom.

3.2.2 A Theory and Method of Discourse Analysis: Language as Saying, Doing, Being

The discourse analysis methodology used in this study is based on Gee's (2011, 2014) theories and methods. Because definitions of both discourse and discourse analysis are so varied and this type of analysis is carried out in such a broad range of fields, it is recommended by many foundational texts on discourse analysis that researchers should adopt a clear theoretical focus and guiding methodology (e.g., Gee, 2014; Lee & Petersen, 2011; Schiffrin, 1994). These scholars have argued that there is no singular "right way" to conduct discourse analysis. Rather, the matter is one of choosing the most appropriate theory and method based on one's theoretical framework and research interests. As Gee (2014) explained, "there are many different approaches to discourse analysis, none of them, including this one, uniquely 'right'. Different approaches fit different issues and questions better or worse than others" (pp. 10-11). Thus, while Gee's (2014) approach does not qualify as a uniquely "right" choice, I chose it, after careful consideration, for a number of reasons.

I first became interested in Gee's approach when reading studies for my literature review (e.g., Moschkovich 2003, 2007) that examined mathematics discourse via the intertwined and complex nature of language and mathematics in bilingual classrooms, an approach that aligned closely with my own research aims (as described in my theoretical framework). Seeing Gee's methodology of discourse analysis applied to studies by renowned scholars that I was using to inform my own work was a good starting point for exploring the applicability of this approach in my own research. Gee's (2014) overall view of language as not just saying things, but also doing things and being things, fits

well with the theoretical frameworks of SCT and of the mathematics education register that were established at the outset of this study in Chapter 2. I needed discourse analysis theory and methods that would allow me to examine the specialized language of interaction in the L2 mathematics classroom, and viewing language as "saying, doing, and being," and analyzing it as such, met this need well. Moreover, Gee (2014) pointed out that language exists beyond the definitions of words, and that words and utterances take their meanings when they are situated in practice (such as, for example, the practice of making meaning in the L2 and/or mathematics classroom), something he refers to as "situated meaning." It is worth noting that this concept is not unique to Gee's (2011, 2014) work (Gee readily acknowledges that this and other ideas in his work are informed and influenced by many other scholars in the field), and other discourse analysis literature also has underscored the importance of situated meaning, particularly that which has its roots in interactional sociolinguistics. As Schiffrin (1994) commented, "implicit in this procedure was a belief that it is the contextualization of an utterance that motivates its use: the contexts in which an utterance occurs explain why it occurs there" (p. 132).

The flexibility inherent in Gee's (2014) approach is a second reason that I chose it. The approach draws from a range of perspectives, including importantly education, and has encouraged researchers to adapt the theory and method to their own purposes. The discourse analysis tools presented in Gee's (2014) approach

> are not meant to be rigid definitions. Rather, they are meant to be "thinking devices" that guide inquiry in regard to specific sorts of data and specific sorts of issues and questions. They are meant to be adapted for the reader's own purposes. They are meant, as well, to be transformed as the reader adapts them to his or her

> own theory of the domain. Of course, if the reader's theory gets too far away from my theory of the domain, the tools will become less and less easily or sensible adaptable and useful. (p. 12)

Thus without straying too far from Gee's guiding methodology, I felt that these kinds of discourse analysis tools could be adapted well for exploring the multifaceted nature of my own research context and needs.

Finally, a third reason was that Gee's (2014) approach has centred on language as used in meaning making in social practices (under the guiding principle of language-in-use) and thus, any discourse analysis using this approach carries with it implications for social practices (e.g., what goes on in the classroom). As Gee (2014) put it: "For me, discourse analysis must have a 'point' . . . [and] have relevance to 'applied' issues" (p. 12). In this study, my aim is to take the information gleaned from the data analysis and use it to respond to my research questions, which are expressly interested in the nature of students' communication in FI mathematics. In turn, the answers to these research questions should have implications for the teaching and learning of both language and content in high school FI mathematics classrooms and perhaps other bilingual mathematics classroom contexts.

3.2.2.1 The building tasks of language and tools of inquiry.

Gee's (2014) discourse analysis theory and methods comprise a number of key ideas that underpin the approach and helped prepare me to undertake the analysis of my data. Gee (2014) has suggested that when we speak (or write, for that matter), we are building seven areas of our reality; these are the seven "building tasks" of language and each

carries with it a key question researchers can ask themselves as they analyze any piece of discourse. The building tasks and their corresponding questions are as follows:

1. Significance: How is this piece of language being used to make certain things significant or not and in what ways?
2. Practices (Activities): What practice (activity) or practices (activities) is this piece of language being used to enact (i.e., get others to recognize as going on)?
3. Identities: What identity or identities is this piece of language being used to enact (i.e., get others to recognize as operative)? What identity or identities is this piece of language attributing to others, and how does this help the speaker or writer enact his or [*sic*] own identity?
4. Relationships: What sort of relationship or relationships is this piece of language seeking to enact with others (present or not)?
5. Politics (the distribution of social goods): What perspective on social goods is this piece of language communicating (i.e., what is being communicated as to what is taken to be "normal," "right," "good," "correct," "proper," "appropriate," "valuable," "the ways things are," the way things ought to be," "high status or low status," "like me or not like me," and so forth)?
6. Connections: How does this piece of language connect or disconnect things; how does it make one thing relevant or irrelevant to another?
7. Sign Systems and Knowledge: How does this piece of language privilege or disprivilege specific sign systems (e.g., Spanish vs. English, technical language vs. everyday language, words vs. images, words vs. equations) or different ways of knowing and believing or claims to knowledge and belief (e.g., science vs. the

humanities; science vs. “commonsense”; biology vs. “creation science”)? (Gee, 2014, pp. 32-36)

And so when researchers prepare to think about and formally analyze any piece of discourse, they can use these guiding questions to think about the building tasks of language and what the tasks are accomplishing in various instances of language-in-use. Moreover, Gee (2011, 2014) introduced readers to a total of 27 discourse analysis “tools,” which he also refers to as “thinking devices.” These tools are ways of asking more specific questions about discourse data that allow researchers to examine more closely the details of any given text. They represent a further focusing, a further narrowing of the research focus and of how to carry out the data analysis in practice. Some tools are more suitable to exploring certain building tasks over others, but, in principle, any of the 27 tools could be applied to exploring any of the seven building tasks and several could be explored simultaneously, which gives researchers a vast array of choices with regard to how to approach their analyses.

At this point in my reading, contemplating the number of ways in which I might analyze my own data, I became overwhelmed, in fact, by the seemingly monumental task ahead of me that was “discourse analysis.” However, as I read on past the chapters that outlined the tools of inquiry, I soon took comfort in Gee’s (2014) subsequent discussion of the elusive notion of an “ideal” discourse analysis:

> Asking and answering [all of] these . . . questions about any one piece of data would lead to a very long analysis indeed. But that is what would constitute a “full” or “ideal” discourse analysis. For the most part, any real discourse analysis only deals with some of the questions. . . . develop[s] in detail only a small part of

> the full picture. However, any discourse analysis needs, at least, to give some consideration, if only as background, to the whole picture. (pp. 140-141)

Thus, I began to tease out the building tasks of language and tools of inquiry that I felt would best guide me toward exploring my own research questions. In so doing, I acknowledge the inherent limitations of leaving some things underdeveloped, while underscoring the advantages of having a focused analysis that best investigates what is relevant to my current study.

3.2.2.2 Practices (activities), sign systems and knowledge, and tools.

Like any theory or theoretical model, it is often difficult to categorize and separate the components; in an extremely complex exercise such as discourse analysis, it is arguably nearly impossible to separate the parts from the whole. That being said, as previously discussed it is useful to try to narrow things down to those elements that might best help explore a given research problem at hand. Looking at my own research questions, I found that the "Practices (Activies)" building task of language was very relevant. Gee (2014) described this building task as "a socially recognized and institutionally or culturally supported endeavour that usually involves sequencing or combining actions in certain specified ways" (p. 32). Thus, this building task is a good one to frame a discourse analysis, such as this one, that is concerned with students' communication during their interactions while engaging in mathematical classroom tasks. A second useful building task to guide my analysis is "Sign Systems and Knowledge," particularly as it pertains to questions of which language (in this case, English or French) is being used and for what potential purpose. Furthermore, this task helps focus the analysis on instances of language-in-use that can be very important for the mathematics classroom, for example,

technical language versus so-called everyday language, words versus images, and words versus equations (Gee, 2014).

In order to explore my research questions via these building tasks of language, I next turned to choosing appropriate tools of inquiry among the 27 outlined in Gee (2011, 2014). To reiterate, any and all of the 27 tools could be applied to any and all of the seven building tasks of language (and in an "ideal" discourse analysis, one would do just that), but a more realistic goal in practice is to choose those tools that will help best explore the research topic at hand. Therefore, the tools of inquiry I have chosen that guide my data analysis include:

- Tool #7: The Doing and Not Just Saying Tool: For any communication, ask not just what the speaker is saying, but what he or she is trying to do, keeping in mind that he or she may be trying to do more than one thing.
- Tool #9: The Why This Way and Not That Way Tool: For any communication, ask why the speaker built and designed with grammar in the way in which he or she did and not in some other way. Always ask how else this could have been said and what the speaker was trying to mean and do by saying it the way in which he or she did, and not in other ways.
- Tool #21: The Sign Systems and Knowledge Building Tool: For any communication, ask how the words and grammar being used privilege or de-privilege specific sign systems (e.g., Spanish vs. English, technical language vs. everyday language, words vs. images, words vs. equations) or different ways of knowing and believing, or claims to knowledge and belief.

- Tool #23: The Situated Meaning Tool: For any communication, ask of words and phrases what situated meanings they have. That is, what specific meanings do listeners have to attribute to these words and phrases given the context and how the context is construed? (Gee, 2011, pp. 195-201).

To clarify, some tools of inquiry have a name that is very similar or identical to one of the seven building tasks of language (e.g., Tool #21: "The Sign Systems and Knowledge Building Tool" and Building Task #7: "Sign Systems and Knowledge"). Others' names stem from sub-branches of one of the seven building tasks and thus have their own unique names.

Gee's (2011, 2014) theory and methods of discourse analysis and their accompanying toolkit provided an important foundation on which to build my data analysis. They provided the guiding lenses, largely in the form of guiding questions, through which I explored my own, more specific, research questions. Gee's (2014) approach has encouraged researchers to also use the tools of inquiry to "engage in their own discourse-related reflections" (p. 144). Therefore, in turning now to my specific research questions and the needs of this particular study, I outline the finer details of my discourse analysis including the units of analysis and the coding schemes. These analysis tools helped me to engage in my own "discourse-related reflections," which in turn helped me to interpret the results of the analysis.

3.2.3 Units of Analysis

As discussed in previous sections of this chapter, *language*, *discourse*, and *discourse analysis* are terms that are difficult to define. There is no real singular, reductive definition; rather, there are many different ways of conceptualizing and

operationalizing these terms—none of which are categorically "right" or "wrong." The analysis we ultimately undertake as researchers depends on our own operating views of these terms and our own research interests. The "unit of analysis" on which a discourse analysis researcher decides to focus depends on how best the research questions would be answered—again, there is no "right" or "wrong" unit of analysis. Furthermore, the researcher must take into account what kinds of discourse are under analysis: Do the data stem from written texts? If so, what kind of texts (e.g., textbooks, personal letters, emails)? Or, do the data represent spoken discourse? If so, what kind of spoken discourse (e.g., interview, interaction)? With regard to units of analysis, a highly linguistic analysis of discourse might focus, for example, on a given data set at the level of the word or, taking an even smaller unit of analysis, morphemes. Other analyses, perhaps still purely linguistic or those moving into realms of applied linguistics (as applied to education, for instance), may analyze discourse by chunking and organizing it into larger units such as phrases, clauses, sentences, microlines, macrolines, stanzas, utterances, or turns, and so on (Gee, 2014; McCarthy, 1991; Schiffrin, 1994). In order to decide how I might best organize and analyze my own discourse data that is, how to determine what my unit of analysis might be, and having established my guiding theory and method for analysis (Gee, 2011, 2014), I turned to studies of discourse analysis that were tailored to contexts that were the most similar to my own—L2 and/or mathematics classroom-based research—for some ideas.

I began by reading about research approaches to discourse analysis in L2 classrooms in general. Within McKay's (2006) chapter on researching L2 classroom discourse, various definitions of discourse and approaches to analysis, taken from several

different fields of research, were offered as an overview. I found that the definitions of discourse and the approaches to discourse analysis that drew from fields related to L2 classroom-based research fit well with my own previously established definitions of discourse: "discourse analysis is concerned with the study of the relationship between language and the contexts in which it is used" (McCarthy, 1991, p. 5); "[discourse analysis is] the study of language in use that extends beyond sentence boundaries" (Celce-Murcia & Olshtain, 2000, p. 4). As these definitions fit nicely with my theoretical frameworks for the study (SCT and the mathematics education register) and with the discourse analysis theory and methods that I had previously established, I was confident that work coming from this field would inform my own with regard to appropriate units of analysis.

I looked for ideas based in the L2 classroom examples about how best to choose my unit of analysis considering that I was analyzing classroom-based student interactions. Within the classroom, there are many different kinds of "talk"—teacher talk, student talk, teacher-student interaction, student-student interaction, and so on. As McCarthy (1991) has explained, some of this talk follows fairly rigid conventions deeply entrenched in typical classroom discourse/Discourse structures, while other types of talk are more fluid:

> Because of the rigid conventions of situations such as teacher talk . . . it is relatively easy to predict who will speak when, who will ask and who will answer, who will interrupt, who will open and close the talk, and so on. But where talk is more casual, and among equals, everyone will have a part to play in controlling and monitoring the discourse, and the picture will look considerably more complicated. (pp. 22-23)

My data consisted of this more "informal" type of talk, between students as they work together, and, as such, were, to borrow McCarthy's word, "complicated." I use the term *informal* carefully, since I acknowledge that all classroom talk follows a certain classroom discourse/Discourse protocol to some extent, thus it follows certain "rules" and therefore might always be considered "formal" to at least some extent. However, the talk between students, even working together in a classroom context, represents the more "casual" talk of the type McCarthy is referring to above (as compared to, for example, more structured teacher-student exchanges).

As discussed in McCarthy (1991), in examining more natural oral interactions, it is useful to reflect on *turn taking*, or *the turn*, as this is one of the key characteristics of this type of discourse. Similarly, McKay (2006) suggested that, "the researcher needs to characterize the actions in the sequence [of the interaction] on a turn-by-turn basis by considering what participants are doing in each turn" (p. 104). As speakers interact, there are linguistic mechanisms for organizing turn taking. For example, speakers take turns when they are selected by the current speaker (*nominated*), or, if no one is selected a speaker may speak of their own accord (*self-selection*). Alternatively, if neither of these happens, the current speaker may continue. There are linguistic devices used for interrupting a turn, with varying degrees of formality and politeness depending on the situation. Furthermore, there are linguistic means for not taking a turn when nominated, which is usually referred to as *back-channelling* (e.g., *mm*, *ah-ha*, *yeah*, *no*, *right*, *sure*). Finally, in turn taking speakers may predict one another's utterances and either complete them or overlap with them. All of this makes for quite a complex undertaking in analyzing natural interactional discourse. As McCarthy has pointed out, "natural

conversation data can often seem chaotic because of back-channel, utterance-completions and overlaps" (p. 127).

My main source of data is oral interactions, during which: (a) there is a sequence of talk and actions; (b) students engage in turn taking; and (c) there are back-chanellings, interruptions, completions, and overlaps. Consequently, the turn seemed an appropriate way to organize, transcribe, and analyze the data. Furthermore, for the purposes of my research questions, organizing the transcripts by participant turns provided an appropriate enough detail in order to achieve the goals of the analysis, while not unnecessarily cluttering up the transcripts with details that would not be pertinent to this particular analysis.

With a plan to focus on turns and the theory and methods of the discourse analysis in place, I returned to the studies from my literature review and explored more closely the data analysis approaches of those that focused on research that was most relevant to my own study. This helped me further refine and focus my discourse analysis, and helped me to elicit salient details from the large amount of data; in other words, my next steps were to develop a scheme for coding the data.

3.2.4 Coding the Data

While working within Gee's (2011, 2014) discourse analysis methodology and using the tools of inquiry related to this approach, I found I needed to refine my analysis in order to tease out data that would be the most relevant to my study. Thinking about how to code my data enabled me to explore ways to reflect on the key questions related to Gee's (2011, 2014) tools of inquiry while simultaneously responding to my more specific research questions related to the study at hand. As will be shown in Chapter 4

with the presentation of the results, I analyzed the data first using the coding frameworks. This analysis enabled me to identify reoccurring and salient themes. Next, I conducted an in-depth analysis of the themes using Gee's tools of inquiry. In other words, for a given instance coded, I engaged in a discourse-related reflection (analysis) by asking myself the questions provided in association with the tools of inquiry. My "responses" to these tools of inquiry questions form the in-depth qualitative narrative analysis that accompanies each of the coding tables.

In this section, I describe how I established and used the coding frameworks. In order to proceed with an analysis that was tailored to my particular research context, I decided to use a combination of a priori codes, which were based on relevant studies from L2 language- and mathematics-classroom-based discourse analysis research, and emergent codes that were established through an inductive approach. Appendix E shows the a priori codes (i.e., the initial codes that were drawn from the literature) with which I began analyzing my data). The finalized coding frameworks used (i.e., the coding frameworks that reflect the changes I made to the initial codes based on emergent findings from the first iterations of data analysis) are shown and explained in detail in the next three sections of this chapter (3.2.4.1, 3.2.4.2, and 3.2.4.3).

3.2.4.1 Coding language-related episodes.

Swain and Lapkin's (1998, 2000) work is very relevant to my own study (in terms of research interests, theoretical framework, and literature review); therefore, I drew from it in order to develop ideas for coding my data. These studies are pertinent to my own due to their SCT framework, their discourse analysis methodology, their similar context (participants were Grade 8 early FI students), their focus on language use in student

collaborative dialogue, and their exploration of student L1 use. Based in a Vygotskian SCT framework, Swain and Lapkin's 1998 study explored the collaborative dialogue that occurred between two adolescent FI students as they worked on two different types of linguistic problem-solving tasks. The authors described the SCT-based view of language that underpinned their study as follows:

> Language becomes a mediating tool by its first having been used by others in order to regulate behaviour, including cognitive behaviour. Through a gradual process of internalization, one comes to be able to use the language of others (and the mental processes that interaction has constructed) to regulate one's own cognitive functioning. . . . In a joint problem-solving activity, what normally remains hidden in individually internalized thought may manifest itself in dialogue. (Swain & Lapkin, 1998, p. 321)

In their discourse analysis, Swain and Lapkin (1998) identified "language-related episodes" that occurred in the students' collaborative dialogue. The authors defined a language-related episode (LRE) as "any part of a dialogue where the students talk about the language they are producing, question their language use, or correct themselves or others" (Swain & Lapkin, 1998, p. 326). Swain and Lapkin's (1998, 2000) analyses of student participants' interactions used the turn as the unit of analysis in order to explore the LREs present in the discourse. This work provided part of the coding schemes I used during my discourse analysis—specifically those related to LREs and L1 use during student collaborative dialogue.

The LREs in Swain and Lapkin's (1998) study were analyzed in terms of three different topic strands: "(a) dialogue as an enactment of mental processes, (b) dialogue as

occasions for L2 learning, and (c) LREs and their implications for classroom-based research" (Swain & Lapkin, 1998, p. 329). But while the authors discussed the LREs along these three lines, they acknowledged that seeing the separation between them was at times challenging because the same dialogue could illustrate, for example, both an enactment of mental process and an occasion for L2 learning. With regard to (a), enactment of mental processes, the authors found that collaborative dialogue mediated L2 learning through the following: generating alternatives, assessing alternatives, and applying rules or extending knowledge to new L2 contexts (Swain & Lapkin, 1998). Topic (b) explored "other" occasions for L2 learning through collaborative dialogue, such as when one student learned and/or adopted a vocabulary term or grammar structure from another and used it later (either during the dialogue or on a posttest item). Throughout their analysis Swain and Lapkin (1998) categorized the LREs into one of two groups, either "lexis-based" or "form-based": The lexis-based LREs involved students seeking French (L2) vocabulary (or choosing among competing vocabulary items, which was part of the particular L2 tasks assigned), whereas the form-based LREs involved students focusing on some aspect of French grammatical structures. With regard to topic strand (c), the authors presented a broader discussion about the high degree of variability in students' performance on the tasks (e.g., in terms of time on task, number of LREs produced), and suggested that teachers need to be aware of how students respond in different ways to various collaboration-based tasks in the classroom. This topic strand did not enter into the a priori coding categories of LREs, however it did serve to inform the presentation and interpretation of some of my own results related to the mathematics task itself, including overall time on task.

Interestingly, in a study from the mathematics education field Barwell (2009c) found a number of similar LREs (although he did not use the LRE terminology in this particular study) occurring in the dialogue of adolescent bilingual learners working together on mathematical word problems. However, in an additional layer due to the mathematics context, these LREs were linked not only to language but also to mathematical content. When students were given the collaborative task of writing their own word problems, Barwell (2009c) found that they paid attention to the linguistic features of the word problems, and that there was strong interaction between the attention paid to language and mathematical understanding. Students engaged in meaning-making activity regarding particular words, pronunciation, spelling, punctuation, and tense, but these language-related exchanges also mediated students' mathematical understanding of the problems. The fact that LREs emerged in key studies from not only the L2 but also the mathematics education field was a strong indication that coding my own data for LREs could add to the research conversation and help elicit responses to the research questions posed in this study.

Based on Swain and Lapkin's (1998, 2000), and Barwell's (2009c), work with LREs, I decided to use similar categories to establish some a priori codes to guide my discourse analysis. I thought I might find similar LREs and instances of L1 use in my own data, due to the fact that, like in these studies, my student participants would be engaged in collaborative problem-solving work in pairs or small groups in an immersion (bilingual) classroom setting. Using the parts of the LRE and L1 coding schemes that I found most relevant to my own research provided a useful starting point for data analysis, and allowed me to explore whether the same types of LREs and instances of L1 use

emerged in this particular setting in the same (or in different) ways as compared to Swain and Lapkin's (1998, 2000) and Barwell's (2009c) studies—thus providing an opportunity for a replication of sorts.

Along with these potential similarities, however, as previously alluded to I also anticipated that some different patterns might emerge for a number of reasons. First, within language, including language registers such as the mathematics register, the lexicon and the grammar are often intertwined in a so-called "lexicogrammar" (Halliday, 1978). Thus, I wondered whether Swain and Lapkin's (1998) lexical LREs and form-related LREs would always be able to be identified as distinct from one another. I decided to include the possibility for a "lexicogrammatical-based" LRE in my a priori codes, in addition to the lexis-based and form-based categories. Second, and of utmost importance, the participants in my study would be engaged in mathematical tasks (a mathematical problem-solving activity based on perimeter and area, described fully in the "materials" section of this chapter), whereas the participants in Swain and Lapkin's (1998, 2000) research were engaged in classic L2 communicative tasks of a linguistic nature, a jigsaw and a dictogloss (Swain & Lapkin, 2001; see also Mackey & Gass, 2005). Therefore, I anticipated that although some of the LREs in my data could be the same as Swain and Lapkin's (1998, 2000), since my participants would be working on tasks assigned in their L2 (French), I also expected that the students might talk about different aspects of language, or talk about it in different ways, due to the mathematical nature of the tasks. Including ideas from Barwell (2009c) as part of the LRE-related coding was done in order to mitigate this potential gap. I began with an initial iteration of data analysis, using the a priori codes. As I got deeper into the data analysis, and through

multiple iterations, I established new emergent categories that would eventually become the finalized coding framework for LREs.

Some of the key retained LRE elements, as well as the emergent codes and changes made to reflect them in the framework, were as follows: As I began my data analysis, I found it necessary and useful to subdivide the categories of LREs with the aim of teasing out some of the finer nuances of language and mathematics. The LREs were first divided into two overarching categories that drew on the literature: meaning-making activity and other occasions for L2 learning. During meaning-making activity in collaborative dialogue, students engage in LREs by attending to language explicitly in the following subcategories: generating alternatives (which also includes vocabulary searches in lexis-based cases); assessing alternatives (i.e., the act of choosing); or applying L2 rules to a new context. Aside from these meaning-making activity LREs, there were those categorized as other occasions for L2 learning. In these LREs, students may not attend explicitly to language, but, rather, they attend to it and use it in a more implicit, indirect manner in the following subcategories: either by correcting themselves or others, or taking up (using) language introduced by others.

The two overarching LRE categories (meaning-making activity and other occasions for L2 learning) and their subcategories (generating alternatives, assessing alternatives, applying rules, correcting, taking up) where then cross-categorized as lexis-based or form-based LREs, which were retained as codes based on the literature; moreover, a lexicogrammatical cross-category was added, which was different from Swain and Lapkin's (1998) work but also taken from the literature (e.g., Halliday, 1978). Lexis-based LREs are those where students attend to lexis when engaging in meaning-

making activity or other occasions for L2 learning (i.e., generating alternatives for items, searching for an item, going about choosing a particular item, or applying lexis rules to a new context). Form-based LREs involve attention to things such as spelling, morphology, syntax, grammar, and verb conjugation. In a lexicogrammatical LRE, students simultaneously attend to both lexis and form; an example of this would be a vocabulary search for the French equivalent of the verb *to fit* and coming up with alternatives such as *aller dans*, *mettre dedans*, or *aller avec assez d'espace* (*to go in*, *to put inside*, *to go with enough space*).

These three categories (lexis, form, and lexicogrammatical) were also subdivided into "strictly" lexis (/form/lexicogrammatical)-based, mathematical-lexis (/form/lexicogrammatical)-based, and non-academic-mathematical-lexis (/form/lexicogrammatical)-based. These codes were defined as follows (using only the lexis-based category as an example in order to reduce text, however the same subcategorization was used for form-based and lexicogrammatical LREs): An episode was considered strictly lexis-based if the students attended to lexis that was unrelated to mathematics (although not necessarily unrelated to the task), for example, a vocabulary search for the term *paper clip*. On the other hand, LREs were considered mathematical-lexis-based if the item to which students were attending was an academically mathematical term, for example, generating alternatives for the term *diagramme à l'échelle* (*scale diagram*). Finally, there were instances where students were attending to a lexical item that was mathematical yet non-academic ("everyday mathematical"), for example, a vocabulary search for the equivalent of the English word *overlap*. I refer to this category as non-academic-mathematical-lexis-based. These kinds of divisions

enabled me to analyze what parts/kinds of language students attend to, how they do so, and when.

These categories, based on the literature as well as salient episodes that emerged during the first iterations of data analysis, formed the framework that was used to code the students' collaborative classroom dialogue for LREs as see in Table 6.

Table 6

Language-Related Episodes (LREs) Occurring in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language

	Lexis-Based	Mathematical-Lexis-Based	Non-Academic-Mathematical-Lexis-Based	Form-Based	Mathematical-Form-Based	Non-Academic-Mathematical-Form-Based	Lexicogrammatical	Mathematical-Lexicogrammatical	Non-Academic-Mathematical-Lexicogrammatical
Meaning-Making Activity									
Generating Alternatives									
Assessing Alternatives									
Applying Rules									
Other Occasions for L2 Learning									
Correction									
Uptake									

Although the LRE coding framework allowed for mathematical communication in some respects, this framework was limited to identifying singular lexical items, or short instances of attention to form or lexicogrammar that involved mathematical items. Moreover, these instances, as the name LRE suggests, highlight communication in which students are attending primarily to language, albeit language used in a mathematical context and sometimes in order to do mathematics. Thus, in order to analyze broader instances of mathematical communication, I identified what I call "mathematics-related episodes" (MREs), and incorporated these into my coding scheme. Identifying these MREs helped highlight and explore the mathematics at work in students' communication.

In order to do this, I considered findings stemming from mathematics education literature based in L2 contexts in determining my a priori codes, and then adjusted these to reflect the emergent themes.

3.2.4.2 Coding mathematical communication.

The coding categories I decided to use for analyzing instances of participants' mathematical communication during their collaborative dialogue stem from key studies by Barwell (e.g., 2005a, 2005b, 2005c, 2009a, 2014) and Moschkovich (e.g., 2002, 2003, 2005, 2007, 2009a) that explored mathematics education registers particularly as they manifest in bilingual mathematics classrooms. Drawing on Gee's approach to discourse analysis (which served to further align this work with my own), Moschkovich (2003, 2007), for example, analyzed two particular features of mathematical discourse that were pertinent to this study: (a) focus of attention and (b) meanings for utterances. Moschkovich (2003, 2007) has noted that there are certain characteristics of speech that serve to focus attention on the mathematics at hand and are generally recognized and valued as academic mathematical discourse practices. These can include, among others, "abstracting, generalizing, and searching for certainty" (Moschkovich, 2007, p. 28). The use of canonical, textbook vocabulary and their definitions is also a highly valued academic mathematical discourse. Thus, coding my own data for this type of language use helped identify and analyze some instances of academic mathematical communication in students' interactions.

Moschkovich (2003, 2007) and others (e.g., Barwell, 2009a) have cautioned, however, against an over-reliance on strictly academic mathematical talk as being indicative of mathematical communication. More precisely, these scholars have

suggested that mathematical communication can stem from and be expressed through everyday experiences, practices, and language. Moreover, they have pointed out that it is important to avoid dichotomising academic mathematical and "everyday" communication. Moschkovich (2007) argued that, "everyday experiences with natural phenomena can be resources for communicating mathematically. For example, climbing hills is an experience that can be a resource for describing the steepness of lines" (p. 28). Furthermore, Barwell (2009a) suggested that students' discussions of personal experiences could seem like "off-task" (i.e., non-mathematical) episodes of talk, but that "they are often implicated in the students' sense making in relation to their word problems [i.e., the mathematics at hand]" (p. 71). With regard to students' use of formal, textbook definitions, which is an example of academic mathematical communication, it has been argued that students' "working" or "stipulative" definitions can be as mathematical as these traditionally recognized formal definitions (Moschkovich, 2003, 2005). Clearly, these ideas echo and link directly to the idea of the mathematics education register that is part (along with SCT) of the theoretical framework of this study. Thus, being open to recognizing mathematical communication in these different forms was important for determining a priori and emergent codes for analyzing the data in my study for mathematical communication acts or MREs.

Like the a priori codes for analyzing LREs in collaborative dialogue based on Swain and Lapkin's (1998, 2000) work, I felt that an a priori coding scheme for mathematical communication would likely serve me well as a starting point in the analysis of my own data. The key similarities between the current study and Barwell's (2009c) and Moschkovich's (2003, 2007) studies are that all are based in classrooms in

which bilingual students are doing mathematics in their L2, all are qualitative in nature, all employ discourse analysis methodology, and all view the mathematics education register as an important theoretical underpinning to the analysis of student discourse.

Despite these similarities, as with the LRE codes I again anticipated that there would be some new aspects to consider with regard to mathematical communication in my own discourse analysis. The different contexts (my own research context, FI, vs. the minority Spanish L1 speakers in mainstream English classes of the Moschkovich studies) are one key example of how I anticipated students' discourse might be different; I anticipated that the different context of this study might cause other more salient discourse features and themes to emerge, when compared to the existing literature. Other differing factors included those related to the students, teachers, and researchers involved, and the nature of the mathematics tasks. In order to help identify "look fors" in the discourse, I also consulted Halliday's (1985, 2009) functional grammar approach. This helped me gain an understanding of what words and phrases I might identify in students' language use as indicators of a particular language function, for example, "expanding."

The finalized coding categories, subcategories, and key words and structures for mathematical communication, based on both the a priori codes taken from the literature as well as the emergent codes from the first iterations of the data analysis, were as follows: generalizing, with subcategories of making claims (keywords "any," "always," "never") and searching for certainty ("probably," "must," "certain," "usually,"); hypothesizing, with subcategories of predicting ("I think that") and stating assumptions ("let's say"); describing, with subcategories of describing a mathematical situation (key words/structures context-dependent) and describing a pattern (key words/structures

context-dependent); representing, with subcategories of using drawings (act of drawing, "this goes here") and using objects (act of using models); and finally, expanding, with subcategories of elaborating (repeat, restate, clarify, refine; keywords "that is," "for example"), extending (add, replace, provide alternative; "and," "or," "but," "instead," "except"), and enhancing (qualify by time, place, manner, cause, condition; "meanwhile," "after," "before," "via," "because," "if . . . then"). As I analyzed the interaction data for instances of mathematical communication, I found that these communication acts often comprised several turns and that multiple categories and subcategories of coding of MREs were often present within a single communication.

In the literature review (Chapter 2) and previously in this section, I discussed how mathematical classroom discourse can be made up of "traditional" academic mathematical vocabulary and forms, as well as "everyday" vocabulary and forms that students use in mathematical ways. In coding the data, I found this to be true. However, whereas I initially thought I might tease out the academic-mathematical from the non-academic ("everyday") mathematical communication, in practice I found this to be an extremely difficult, if not impossible, enterprise. In the coding of the LREs as described in the previous section, it was possible to separate "regular" language-based episodes, from (academic) mathematics-based episodes, from non-academic-mathematics-based episodes, because these episodes mainly revolved around a single lexical item, form, or lexicogrammatical issue (e.g., *paper clip*, *scale diagram*, *fit*). In coding mathematical discourse however, the mathematical communication acts involved both academic and non-academic mathematical language intertwined to such an extent that it became futile to try to separate them; the dichotomy did not seem natural. Indeed, as mentioned earlier

in this section, scholars such as Barwell (e.g., 2009a) and Moschkovich (2003, 2007) have cautioned against such a dichotomization. Therefore, in each act of mathematical communication, where MREs occurred, it should be understood that students used various combinations of linguistic resources (lexis, form, and lexicogrammar) from both the academic and non-academic mathematics registers; they also used non-mathematical linguistic resources (lexis, form, and lexicogrammar). I found this view of mathematical communication to be an appropriate approach to this data analysis as this is, after all, the essence of the mathematics education register (see Chapter 2). Several coded LREs and MREs were often embedded within a larger instance of mathematical communication (i.e., pertaining to the same topic).

With these considerations in mind, the finalized coding framework for MREs, based on both the predetermined a priori codes and the emergent codes from the first iterations of data analysis are shown in Table 7.

Table 7

Coding Framework for Mathematics-Related Episodes (MREs) in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language

Generalizing	Making Claims	Searching for Certainty	
Hypothesizing	Predicting	Stating Assumptions	
Describing	A Mathematical Situation	A Pattern	
Representing	Using Drawings	Using Objects	
Expanding	Elaborate (Repeat, Restate, Clarify, Refine)	Extend (Add, Replace, Provide Alternative)	Enhance (Time, Place, Manner, Cause, Condition)

Finally, after working with the coding frameworks for LREs and MREs, I established a coding scheme for students' use of the L1. The L1 use framework adapted ideas and categories from both the LRE and MRE frameworks, incorporated other categories from the literature, and introduced some emergent ideas as well.

3.2.4.3 Coding students' use of the L1.

In studying L1 use or codeswitching more broadly, there are a number of approaches one could take. Codeswitching is a widely studied phenomenon in sociolinguistics in particular, and the ways in which interlocutors codeswitch for sociolinguistic reasons have been well documented (see Chapter 2). Therefore, to narrow down my approach to coding I returned to the studies most relevant to my own research context, namely, those based in L2 and/or mathematics classrooms exploring L1 use in bilingual students. Two in particular influenced the a priori codes: Swain and Lapkin (2000), from the L2 education field, and Moschkovich (2005), based in mathematics education.

While Swain and Lapkin's previously cited 1998 study focused mainly on LREs as they related to L2 use and learning in collaborative dialogue, in a subsequent study based on the same data set as their 1998 work Swain and Lapkin (2000) focused their analysis on instances of L1 use. Working from the same SCT framework, the authors viewed the L1 as a cognitive tool and examined the purposes of its use in students' collaborative dialogue as they worked through the linguistic problem-solving tasks (i.e., the dictogloss and jigsaw) in their L2. Swain and Lapkin (2000) determined that students used their L1 (English) for three main purposes. In addition, each of these three main categories of use was subdivided into more specific detail:

1. Moving the task along
    (a) sequencing (figuring the order of events)
    (b) retrieving semantic information; understanding pieces of information; developing an understanding of the story
    (c) task management
2. Focusing attention
    (a) vocabulary search
    (b) focus on form; explanation; framing; retrieving grammatical information
3. Interpersonal interaction
    (a) off task (includes L1 vernacular use)
    (b) disagreement (Swain & Lapkin, 2000, pp. 257-258)

In exploring these categories for my own analysis of students' L1 use, I found that, on one hand, many would likely be pertinent. On the other hand, a number seemed specific to the particular L2 tasks used in Swain and Lapkin's (1998, 2000) study (i.e., the dictogloss and jigsaw)—for instance, 1. (a) "sequencing"—and therefore I knew these might be less likely to emerge as relevant in my own data coding. Overall, however, many of these L1 use categories helped form the a priori codes for this study.

Moschkovich (2005) has also examined students' L1 (Spanish) use during collaborative problem-solving tasks in the L2 but, in this case, the tasks were mathematical (i.e., mathematics problems) rather than linguistic (i.e., a dictogloss or jigsaw). Moschkovich (2005) found that bilingual learners used their L1 in mathematics to: (a) provide a missing L2 vocabulary item; (b) explain a concept; (c) justify an answer;

(d) describe mathematical situations; and (e) elaborate, expand and provide additional information. Furthermore, from a more social standpoint, students' L1 use was found to: (a) reflect a level of communicative competence; (b) reflect community norms; (c) provide stylistic switches in the conversation (to add colour, emphasis, etc.); and (d) relate to memory and to routines. These coding categories reflecting codeswitching and L1 use based specifically in a mathematics classroom context enabled me to enhance and build upon the L1 a priori codes taken from Swain and Lapkin (2000). I began with an initial a priori coding framework and, as with the LREs and MREs, I eventually made adjustments to these frameworks to reflect emergent issues from the first iterations of data analysis.

In coding the students' classroom interactions for L1 use, two main issues arose during the first iterations of data analysis. First, I used the post hoc adjustments I had made to the LRE and MRE coding schemes, reflecting emergent codes in those categories (e.g., the addition of mathematical categories to the LREs, the expansion categories for the MREs), to modify the a priori L1 codes. Second, when it came to coding "L1 use" the exercise proved challenging. The main challenge arose in the process of determining what exactly constitutes "L1 use," in order to classify a turn as L1, L2, or codeswitched. It was important to establish coding criteria in order to aim for consistency in the coding, and to reflect the overall context of the discourse (i.e., secondary FI students in the school classroom) and the framework of language in use.

One area that proved in particular need of attention was the students' use of discourse markers because these markers very often appeared in the students' L1, even though the remainder of the turn was uttered in the L2. However, I had misgivings about

coding a turn as codeswitched due to the presence of L1 discourse markers alone. For example, coding a turn such as, "**Yeah**, *et puis cinquante-et-un sur l'autre*" as "codeswitched" did not seem to best reflect the student's overall communicative intent with regard to language of communication. I decided to investigate discourse markers in more detail in order to help me to make informed, useful decisions regarding the coding of L1 use.

Discourse markers are a phenomenon largely studied by linguists and sociolinguists (rather than applied linguists) and defined by *The Concise Oxford Dictionary of Linguistics* (Matthews, 2014) as, "any of a variety of units whose function is within a larger discourse rather than an individual sentence or clause;" the dictionary entry cited *well* as an example in, "*Well what if he is?*" (p. 109). A further investigation into discourse markers revealed a complex phenomenon that has been and continues to be studied from several different angles and with different research goals (see, e.g., Schiffrin, 2001, for an overview of many of the key works on discourse markers stemming from the linguistics field). There are many different categories of discourse markers, which are used in different ways for different purposes across written and oral discourse, and within oral interaction specifically. As Schiffrin (2001) explained, discourse markers are further complicated when comparing across languages, as they are often directly borrowed from one language and used in another (taking on the same meaning and remaining in the original language, e.g., French *OK* borrowed from English and used exactly the same way); or have equivalent translations (that may or may not mean the same thing, e.g., *bien* = *well* [organizational]). Moreover, even in the same language, discourse markers can mean different things in different contexts, and these nuances may or may not carry

over to another language (e.g., syntax, pronunciation, and meaning all differentiate the discourse marker uses of *anyway*, which in turn are difficult to translate to a French equivalent—as an English/French bilingual myself I have at times used English *anyway* in my spoken French). Discourse markers are heavily used in natural oral interaction speech and were therefore, not surprisingly, present in the oral interaction data collected in this study.

For the purposes of this analysis, I was concerned mainly with discourse markers associated with interactive speech, in particular those related to back channelling and hedging, when it came to coding decisions regarding L1 use, for example, markers such as *yeah*, *uh-huh*, *OK*, *like*, *well*, *oh*, *um*, and so on. A search of the literature quickly cleared up questions I had regarding the language (i.e., English or French) of *OK*, which was one of the most heavily used markers, since it has been argued that *OK* can be considered a French discourse marker, particularly in the Canadian context (see, e.g., Dostie, 2009). With regard to the other discourse markers and the coding of L1 use overall, I decided on the following guidelines after some study of the literature:

- English discourse markers were considered as being part of the overall language of the turn. For example, on one hand, the turn, "*OK, est-ce qu'on devait tracer après, oh, tu as des, ça c'est intelligent, OK. Um, qu'est-ce qu'on fait?*" was coded as an L2 turn even though it contains English discourse markers (backchannelling) *oh* and *um*. On the other hand, the turns, "**Oh, so each side has to have fifty, right?**" and "**Um, this doesn't fit**," which also contain the markers *oh* and *um*, respectively, were each coded as L1 turns. The turn, "*Yeah tu peux compter si tu veux, mais ça va être soixante-six.* **No. Did you count the green**

one?" was coded as codeswitched, not because of the discourse marker *yeah* alone, but because of the last two sentences of the turn, which were clearly uttered in the L1.

- French discourse markers, for example *donc*, *comme*, *alors*, *bien*, and so on, which were also present in the data, were treated in the same manner as English discourse markers in terms of L1/L2/codeswitched coding (see first bullet point).
- Turns were coded as L1 if all words (lexis) in the turn were clearly English (bearing in mind the discourse markers). For example, "**Give it back. I'll make another one**" was coded as an L1 turn.
- Turns were coded as L2 if all words (lexis) in the turn were clearly French (bearing in mind the discourse markers). "*Oui, parce qu'il n'y a pas d'espace*" was an example of an L2 turn.
- Codeswitched turns were coded as such if any words (lexis) were uttered in the language other than the main language of the turn. Sometimes, the codeswitch was a complete sentence, other times a few words, and still other times a single word. For example, "**No, it's not. This is mine!** *Quel est le maximum? Cinquante fois cinquante c'est vingt-cinq mille*"; "*Mais il besoin d'être* **straight**. *Comme*, **tight** *et pas* **loose**. *Donc on va avoir peu d'espace*"; "*Qu'est-ce que c'est* **mesurements**?"; and "**What does** *échelle* **mean?**" were all coded as codeswitched turns.
- Turns that were in the L2 but that contained syntax, form, and/or lexicogrammatical errors and/or errors that were obviously influenced by the L1 were counted nonetheless as L2 turns (rather than codeswitched turns); for

example, "*J'aime mais on doit couper parce que tu as coupé cette un*" (*cet un = this one*); and "*On devrait commencer ça demain parce que ça regarde compliqué et je ne veux pas, comme, finir dans le milieu de le*" (*ça regarde compliqué = it looks complicated*, *dans le milieu de le* = in the middle of it) were both coded as L2 turns. The same issue did not arise in the opposite direction (i.e., L1 turns did not contain L2-influenced erroneous syntax, forms, lexicogrammar).

- Attempts at idiomatic expressions in the L2 that were erroneous were nonetheless counted as L2 turns, such as, for example, "*Tu es méchante à les yeux*" (= *You're hard on the eyes*) and "*J'essaie de donner toi un tête à haut*" (= *I'm trying to give you a heads-up*). The same issue did not arise in the opposite direction (i.e., L1 turns did not contain idiomatic expressions that were erroneously influenced by the L2).

As with the other coding frameworks used in this study (i.e., for LREs and MREs), the coding framework and decisions regarding students' L1 use are not the only valid options. However, based on the literature and on the research aims of this study, this framework was useful in order to analyze L1 use in a way that was pertinent and reflective of the context. The finalized coding frameworks for students' L1 use are shown in Table 8, Table 9, and Table 10. As seen from the Tables, in the end I used three frameworks for coding L1 use: one was embedded within the LRE coding framework, another in the MRE framework, and the third was based on "other" and/or task-related uses of the L1.

Table 8

Coding Framework for Language-Related Episodes (LREs) Occurring With Use of the First Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language

	Lexis-Based	Mathematical-Lexis-Based	Non-Academic-Mathematical-Lexis-Based	Form-Based	Mathematical-Form-Based	Non-Academic-Mathematical-Form-Based	Lexicogrammatical	Mathematical-Lexicogrammatical	Non-Academic-Mathematical-Lexicogrammatical
Meaning-Making Activity									
Generating Alternatives									
Assessing Alternatives									
Applying Rules									
Other Occasions for L2 Learning									
Correction									
Uptake									

Table 9

Coding Framework for Mathematics-Related Episodes (MREs) Occurring With Use of the First Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language

Generalizing	Making Claims	Searching for Certainty	
Hypothesizing	Predicting	Stating Assumptions	
Describing	A Mathematical Situation	A Pattern	
Representing	Using Drawings	Using Objects	
Expanding	Elaborate (Repeat, Restate, Clarify, Refine)	Extend (Add, Replace, Provide Alternative)	Enhance (Time, Place, Manner, Cause, Condition)

Table 10

Coding Framework for "Other" and Task-Related Use of the First Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language

Moving the Task Along
Understanding the Problem
"Wait"
Task Management
Interpersonal Interaction
"Off Task"
Vernacular
Disagreements
Humour, Feelings
Communicative Competence

In the L1 coding framework, "off task" is in scare quotes to indicate that separating "off task" from "on task" work in the mathematics classroom is problematic. Elsewhere (e.g., Culligan & Wagner, 2016), I have begun to investigate how students' multiple storylines (Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015; see also Gee's, 2014, notion of "figured worlds") are at play and interact in the mathematics classroom, thus affecting and influencing the mathematics and the discourse.

Overall, these frameworks were established having explored coding possibilities from several studies that proved to be among the most relevant for my own work, and then making adjustments to these a priori codes during the first iterations of data analysis. The tables in this section show my adaptation of the codes taken from the literature (sources of particular importance included: Barwell, 2009a, 2009c; Halliday, 1985; Moschkovich, 2002, 2007; Swain & Lapkin, 1998, 2000, 2013), as well as the identification of new observations that emerged as relevant to this particular study—an

important part of qualitative analysis (Bogdan & Knopp Biklen, 2007; Creswell, 2003; Mackey & Gass, 2005; McKay, 2006; Merriam, 2009). The coding frameworks were used to analyze students' collaborative dialogue as they worked through the mathematics problem-solving activity in their L2.

Finally, a remark on the categorization evident in the coding frameworks: The analysis of discourse using codes represents a way for researchers to attempt to make sense of, understand, and interpret the exceedingly complex phenomenon of student interaction discourse (McCarthy, 1991; McKay, 2006). Although I used these frameworks to analyze the discourse, I wish to highlight what I believe to be the oftentimes-integrated nature of the categories (some categories more than others). In some cases, there were overlapping codes: For example, "expanding" by way of "repeating" could be simultaneously an example of an MRE, with embedded L1 use (if part of the MRE, for example, a repeated lexical item, were to occur in the L1). As with all models of complex phenomenon, I am cognizant of the somewhat artificial nature of the divisions between the different categories in the framework. In particular, I believe there is substantial crossover in terms of the language-mathematics distinction, and I remained open to such possible blurring of lines as I approached my data analysis. In essence, as evidenced in my theoretical approach, I view language and mathematics as intertwined and therefore an attempt to separate the two categories in many ways proves difficult at best. As Barwell (2009c) once remarked, "language learning is fully integrated with mathematics learning" (p. 77). Although the complexity of this type of analysis presented a challenge, I believe it is what strengthens the potential of the work for contributing in a real way to understanding both the linguistic and the mathematical

episodes at work in the bilingual classroom. The messiness of this type of discourse analysis is discussed in more detail in the next chapter (Chapter 4, Results). Before concluding this chapter, however, it is necessary to describe how, despite the messiness inherent in this type of research study, I worked to build validity in the analysis.

3.2.5 Validity and Triangulation

As with all qualitative research, the term *validity* needs to be interpreted appropriately in the context of this study. Specifically, it needs to be differentiated from reliability and generalizability, which are cornerstones of quantitative research but play little to no role in qualitative research (Creswell, 2003). Distinct from quantitative approaches, validity in a qualitative study is "measured" by the authenticity and credibility of its findings (Creswell, 2003, Merriam, 2009). In fact, the term itself is somewhat contentious among qualitative researchers (see Creswell, 2007) who have argued that it is too difficult to shed the positivistic notions associated with it. Here, I use the term *validation* in Creswell's (2007) qualitative research sense:

> [Validation is] an attempt to assess the "accuracy" of the findings, as best described by the researcher and the participants. This view also suggests that any report of research is a representation by the author. I also view validation as a distinct strength of qualitative research in that the account made through extensive time spent in the field, the detailed thick description, and the closeness of the researcher to participants in the study all add to the value or accuracy of a study. I use the term "validation" to emphasize a process (see Angen, 2000), rather than "verification" (which has quantitative overtones) . . . My framework for thinking about validation in qualitative research is to suggest that researchers employ

> accepted strategies to document the "accuracy" of their studies. These I call "validation strategies." (pp. 206-207)

According to Creswell (2003, 2007; see also Merriam, 2009), some such strategies include: prolonged engagement in the field; triangulation; peer review; negative case analysis; clarifying researcher bias; member checking; rich, thick description; and external audits. Creswell (2007) recommended that researchers engage in at least two of these possible validation strategies, and suggested that triangulating among different data sources, member checking, as well as writing with detailed thick description are among the most easy, popular, and cost effective ones. In this study, I relied largely on data triangulation as a validation strategy, but I also used clarification of bias, thick description, member checking, engagement in the field, and peer (and supervisor) debriefing in order to build a valid qualitative study. Here, I describe first my main validation strategy, that is, triangulation, and then go on to explain how the other strategies were used as well.

As just pointed out, in this study, one of the main ways in which I strove to enhance validity was by triangulating the data. *Triangulation* can be defined and accomplished in different ways, therefore it is important to explain precisely what was undertaken in a given study rather than simply saying something general like "data were triangulated" (see Bogden & Knopp Biklen, 2009, for a discussion). In my research, I triangulated my data mainly by using multiple methods of data collection (Creswell, 2003, 2007, Merriam, 2009). As described in the Data Collection section of this chapter, the main source of data for this study was audio recordings of students' interactions in the classroom. However, I also collected supplemental data by conducting post hoc

interviews (based in part on stimuli from the main data collection sessions), collecting students' written work, taking fieldnotes and making observations. To a lesser degree, data were also triangulated by collecting them from different sources—not only did students participate in this study, but their classroom teacher was also an active participant, contributing data by way of being observed in my fieldnotes and participating in many informal as well as one formal interview. In the data analysis, for example, when I heard something of interest in the classroom interaction audio data, I was able to follow up on the point by examining the corresponding part(s) of the written text that was produced, the discussion during follow-up interviews, and sometimes the fieldnotes. Together, the multiple data sources help to deepen, support, and build interpretations and explanations of the phenomenon in question. Similarly, when I noticed something of interest during my classroom observations and took note of it in my fieldnotes, I was able to explore it through informal and/or formal interviewing with the teacher and/or students. With the complexities inherent in classroom-based research contexts and in phenomena like interaction discourse, the goal and bottom line of data triangulation is to "have multiple sources on which to build an interpretation of what is being studied" (McKay, 2006, p. 79).

Having multiple methods and sources of data were a key way in which I found I could build thick, rich description into the analysis and discussion of the results of this study, which is another validation strategy. An extensive theoretical framework and literature review from which I could draw for potential areas of corroboration (or conflict) also helped to fuel the descriptive writing of the analysis and results. With regard to discourse analysis in particular, Gee (2014) suggested that validity is made up

of four elements: convergence, agreement, coverage, and linguistic details. I used these elements to guide me as I analyzed the data, reported the results, and discussed these results. Having rich data collected from multiple methods and sources allowed me to address Gee's (2014) four elements in the writing-up of this research.

Finally, additional validation strategies such as clarification of bias, member checking, engagement in the field, and peer (and supervisor) debriefing were also used in this research. In the Introduction chapter of this dissertation, I devote some space to writing about my own positioning as a researcher in this study. In this strategy, "the researcher comments on past experiences, biases, prejudices, and orientations that have likely shaped the interpretation and approach to the study" (Creswell, 2007, p. 208). Such acknowledgement is a part of qualitative research and rather than try to strive for an impartiality that is in reality unachievable, a better strategy is to reveal ourselves as researchers and then remain open to emergent and unanticipated findings. This links to another important validation strategy, engagement in the field, which helped allow the salient themes to emerge (Creswell, 2007). Of particular importance in ethnographic study but in many other types of qualitative research as well, engagement in the field can also help to build trust with participants, and provide opportunities to check for misinformation. In this vein, member checking (Creswell, 2007) was also done to some degree throughout this project. The teacher, in particular, was consulted regarding observations stemming from the fieldnotes as well as overall impressions of the discourse analysis. Student participants were able to reflect on or clarify things said and/or written during the classroom activities during their follow-up interviews. Finally, since this is a doctoral study, peer and, especially, supervisor debriefing played a role in building the

validity of the work. At every major step of the way throughout the research, my colleagues, committee members, and most especially my supervisor acted in this role described by Creswell (2007): "an individual who keeps the researcher honest; asks hard questions about methods, meanings, and interpretations; and provides the researcher with the opportunity for catharsis by sympathetically listening to the researcher's feelings" (p. 208).

Thus, despite the limitations that can be associated with qualitative discourse analysis (which will be discussed in full detail nearer the end of this dissertation in the Conclusion), several key strategies were used in this research study, including prolonged engagement in the field; triangulation; peer review; clarifying researcher bias; member checking; and rich, thick description with the aim of building understandings and interpretations that are valid and "accurate."

3.3 Conclusion

Exploring language and mathematics in classrooms is no easy task. As Pimm (2010) once reflected, the "dual track" of a researcher working in mathematics and language has its "many crossovers, intersections, chicanes, and occasional collisions" (p. xi). Language represents an extremely complex social and cognitive phenomenon, and mathematics for its part is increasingly recognized as not just a cognitive enterprise but a social one as well. Moreover, the actions of using language and doing mathematics are intertwined and occur simultaneously in the mathematics classroom. This phenomenon is perhaps most noticeable in bilingual mathematics classrooms, particularly those whose program aims are stated explicitly as to teach both language and content, such as FI for example.

Grade 9 mathematics students and their classroom teacher participated in this study (along with Grade 10 students during a pilot round). Students completed a mathematics problem-solving task in their L2, during which they worked collaboratively in pairs or small groups of three. Student data were collected via multiple sources, including by way of audio recordings of their classroom dialogue, written work, post hoc interviews, and researcher fieldnotes. The teacher was involved in and contributed to the various stages of the research (from planning to action to reflection), and also participated in interviews (formal and informal).

This study is a qualitative one and, as such, it aims to describe and understand thoroughly and richly the phenomena under investigation. The overarching analysis is guided by the traditions of qualitative research, particularly as these apply to education contexts (e.g., Bogdan & Knopp Biklen, 2007; see also, e.g., Creswell, 2003; Merriam, 2009). Discourse analysis methodology, informed by key tenets of SCT and the mathematics education register, provides the overarching methodological approach to studying language and mathematics. More specifically, a theory of discourse analysis that views discourse (language) as social, situated, and tied to context (Gee, 2014), offers the lens through which the main source of data (audio recordings of students' interactions) is explored. Several tools of inquiry (guiding questions) further guide the overarching methodological approach to the discourse analysis (Gee, 2011).

A number of a priori codes were used in order to make sense of the data (Barwell, 2009c; Halliday, 1985; Moschkovich, 2002, 2007; Swain & Lapkin, 1998, 2000). The qualitative, inductive methodology also allowed other coding categories to emerge through the analysis. Supplemental data sources from different data-collecting techniques

in the form of post hoc interviews, written texts, and fieldnotes triangulated the codes and themes emerging from the interaction recordings and helped to build on and support the descriptions and interpretations made during the analysis.

In the next chapter, Chapter 4, I present the results of the discourse analysis undertaken in this study. The writing of the results includes some descriptive quantitative reporting; however, overall the chapter is presented in keeping with the qualitative tradition. Results are reported such that they offer thick description and interpretation of salient emergent themes that address the research questions.

4.0 RESULTS

The results of this discourse analysis are presented within an overarching framework based primarily on Gee's (2014) "building tasks of language" as described in the methodology in Chapter 3. In particular, the analysis takes into account building tasks #2 (activities) and #7 (sign systems). In other words, the results stem from a discourse analysis that broadly seeks to explore what activities are enacted by the language being used and how different sign systems (e.g., different languages, different registers, words, images, equations) are used within the discourse. Within this broader framework, the results are presented so as to respond to the more specific research questions posed in Chapter 2, by using coding frameworks and Gee's (2011) discourse tools of inquiry.

This results chapter begins with an introductory section that describes overarching themes that emerged from the discourse analysis, more specifically those related to the mathematics problem-solving task itself, in order to situate the reader within the particular context of this high school FI mathematics classroom. This is in line with key tenets of qualitative research, sociocultural views of language and mathematics in educational contexts, and the guiding methodology of the study as laid out in Chapter 3. These themes emerged primarily from the main source of data, that is, the transcripts of the classroom interactions, but they are triangulated by data from other sources such as the students' written work and their follow-up interviews, as well as the researcher fieldnotes.

Next, other key results of this study are presented in three different sections according to the three specific research questions posed in Chapter 2, namely:

1. How do FI students at the secondary level attend to language, that is, what language-related episodes emerge, while working collaboratively during mathematical problem solving in their L2 (French)?
2. How do these students communicate mathematically while working collaboratively during mathematical problem solving in their L2 (French)?
3. When and how do these students use their L1 (English) while working collaboratively during mathematical problem solving in their L2 (French)?

To address each of these three research questions, the presentation of the results follows a two-pronged approach in each section. First, I provide descriptive quantitative data stemming from the coding analysis described in the methodology in Chapter 3, in the form of tables (tallies, percentages) and graphs. These quantitative data allow me to identify salient LREs and MREs, as well as episodes of L1 use. Next, drawing from these quantitative findings, I illustrate and elucidate the most pertinent coded episodes through thick description highly qualitative in nature: Excerpts from transcripts of students' classroom-based collaborative dialogue are presented as the main data. These excerpts were analyzed using Gee's (2011) tools of inquiry for discourse analysis as described in the methodology in Chapter 3, in order to explore more deeply the LREs and MREs that emerged during the coding. In particular, these qualitative descriptions use the "doing and not just saying tool" (analyzing not just what speakers are saying but also what they are trying to do); the "why this way and not that way tool" (analyzing why the speakers use language [lexis, form] in the ways in which they do); the "sign systems and knowledge building tool" (analyzing how and why different sign systems, i.e., languages, are used); and the situated meaning tool (analyzing the specific meanings of words and phrases

given the context) to elucidate the coded episodes. These transcript excerpts from the classroom-based audio data are then triangulated and thus further enriched by related passages from students' written work and follow-up interviews.

Throughout Chapter 4, all data are presented in their original language, whether French, English, or codeswitched. As described in Chapter 3, **bold** type indicates use of English (see that chapter for a complete list of transcript conventions). When French or codeswitched excerpts and examples are given, my translations follow immediately in parentheses. In all instances, participants' data (transcripts of oral data and written text) are presented in their original form verbatim, including participants' errors. In the English translated excerpts, it was difficult to convey exactly the same types of errors that occurred in the original French version therefore I was most often not able to do so. Arguably, that exercise would entail considerable time, effort, and expertise beyond the scope of this dissertation. Thus, although some French errors are reflected in the English translations (most often syntax errors, as these are the most straightforward to duplicate), it should be noted that the English translations should be considered interpretative translations (which aim to convey original versions as accurately as possible within reason) and not verbatim transcriptions.

The key results presented in this study and the excerpts used to illustrate them should not be taken to be the only possible interpretations and examples. Rather, the themes and excerpts represent my choices of what I found to be the most illustrative and salient with regard to the research questions posed and the theoretical framework of this study. Thus, while the main discussion and interpretation of the results are presented in Chapter 5, I acknowledge that, due to the inherent characteristics of this qualitative

research, the presentation of the results here in Chapter 4 is reflective of a certain (unavoidable) level of interpretation on my part, as the researcher.

4.1 Results Related to the Mathematics Problem-Solving Task

As described in Chapter 3, the stimulus for the data collection was a mathematics activity (Planning a Playground) that was hands-on in nature, adhered to several guidelines for good problem-solving tasks, and connected to a number of general and specific curriculum outcomes for Grade 9 mathematics. In this section, I present results related to three key interrelated findings that emerged with regard to the problem-solving task itself: student engagement, quantity and nature of student discourse, and mathematics problem-solving strategies.

4.1.1 Student Engagement

Overall, there was a high level of student engagement in the task over the three class periods it took to complete it (or, for two of the groups, two class periods). Students' engagement was judged to be "high" overall, based on three interrelated findings that weave together to tell the story of the unfolding of the task. First, students' physical engagement with/use of the hands-on materials (large-scale paper models and paper clips fence; small-scale diagrams using graph paper, rulers, and compasses; answering the written questions on the worksheet) appeared enthusiastic and they seemed interested in working to ensure their models were working well (researcher fieldnotes, March 17, 18, 22). Second, most students were observed to be able to access the task, presumably due to its emphasis on problem solving rather than strictly on procedural knowledge (researcher fieldnotes/informal interview with participating teacher, March 17). And finally, students' positive feedback regarding the activity as given in their follow-up interviews confirms

the observations regarding task engagement and access that were noted during the in-class portion of the activity.

The hands-on element, which the participating teacher, Mme Nathalie, had acknowledged was largely absent from her mathematics classes (see Chapter 3), seemed to excite the students, as did the move from the regular classroom to the library: "Students were excited about being in the library and about using the giant models of paper clips and brown paper; Students were very engaged in the activity" (researcher fieldnotes, March 17). During the activity, the teacher also remarked to me that the opportunity to get hands-on with some "low-tech" mathematics tools seemed to motivate students, particularly those who might otherwise be unmotivated: "Teacher noted that some students who were not normally engaged in mathematics class were very engaged in this activity (e.g., using the compass, rulers, scissors)" (researcher fieldnotes, March 18). Two students in particular who were categorized as such by the teacher were Dan and Zac, partners in group D4. Indeed, in their follow-up interview, Zac, in particular, expressed his thoughts on the mathematics task, which seem to confirm the teacher's observations:

> Zac: *C'était bon comme de faire toutes les maths en classe et c'était plus interactive, parce que j'aime être debout et marcher au lieu d'être assis aux chaises. C'était pas un jeu mais c'était plus, c'est plus amusant que juste résoudre des problèmes sur le Smart Board.*
>
> (Zac: It was good like to do all the math in class and it was more interactive, because I like being up and walking instead of sitting in chairs. It wasn't a game but it was more, it was more fun than just solving problems on the Smart Board.)

The teacher noted that the task appeared to provide opportunities for success regardless of mathematics achievement, which she felt was due not only to the hands-on element but also to the type of task (i.e., problem-solving).

Mme Nathalie noted that the task seemed to allow students who were not typical "high achievers" to take advantage of their problem-solving skills that were not necessarily linked to procedural skills (researcher fieldnotes, March 18). In fact, some of the lower achieving students were at times among the first to figure out that the maximum area for a perimeter of 200 is given by a square measuring 50 x 50 (researcher fieldnotes, March 17). For instance, the three members of group K4 (Mae, Max, and Mya), who were some of the lowest achieving participants, were among the first to find the square as the figure with the maximum area (researcher fieldnotes, March 17) by using the guess-and-check problem-solving strategy and immediately beginning with the reasonable guess of 50 for each side. As Mae, an LFI student, explained in her written work:

> *Nous avons trouvé les dimensions parce-ce que nous pensons de le fait qu'il y a 200 pieds de cloture et 4 côtés donc si nous faitons 4 côtés de 50 pieds nous avons rendez au 200.*
>
> (We found the dimensions because we think of the fact that there is 200 feet of fence and 4 sides so if we do 4 sides of 50 feet we got to 200.)

Thus, the activity, in part due to its hands-on, visual elements, seemed to not only engage students who typically were unmotivated, but also allowed those and other students to access mathematics when they otherwise might not be (or feel) able.

The task seemed to engage most of the higher-achieving mathematics students as well, for at least some of the same reasons. Most of these students also expressed that they enjoyed the activity—the hands-on aspect as well as the discussion-based, problem-solving approach. Liz, an LFI student from group L4 whose mathematics marks were in the 90s, for example, said in her follow-up interview:

> *J'aime la partie, comme, écouter des différentes idées et, comme, placer des choses et . . . um . . . tester les différentes façons de mettre des choses ensemble.*
>
> (I like the part, like, listening to different ideas and, like, placing the things and . . . um . . . testing the different ways to put the things together.)

Liz and her partner, Sue (an EFI student), were observed (researcher fieldnotes, March 17, 18) to work well with the hands-on models (large-scale and small-scale) and engaged in effective discussion as later confirmed in the classroom-based audio recordings.

The results show that the majority of students, both lower-achieving and higher-achieving, were generally engaged in the mathematics task and made statements to this effect during their follow-up interviews. Nonetheless, this was not true of all of the groups. Data showing the less-favourable student perceptions of the mathematics task will be presented next, as they connect strongly to results related to the quantity and nature of student discourse produced in response to the task.

4.1.2 Quantity and Nature of Student Discourse

Detailed results relating to the specific (coded) content of students' discourse produced over the course of the classroom activity will be presented in subsequent sections of this chapter. However, with regard to the task-related results in particular, the mathematics problem-solving activity succeeded in prompting students, rather than the

classroom teacher or me, to produce the vast majority of discourse in the classroom. This is because they interacted mainly with each other, rather than with either their classroom teacher or me. The teacher and I intervened when asked by the students to do so, or when we judged an intervention to be necessary (e.g., for task management, affective, or pedagogical purposes). However, the mathematics problem-solving task we chose allowed students' interactions to dominate the discourse of the classroom for approximately 2.5 to 3 days. Table 11 shows the data related to the total number of student turns and teacher turns spoken in each participant group, for the 3-day activity.

Table 11

Participant Groups, Number of Turns, and Percentage of Turns During the Mathematics Problem-Solving Task

Pseu-donym	Group (Letter, Period)	Program (Early or Late)	Total # of Turns	Total # of Student Turns by Student	Total # of Student Turns (%)	Total # of Teacher Turns (%)
Mac	E1	EFI	133	50	111 (83.5)	22 (16.5)
Mike	E1	EFI		61		
Jane	M1	EFI	534	252	515 (96.4)	19 (3.6)
Beth	M1	EFI		263		
Dan	D4	EFI	239	116	224 (93.7)	15 (6.3)
Zac	D4	EFI		108		
Kim	H4	LFI	770	338	716 (93.0)	54 (7.0)
Eve	H4	LFI		378		
Amy	E4	EFI	558	209	549 (98.4)	9 (1.6)
Hali	E4	EFI		210		
Ella[a]	E4	LFI		130		
Scot	L1	EFI	685	318	639 (93.3)	46 (6.7)
Ava	L1	LFI		321		
Sue	L4	EFI	232	114	223 (96.1)	9 (3.9)
Liz	L4	LFI		109		
Sara	H1	EFI	901	218	862 (95.7)	39 (4.3)
Brit	H1	LFI		286		
Bea	H1	EFI		358		
Mae	K4	LFI	223	91	214 (96.0)	9 (4.0)
Max	K4	EFI		52		
Mya	K4	LFI		71		

Clearly, student talk monopolized the discourse of the mathematics classroom on these days, with student turns accounting for a low of 83.5% to a high of 98.4% of the total turns. With regard to length of turns (i.e., in number of words), patterns were difficult to identify. For every participant, the shortest turn was of one word in length. Beth (group M1) uttered the longest turn, at 126 words (with some overlapping "hmm" and "uh huh" words/sounds uttered by Jane). Among the other participants, length of longest turn ranged from 23 to 100 words. Language of the turn did not seem to affect the length and vice versa, for groups who tended to speak mainly in the L2: these participants uttered their longest and shortest turns in the L2. For those who spoke mainly in the L1, the longest turns tended to be uttered in the L1, while turns in the L2 (if any) tended to be short (although not always). Students who codeswitched more frequently tended to do so in their longest turns (although not always) and in their shortest turns (although not always). More detail about language use (L1, L2, and codeswitching) is presented in the section dealing with research question #3.

Investigating student talk in terms of "time on task" with regard to mathematics was not an explicit aim of this study. However, as just noted, "time on task" as it relates to students' use of the target L2 (French) versus the L1 (English) enters into consideration in response to research question #3 and therefore will be presented in detail in that section of this chapter. Nonetheless, as part of contextualizing the results, it is useful to present results in this section related to "time on task" mathematics, since this relates to quantity and nature of discourse as prompted by the task itself. In calculating the number of student turns that could be coded as "off task" in relation to mathematics, a number of considerations were taken into account. First, students often uttered a few

turns (i.e., 1 to 3) that were interspersed within a longer "on-task" interaction and that I did not consider to interrupt the flow or intention of otherwise mathematically "on-task" talk; therefore, I counted "off-task episodes" as comprising 4 or more turns, which seemed to accurately reflect the overall nature and intention of those interactions (i.e., as being "off-task"). Second, as mentioned in Chapter 3, I find it difficult to make adequate judgement calls about what constitutes "off-task" mathematics, since the storylines (Herbel-Eisenmann et al., 2015) and figured worlds (Gee's, 2014) that students bring into the classroom intersect with and affect the mathematics at hand (this is discussed in more detail in the final chapter of this dissertation, Chapter 5). However, for the purposes of the reporting of these results, an episode was considered "off task" if it had no clear links to either the mathematics task at hand or the activities of the mathematics classroom. Third, "off-task" talk was difficult to transcribe, and therefore document, at times because some groups engaged in it more covertly than others. For example, some participants would sometimes whisper or cover the recording device, therefore making it impossible to hear what was being said (the conversation could have been "off task"). One group (D4) stopped the recording device on day 2 of the activity and spent the subsequent 20 minutes of the class engaged in off-task behaviour and, presumably, off-task discussions (researcher fieldnotes, March 18). Finally, group dynamics (e.g., how well the members knew each other) and the differences in individual characteristics (e.g., overall level of verbosity and personality of the participants) also affected the occurrence of "off-task" episodes but not necessarily the quality of the "on-task" mathematics (i.e., two chatty students who know each other well might go "off task" more often, but might also engage in just as much "on task" mathematics as another, less-chatty pair). With

these considerations in mind, Table 12 shows the results related to students' "on-task" and "off-task" talk with regard to mathematics. As noted earlier in this section, "time on task" in terms of L1 and L2 will be addressed in a later section of this chapter as it relates to research question #3.

Table 12

Total Number of Mathematically "Off-Task" Episodes, and Number and Percentage of Mathematically "Off-Task" Turns in Comparison to Total Number of Student Turns During the Mathematics Problem-Solving Task

Pseu-donym	Group (Letter, Period)	Program (Early or Late)	Total # of Student Turns	Total # of Mathematically "Off-Task" Episodes	Total # of Turns in Mathematically "Off-Task" Episodes (% of total turns)
Mac Mike	E1 E1	EFI EFI	111	0	0 (0.0)
Jane Beth	M1 M1	EFI EFI	515	2	17 (3.3)
Dan Zac	D4 D4	EFI EFI	224	0	0 (0.0)
Kim Eve	H4 H4	LFI LFI	716	12	121 (16.9)
Amy Hali Ella[a]	E4 E4 E4	EFI EFI LFI	549	10	174 (31.7)
Scot Ava	L1 L1	EFI LFI	639	5	67 (10.5)
Sue Liz	L4 L4	EFI LFI	223	0	0 (0.0)
Sara Brit Bea	H1 H1 H1	EFI LFI EFI	862	8	102 (11.8)
Mae Max Mya	K4 K4 K4	LFI EFI LFI	214	1	9 (4.2)

Overall, the results based on analysis of the transcripts of the classroom interactions show that students were mainly "on task," that is, their interactions dealt mostly with the mathematics task at hand and/or with other aspects related to the general

activity of the mathematics classroom. This is corroborated by informal interview data from the participating teacher, and in researcher fieldnotes where it was noted that the participating teacher and I observed that, overall, most students' behaviours and discussions appeared to be mathematically "on task" over the course the mathematics activity (researcher fieldnotes, March 17, 18, 22).

Typical of any classroom, however, in which students' learning preferences, goals, and needs vary (sometimes significantly) from one to the other, the Planning a Playground activity did not appeal to all students. While most participants completed the activity, it was apparent during the classroom observations (researcher fieldnotes March 17, 18) and from the follow-up interviews that some students did not appreciate the activity as much as others and that some went "off task" more than others. This was evident in their lower levels of engagement in the task as well as the nature (and/or quantity) of the discourse they produced. The three participants who seemed to enjoy the activity the least were all in the same group (H1) and made the following comments during the classroom-based audio recordings, which expressed their displeasure and/or frustration at various times during the activity: "**This is dirty and gross," "Oh, fuck! Wait, can you, like, hold that piece of paper? . . . Oh my God!**" and "**This is really hard.**" This represents somewhat of a pattern of disengagement or demotivation for this particular group. Its members, Sara, Brit, and Bea, had some of the higher numbers of student turns (218, 286, and 358, respectively, for a total of 862 for the group) among all of the participants. However, in contrast to all other groups but one (group E1, Mac and Mike), H1's members also chose to speak in English most of the time—86.7% of the group's turns were in English and a further 7.1% were codeswitched, for a total of only

6.2% of turns spoken in French—which could explain why they spoke more than most of the other groups (more details regarding target language-related time on task is discussed later in this chapter in reference to research question #3). Approximately 11.8% of the groups' turns were found to be mathematically "off-task," which is third highest of all the pairs/groups. It bears mentioning that utterances such as the three shown above were counted as on task, because they related or referred directly to the mathematics task at hand. Also with regard to group H1's pattern of disengagement, this group, far more than any other, seemed distracted by the presence of the recording device as evidenced by their (particularly Sara's) frequent episodes of talking into it in a "Blair Witch Project"-style narrative, for example: "**It's day one in the library. We're stranded here. Seventeen more minutes. We don't have much time**."

Despite what appeared to be group H1's frustrations with the playground activity in particular, and perhaps its overall disenchantment with mathematics and FI in general, the group's comments cannot be dismissed as complete outliers. Other participants in other groups also brought to light some concerns or alternate preferences they had with regard to hands-on, interaction-based mathematics tasks such as the playground activity, even when their overall feedback about the activity was positive. Returning to the interview with Liz for example, cited in the previous section describing how she liked the interactive discussion-based nature of the task, she went on to describe some challenges that can go along with working with a partner. Her partner, Sue, was an EFI student with high mathematics achievement (marks in the high 90s) and similarly expressed her own concerns with discussion-based mathematics tasks, even though she had also stated that she enjoyed the task overall (MC is Madame Culligan, i.e., me):

MC: *Est-ce que vous avez tendance à discuter beaucoup quand vous faites les mathématiques? Ou est-ce que vous préférez juste travailler individuellement?*

Liz: *J'aime les deux. Comme, quand tu es individuellement c'est plus vite et tu sais quoi faire, mais quand tu travailles avec les autres personnes tu peux avoir des autres idées et des autres, comme, perspectives donc ça aide aussi.*

Sue: *Je trouve que je juste comprends beaucoup de choses parce que je dois pas vraiment discuter pour comprendre. Donc je pense si j'avais de la difficulté à comprendre je discuterais plus.*

(MC: Do you tend to discuss a lot when you're doing mathematics? Or do you prefer just working individually?

Liz: I like both. Like, when you are individually it's faster and you know what to do, but when you work with other people you can have other ideas and other, like, perspectives so that helps too.

Sue: I find that I just understand a lot because I don't really discuss to understand. So I think if I had difficulty understanding I would discuss more.)

With regard to quantity and nature of discourse, Sue and Liz produced some of the lower numbers of student turns (with 114 and 109 respectively, for a total of 223 for the pair) but with no mathematically "off-task" episodes, which supports the statements they made in their follow-up interview.

In a different interview, with group E4, Ella, a very high achieving LFI mathematics student with marks of 98% and 100% in semesters 1 and 2 respectively, stated that the "certainty" of mathematics was why she preferred it to the more "subjective" school subjects. For Ella, discussion of a subjective nature, and therefore a

task such as the playground activity, was not a preferred component of doing mathematics:

> *En mathématiques tu peux avoir un 100%, donc si tu fais une équation, là il y a une réponse, si la réponse n'est pas correcte c'est pas correct, mais en anglais si tu écris une histoire ça fait, ça ne peut pas être 100%.*
>
> (In mathematics you can have 100%, so if you do an equation, there is an answer, if the answer isn't right it's not right, but in English if you write a story it's like, it can't be 100%.)

Ella's group (E4) had the highest number of mathematically "off-task" turns at approximately 31.7% of the total number of student turns.

Student engagement in the mathematics problem-solving task and the discourse they produced are closely connected. Moreover, these two elements are enacted within the mathematics embedded in the task itself. Therefore, an important factor in contextualizing the remainder of this results chapter also lies in providing an overview of how the "mathematics" itself unfolded, in particular, connecting back to the problem-solving strategies presented in Chapter 3 in order to look at how the student pairs/groups handled the task overall.

4.1.3 Mathematics Problem-Solving Strategies

In Chapter 3, part of the presentation of the materials for the study included a discussion of potential problem-solving strategies students might use in order to work through the mathematics task. The nature of the task along with the written follow-up questions on the worksheet provide the framework within which students produced their collaborative dialogue. Table 13 shows the problem-solving strategies and final solutions

for each participant pair/group, as identified via analysis of the audio recordings, written work, and follow-up interviews.

Table 13

Program, Academic Achievement, Problem-Solving Strategies, and Final Solutions for Each Participant Group

Pseudonym	Group (Letter, Period)	Program (Early or Late)	FILA 9 (Mark in %)	Math 9A, 9B (Marks in %)	Problem-Solving Strategy(ies) Observed	Final Solution
Mac	E1	EFI	81	95, 99	Symmetry, Guess & Check, Using a Model (50 x 50); Scale 1:2	50 x 50, although during modelling discovered equipment did not fit, discussed 51 x 49 but changed it to 50 x 50 "because it was too much to write to explain"
Mike	E1	EFI	78	81, 97		
Scot	L1	EFI	85	91, 92	Guess & Check, Symmetry, Using a Model (66 x 34, 53 x 47, 52 x 48, 50 x 50); Scale 1:2	51 x 49, found 50 x 50 to be the maximum area initially and were successful in determining the necessary adjustment for the equipment
Ava	L1	LFI	71	76, 70		
Jane	M1	EFI	76	90, 94	Guess & Check, Using a Model (63 x 37, 50 x 50, 51 x 49); Scale 1:2	50 x 50, although during modelling/drawing discovered equipment did not fit, discussed 51 x 49 but changed it to 50 x 50 "because that was the maximum area therefore 51 x 49 could not be right"
Beth	M1	EFI	92	87, 93		
Sara	H1	EFI	81	82, 88	Symmetry, Guess & Check, Using a Drawing (50 x 50); Scale 1:2	50 x 50, although they acknowledged their drawing was "off" but moved ahead because it
Brit	H1	LFI	65	60, 72		
Bea	H1	EFI	72	84, 85		

						was “no big deal”
Pseudonym	Group (Letter, Period)	Program (Early or Late)	FILA 9 (Mark in %)	Math 9A, 9B (Marks in %)	Problem-Solving Strategy(ies) Observed	Final Solution
Kim	H4	LFI	84	82, 87	Guess & Check, Using a Model (60 x 30, 50 x 50); Scale 1:1	50 x 50, although during modelling/drawing discovered equipment did not fit, kept 50 x 50 because it was “good enough”
Eve	H4	LFI	90	94, 94		
Amy	E4	EFI	87	86, 91	Guess & Check, Symmetry, Using a Drawing; 50 x 50; Scale 1:2	50 x 50, although during modelling/drawing had problems with equipment not fitting, ignored
Hali	E4	EFI	89	93, 93		
Ella	E4	LFI	100	98, 100		
Dan	D4	EFI	72	85, 90	Symmetry (square immediately), Using a Drawing; Scale 1:1	50 x 50, although during drawing had problems with equipment not fitting, ignored
Zac	D4	EFI	82	92, 92		
Sue	L4	EFI	96	97, 97	Guess & Check, Symmetry; Using a Model (50 x 50, 52 x 48); Scale 1:2	52 x 48, found 50 x 50 to be the maximum area initially and were successful in determining the necessary adjustment for the equipment
Liz	L4	LFI	93	94, 94		
Mae	K4	LFI	85	72, 77	Guess & Check, Symmetry, 50 x 50; Scale 1:1	50 x 50, no acknowledgement of fit problems
Max	K4	EFI	70	87, 87		
Mya	K4	LFI	83	86, 91		

As shown in Table 13, the pairs/groups’ performance on the task varied in terms of problem solving, choice of scale, and final solutions.

To illustrate the most successful completion of the task of all the pairs/groups, accomplished by Liz (LFI) and Sue (EFI) of group L4, these qualitative data highlight the

key problem-solving strategies and final solutions from their collaborative dialogue and written work. Mathematically speaking, this pair achieved all of the targets for the problem-solving task. While working with the large-scale models on day 1, Liz and Sue had the following exchange:

37 Sue: *Donc, il faut. On peut faire plus étroit. On peut faire comme sept? Est-ce que ça va?* (So, we have to. We can do straighter. We can do like seven? Does that work?

38 Liz: *Combien de (?) est-ce que tu fais?* (How many [?] are you doing?)

39 Sue: *Je ne sais pas. C'est cinquante-deux et quarante-huit*. (I don't know. It's fifty-two and forty-eight.)

40 Liz: *Cinquante-deux?* (Fifty-two?)

41 Sue: *Je pense. Cinquante-deux par quarante* . . . (I think. Fifty-two by forty . . .)

42 Liz: *Oui, je pense.* (Yes, I think.)

43 Sue: *Comme ça?* (Like this?)

44 Liz: *Oui.* (Yes.)

45 Sue: *Je pense donc c'est cinquante-deux par quarante-huit. Je pense ça c'est tout bon.* (I think then that it's fifty-two by forty eight. I think that it's all good.)

46 Liz: *Oui. OK donc ça va être la même chose il faut juste changer les nombres.* (Yes. OK so it's going to be the same thing we just have to change the numbers.)

47 Sue: *Je pense que c'est car l'équipement est plus long que c'est large.* (I think it's because the equipment is longer than it is wide.)

48 Liz: *Oui.* (Yes.)

In this dialogue, Sue took the lead on making the adjustments to the dimensions (turns 39, 41, 45) but looked to Liz often for affirmation, which Liz supplied (turns 42, 44, 46, 48). The pair was able to adjust their correct finding for the initial maximum area (from 50 x 50 to 52 x 48) to reflect the real-life mathematical problem of fitting the equipment.

Their explanation was given as follows in their written work, which was clearly based on their collaborative dialogue that had occurred in class:

> Liz: *L'équipement est plus long que c'est large dans la façon qu'on avait positionné donc on avait besoin a changer 50 x 50 a 52 x 48.* (The equipment is longer that it is wide the way in which we positioned it so we had to change 50 x 50 to 52 x 48.)
>
> Sue: *L'équipement est plus long que c'est large, donc le cour de récréation doit aussi être plus long que c'est large.* (The equipment is longer that it is wide, so the playground also has to be longer than it is wide.)

Moreover, Liz and Sue chose an appropriate scale factor for their scale diagram (Figure 6): The graph paper measured only 34 squares across at its longest, therefore a 1:1 scale (1 square = 1 foot) would become quite cumbersome, requiring students to try to tape four pieces of graph paper together and resulting in more inaccuracies and errors. Anything smaller than a 1:2 factor would become too small to work with, therefore the scale of 1 square = 2 feet was ideal.

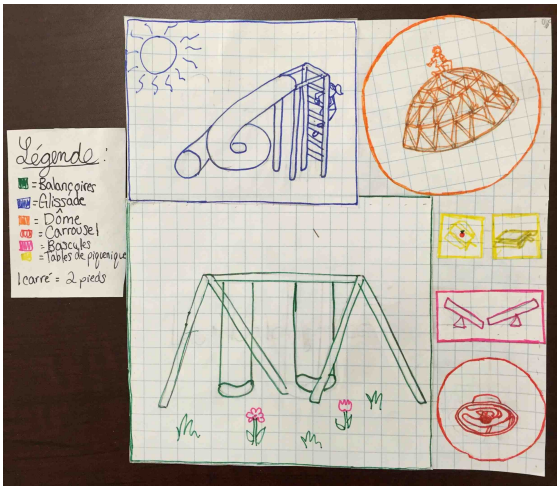

Figure 6. Liz and Sue's scale diagram.

Finally, in the post-hoc interview, I was able to pursue Liz and Sue's reflections on their problem-solving strategies and thought processes by asking them about their work on the mathematics task, using their written work as a stimulus:

> Sue: *J'ai, je commence avec ça (le carré) parce que je pouvais faire dans ma tête, donc je pouvais savoir où je commençais, puis essayer les autres pour savoir s'ils étaient tous plus petits.* (I had, I start with that [the square] because I could do in my head, so I could know where I started, and try the others to know if they were all smaller.)
>
> MC: *Alors avez-vous commencé tout de suite avec le carré?* (So did you start right away with the square?)
>
> Sue: *Je pense que oui.* (I think so.)
>
> MC: *Et ensuite vous avez vérifié quelques autres?* (And then you checked a few others?)
>
> Sue: *Oui, on n'avait pas, comme, fait tout le travail et après ça dit, comme, « oh, comme, on a oublié que ça c'est plus grand ». C'était pour juste, comme, vérifier*

on a la vraie forme. (Yes, we didn't, like, do all the work and then after that say, like, "oh, like, we forgot that that is the biggest." It was to just, like, verify we had the true shape.)

MC: *As-tu pensé au carré tout de suite au début toi aussi, Liz?* (Did you think of the square right away in the beginning too, Liz?)

Liz: *Oui parce que c'est une forme simple et c'est aussi facile à déterminer l'aire, donc, oui.* (Yes because it's a simple shape and it's also easy to find the area, so, yes.)

MC: *Et après ça je peux voir ici à côté, vous êtes allées à quelques rectangles après ça pour voir. Avez-vous beaucoup pensé à ça? Ce phénomène que l'aire maximale ne peut quand même pas accommoder tout l'équipement?* (And after that I can see here on the side, you went to a few rectangles after that to see. Did you think about that a lot? This phenomenon that the maximum area actually can't accommodate all the equipment?)

Sue: *Oui, j'ai pas pensé avant que, je n'aurais jamais pensé à ça si je n'avais pas fait. Je dirais juste « oui c'est la plus grande donc toutes les choses vont aller ».* (Yes, I didn't think before that, I would have never thought of that if I hadn't done it. I would just say, "yes, that's the biggest so all the things are going to go.")

Liz: *Oui, c'est parce qu'il y a les différentes formes, et des cercles et des carrées. Toute l'espace n'est pas occupée mais tu ne peux pas utiliser des petits espaces comme, ici.* (Yes, it's because there are the different shapes, and the circles and the squares. All the space isn't occupied but you can't use the little spaces like, here.)

Sue: *Et (le carré) c'est comme ju::ste pas assez grand.* (And [the square] it's like ju::st not big enough.)

The audio recordings of the in-class collaborative dialogue, the written work, and the audio recordings of the follow-up interview provide a rich, detailed description of this group's problem-solving processes and final solutions. In this case, the students not only successfully accomplished the various parts of the mathematics problem-solving task but also provided clear evidence of their success (via their dialogue, written work, and follow-up interview reflections). For group L1 (Scot and Ava), the trajectory was similar (the main notable difference being that their final dimensions were 51 x 49, a small mis-measurement due to margin of error).

Not all cases were as easy to follow with regard to students' strategies and final products. Mathematically speaking, all the remaining pairs/groups were able to succeed in constructing a scale diagram (although two used a 1:1 scale, which was not the most efficient choice). All the remaining pairs/groups found success in determining that the maximum area for a given perimeter of 200 is created with a 50 x 50 square with A = 2500. Students confronted the next phase of the problem with varying degrees of achievement. Group K4 never discussed having a problem with fitting the equipment in their square. Group M1 discussed the fit problem, thought that dimensions of 51 x 49 would work better, but ultimately could not get beyond the logic that the equipment had to fit into the square because it was the figure with the maximum area (thus choosing to keep the 50 x 50 fence even though it did not work). In two other groups (E4 and D4), the fit problem was discussed but then ignored (i.e., the group allowed the paper models of the equipment to overlap each other and/or the paper clip fence; and/or the group allowed

the lines representing the pieces of equipment and/or the fence were allowed to overlap on the scale diagram drawing). And, in the remaining three groups (H1, H4, and E1), students chose not to adjust their dimensions because they felt it was "no big deal" (H1), "good enough" (H4), or "a lot to write" (E1). These types of discourses (or perhaps more accurately, Discourses) illustrate the complexity of "doing mathematics" within the classroom context; it is not a disembodied enterprise. For instance, Mac and Mike (both EFI students, group E1) discussed the following during their in-class collaboration on day 1:

39 Mike: **Yeah. So put everything in here. To accommodate all the equipment it needs to be changed to forty-nine by fifty-one. Because of the size that the swing set needs. You need a rectangle because it's, you can't put them in a square because they cross the outside fence.**

40 Mac: **Yeah, I don't know what you're writing. "You can't put them in a square because . . ."**

41 Mike: **Because of the size of the** *dôme* **and because of the size of the** *balançoire*. **The** *dôme* **has to go in the corner, and the** *balançoire* **needs almost the whole side-**

42 Mac: **So that only-**

43 Mike: **So that only leaves room for the** *glissade* **and the, um,** *bascules*. **So you have to change it to forty-nine by fifty-one to make everything fit. But that is a lot to write and I don't want to write it.**

44 Mac: **How about we did**, *« Toute l'équipement, l'aire a changé à quarante-neuf par cinquante-et-un parce que, ah, les balançoires sont presque un côté,*

et le dôme doit être dans le coin, donc ça laisse un petit peu d'espace pour le glissade et les bascules. » ("All the equipment, the area changed to forty-nine by fifty-one because, ah, the swings are almost one side, and the dome has to be in the corner, so it leaves a little bit of space for the slide and the teeter-totters.")

Clearly, Mac and Mike, particularly Mike, had figured out during this collaborative dialogue that the 50 x 50 square was not working during their large-scale modelling of the problem (turns 39, 41, 43). Mike gave an oral explanation for why that was the case (turn 39) but then stated that he did not want to write it all out (turn 43). Mac built on Mike's oral explanation by providing a written alternative (turn 44), which, incidentally was given in the L2, French, whereas the rest of the exchange occurred in the L1 (English, except for the names of the playground equipment items, which were given in French).

This pair was clearly able to understand the task and provide explanations for the results they were noticing through their collaborative dialogue. However, on their written work, only the first two questions were answered, which only addressed the purely mathematical result of maximum area (50 x 50). The questions regarding the necessary adjustments made due to the equipment were left blank. Thus, if based on written output alone, there is no evidence of group E1's problem-solving strategies or their understanding of the task; the audio data, however, shed more light on how they undertook the task and their choice to leave the question sheet blank. I was unable to elicit much more information during the follow-up interview, as the following excerpt illustrates:

MC: *Alors vous m'avez dit que vous n'avez pas beaucoup discuté de votre travail. Je vois que vous avez trouvé un carré pour votre clôture, 50 par 50, pouvez-vous me dire, ça c'est les questions là à l'autre page. Comment est-ce que vous êtes arrivés à cette réponse-là? Pouvez-vous m'en parler de ça? Comment avez-vous trouvé ce carré-là?*

Mac: *Je oublie.*

MC: *Tu oublies?*

Mac: *Oui.*

MC: *Est-ce que le carré a été la première figure que vous avez essayé?*

Mac: *Oui.*

MC: *Oui. Pourquoi? Pourquoi avez-vous commencé avec le carré?*

Mac: *Parce que c'est un carré.*

Mike: *C'est comme, c'est pas, c'est comme,* ***even****. Je ne sais pas en français, ça.*

(MC: So you told me that you didn't discuss your work a lot. I see that you found a square as your fence, 50 by 50, can you tell me, that is the questions there on the other page. How did you come up with that answer? Can you talk to me about that? . . . How did you find that square?

Mac: I forget.

MC: You forget?

Mac: Yes.

MC: Was the square the first shape you tried?

Mac: Yes.

MC: Yes. Why? Why did you start with the square?

Mac: Because it's a square.

Mike: It's like, it's not, it's like, **even**. I don't know in French, that.)

In addition to the key mathematical concepts of finding maximum area, choosing appropriate scale factors and drawing scale diagrams, and understanding the real-life problem of adjusting the fence dimensions to accommodate the equipment, another key mathematical idea emerged during the task. For some students, a new mathematical discovery was made: that changing the dimensions of the rectangle given a fixed total perimeter would change the resulting area of the given figure. This discovery was clearly identified in the collaborative dialogues of two groups, H4 and M1. For example, in group H4, Eve (LFI) thought that the areas would be the same regardless of the dimensions, whereas her partner, Kim (LFI) expressed her disagreement:

6 Eve: *Le périmètre est deux cent? . . . . Uh, c'est comme, c'est comme à peu près trente. C'est trente, soixante. . . .* (The perimeter is two hundred? . . . . Uh, it's like, it's like about thirty. It's thirty, sixty.)

11 Kim: *OK.*

12 Eve: *Mais l'aires sont les même. Si, non-* (But the areas are the same. If, no-)

13 Kim: *Non.* (No.)

14 Eve: *C'est le même si tu changes le-* (It's the same if you change the-)

15 Kim: *Non.* (No.)

At one point, Eve seemed to hedge her claim that the areas of the rectangles would be the same regardless of the dimensions (turn 12, "*Si, non-*"), and Kim was adamant that they would not (turn 13). Despite her apparent hedging, Eve restated the idea (turn 14), but

was again immediately shut down by Kim (turn 15). Eventually, the pair worked out that the different dimensions do result in different areas, moving on to the 50 x 50 configuration during the large-scale modelling phase and thus finding the maximum area. They subsequently had problems fitting the equipment into the square, but decided things were "good enough" and did not change the dimensions, staying with the 50 x 50 measurements.

On their written work, Kim and Eve each produced the following:

Kim: *A = 50 x 50 = 2500 p^2. On a fait 200 ÷ 4 = 50.*

Eve: *A = 50 x 50 = 2500 p^2. Ont fait 200 ÷ 4 pour trouvé 50 par 50.*

(Kim: A = 50 x 50 = 2500 p^2. We did 200 ÷ 4 = 50.

Eve: A = 50 x 50 = 2500 p^2. We do 200 ÷ 4 to found 50 by 50.)

Thus, on the written work these students each showed their final solution for maximum area, but neglected to explain their discovery (or at least, Eve's discovery) that different dimensions produce different areas, and that they had initially tried a different configuration (i.e., 30 x 60). However, during their follow-up interview the students reflected more on their process:

MC: *Comme, vous avez décidé de faire un carré de cinquante par cinquante, comment est-ce que vous avez décidé de . . . ?*

Eve: *Il y a les deux cent trombones et on a juste pensé comme, qu'il doit avoir quatre côtés donc on peut juste . . .*

Kim: *Oui ça c'est la meilleur, je dit « on peut juste diviser par quatre » donc on fait cela. Il fait, comme, des rectangles et il y a pas d'espace pour l'autres choses de mettre dedans.*

Eve: *Oui et on sait il y a d'autres options que ça, mais aussi c'est que si il y a un rectangle comme ça, le grand cercle et le grand carré ne pas mettre ensemble, donc on pense ça c'est le meilleur choix.*

(MC: Like, you decided to do a fifty by fifty square, how did you decide to . . . ?)

Eve: There are the two hundred paper clips and we just thought, like, there have to be four sides so we can just . . .

Kim: Yes that was the best, I say, "we can just divide by four" so we do that. They make, like, rectangles and there is no space for other things to go in.

Eve: Yes and we know there are options other than that, but also it's that if there is a rectangle like that, the big circle and the big square can't go together, so we think that is the best choice.)

In the interview, Eve and Kim connected the problem of fitting the large-scale paper models into their small rectangular fence to their eventual discovery of the maximum area contained within the square.

The same type of mathematical discovery was made in Beth and Jane's group (M1, both EFI students), during the following exchange:

20 Beth: *OK, donc, ça devrait être trente-sept mais est-ce que c'est le, l'aire maximale? Ça va donner le même nombre, mais je ne fais pas si, ça donne le même nombre si tu multiplies mais, ce n'est ce, ce n'est pas . . . OK, donc, est-ce que . . . ?* (OK, so, it should be thirty-seven but is that the, the maximum area? It will give the same number, but I don't do it if, it gives the same number if you multiply but, it's, it's not . . . OK, so, is it . . .?)

21 Jane: *Je pense que ça serait bon si on mis tout les* . . . (I think that that would be good if we put all the . . .)

22 Beth: *Parce que quand même c'est le même, c'est le même nombre.* (Because after all it's the same, it's the same number.)

23 Jane: *Ou:::::i?* ((uncertain)) (Ye::::::s?)

24 Beth: Multiplie. *Donc ça devrait être dans la maximale.* (Multiply. So it should be in the maximum.

25 Jane: *Ou::i.* (Ye::s.)

At the beginning of this exchange, Beth began by talking things through, grappling with her own thoughts about the mathematical situation of changing dimensions within a fixed perimeter and how that might affect the given area (turn 20). Jane expressed her own uncertainty about the topic (turns 21, 23, 25). Later, around turn 60, the pair began representing the playground with the large-scale paper models and realized that the model pieces of playground equipment would not fit within their 63 x 37 paper clip fence:

62 Beth: *Um, est-ce que cette partie est dans notre . . . Je ne pense pas que ça va être* . . . (Um, is this part in our . . . I don't think it's going to be . . .)

63 Jane: *Assez d'espace?* (Enough space?)

64 Beth: *Oui, c'est* . . . (Yes, it's . . .)

65 Jane: *Je pense pas que ça va.* **It's OK.** *Non, c'est pas. Oh, oh!* (I think it's all right. **It's OK.** No, it's not. Oh, oh!)

66 Beth: *Ça ne va pas dans le coin. Donc on a besoin de changer notre balançoire. Tu peux faire (?). Oh, non. Je pense pas que ça* **fit**. *Donc on a besoin de changer notre dimension parce que cette . . . trop espace . . .*

n'est pas. (It's not working in the corner. So we have to change our swings. You can do (?). Oh, no. I don't think it **fits**. So we have to change our dimension because this . . . too much space . . . it isn't.)

67 Jane: *Ç'a pas vraiment fonctionné.* (It didn't really work.)

68 Beth: *Um, donc* . . . (Um, so . . .)

69 Jane: *Les balançoires sont gigantesques.* (The swings are gigantic.)

70 Beth: *OK, on va changer nos dimensions.* (OK, we're going to change our dimensions.)

Thus, through the modelling phase Beth and Jane discovered that, indeed, changing the dimensions did have an effect on the area.

Beth explained their process on her written work (which she wrote on behalf of the pair), placing emphasis on how the pair used the hands-on models to find their dimensions:

On a trouvée que si on utilise plusieurs façons avec l'équipement, on peut trouvée comment grand ça besoin d'être. Si on n'a pas mit l'équipement en avance, ça pourrait pas fonctionner.

(We found that if we use many ways with the equipment, we can find how big it needs to be. If we did not put the equipment first, it would not work.)

In addition, both Beth and Jane reflected on their mathematical problem-solving processes during the follow-up interview:

Jane: *Oui, on pensait que parce que c'était comme deux cent trombones que ce serait comme le même air que n'importe quoi qu'on faire.*

Beth: *Ce n'est pas le cas. Comme si tu veux mettre . . . comme, comme si tu fais*

une boîte et tu veux mettre le plus de choses possibles dedans, tu as besoin de mettre dans un spécifique ordre ou comme spécifique mesure, ou tu vas comme gaspiller l'espace.

Jane: *Comme je pensais si c'était, comme, tous les mêmes périmètres ça serait comme les mêmes aires, mais, oui.*

(Jane: Yes, we thought that because it was like two hundred paper clips that it would be like the same area as anything else we would do.

Beth: It is not the case. Like if you want to . . . like, like if you do a box and you want to put as many things as possible in it, you have to put in a specific order or like a specific measure, or you're going to like waste space.

Jane: Like I thought if it was, like, all the same perimeters it would be like the same areas, but, yeah.)

Thus, although Beth and Jane did not ultimately change their dimensions to the correct 52 x 48 configuration, they both made important mathematical discoveries about the link between fixed perimeter, dimensions, and area, as expressed through their collaborative dialogue, written work, and follow-up interview.

The nature of the mathematics problem-solving task used in this study and its effects on student engagement and the quantity (and nature, i.e., "on-task" vs. "off-task") of discourse produced were not topics of exploration laid out in the research questions per se. Neither was the type of problem-solving strategies that might be used in order to complete the task. However, it became clear during both the data collection and analysis phases of this study that the choice of task had a substantial effect on student participation in and perception of the task, and on the mathematics that took place within

it. It also, most importantly, had an effect on the kinds of discourses that participants produced and provided the context within which their interactions (and written work and follow-up interviews) took place. The task itself required the use of particular types of problem-solving strategies and vocabulary, and resulted in particular solutions. Because the problem-solving task was based on mathematical content, language was produced and situated in an overarching context of "doing mathematics." Consequently, I found it important and useful to devote space in this first section of the chapter to presenting the key emergent themes regarding these task-related results. Next, I present results related in a more direct way to the three research questions that were posed at the outset of this inquiry.

4.2 Research Question 1: Language-Related Episodes

In this section of Chapter 4, I present the results of the discourse analysis that relate to research question #1: How do FI students at the secondary level attend to language, that is, what language-related episodes (LREs) emerge, while working collaboratively during mathematical problem solving in their L2 (French)?

In the reporting of these results, two student groups are omitted: group E1 (Mac and Mike) and group H1 (Bea, Brit, and Sara); these same two groups are also omitted from the results related to research question #2. This omission is due to the fact very few of the group members' turns occurred in the L2. For instance, only approximately 22.5% of group E1's turns occurred in French; for group H1, only 6.3% of turns occurred in French. Since the aim of research questions #1 and #2 is to investigate how students attend to language and communicate mathematically in their L2, these two data samples, which comprise mainly English (or codeswitched) turns, were omitted. Groups E1 and

H1 represent outliers in the data with regard to this phenomenon, as, for the other groups, percentage of turns in French ranged from 56.0% to 95.5%, with a mean percentage of 76.7%. These two groups were, however, included in the reporting of task-related results in the previous section and they will be addressed in the section of this chapter devoted to research question #3.

Overall, for the seven student groups analyzed, results show that there were LREs present in students' collaborative dialogue as they worked on mathematical problem solving in their L2. The salient LREs for each student group were coded as shown in Table 14. Table 14 shows only the LREs occurring completely in the L2. There were many instances of LREs in which the L1 was used, and these will be reported on in response to research question #3. Thus, with regard to individual participant pairs/groups, they produced the following total number of LREs in the L2: M1 = 18, L1 = 14, H4 = 11, L4 = 8, E4 = 7, K4 = 6, D4 = 5.

Regarding the total number of L2 LREs produced by the participants for each category, the results are reported in Table 15. As per Table 15, a total of 69 LREs were identified that occurred entirely in students' L2; of these, 35 (50.7%) were lexis-based, 18 (26.1%) were form-based, and 16 (23.2%) were lexicogrammatical. Overall (for lexis-based, form-based, and lexicogrammatical LREs), 39 (56.5%) were unrelated to mathematics, while 30 (43.5%) were mathematical (either academic or non-academic).

Table 14

Language-Related Episodes (LREs) Occurring in the Second Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (by Student Group)

	Lexis-Based	Mathematical-Lexis-Based	Non-Academic-Mathematical-Lexis-Based	Form-Based	Mathematical-Form-Based	Non-Academic-Mathematical-Form-Based	Lexicogrammatical	Mathematical-Lexicogrammatical	Non-Academic-Mathematical-Lexicogrammatical
Meaning-Making Activity									
Generating Alternatives	M1(4); L1(2); D4	K4(2); L4(2); H4; M1		M1(2)	H4; K4; M1	L1			L1(3); M1
Assessing Alternatives	M1(3); L1	H4; L4		M1(2)	H4; M1	L1			L1(2); M1
Applying Rules to New L2 Context	L1(2)								
Other Occasions for L2 Learning									
Correction	H4(4); L4(4); M1(2); K4	L4		E4(5); H4	L1		H4; K4		D4(2); E4; L1
Uptake	E4; K4						H4		D4(2)

Note. When more than one occurrence was coded for a particular pair/group, the number of occurrences is given in parentheses.

Although LREs were interspersed throughout the discourse over the course of the 3-day mathematics activity, higher concentrations of LREs were observed during two phases in particular: (a) when students were faced with trying to work through and explain why the playground equipment was or was not fitting into their particular fenced perimeter and (b) when students were discussing and then responding in written form to the questions on the task worksheet—especially those that asked them to explain their process and/or reasoning with regard to various aspects of the mathematics problem-solving task (e.g., why there are differences between maximum possible area of an empty playground vs.

one with all the equipment, reasons for choice of scale, reasons for why their playground layout worked).

Table 15

Language-Related Episodes (LREs) Occurring in the Second Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (Totals)

	Lexis-Based	Mathematical-Lexis-Based	Non-Academic-Mathematical-Lexis-Based	Form-Based	Mathematical-Form-Based	Non-Academic-Mathematical-Form-Based	Lexicogrammatical	Mathematical-Lexicogrammatical	Non-Academic-Mathematical-Lexicogrammatical
Meaning-Making Activity									
Generating Alternatives	7	6		2	3	1			4
Assessing Alternatives	4	2		2	2	1			3
Applying Rules to New L2 Context	2								
Other Occasions for L2 Learning									
Correction	11	1		6	1		2		4
Uptake	2						1		2

Of the 35 L2 LREs that were lexis-based, most fell into the category of Generating Alternatives. A total of 13 LREs were coded as Generating Alternatives: 7 of these were Non-Mathematical-Lexis-Based; while 6 were Academic-Mathematical-Lexis-Based. Further, in 6 of those 13 cases of Generating Alternatives, follow-ups could be traced directly to students' act of choosing a particular lexical item within the oral dialogue and thus coded as Assessing Alternatives. Of these 6 instances of Assessing Alternatives, 4 were Non-Mathematical-Lexis-Based and 2 were Mathematical-Lexis-Based. The next highest instance of L2 lexis-based LREs fell into the Other Occasions for L2 Learning category, with students correcting themselves or others on 13 occasions

(12 Non-Mathematical-Lexis-Based, 1 Mathematical-Lexis-Based). Further with regard to lexis-based L2 LREs, students were observed Applying Rules to a New L2 Context in 2 cases (both Non-Mathematical-Lexis-Based) and lexis-based Uptake was also coded twice (both Non-Mathematical-Lexis-Based).

For LREs overall, that is, types of Meaning-Making Activity and Other Occasions for L2 Learning across the board (when adding together the cross-categories of Lexis-Based, Mathematical-Lexis-Based, Non-Academic-Mathematical-Lexis-Based, Form-Based, Mathematical-Form-Based, Non-Academic-Mathematical-Form-Based, Lexicogrammatical, Mathematical-Lexicogrammatical, Non-Academic-Mathematical-Lexicogrammatical), the subcategory of Correction (which falls under the Other Occasions for L2 Learning category) has the highest number of occurrences, with a total of 24 LREs. Generating Alternatives (under the category of Meaning Making) is a close second, with a total of 23.

In order to illustrate the most striking results, these descriptive quantitative data are augmented with qualitative examples. The narrative of the results is guided by the tools of inquiry as described in Chapter 3 and at the outset of this chapter.

I begin with an excerpt showing an example of the most frequently occurring LRE with regard to meaning-making activity in students' collaborative dialogue: In this excerpt from the classroom-based audio data, Beth and Jane (group M1, both EFI students) generate alternatives for a lexis-based LRE on day 1 of the activity:

111 Jane : *On peut juste, comme, additionner, comme, les choses,* **or whatever**.
(We can just, like, add, like, the things, **or whatever**.)

112 Beth : *Les choses?* (The things?)

113 Jane : *Les choses.* ((laughs)) *Comme un chose à chaque chose*. (The things. ((laughs)) Like one thing to each thing.)

114 Beth : *Des trombones?* (Paper clips?)

115 Jane : *Oui! Oui. Oh! Les trombones!* (Yes! Yes. Oh! The paper clips!)

In this collaborative dialogue, Jane replaced the missing lexical item (*trombones*) with "*choses*," an act which she drew attention to by adding "or whatever" immediately following the use of "*choses*" (turn 111) and also by adding emphasis to "*chose*" (indicated in the transcript with underlining, turn 113). Her uncertainty about the item was also underscored by the use of back channelling ("*comme*") and hedging ("or whatever"). Her partner, Beth, was sensitive to these signals, thus responding with "*choses?*" (turn 112). Beth then supplied an alternative to "*choses*," the correct lexical item "*trombones*", which was assessed by Jane as she excitedly (indicated with exclamation marks) confirmed the choice, "*Oui! Oui. Oh! Les trombones!*" (turn 115). In this example, the lexical item was non-mathematical, but was directly related to the mathematics task at hand (paper clips). Here, the students not only generated alternatives as they engaged in a vocabulary search, but they also assessed the alternatives by acknowledging the correct choice.

In this example with Beth and Jane, doing the task, rather than answering the written questions on the task worksheet, prompted the LRE. Consequently, there is no appearance of the term *trombone* in the students' written work. In the follow-up interview, however, it reappeared when I asked Beth and Jane (among other things) about how they accomplished the task and what they enjoyed about the task:

Jane: *C'est le même nombre de . . . qu'est-ce que c'est, le mot pour* **paper clip** . . . *c'est quoi . . . ?* (It's the same number of . . . what is the word for ***paper clip*** . . . what is it . . . ?)

Beth: *U:::::::h . . .*

MC: *C'est bizarre, ça resemble à un . . .* (It's weird, it looks like . . . ((makes gesture that shows the trombone-like shape of a paper clip))

Beth: *[Oh, un trombone.]* ([Oh, a paper clip.])

Jane : *[Un trombone!]* ([A paper clip!])

MC: *Oui!* (Yes!)

Jane: *Donc, qu'est-ce que je disais?* (So, what was I saying?)

MC: *Qu'il fallait, il y avait plusieurs différentes façons et . . .* (That you had to, there were many different ways and . . .)

Jane: *Oui, comme juste parce que c'était le même nombre de trombones, c'est comme aussi, comme, les différentes aires à la façon que tu fais.* (Yes, like just because it was the same number of paper clips, it's like also, like, the different areas in the way you do it.)

In the follow-up interview, Jane was describing what she enjoyed about the activity and, since she enjoyed the modelling with the paper clips, needed to retrieve the lexical item. Beth, who had supplied the item during the in-class collaborative dialogue, initially was unable to do so again during this interview 15 days post hoc. With scaffolding from me, via a verbal and gestural "hint" (i.e., drawing attention to the shape of the paper clip being like that of a trombone musical instrument), the item was retrieved simultaneously

by both Jane and Beth (indicated by overlapping speech in square brackets) and uptake occurred (*trombone* was used by Jane on two more occasions later in the interview).

In another example of meaning-making activity through the generation and assessment of alternatives, the students of group K4, Max (EFI), Mae (LFI), and Mya (LFI), engaged in an LRE that, this time, was mathematical-lexis-based, on day 1 of the activity:

78 Mae : *OK, qu'est-ce que-* (OK, what is-)

79 Mya : *Notre échelle-* (Our scale-)

80 Mae : *Oui, qu'est-ce que…?* (Yes, what is . . . ?)

81 Mya : *Est-ce que ça c'est qu'est-ce qu'on fait première semestre*? (Is that what we did first semester?)

82 Mae : **I dunno. (I dunno.)**

83 Mya : *Les facteurs d'échelle et* . . . (Scale factors and . . .) . . .

86 Max : *Hmmm.*

87 Mya : *OK.*

88 Mae : *Yeah. Est-ce que je peux, comme, demander?* (Yeah. Can I, like, ask?)

89 Mya : *On ne peut pas aller plus loin parce que j'oublier qu'est-ce que échelle* . . . (We can't go any further because I forget what is scale . . .)

90 Max : *Elle a juste, OK.* (She just, OK.)

91 Mae : *Diagramme à l'échelle. Est-ce que c'est comme . . . ?* (Scale diagram. Is it like . . . ?)

92 Max : *Est-ce qu'on va juste dessine ces?* (Are we just going to draw these?)

93 Mya : *Mae?* . . .

95 Mae : *Une réduction,* **like**, *comme, si c'est six mètres, si c'est comme cinq mètres, c'est, comme, plus petit?* (A reduction, **like,** like, if it's six metres, if it's like five metres, it's, like, smaller?)

In this exchange, students worked collaboratively through the vocabulary search of the academic mathematical item *échelle*. Students, particularly Mae and Mya, used prior knowledge ("*Est-ce que ça c'est qu'est-ce qu'on fait première semestre*?") and generated lexical alternatives ("*facteurs d'échelle*," "*diagramme à l'échelle*," "*réduction*," "*plus petit*") to help make meaning of the term. The exercise appears to have posed a significant challenge, since at one point (turn 88), Mae suggested that perhaps she should ask the teacher for help. This suggestion was not taken up, however, and, ultimately, it was Mae who provided many of the alternatives and finally an expanded definition of the mathematics-based lexical item in question (turn 95). Following this utterance, the group immediately began working on the scale diagram in question, therefore, although no verbal utterance of the "assessment" can be traced, the definition was indirectly chosen as correct or acceptable.

In students' written work on the task sheet questions regarding scale, it is clear that this meaning-making dialogue influenced their writing:

Max: *1 carré = 1 pieds, parce que c'est le plus facile de utiliser.* (1 square = 1 foot, because it's the easiest to use.)

Mya: *Ont utilise 1 carré pour un pied parce qu'il crée un taille raisonnable. Il ne pas plus grand ou plus petit.* (We used 1 square for 1 foot because it makes a reasonable size. It is not bigger or smaller.)

Mae: *1 carré = 1 pieds, parce que c'est la plus facile de utiliser et c'est le plus*

facile à comprendre. (1 square = 1 foot, because it's the easiest to use and it's the easiest to understand.)

In the written work, the students described their scale diagram as being a reduction in size of the actual playground measurements by using mathematical notation: *1 carré = 1 pied*. This succinct notation reflected the meaning making achieved through the LRE by generating alternative lexical items such as "*facteurs d'échelle*," "*diagramme à l'échelle*," "*réduction*," and "*plus petit*." In addition to the written mathematical notation, the production of the scale diagram itself (Figure 7) provided "written" mathematical follow-up of the mathematics-lexis-based LRE.

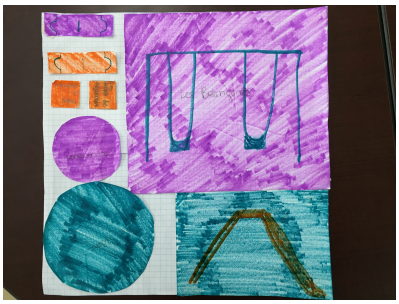

Figure 7. Mae, Max, and Mya's scale diagram.

The next example shows an occurrence of an LRE coded as meaning-making activity, in a non-academic-mathematical-lexicogrammatical-based episode, in which Ava (LFI) and Scot (EFI), of group L1, were generating alternatives to explain how their pieces of playground equipment fit well within their fenced perimeter. They were working on their responses to written questions on the task worksheet on day 3:

23 Scot: *Oh. « Pourquoi avez-vous choisi cet emplacement d'objets en particulier? » Um, les objets pouvaient comme, aller ensemble bien et ne prenaient pas trop d'espace.* **Kind of, right?** (Oh. "Why did you choose this

24 particular placement of objects?" Um, the objects could like, go together well and didn't take up too much space. **Kind of, right?**)

25 Ava: *Et aussi il y a d'autre espace pour juste* . . . (And also there's other space for just . . .)

26 Scot: *Oui.* (Yes.)

27 Ava: *Parce que les objets plusse petits, l'aire-* (Because the smaller objects, the area-)

28 Scot: *On pourrait dire qu'il y a de l'espace extra dans C, dans « pourquoi c'est le meilleur »?* (We could say that there's extra space in C, in "why is it the best"?)

In this excerpt, lexis and form intermingled in a search for how to convey, in the L2, the essence of fit. The idea became communicated via expressions, for instance, Scot's "*les objets pouvaient aller ensemble bien*." This expression involves lexis (*aller*, *ensemble*, *bien*) and form, namely a verb conjugation (*pouvaient aller*, which is correctly conjugated with the subject, *les objets*) and a verb-adverb-modifier combination (*pouvaient aller ensemble bien*, which appears to represent an incorrect direct translation of *went together well*). This was built upon by Ava's "*il y a d'autre espace*," which similarly involved lexicogrammatical constructs (lexis: *espace*, form: *il y a d'autre*). The collaborative dialogue did not involve the introduction of academic mathematical lexis (except for when Ava utters "*aire*" and is immediately cut off by Scot) in the vocabulary search, however, the dialogue is nonetheless mathematical in its content; thus, it is an example of non-academic-mathematical language.

Because this collaborative dialogue related directly to questions on the worksheet, the results, in other words, the assessment of the alternatives, can be traced to the final written product:

> Scot: *Si tu fait 50 x 50, il y a des objects qui ne peut pas être mis en dedans à cause de leur longueur ou largeur. . . . Il y a de l'espace entre les objets et sa peut contenir tout les équipments.* (If you do 50 x 50, there are objects that cannot be put inside because of their length or width. . . . There is space between the objects and it can contain all the equipment.)
>
> Ava: *Quand tu fair 50 par 50, il y a un* ***overlap****, et pas d'espace pour l'equipment. . . . Il y a l'espace entre l'equipment.* (When you do 50 by 50, there is an **overlap**, and no space for the equipment. . . . There is space between the equipment.)

In the written response, some of the same lexicogrammatical elements reappear (e.g., *il y a de l'espace*) while others have been changed once put into written form (e.g., "*les objets pouvaient aller ensemble bien,*" from the oral interaction, seems to have become, "*il y a des objects qui ne peut pas être mis en dedans*," in the written response). As in the oral interaction, the written work uses non-academic-mathematical lexicogrammar to convey the mathematical ideas. Also of note, Ava introduced an L1 lexical item in her written work, namely *overlap*, which had previously been uttered by Scot (also in the L1) on day 1 of the in-class activity while trying to fit the pieces of playground equipment into the fence. (Instances of LREs in which the L1 is used in students' collaborative dialogue are discussed further in response to research question #3.)

A final qualitative example showcases the most frequently occurring type of LRE, correction, which falls under the overarching category of other occasions for L2 learning. Within correction, lexis-based corrections were the most used, followed by non-academic-mathematical-lexicogrammatical corrections. In this excerpt from the classroom audio recordings, Eve and Kim (both LFI students, group H4) were working through the mathematics task on day 1:

103 Eve: *Ton grand efface chose.* (Your big erase thing.)

104 Kim: *Effaceur.* (Eraser.)

In this brief exchange, Kim provided a lexis-based correction for her partner. As with many other lexis-based LREs, although the lexical item is not itself mathematical (neither academic nor non-academic), it is "on task" in the sense that it was used in order for the students to complete the mathematics work at hand. The term *effaceur* was not used again in the dialogue (nor was it used in the written work or in the follow-up interview) so it is impossible to determine uptake of the correction.

Other times, corrections were followed by an uptake, such as in this exchange in group D4, between Dan (EFI) and Zac (EFI):

103 Dan: *C'est demi.* (It's half.)

104 Zac: *C'est un demi là.* (It's a half there.)

105 Dan: *Oui, ça c'est plus de un demi.* (Yes, that is more than a half.)

In this brief collaborative exchange, Zac provided a correction for Dan (correcting the direct translation of English, "it's half"/"*c'est demi*" to "*c'est un demi*"), which Dan then reproduced in the next turn. This instance of LRE was coded under other occasions for

L2 learning, correction and uptake, mathematical-lexicogrammatical: there was attention to mathematical lexis (*demi*) and attention to form (*un*).

Although attention was paid to lexis and lexicogrammar far more often than to form alone, in both the meaning-making activity LREs and the other occasions for L2 learning LREs, one group in particular, E4, with Amy (EFI), Ella (LFI), and Hali (EFI), made corrections to form 5 times (plus one other correction to non-academic-mathematical-lexicogrammar). The group engaged in both other- and self-correction of form. For example:

77 Amy: *Um, je vais faire les premiers uns, si tu veux.* (Um, I'll do the first ones, if you want.)

78 Hali: *Ok. Je peux faire, je vais faire les derniers.* (Ok. I can do, I'll do the last.)

In this dialogue, Hali corrected Amy, who had uttered an incorrect direct translation of English *the first ones* ("*les premiers uns*"). Hali used a different term in the parallel structure, "*derniers*" to refer to her own work, however she provided the correct form by omitting the "*uns*". As with many of the other-corrections, in this case the form was not used again in the dialogue (nor in the written work or follow-up interviews) therefore it was impossible to track uptake. Moreover, a number of the corrections were self-corrections rather than other-corrections, as in these unrelated turns:

Ella: *OK, le, c'est la glissade* . . . (OK, it's slide, it's the slide . . .)

Hali: *C'est cool, il y a des rubans, du ruban.* (It's cool, there are tapes, there is tape.)

Amy: *Est-ce qu'on devrait colorier sur cette côté? Alors tu ne voirais, voyerais . . . ?* (Should we colour on this side? So you won't saw, see . . . ?)

Here, the LREs were self-initiated and self-fulfilled. They were not collaborative per se, however they occurred within the context of the collaborative dialogue necessary to complete the mathematics task.

These qualitative examples showcase some of the salient results that emerge from the coding of LREs occurring in the L2 in students' collaborative dialogue during mathematical problem solving. A significant number of LREs also occurred with the use of students' L1, and a majority of these were lexis-oriented (i.e., lexis-based, mathematical-lexis-based, non-academic-mathematical-lexis-based, or non-academic-mathematical-lexicogrammatical). These L1 LREs will be looked at more closely in response to research question #3. First, though, a look at research question #2 will shed light on the mathematical communication that occurred during students' collaborative problem solving. Although not highlighted directly, it is likely evident already from the reading of this past section that mathematical communication did occur during the in-class collaborative dialogue, and it often contained smaller units of LREs and MREs within it.

4.3 Research Question 2: Mathematics-Related Episodes

In this section, I present results that pertain to research question #2, namely: How do FI students at the secondary level communicate mathematically while working collaboratively during mathematical problem solving in their L2 (French)?

Table 16 shows the salient MREs that occurred strictly in the L2 in students' collaborative dialogue during mathematical problem solving. Some MREs involved the

use of students' L1, and these cases will be reported on with respect to research question #3.

Table 16

Mathematics-Related Episodes (MREs) Occurring in the Second Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (by Group)

	Making Claims	Searching for Certainty	
Generalizing	L4(2); H4; K4; M1	H4; L4	
	Predicting	Stating Assumptions	
Hypothesizing	M1(2); L4	E4	
	A Mathematical Situation	A Pattern	
Describing	M1(8); E4(7); H4(7); L1(6); D4(5); K4(2); L4		
	Using Drawings	Using Objects	
Representing	L1(4); D4; H4; M1	M1(4); L1(3); H4(2); L4(2); E4; K4	
	Elaborate (Repeat, Restate, Clarify, Refine)	Extend (Add, Replace, Provide Alternative)	Enhance (Time, Place, Manner, Cause, Condition)
Expanding	E4(4); L1(3); K4(2); M1(2); D4; H4; L4	L1(4); M1(4); E4(2); H4(2); D4; K4; L4	M1(8); E4(5); L1(4); H4(3); D4(2); L4(2); K4

Note. When more than one occurrence was coded for a particular pair/group, the number of occurrences is given in parentheses.

Thus, individual participant groups produced the following total number of MREs in the L2: M1 = 30, L1 = 24, E4 = 20, H4 = 18, L4 = 11, D4 = 10, K4 = 8.

With regard to the total number of L2 MREs produced by the participants for each category, the results are reported in Table 17.

Table 17

Mathematics-Related Episodes (MREs) Occurring in the Second Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (Totals)

	Making Claims	Searching for Certainty	
Generalizing	5	2	
	Predicting	Stating Assumptions	
Hypothesizing	3	1	
	A Mathematical Situation	A Pattern	
Describing	36	0	
	Using Drawings	Using Objects	
Representing	7	13	
	Elaborate (Repeat, Restate, Clarify, Refine)	Extend (Add, Replace, Provide Alternative)	Enhance (Time, Place, Manner, Cause, Condition)
Expanding	14	15	25

As indicated in Table 17, a total of 121 MREs were identified that occurred entirely in students' L2; of these, 54 (44.6%) involved some form of expanding, 36 (29.8%) involved describing, 20 (16.5%) involved representing, 7 involved generalizing (5.8%), and 4 involved hypothesizing (3.3%). Like the LREs, MREs were also interspersed throughout the discourse over the span of the 3-day mathematics activity, and higher concentrations of MREs were observed during the same two phases in particular: (a) when students were faced with trying to work through and explain why the playground equipment was or was not fitting into their particular fenced perimeter and (b) when students were discussing and then responding in written form to the questions on the task worksheet—especially those that asked them to explain their process and/or reasoning with regard to various aspects of the mathematics problem-solving task (e.g., why there are differences between maximum possible area of an empty playground vs. one with all

the equipment, reasons for choice of scale, reasons for why their playground layout worked).

As was the case for research question #1 and the LREs, in order to illustrate the most striking results, these descriptive quantitative data for the MREs are augmented with qualitative examples. I begin with an excerpt from group E4 on day 2 (Amy, EFI; Hali, EFI; and Ella, LFI), in which the group discussed how to construct their scale diagram on graph paper:

30 Amy: *Non, il n'est pas assez grand maintenant car il y a seulement trente-deux.* (No, it's not big enough now because there's only thirty-two.)

31 Ella: *Est-ce qu'il y a cinquante carreaux là?* (Are there fifty squares there?)

32 Amy: *Non.* (No.)

33 Ella: *Donc t'as besoin de quatre?* (So you need four?)

34 Amy: *Oui! Papiers.* (Yes! Papers.)

35 Hali: **What?!**

36 Amy: *On besoin de quatre car c'est pas-* (We need four because it's not-)

37 Hali: *Quatre? Pièces de papier?* (Four? Pieces of paper?)

38 Amy: *Oui! On n'a pas pensé de ceci.* (Yes! We didn't think of this.)

39 Hali: *Quoi?! Mais est-ce que c'est juste cinquante par cinquante?* (What?! But is it just fifty by fifty?)

40 Amy: *Oui. Car c'est seulement trente-trois par là et c'est, comme, quarante par là. Alors on peut utiliser, comme.* (Yes. Because it's only thirty-three there and it's, like, forty there. So we can use, like.)

41 Hali: *Est-ce que tu peux couper tout ça qu'on a en deux?* (Can you cut everything that we have in two?)

42 Amy: *Oui, hm hm. En deux. Ça veut dire que tu dois refaire les cercles. Le diamètre va changer.* (Yes, hm, hm. In two. That means you need to re-do the circles. The diameter is going to change.)

43 Hali: *Dedans les cercles? On va faire vingt-cinq sur vingt cinq.* (Inside the circles? We're going to do twenty-five on twenty-five.)

During this mathematical communication act, more than one MRE emerged. First, there is an example of representing using drawings. In turn 30, Amy pointed out to the group that their planned scale of 1 square = 1 foot would not fit onto one piece of graph paper, since the paper itself only measured approximately 33 x 40 squares and their fence measured 50 x 50. Thus, when it came time to actually begin the scale diagram, the mathematical and practical necessity of choosing an appropriate scale was brought to the forefront of the problem and the dialogue. Ella and Hali both asked questions in order to clarify the situation (turns 31, 39), thus contributing to expansion by way of elaboration (i.e., clarifying). Amy responded to the clarifying questions with further expansion, in the form of an enhancement (i.e., turn 40, *"'car'" c'est seulement trente-trois par là et c'est, comme, quarante par là"*). The representing and expanding MREs are nestled within an MRE in which students are describing a mathematical situation (i.e., the need to adjust the scale factor or else have to use four pieces of graph paper taped together), which culminated in the students' solution to the mathematical scale problem they were facing (turns 41, 42, 43, "*Est-ce que tu peux couper tout ça qu'on a en deux?*").

The group's reasoning through the scale factor problem during their in-class collaborative dialogue was echoed in their written responses to the worksheet questions asking what scale they used and why, particularly for Amy and Hali, who were more involved in the collaborative dialogue:

Amy: *2' par 1 carré. Nous avons fais alors que tout est sur un page.*

Hali: *2' = 1 carré. Nous avons fais alors que tout est sur page.*

Ella: *1/2. (Un carré pour 2 pieds)*

(Amy: 2' by 1 square. We did it so that everything is on one page.

Hali: 2' = 1 square. We did it so that everything is on page.

Ella: 1/2. [One square for 2 feet])

When asked about their process during the follow-up interview, Hali said:

On a fait un carré égale à un pied et c'était vraiment gros. Mais on a juste divisé. On a essayé un et on ne pouvait pas comme mis les deux (feuilles) ensemble. Je suis comme, « on fait pas ça ».

(We did one square equal to one foot and it was really big. But we just divided. We tried one and we couldn't like put the two [papers] together. I'm like, "we're not doing that.")

This example illustrates how various MREs coded in the framework—in this case representing using drawings, describing a mathematical situation, and expanding using elaboration (clarification) and enhancement (qualify by cause)—oftentimes comprise instances of mathematical communication regarding a single topic or issue. Furthermore, while individual lexical items (/forms/lexicogrammar) within MREs can be identified as academic-mathematical (e.g., *le diamètre*, *vingt-cinq sur vingt-cinq*) or non-academic-

mathematical (e.g., *couper tout ça en deux*), the academic and non-academic mathematical discourses are intertwined throughout the MREs and the mathematical communication episode more broadly. Reiterated in the written work and the follow-up interview, when working on hands-on problems that connect to the need for practical problem solving, students combine the more academic elements of mathematics (e.g., *2' = 1 carré*; *on a juste divisé*) with "everyday" language in order to explain their mathematical processes (e.g., *nous avons fais alors que tout est sur page*; *je suis comme, « on fait pas ça »*).

In another example, the members of group L1, Ava (LFI) and Scot (EFI) discussed the problem they were having with fitting the equipment into their 50 x 50 square paper clip fence on day 2:

11 Scot: *Comme, parce que ça c'est cinquante par cinquante mais il n'y a pas assez jusqu'à là, alors est-ce qu'on peut faire comme, quarante-neuf . . . là, par cinquante-et-un là.* (Like, because that's fifty by fifty but there isn't enough up to there, so can we do like, forty-nine . . . there, by fifty-one there.)

12 Ava: *Je . . . pense?* (I . . . think?)

13 Scot: *On peut, comme, essayer? On a besoin de changer des choses aussi.* (We can, like, try? We have to change some things too.)

14 Ava: *Je ne sais pas si ça marche.* (I don't know if it works.)

15 Scot: *Mais cinquante par cinquante ne va pas fonctionner.* (But fifty by fifty will not work.)

16 Ava: *Oui.* (Right.)

17 Scot: *Alors.* (So.)

From there, Ava and Scot went on to adjust their large-scale model and then eventually draw their scale diagram with dimensions of 51 x 49. Scot initiated the MRE description of this mathematical situation (turn 11) with reference to the initial large-scale model, which was not able to accommodate the playground equipment. Thus, the MRE is both a description of a mathematical situation and a case of representing using objects. This turn also illustrates an MRE of expanding: First the expansion was made by an extension, marked where Scot used "*mais*" to add something new to the description (i.e., *mais il n'y a pas assez jusqu'à là*). Next, Scot extended further by adding another new facet to the description, which was marked by "*alors*," where he offered something they could do instead thus providing an alternative (i.e., *alors est-ce qu'on peut faire comme, quarante-neuf . . . là, par cinquante-et-un là*). When Ava questioned his alternative (turn 14, *je ne sais pas si ça marche*), Scot provided further expansion by restating his initial extension: "*mais cinquante par cinquante ne va pas fonctionner*" (turn 15). Together, Ava and Scot decided that, although they were not sure that 51 x 49 would be an appropriate design, they would at least try it since they were convinced that 50 x 50 was most certainly not the correct way to proceed.

Indeed, the pair followed up with their problem-solving strategy with the hands-on models on their written work, where they showed that the maximum area for a perimeter of 200 is given by a square measuring 50 x 50, but also that they had changed their fencing to 51 x 49 due to the placement of the equipment (Figure 8).

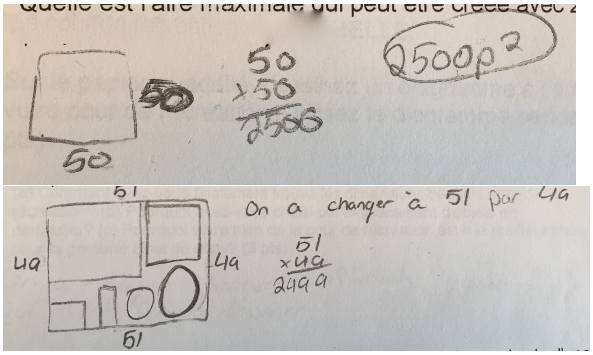

Figure 8. Ava and Scot's written work and sketches.

Ava: *On a essayer plusier different dimensions qui ne march pas.*

Scot: *On a essayer plusieurs différents grandeurs qui n'ont pas fonctionner.*

(Ava: We tried many different dimensions that do not work.

Scot: We tried many different sizes that did not work.)

In their follow-up interview, Ava and Scot reflected on how they worked through the equipment problem, eventually changing their dimensions, and how the hands-on modelling required them to "think more":

Scot: *Comme, utiliser tes mains pour le faire, parce que ça, c'est plus intéressant que juste du papier, et ça te fait penser plus. Comme, si tu juste fais sur le papier tu vas juste faire ce qui est là, et tu ne vraiment pense pas à ce que tu fais. Tu juste fais pour le faire.*

Ava: *Oui parce que pour, like, vraiment regarder comment on a fonctionné, comme juste faire sur le papier tu peux, like, faire des petits erreurs et c'est pas fonctionne bien. C'est comme, plus d'aide que juste écrire sur le papier. . . .*

Scot: *Je pense que avoir, comme, les trombones et tout, ça aidait parce qu'on a essayé plusieurs différentes grandeurs avec les trombones, et si on avait besoin de*

redessiner et effacer beaucoup de fois on seraient ennuyés. On ne vouleraient pas faire.

Ava: . . . *Je pense qu'on a bougé comme, cinq, six fois avant de trouver une,* **and then** *ça ne marche pas, on a bougé un plus et ça marche. . . . On a commencé avec un rectangle puis on a commercé à comme pousser . . .*

Scot: *Pousser pour que ça faisait un carré.*

Ava: *Carré parfait et elle a cassé un peu le carré, mais un peu, je ne sais pas pour dire. C'est comme un peu plus longue à un côté, plus courte l'autre. C'est pas un carré parfait mais je ne sais pas comment dire.*

(Scot: Like, using your hands to do it, because that, it's more interesting than just paper, and it makes you think more. Like, if you just do it on paper you're going to just do what's there, and you're not really thinking about what you're doing. You're just doing it to do it.

Ava: Yes because to, like, really see how we worked, like just doing it on paper you can, like, make small errors and it doesn't work well. It's like, more help than just writing on paper. . . .

Scot: I think that having, like, the paper clips and everything, it helped because we tried many different sizes with the paper clips, and if we had to redraw and erase a lot of times we would be bored. We wouldn't want to do it.

Ava: . . . I think we moved like, five, six times before finding one, **and then** it doesn't work, we moved it one more and it works. . . . We started with a rectangle and then we started to like push . . .

Scot: Push so that it made a square.

Ava: Perfect square and it broke the square a bit, but a bit, I don't know for to say it. It was like a bit longer on one side, shorter the other. It's not a perfect square but I don't know how to say.)

For this instance of mathematical communication that occurred during students' collaborative in-class dialogue, the MREs (representing, describing, expanding) combine to show how the problem was solved. Ava and Scot' written work used mathematical notation as well as written language to show their problem solving. In the follow-up interview, the students were able to reflect on what happened during these MREs, which paints an even clearer picture of their use of mathematics and language during the various episodes.

Through these qualitative data, the descriptive quantitative results are elaborated and elucidated. Clearly, over the course of the 3-day mathematics task students engaged in collaborative dialogues in which they communicated mathematically and in which MREs occurred. In response to the problem-solving task and to the written questions, students produced high numbers of MREs that fell under the categories of describing and expanding. The students used lexis/form/lexicogrammatical elements from both academic and non-academic registers in order to express their mathematical ideas in their communication with each other. In most cases, the MREs from the oral interaction data were augmented and further clarified by students' written work as well as by their reflections during the follow-up interviews. The results in this section show only those MREs that occurred in the L2. As with the LREs, a number of MREs also occurred with the use of students' L1 (these also fell most often into the category of describing and expanding). Further, both LREs and MREs often occurred within a larger instance of

mathematical communication regarding a particular aspect of the problem-solving task. The L1 MREs and LREs are explored next, in response to research question #3.

4.4 Research Question 3: Use of the First Language

As noted already in the previous sections responding to research questions 1 and 2, students used their L1 (English) within some of the LREs and MREs and these data will be reported on in this section. The L1 was also used for other purposes, many of which were task-related, and there were also some overall trends that emerged with regard to L1 use that will be addressed.

First, the descriptive quantitative data in Table 18 provide an overview of L1 use for each student in the participant groups, with comparisons to their demographic data. A number of results regarding overall L1 use emerge from the data. With regard to range, the student who had the highest number of L1 turns was Brit (LFI), with 96.9%. Overall, Brit's group, H1, comprising herself, Sara (EFI) and Bea (EFI), also had the highest percentage of L1 turns per group, with 86.7%. When codeswitched turns are factored into the scenario, and looking strictly at L2 turns, results show Brit (LFI) with the lowest percentage of all participants, at 1.0%. Likewise, the H1 group had the lowest overall group percentage of L2 turns with 7.1%. Together, Bea, Brit, and Sara also produced the highest total number of turns (irrespective of language) of any group, with 901 (358, 286, and 218 respectively).

Table 18

Program, Academic Achievement, English Language Proficiency Achievement (ELPA) Results, and Turns by Language for Each Participant

Pseu-donym	Group (Letter, Period)	Program (Early or Late)	FILA 9 (Mark in %)	Math 9A, 9B (Marks in %)	ELPA Re-sult	Total # of Turns	# of French Turns (%)	# of English Turns (%)	# of CS Turns (%)
Mac	E1	EFI	81	95, 99	AA	50	10 (20.0)	24 (48.0)	16 (32.0)
Mike	E1	EFI	78	81, 97	BAA	61	15 (24.6)	26 (42.6)	20 (32.8)
Jane	M1	EFI	76	90, 94	AA	252	190 (75.4)	22 (8.7)	40 (15.9)
Beth	M1	EFI	92	87, 93	AA	263	224 (85.2)	16 (6.1)	23 (8.7)
Dan	D4	EFI	72	85, 90	AA	116	83 (71.6)	13 (11.2)	20 (17.2)
Zac	D4	EFI	82	92, 92	AA	108	70 (64.8)	12 (11.1)	26 (24.1)
Kim	H4	LFI	84	82, 87	AA	338	231 (68.3)	37 (10.9)	70 (20.7)
Eve	H4	LFI	90	94, 94	SA	378	299 (79.1)	18 (4.8)	61 (16.1)
Amy	E4	EFI	87	86, 91	AA	209	183 (87.6)	7 (3.3)	19 (9.1)
Hali	E4	EFI	89	93, 93	AA	210	183 (87.1)	4 (1.9)	23 (11.0)
Ella	E4	LFI	100	98, 100	AA	130	120 (92.3)	2 (1.5)	8 (6.2)
Scot	L1	EFI	85	91, 92	AA	318	149 (46.9)	72 (22.6)	97 (30.5)
Ava	L1	LFI	71	76, 70	AA	321	209 (65.1)	81 (25.2)	31 (9.7)
Sue	L4	EFI	96	97, 97	SA	114	109 (95.6)	0 (0.0)	5 (4.4)
Liz	L4	LFI	93	94, 94	SA	109	104 (95.4)	1 (0.9)	4 (3.7)
Sara	H1	EFI	81	82, 88	AA	218	16 (7.3)	178 (81.7)	24 (11.0)
Brit	H1	LFI	65	60, 72	AA	286	3 (1.0)	277 (96.9)	6 (2.1)
Bea	H1	EFI	72	84, 85	AA	358	35 (9.8)	292 (81.6)	31 (8.7)
Mae	K4	LFI	85	72, 77	AA	91	64 (70.3)	10 (11.0)	17 (18.7)
Max	K4	EFI	70	87, 87	AA	52	35 (67.3)	15 (28.8)	2 (3.8)
Mya	K4	LFI	83	86, 91	AA	71	60 (84.5)	7 (9.9)	11 (15.5)

Note. CS = Codeswitched; ELPA = English Language Proficiency Assessment; FILA = French Immersion Language Arts.

Results show that reasons for this are multifaceted. Brit, who uttered the lowest percentage of L2 turns of all the participants at 1.0%, was an LFI student who likely had a lower French proficiency level relatively speaking—her FILA 9 mark was 65%. She also had the lowest mathematics mark of the participants, with a two-semester average of 66%. She had successfully completed her ELPA with a score of AA (average achievement). Brit's lower achievement in both her FILA 9 and FI Math 9 courses could point toward her increased use of the L1 as a sign of being unable to cope with the linguistic and/or mathematical demands of the task. For example, when asked about what

it was like to explain mathematics in French in her follow-up interview, Brit stated: "**It's hard to say what you're thinking about the questions to the teacher because I don't know a lot of French words. It's difficult to get her to understand what I'm thinking.**" There are also the socioaffective and motivational factors that must be considered. Brit self-reported as being "not smart" when asked about her contributions to the group during the mathematics task:

> MC: **What about you Brit? Did you have a lot of input or did you, sort of, what was your style in the group?**
>
> Brit: **I don't know.**
>
> MC: **That's OK, you don't have to answer anything. I just want to make sure that you know you can, anytime you want to.**
>
> Brit: **I don't know. I'm not smart.**

Concerning group H1's overall tendency to speak in English, I described in the first section of this chapter, regarding task-related results, how this group seemed the least motivated in general in terms of both French and mathematics. Indeed, the following exchange occurred 8 turns into the dialogue on day 1 of the in-class activity:

> 8 Sara: **We're supposed to be talking in French!**
>
> 9 Bea: **Oh, who cares? She said she's interested in, like-**
>
> 10 Brit: **She said she's interested in, she wants to listen to us work in French.**
>
> 11 Bea: **I don't work in French.**

Bea, who stated definitively in turn 11, "I don't work in French" also had one of the lower FILA 9 marks (fourth lowest of all the participants), at 72%, but a higher Math 9

average than Brit, of 84.5%. Bea, an EFI student, uttered the highest number of L2 turns in the group, with 9.8%, despite her statement in turn 11. Sara, for her part, had the highest FILA 9 and Math 9 averages of the group, at 81% and 85% respectively, but uttered only 7.3% of her turns in French. Whereas Brit had expressed her difficulty with finding the words in French to explain herself in mathematics, Sara, an EFI student, stated in her follow-up interview: "*C'est pas, comme, difficile parce que c'était toujours en français*" ("It's not, like, difficult because it's always been in French"). However, she later added, "**Just, like, it's [English is] our natural language. French is, like, easy too, but I just find I will speak English with the French. Have the main, like,** *puissance et*, **like**, *polynôme*, **the rest doesn't really matter**." Here, Sara painted a complicated picture of how she thought French was easy, yet she often spoke in English (her "natural" language), but nonetheless kept the academic-mathematical lexis (e.g., *puissance*, *polynôme*) in the L2, French; Sara uttered the highest number of codeswitched turns in the group, with 11.0%.

At the other end of the range, Sue (EFI) had the lowest number of L1 turns of all the participants, since she uttered precisely none. She did have 4.4% codeswitched turns. Sue also had the highest percentage of L2 turns, with 95.6%. As group L4, Sue and her partner, Liz (LFI), had the lowest percentage of L1 turns at 0.4%, and the highest percentage of L2 turns, with 95.5%. Together, Liz and Sue produced the third lowest (out of nine) total number of turns among the groups, with 223 (104 and 109, respectively).

Similar to the group with the lowest number of turns (group H1), the reasons for this seem to be varied. In contrast to the members of group H1, group L4's members had the second and third highest FILA 9 marks among all participants, with Liz's mark at

93% and Sue's at 96%. Sue was tied (with Mac of group E1) for second highest mathematics average, with 97%, and Liz was tied at third place (with Eve of group H4) at 94%. Liz and Sue were able to work through the problem-solving task, both mathematically and linguistically, without being overburdened by the cognitive load and thus were able to navigate almost the entire task in their L2. As with the H4 group, for the L4 group, socioaffective and motivational factors seemed to enter into play but with the opposite effects. Both Liz and Sue appeared motivated to speak in French and to accomplish the mathematics task to the best of their abilities—they took the task seriously, and wanted to do well (researcher fieldnotes, March 17). During the interview, I was able to follow up with what I had noticed during the in-class activity. Both Liz and Sue described how doing mathematics in French did not pose a problem:

> MC: *Qu'est-ce que vous pensez de faire les mathématiques en particulier en français? Est-ce que c'est la même chose que tous les autres sujets? Est-ce qu'il y a une différence?*
>
> Sue: *Je pense que c'est mieux si tu ne changes pas [en anglais] parce que lorsque tu apprendre tous les termes en français c'est difficile. Je suis comme, « qu'est-ce ça veut dire ? » parce que tu entends en anglais et tu ne sais pas. Mais, pour la plupart ils sonnent presque le même et tu peux deviner. Mais je ne pense pas que c'est plus difficile que . . . en anglais. Je ne pense pas.*
>
> Liz: *Oui, comme, tous les professeurs dit, comme, la langue, ou la math est comme une langue universelle. Comme, tout le monde peut comprendre, comme quand tu voir sur papier, mais, des termes sont un peu différents.*
>
> (MC: What do you think about doing math in particular in French? Is it the same

thing as the other subjects? Is there a difference?

Sue: I think it's better if you don't change [into English] because when you learn all the terms in French it's difficult. I'm like, "what does that mean?" because you hear it in English and you don't know. But for the most part they sound almost the same and you can guess. But I don't think it's more difficult than . . . in English. I don't think so.

Liz: Yes, like, all the teachers say, like, language, or math is like a universal language. Like, everyone can understand, like, when you see on paper, but, the terms are a bit different.)

Despite their very high percentage of L2 turns (95.5% as a group) and what they reported in the previous interview excerpt, Liz and Sue still alluded to the other factors at play when it came to their language use in the classroom:

MC: *Autre que le vocabulaire en mathématiques, comme, tout cette terminologie, par exemple, mettons que vous avez une activité comme ceci, où il faut discuter un petit peu de comment le faire . . . . Est-ce que tout ça aussi ça se passe bien en français?*

Sue: *Je trouve que c'est plus difficile parce que la plupart du temps avec tes amis tu parles en anglais, donc tu es habituée à parler en anglais.*

Liz: *Oui. C'est comme un peu . . . bizarre. Oui, c'est comme pas . . . naturel. On peut le faire mais c'est plus naturel de parler en anglais.*

(MC: Other than mathematics vocabulary, like, all this terminology, for example, say you have an activity like this one, where you need to discuss a bit how to do it . . . . Does all that go well in French?

Sue: I find that it's more difficult because most of the time with your friends you speak in English, so you're used to speaking in English.

Liz: Yes. It's like a bit . . . bizarre. Yes, it's like not . . . natural. We can do it but it's more natural to speak in English.)

In this group with the highest number of L2 turns, themes similar to those expressed by the group with the lowest number of L2 turns were brought to the forefront of the interview discussions, including the differences between mathematics vocabulary and "natural" language. Both the group with the most L2 turns and the group with the fewest L2 turns had a mixture of EFI and LFI members.

To further illustrate the results of socioaffective and motivational factors, it is useful to examine the only group that was composed of LFI students only, which was group H4 with Eve and Kim. These students uttered 74.0% of their turns in the L2. A further 18.3% of the group's turns were codeswitched, which means that only 7.7% of the turns were uttered in the L1. Regarding the students' achievement, Eve had marks in the mid-high range for FILA 9 (90%) and a high average in Math 9 (94%). Kim had mid-range FILA 9 (84%) and Math 9 (84.5%) performance. As with the other two groups discussed in detail here (i.e., the lowest and highest L1 use groups), the academic performance of the students only provides partial information as to when and why the L1 was used (or not) during the collaborative dialogue. In their follow-up interview, Eve and Kim reflected on their language use. During the in-class mathematics activity I had noticed that Eve, in particular, seemed very motivated to speak the L2 whenever possible. She commented:

En 6e année j'aime, comme, apprendre un nouveau langue parce que je pense que c'est, comme, cool pour parler une autre langue que l'anglais mais c'est difficulte parce que il y a beaucoup de **grammar** *et quelque chose comme ça et c'est comme, « tu peux pas dit ça » ou « tu besoin de mettre ça à la fin de cette mot » mais . . . je pense que c'est bien.*

(In Grade 6 I like, like, learn a new language because I think it's, like, cool for speaking another language than English but it's difficult because there is a lot of **grammar** and something like that and it's like, "you can't said that" or "you need put that at the end of this word" but . . . I think it's good.)

With regard to learning mathematics, in particular, in their L2, the pair discussed the following:

Eve: *Je pense que les mathématiques sont pas difficiles parce que on a appris tous les mots en français pas en anglais. Parce que en 6e année, ça c'est quand, vraiment, tu apprendre beaucoup de maths donc je sais tous les mots en français. Si je vais, um, transfère en anglais je ne peux- je pense que c'est difficile parce que tu es comme, « qu'est-ce que ça c'est ?»*

Kim: *Tu connais tout les choses en français, pas en anglais, donc c'est difficile de-*

Eve: *Mais aussi il y a des questions de mots, comme, on peut, on sait pas le mot. Comme, en anglais- c'est difficile-*

Kim: **Like, a word problem**. *Oui il y a des mots en français que on sait pas donc c'est un peu difficile avec ça.*

MC: *Les petits mots ici et là. Est-ce que ce sont les mots de mathématiques?*

Eve: *Non.*

(Eve: I think math is not difficult because we learned the words in French not in English. Because in Grade 6, that's when, really, you learn a lot of math so I know all the words in French. If I will, like, um, transfer in English, I can't- I think it's difficult because you're like, "what is that?"

Kim: You know all the things in French, not in English, so it's difficult to-

Eve: But there is also the word questions, like, we can, we don't know the word. Like, in English- it's difficult-

Kim: **Like, a word problem.** Yes there are words in French that we don't know so it's a bit difficult with that.

MC: Little words here and there. Are they mathematics words?

Eve: No.)

Similar to both the high L1 and low L1 use groups, this group reflected on their language use by underscoring the differences between the academic-mathematical language (mathematics "words" and "things) and the remainder of the language that goes into, for example, "a word problem." Despite being LFI students and thus having had only 3 years of schooling in the L2, these students spoke relatively few turns entirely in the L1 (7.7%). Again, similar to the other groups, this would appear to be linked to socioaffective factors, such as the perception that it is, to use Eve's word, "cool" to speak another language. Eve had also discussed in the interview how she had travelled to Europe and enjoyed trying to learn a different language while there, and she has also alluded to travelling to Quebec during an "off-task" communication act in the in-class dialogue.

Regarding overall L1 use, there were clear variations in language use among each participant individually and among the participant groups. Despite these variations, however, the results show that, overall, most students spoke French most of the time over the course of the 3-day mathematics activity. When all students are included, 60.3% of turns were L2, 27.5% of turns were L1, and 13.7% were codeswitched. If the maximum and minimum values for L1 and L2 turns for individual students are discarded (i.e., Liz and Brit's data sets are omitted), then overall, students' turns were 73.3% L2, 12.3% L1, and 16.3% codeswitched.

In the reporting of the remainder of the results related to research question 3, two groups are omitted: E1 (Mac and Mike) and H1 (Bea, Brit, and Sara), for the same reasons described for research questions 1 and 2—they uttered very few L2 turns relative to the remainder of the groups in the data set (i.e., under 50% of turns were L2 turns). With breakdowns by participant groups, Table 19 shows students' L1 use as it related to LREs, Table 20 shows the information for MREs, and Table 21 shows L1 use related to "other" purposes. Tables 22, 23, and 24 show the same information, but by number of instances of L1 use (rather than breakdown by group).

Table 19

Language-Related Episodes (LREs) Occurring With Use of the First Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (by Student Group)

	Lexis-Based	Mathematical-Lexis-Based	Non-Academic-Mathematical-Lexis-Based	Form-Based	Mathematical-Form-Based	Non-Academic-Mathematical-Form-Based	Lexicogrammatical	Mathematical-Lexicogrammatical	Non-Academic-Mathematical-Lexicogrammatical
Meaning-Making Activity									
Generating Alternatives	L1(16); H4(5); M1(4); E4(4); D4(2); K4; L4	E4(4); H4(2); L1(2); D4; K4	E4(2); L1(2); M1; D4; H4; L4	M1(2)	H4	L1(2)	L1(3)		L1(7); H4(2); M1; D4; E4; K4; L4
Assessing Alternatives	H4(4); D4(2); M1	E4(4)	L1(2); M1		L1(2)				E4; H4; L4
Applying Rules to New L2 Context	H4								L1
Other Occasions for L2 Learning									
Correction	H4(2); L1	L1					H4(2)		
Uptake	H4								

Table 20

Language-Related Episodes (LREs) Occurring With Use of the First Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (by Total Number of Instances of First Language Use)

	Lexis-Based	Mathematical-Lexis-Based	Non-Academic-Mathematical-Lexis-Based	Form-Based	Mathematical-Form-Based	Non-Academic-Mathematical-Form-Based	Lexicogrammatical	Mathematical-Lexicogrammatical	Non-Academic-Mathematical-Lexicogrammatical
Meaning-Making Activity									
Generating Alternatives	33	10	8	2	1	2	3		14
Assessing Alternatives	7	4	3		2				3
Applying Rules to New L2 Context	1								1
Other Occasions for L2 Learning									
Correction	3	1					2		
Uptake	1								

Table 21

Mathematics-Related Episodes (MREs) with Use of the First Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (by Group)

	Making Claims	Searching for Certainty	
Generalizing			
	Predicting	Stating Assumptions	
Hypothesizing	L1(2)		
	A Mathematical Situation	A Pattern	
Describing	L1		
	Using Drawings	Using Objects	
Representing			
	Elaborate (Repeat, Restate, Clarify, Refine)	Extend (Add, Replace, Provide Alternative)	Enhance (Time, Place, Manner, Cause, Condition)
Expanding	L1(4); H4(4); M1	L1; M1	L1(3); D4; E4

Table 22

Mathematics-Related Episodes (MREs) with Use of the First Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (by Total Number of Instances of First Language Use)

	Making Claims	Searching for Certainty	
Generalizing			
	Predicting	Stating Assumptions	
Hypothesizing	2		
	A Mathematical Situation	A Pattern	
Describing	1		
	Using Drawings	Using Objects	
Representing			
	Elaborate (Repeat, Restate, Clarify, Refine)	Extend (Add, Replace, Provide Alternative)	Enhance (Time, Place, Manner, Cause, Condition)
Expanding	9	2	5

Table 23

Task-Related and Interpersonal-Related Uses of the First Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (by Group)

Moving the Task Along	
Understanding the Problem (retrieving info, understanding a directive)	E4(3); M1
Ask for "Wait" Time	L1(7); E4(7); K4(7); M1(5); D4(4); H4(4); L4
Task Management (equipment management, following steps)	H4(13); L1(6); M1(5); D4(4); K4(2)
Interpersonal Interaction	
"Off Task"	H4(7); L1(5); E4(2); K4(2); D4
Vernacular	H4(16); E4(8); D4(7); L1(5); K4(2); M1(2); L4
Disagreements	H4(8); D4(2); L1(2); K4(2); M1(2)
Jokes or Feelings	M1(10); H4(7); L1(2); D4; E4; K4; L4
Communicative Competence	E4(5); H4(2); D4; M1

Table 24

Task-Related and Interpersonal-Related Uses of the First Language in Students' Collaborative Dialogue During Mathematical Problem Solving in the Second Language (by Total Number of Instances of First Language Use)

Moving the Task Along	
Understanding the Problem (retrieving info, understanding a directive)	4
Ask for "Wait" Time	35
Task Management (equipment management, following steps)	30
Interpersonal Interaction	
"Off Task"	17
Vernacular	41
Disagreements	16
Jokes or Feelings	23
Communicative Competence	9

An examination of L1 turns among the seven groups reveals that students used the L1 to serve a variety of purposes. Use of the L1 often related to LREs, especially lexical searches or attention to lexis by way of generating alternatives (either lexical, mathematical, or non-academic-mathematical), which totalled 51 instances. A further 17 instances of L1 were lexicogrammatical in nature, totalling 68 cases altogether that were either lexical or lexicogrammatical generation of alternatives/searches. Other times, the L1 use was embedded within an MRE, particularly those related to expansion, which totalled 16 cases altogether, and especially repeating and restating (9/16 instances). Aside from LRE- and MRE-related purposes, the L1 was most often used in order to move the mathematics task along. Indeed, if all subheadings are taken together, then moving the task along accounts for 69 instances of L1 use. Within this overarching category, asking

for “wait” time (i.e., students uttered the word “wait” in order to reset the task, gather thoughts, or work out some mathematic) proved to be a very common use of the L1 with 35 instances. Use of the L1 occurred most often in the interpersonal interaction category, with 106 instances. The highest number occurred when students were using vernacular (41/106) or expressing jokes or feelings (23/106).

The “other” uses of the L1 (e.g., those related to moving the task along, use of vernacular, and expressing feelings) relate to the notion of “natural” language that a number of students discussed in the follow-up interviews (as already described earlier in this section). Students used the L1 to facilitate the “business” of the task and to express their feelings (including using vernacular expressions) presumably because it was quicker, easier, and/or felt more “natural.” With regard to the category of moving the task along, and task management in particular, there were many cases in which students used the L2, rather than the L1, in their dialogue. Regarding feelings and vernacular, there were fewer examples of the L2 being used, however this does not imply that examples in the L2 were non-existent. Thus, when it comes to L1 used for these purposes, an important finding is that using the L1 often appears to be a choice and does not automatically imply a lexical (or lexicogrammatical) gap.

Using the same format as the other sections of this chapter, these descriptive quantitative data will now be augmented with detailed qualitative examples by reflecting on the questions posed through the discourse analysis tools of inquiry. Indeed, one of the most salient, fascinating, and complex results of this study emerged in relation to this third research question exploring L1 use; moreover, this particular result was common to each of the seven student participant groups. By exploring in detail the discourse

surrounding the L2 equivalent of the L1 notion of *fit*, several simultaneous uses of the L1 can be elucidated.

The notion of *fit* emerged as a direct implication of the particular mathematics problem-solving task chosen as the materials for this study. In order to address the problem at hand, students discussed: how to *fit* the equipment into the fence, whether or not the equipment would *fit* into a given fence, and what to do when something did not *fit*. They used *fit* as an explanation for why their particular playground designs were appropriate (e.g., "everything fits"), and they used it as a way to assess whether their design was working (e.g., "does it fit?"). *Fit* is a deceptively complex word—a search in *Merriam-Webster Online Dictionary* reveals several definitions, many of which were unrelated to this particular context (although, arguably, they could have been, e.g., *fit* as adjective, "put into a suitable state;" *fit* as noun, "an emotional reaction"). In the context of this mathematical problem-solving activity, students in this study used *fit* in the following ways:

- verb: to conform correctly to the shape or size of,
- verb: to insert or adjust until correctly in place,
- verb: to make a place or room for,
- intransitive verb: to conform to a particular shape or size,
- intransitive verb: to be accommodated, and/or
- noun: the degree of closeness between surfaces in an assembly of parts. (Fit, 2017)

Students' utterances during the collaborative dialogue included expressions such as, "*Qu'est-ce que c'est* **fit** *en français?*" ("What is ***fit*** in French?"); "*Je ne pense pas que ça*

fit" ("I don't think it **fits**"); "*seize ne* **fit** *pas là*" ("sixteen doesn't **fit** there"); "*on peut juste essayer et tout va* **fitter**" ("we can just try and everything will **fit**"); "*toutes les choses* ***fit***" ("all the things **fit**"); and "*il n'y a pas un qui ne* **fit** *pas*" ("there isn't one that doesn't fit").

Turns containing the L1 word *fit* were counted as codeswitched in the tallies, since, technically speaking, *fit* is not recognized as a French word in French dictionaries. However, it is clear from these examples that students often intended for the turns that included *fit* to be L2 turns, as *fit* was in many cases the lone L1 word uttered in those turns. In the example, "*on peut juste essayer et tout va* **fitter**," the student, in this case Scot (group L1) clearly engaged in an LRE in which he applied rules to a new context, transforming English *fit* into a French verb ending in *-er*, namely *fitter*. The invented verb is correctly conjugated in future tense, "*tout va fitter*." Arguably, several other examples in which fit is used in a lexicogrammatical sense—such as, "*Je ne pense pas que ça* **fitte**," "*seize ne* **fitte** *pas là*," "*toutes les choses* ***fittent***," and "*il n'y a pas un qui ne* **fitte** *pas*"—could also be conjugated correctly as invented *-er* verbs as I have illustrated here, however with only oral data it was in these cases less clear as to the students' intentions (i.e., clearly L1 English *fit* or L2 invented French *-er* verb *fitter*). In these cases, the L1 uses were classified as other examples of LREs, usually generating lexical alternatives, when analyzed in their larger discursive context.

The online *Larousse dictionnaire anglais-francais enligne* recognizes the various English definitions of *fit* (including those noted above). In many cases, the *Larousse English-French Dictionary* does not provide single-word translations for *fit*, but, rather, provides translations of expressions based on the various meanings. For example, for *fit*

as an intransitive verb meaning "be of the correct size," *Larousse* offers the following translations (not all of which relate to the mathematical context of the problem-solving task at hand):

- the dress doesn't fit (*la robe ne lui/me etc. va pas*)
- this lid/key doesn't fit (*ce couvercle/cette clé n'est pas le bon/la bonne*)
- the key won't fit in the lock (*la clé n'entre pas dans la serrure*)
- do these pieces fit together? (*est-ce que ces morceaux vont ensemble?*)
- it won't fit (*cela n'ira pas*)
- we won't all fit round one table (*nous ne tiendrons pas tous autour d'une table*)
- cut the pieces to fit (*couper les morceaux aux mesures adéquates*)

The transitive phrasal verb *fit into* corresponds perhaps most closely with a direct translation:

- to fit something into something (*faire entrer* or *tenir quelque chose dans quelque chose*). (Fit, n.d.)

Indeed, I recently (May, 2017) presented the "fit example" at a colloquium on oral didactics, at the *Association francophone pour le savoir* (ACFAS) in Montreal, at which several members of the audience who were Quebecois francophone speakers informed me that they, themselves, would have a difficult time finding an equivalent translation of the English *fit* in the particular context of this mathematics problem. They also indicated that they often borrow the English word *fit* and use it in sentences like "*Ça ne fit pas bien*" ("It does not fit well").

To illustrate a full analysis of the L1 use involving *fit*, I begin with the following excerpt from the collaborative dialogue of Liz (LFI) and Sue (EFI) of group L4:

26 Liz: *OK. « Pourquoi avez-vous choisi cet emplacement d'objets en particulier ? » Je pense que c'est la seule façon que toutes les choses fait mais.* (OK. "Why did you chose this arrangement of objects in particular?" I think that it's the only way that all the things do but.)

27 Sue: *Je ne sais pas. Parce qu'il y a des enfants (?).* (I don't know. Because there are children [?].)

28 Liz: *Oui, et les couleurs.* (Yes, and the colours.)

29 Sue: *Oui, je ne sais pas qu'est-ce qu'on est supposé de mettre. . . .* (Yes, I don't know what we're supposed to put. . . .)

35 Sue: *OK. Est-ce qu'on peut prendre de (?)? Veux-tu qu'est-ce que j'ai maintenant? « Parce que toutes les objets vont dans la cour et . . . . »* (OK. Can we take some (?)? Do you want what I have now? "Because all the objects go into the the playground and . . . .")

36 Liz: *« Parce que tout les objets . . . »* ("Because all the objects . . . ")

37 Sue: *J'ai dit, « vont dans la cour », parce qu'il n'y a pas un qui ne* **fit** *pas.* (I said, "go into the playground," because there isn't one that doesn't **fit**.)

38 Liz: *On sait qu'essaye de dise.* (We know what we try to say.)

In this collaborative dialogue, Liz and Sue are working through the writing portion of the mathematical task, in which they are responding to a worksheet question asking why they chose their particular design for the playground. This is the first point over the course of the 3-day task, and the only time, at which Liz and Sue (Sue, specifically) use the L1 word, "*fit*." The need to put their explanation into writing prompted Liz and Sue to make decisions about precisely how to express their ideas about fit. In turn 26, Liz makes an

initial attempt by using "*toutes les choses fait*" ("all the things do"). While not explicitly rejected (out loud) by Sue, the suggestion was not taken up as Sue later (turn 35) offered an alternative in her writing, using, "*toutes les objets vont dans la cour*" ("all the objects go into the playground"). She asked Liz, "do you want what I have now?" (turn 35), which was affirmative since Liz began writing the response in the subsequent turn (turn 36), indicating that Liz had assessed Sue's alternative to be correct. Liz, however, casted some doubt on the final choice, when she remarked the end of the exchange, "we know what we try to say" (turn 38). This indicated that she was perhaps not entirely satisfied that "*vont dans*" was conveying the same meaning as "*fit*." The collaborative dialogue shows an example of an L1-based LRE of meaning-making activity, in which the students generated and assessed alternatives to fit, which was sub-coded as, non-academic-mathematical-lexicogrammatical.

The LRE also coincided with at least three MREs, in which the students were searching for certainty, and expanding on their explanations, in particular with the use of elaboration (restate) and enhancement (qualify by cause). For example, Liz begins to provide explanation for their design with an MRE in which no L1 is used, namely, "*Je pense que c'est la seule façon que toutes les choses fait mais*" (turn 26). Here, Liz was generalizing, searching for certainty, by explaining that their configuration was the only way possible since it was the only way in which "all the things do (fit)". Later, in turn 35, Sue provided an enhancement with, "*parce que toutes les objets vont dans la cour*" (signalled by the use of *parce que*) as well as an elaboration with a restate of "*les choses fait*" as, "*les objets vont dans*". Finally, Sue continued the collaborative dialogue and closed this mathematical communication act with another enhancement and elaboration,

this time as a means of clarification with a restate, finally using the L1 "**fit**" in turn 37 ("*parce qu'il n'y a pas un qui ne* **fit** *pas*").

At that point, Liz and Sue moved onto finishing their written responses, thus the final assessment of the alternatives to *fit* was able to be tracked. The final written products were as follows:

> Liz: *Tu devrais choisir notre plan parce que tout les objets vont dans le cour avec assez de place.*
>
> Sue: *Tu devrais choisir notre plan de cour parce que toutes les objets vont dans le cour avec assez d'espace.*
>
> (Liz: You should choose our design because all the objects go into the playground with enough room.
>
> Sue: You should choose our playground design because all the objects go into the playground with enough space.)

Furthermore, in the post hoc interview, I was able to use these written responses as a stimulus for follow-up. Specifically, the question of "fit" emerged during our discussion of the scale diagram. There is evidence of uptake of "*vont dans*" 14 days post hoc, when "*où toutes les choses vont*" is used:

> MC: *Et vous avez choisi une échelle. Alors vous avez choisi « un carré égal deux pieds ». Avez-vous discuté de pourquoi vous avez choisi cette échelle-là?*
>
> Sue: Je ne voulais pas que ça soit trop grand et difficile, ah, parce que si tu fais une erreur plus grand ça prend plus de temps, donc si tu fais ceci c'est plus facile à voir où toutes les choses vont je pense.

(MC: And you chose a scale. So you chose "one square equals two feet." Did you discuss why you chose that particular scale?

Sue: I didn't want it to be too big and difficult, ah, because if you make a bigger mistake it takes more time, so if you do this it's easier to see where all the things go I think.)

Together, the oral collaborative dialogue, the written work, and the follow-up interview help to build an understanding of the complex ways in which the L1 can be used during an L2 mathematics task.

In response to research question 3, descriptive quantitative data give an overall view of the instances of L1 use that occurred during students' collaborative dialogue while problem solving in their L2. Use of the L1 fell into a number of the predetermined coding categories in the frameworks used for analyzing LREs and MREs. Of particular salience were LREs related to generating alternatives for lexical items and lexical searches, and MREs related to expanding on explanations. In addition, a number of "other" instances of L1 use were coded, such as those pertaining to moving the task along and interpersonal communication.

4.5 Conclusion

This chapter presented the results of the data analysis. The main source of data was the audio recordings of students' collaborative dialogues in the classroom as they worked through the mathematics problem-solving task in their L2. These audio data (transcribed into written form) became the core of the discourse analysis. The introductory section of the chapter presented results that emerged organically from the data analysis related to context. While the themes were not named for investigation

specifically in the research questions, these task-related results proved important for contextualizing the remainder of the results chapter and for providing the rich, thick description necessary for a thorough qualitative analysis. The subsequent sections of the chapter were organized in response to the three research questions guiding the study. The collaborative dialogue discourse was analyzed using coding frameworks that related to LREs, MREs, and L1 use. Descriptive quantitative results were elaborated with in-depth analyses of transcript excerpts, which employed the discourse tools of inquiry in order to do so. In the next and final chapter of this dissertation, Chapter 5, I present the discussion of these results, which includes interpretation of the findings and how they relate to previous research and theoretical constructs in the fields. Chapter 5 also includes a summary of the study, recommendations for educators, and suggestions for future research.

5.0 DISCUSSION AND CONCLUSION

In this fifth and final chapter, I present a discussion of the results of the study and conclude the dissertation. I begin the discussion with an interpretation of the results presented in Chapter 4. I interpret the findings by making connections to previous research in the field while underscoring new understandings taken from the present study. Theoretical and practical implications also form part of the discussion, as well as suggestions for future research. Chapter 5 closes with a summary of the study, and concluding reflections on its significance.

5.1 Interpretation of Results

As seen in Chapter 4, the results of this study were grouped into four main sections. First was an introductory section that aimed to contextualize the subsequent results and involved emergent findings related to the mathematics problem-solving task. One of the key unanticipated findings of this study was the importance of the task in terms of student engagement and discourse production. Indeed, when the participating teacher and I first attempted the data collection in the pilot round, the procedure-based worksheet proved to elicit minimal discourse. The lessons learned in the pilot round led to choosing a different problem for the next round of data collection, in particular, one that adhered to the tenets of good problem-solving tasks, as outlined in seminal problem-solving research such as Polya (1957; Polya & Conway, 2004) and reiterated in more recent foundational documents for mathematics education such as the NCTM *Principles and Standards* and related documents (2000, 2010), and the province of New Brunswick mathematics curriculum documents (2011, 2013). In turn, when working on the Planning a Playground problem-solving task, students produced more discourse and discourse that

was more collaborative in nature. The NCTM in particular has been clear about the fundamental need to choose good quality mathematics problems, that is, true problem-solving tasks, in order to promote mathematical communication in the classroom (NCTM 2000, 2010). The influence of task selection on the discourse produced in L2 classrooms has also been highlighted in the literature. For example, Swain and Lapkin's (1998, 1999) initial work on LREs was foundational in the design of the present study. In their study, the authors used two different tasks to elicit student collaborative dialogue: a dictogloss and a jigsaw. With regard to the tasks themselves, the authors described how each task caused the students to produce different kinds of discourses related to three areas: nature of the stimulus (in this case, auditory vs. visual); whether or not a model is provided (in this case, one task had a model, the other did not); and the different cognitive demands associated with the different tasks (Swain & Lapkin, 1999). Indeed, other researchers in L2 education who are interested in task-based learning have underscored the importance of choosing good tasks in order to promote meaningful communication in the L2 classroom, and how the choice of task affects the resulting communication (Lee, 2000). In this study, a similar phenomenon occurred in that there were differences between the discourses produced by the students who used the procedural mathematics task (Grade 10 pilot round data) and those who used the true problem-solving task (Grade 9 data). One key difference was that substantially more collaborative dialogue, LREs, and MREs occurred when the problem-solving task was used as a stimulus. While there could be other factors at play (e.g., different students, different grade levels), I would argue that at least part (and possibly a great part) of the difference stemmed from the nature of the tasks. In order for students to reap the potential benefits from mathematical

communication described by the NCTM *Standards* (2000) and the New Brunswick *Mathematics Curriculum* (2011), such as deeper conceptual understanding of the content and the ability to explain their mathematical thinking, they first need to be motivated to communicate. Likewise, in order for students to use language in meaningful ways, thus learning the language (Long, 2007; Swain, 2000, 2008, 2010), they need to communicate about something. If students are to engage in collaborative dialogue with each other that is meaningful on both a mathematical and a linguistic level, then findings from this study, supported by others taken independently from the two fields, suggest that choosing an effective mathematics problem-solving task is an important first step.

Unlike the procedural worksheet, the mathematics problem-solving task resulted in students producing the vast majority of the classroom discourse over the three class periods of the task. The amount of teacher discourse was greatly reduced, which was key in a study in which the research goal was to explore students' collaborative dialogue. This is not to say that effective teacher talk has no place in the mathematics classroom, but that when the pedagogical goal (and, in this case, the research goal) is to encourage interaction in the form of collaborative dialogue between students, it is necessary to choose tasks conducive to those aims. This result was in contrast to most classrooms at the secondary level, including mathematics classrooms, where seminal research has reported that teachers typically talk anywhere from 55% (for higher achieving classes) to 80% (for lower achieving classes) of the time (Flanders, 1970, cited in Fisher, Frey, & Rothenberg, 2008).

With regard to how much of the student talk was "on task," as noted in Chapter 3, I suggest that it is difficult and perhaps counter-productive to try to make judgements as

to what is or is not part of the mathematics at hand. Other scholars have explored these so-called "non-mathematical" storylines by calling into question whether they are, indeed, "non-mathematical" (Barwell, 2009a; Herbel-Eisenmann et al., 2015), since they often affect the mathematics and certainly the communication that is occurring in the classroom. Elsewhere, I have begun exploring how mathematics is arguably more meaningful (and/or powerful) when it becomes intertwined with "real life" elements or storylines (Culligan & Wagner, 2016). In this study, the so-called "off task" communication affected the mathematics in different ways—for example, when a student was having a social problem with another student at lunchtime, she had difficulty focusing on the mathematics task. From an L2 perspective, this kind of "off task" communication can be quite meaningful and it is often where students feel compelled (or required) to use their L1. In more recent work from the L2 field, for example, Swain and Lapkin (2013) have similarly questioned their own previous categorization of talk as "off task", since this kind of discourse is interpersonal and thus affects the "doing" of the task. Furthermore, from an L2 learning perspective, allowing students the opportunity to engage in these kinds of "off-task" communication acts could lead to increased opportunities for language output, and language output is a key part of language learning (Swain, 1985). Even when the output is about interpersonal communication or other storylines, it is precisely these kinds of language practices that L2 students need to be able to learn to navigate: Students in this study reported that while talking mathematics in their L2 was fine, it was more "natural" to use their L1 for the "in-between" talk. These observations are supported elsewhere in the literature from both L2 education and bilingual mathematics classroom research (e.g., Swain & Lapkin, 2013; Moschkovich, 2005).

Thus, in contrast to the Grade 10 piloting round of data collection, in which a procedural-style worksheet was used as a stimulus, the Grade 9 data collection resulted in higher numbers of participants (presumably at least partly due to their intrigue in the promise of hands-on, interactive mathematics tasks), namely, 22 versus five; a higher quantity of discourse per participant (as measured in number of turns per student); and a higher quality of discourse (as measured in instances of language- and mathematics-related episodes). The nature of the playground activity required students to interact as well as to attempt mathematical explanations. Further, the fact that the problem could be approached from a variety of strategies (e.g., guess-and-check; hands-on modelling; calculations) appeared to offer all students the opportunity to participate. Although there were "off task" communication acts and these can be valuable both mathematically and linguistically, most of the student interactions in the classroom were related to the mathematics task at hand. That being said, students' final written solutions were affected by their abilities to problem-solve; their willingness (or not) to apply "real-life" adjustments to a purely mathematical result; their abilities and/or willingness (or not) to be precise; and their willingness and/or motivation to make changes to initial results and find explanations for these changes. Moreover, in an L2 mathematics class, socioaffective factors related to motivation and identity, in other words, their storylines (Herbel-Eisenmann et al., 2015) and figured worlds (Gee, 2014), along with the Discourses enacted (Gee, 2014), also enter into consideration. This mathematics task was happening in a real classroom, with real students. Therefore, the factors at play within any given student group are complex.

The basic result that LREs were able to be observed and coded in students' collaborative dialogue was an important finding because, unlike Swain and Lapkin's (1998) study, in which students were engaged in L2 language-based tasks that were specifically designed to elicit attention to language in terms of both lexis and form (i.e., the dictogloss and jigsaw tasks), in this study the tasks were mathematical in nature. Thus, despite the content of the task and the focus of the problem-solving activity being mathematical (rather than linguistic), students not only used but also attended to language. This supports the overarching theoretical stance of this study, which is based in the SCT idea that language is an important cognitive tool in any learning activity (Lantolf, 2000; Swain, 2008, 2010; Swain et al., 2011; Vygotsky, 1962, 1978; Wertsch, 1985, 1993) and that "doing" mathematics involves a mathematics register (Halliday, 1978, Pimm, 1987) or, more specifically, a mathematics education register (Barwell, 2005b, 2007; Barwell et al., 2005; Moschkovich, 2003, 2007, 2010). Several a priori coding categories were retained based on Swain and Lapkin's (1998) work on LREs in the L2 field. However, initial iterations of data analysis revealed that it was necessary to adjust the coding categories so that the finalized LRE coding framework was more reflective of the mathematical content of the problem-solving task stimulus. Thus, although the FI mathematics classroom is indeed an appropriate place for LREs to occur and a place in which they do occur, researchers' and educators' ideas of what those LREs might look like need to be broadened so as to recognize and value the oftentimes mathematical nature of the lexis, form, and lexicogrammar. The majority of the LREs were lexis-based (or lexicogrammatical), both those that occurred using only students' L2 and those in which students used the L1. Importantly, students attended to "strictly" lexis-based items,

academic-mathematical items, and non-academic-mathematical items. This suggests that students use a variety of linguistic resources from the "everyday" and the "academic" mathematical repertoires in order to enact the mathematics education register and engage in mathematical discourse, which echoes previous research from the mathematics education field that also suggests this may be the case especially for students working in their L2 due to their need to bring in all their available resources to deal with the cognitive load (Barwell, 2009a, 2009b, 2009c; Moschkovich, 1999, 2002, 2009a, 2009b, 2010). The dichotomization of "everyday" and "academic mathematical" talk is a tension acknowledged by Barwell (2010) in L2 mathematics classrooms, therefore seeing the mathematics education register as a blend of these discourses would serve to ease this tension.

In this study, MREs were defined as "mathematics-related episodes" of dialogue; therefore, the MREs were never separated from language, an approach that is once again faithful to the overarching theoretical framework of this study. The high numbers of MREs in the areas of describing a mathematical situation and expanding (elaborating, extending, enhancing) appear to be directly related to the nature of the mathematics task at hand, which had a focus on explanation of a mathematical phenomenon and of mathematical choices. Thus, students explained using various strategies as coded in the MRE framework, but relied mainly on modes of description and expansion in order to do so. During MREs in particular, it was especially difficult to separate non-academic from academic mathematical discourse. While it was usually possible to do so for LREs that involved a single lexical item or form, MREs typically comprised non-academic and academic mathematical discourse to such an extent that it was counter-productive to try

to untangle them. The broader mathematics communication acts (i.e., about a single topic) were oftentimes composed of multiple MREs and LREs, thus pointing again toward the integrated nature of the mathematics education register (Barwell, 2005b, 2007; Barwell et al., 2005; Moschkovich, 2003, 2007, 2010). The hands-on and visual/graphical components of the mathematics problem-solving task also provided students a way to engage in MREs by using these elements as language supports, which has been suggested as important in L2 mathematics contexts (Moschkovich, 2009a).

Other support upon which students drew included their use of the L1 during the collaborative mathematics problem-solving task. The findings in this study lend support to those conducted in L2 classrooms and bilingual mathematics classrooms. For these participants, certain LREs and MREs contained more L1 use than others, and these areas generally matched those identified in both the L2 and mathematics education literature, for example lexical searches (Swain & Lapkin, 2000), to ease cognitive load (Adler, 1998; Behan et al., 1997; McMillan & Turnbull, 2009; Setati et al., 2002; Swain & Lapkin, 2000, 2013; Turnbull, 2001; Turnbull et al., 2011), for task organization (Swain & Lapkin, 2000, 2013) and for a variety of interpersonal and/or sociolinguistic purposes (Moschkovich, 2005; Swain & Lapkin, 2000, 2013). This study was able to code instances of L1 use using a priori codes taken from L2 and mathematics education literature, an undertaking which again points to the simultaneous uses of language and mathematics in the L2 content classroom. Through their various uses of the L1, students were able to further their L2 language use as well as their attention to content, particularly during times when they were able to use L1 lexical resources and/or when the cognitive exercise of explaining their solutions became heavy (recall the example of "to

fit"). Students' use of the L1 in these various ways also lends support to the SCT view that language, including bilingual students' L1, acts as a cognitive tool and resource (Vygotsky, 1962, 1978). In addition, it reinforces arguments made in more recent years by L2 researchers claiming that some judicious use of the L1 in L2 classrooms can further L2 learning and help scaffold tasks and content learning (e.g., Behan et al., 1997; Cummins, 2007; Levine, 2011, Swain, Kirkpatrick, et al., 2011; Swain & Lapkin, 2000, 2013; Turnbull & Dailey-O'Cain, 2009). The "fit" example, in particular, in this study also illustrates how the L1 can serve as a resource, rather than simply be indicative of an L2 deficiency. The complexity of expressing the English *fit* in L2 French is not to be underestimated. Since students are bilinguals and thus have a common underlying proficiency (Cummins, 1981), it is arguably natural for them to call upon their L1 resource to supply them with a word that so aptly described the particular context. Like those researchers working in L2 education contexts noted previously, scholars in L2 mathematics contexts have similarly cautioned against a deficit-oriented approach to viewing bilingual students' L1 use (e.g., Moschkovich, 2005). In this study, the notion of *fit* became an intersection of lexical and mathematical interest for the participants, and illustrates nicely the complexities of the intertwined nature of the language and the mathematics, the academic and the "everyday."

Despite the appearance of instances of L1 use in students' collaborative dialogue, however, the majority of students used the L2 the majority of the time. This is an important finding since teachers' guilt regarding students' (real or perceived) use of the L1 when allowed to interact and collaborate with each other in class is a reported source of anxiety and a potential reason for limiting these types of tasks (Culligan, 2008, 2015;

McMillan & Turnbull, 2009; Swain, Kirkpatrick, et al., 2011). For teachers interested in providing students the opportunity to engage with each other in collaborative problem solving, I would suggest that based on this study, they should feel comfortable and encouraged to do so. The majority of students do use the target language most of the time and the majority complete the task and take valuable mathematics and language away from it.

For both LREs and MREs, episodes were concentrated around two pivotal parts of the mathematics problem-solving task: the point at which students were trying to determine the correct fit for the playground equipment, and later when they were required to provide written responses (explanations) of their solutions. This again points toward the importance of providing students with a true problem-solving scenario in order to cultivate communication (NCTM, 2000, 2010). It also suggests that providing students with the opportunity to engage in follow-up to oral tasks, such as a written task, is an important component for both the oral dialogue and the deepening of understanding of the language and the mathematical content involved in the task. Moreover, a written production component may serve to push students' L2 use even when the language task proves challenging. Recall, for example, in this study, partners Mac and Mike (group E1), who spoke French in only 22.5% of their turns. One key point in the task at which they did use the L2 was when they needed to formulate their written responses. For groups who used the L2 more regularly, for example, Liz and Sue, who uttered only 1 English turn between them during the entire activity, the formulation of the written responses also provided focused opportunity to extend their L2 use (e.g., figuring out the French equivalent to *fit*). This supports research from the L2 field, which has similarly suggested

that a written component is important for similar kinds of reasons, even in primarily communicative tasks (e.g., Swain, 2001; Swain & Lapkin, 2013).

Finally, the LREs and MREs (whether in the L2 or the L1) occurred during collaborative dialogue. This is a key finding since much of the research on collaborative dialogue as a form of languaging has been conducted in L2 language classrooms (i.e., not L2 content classrooms). Importantly, the results of this study show that many of the concepts of SCT for L2 learning in language classes also apply to the FI mathematics classroom in this study and likely to other L2 mathematics/content classrooms, such as: the social nature of learning, the key role of language in learning, the importance of collaborative dialogue more specifically, the importance of the co-construction of knowledge in the ZPD (Swain, 2000, 2008, 2010; Swain, Kinnear, et al., 2011), and the ability of peers to act as expert other (e.g., Donato, 1994, 2000; Gutiérrez, 2007; Lantolf, 2000; Lantolf & Appel, 1994; Lantolf & Pavlenko, 1995; Ohta, 2000; Swain, 2000; Swain & Lapkin, 1998, 2000).

The interpretation of the results presented in Chapter 4 in light of previous research illustrates how the present study builds on key pertinent literature from the L2 education and mathematics education fields, particularly those whose contexts and methodological approaches are tied most closely to those of the present study: the secondary FI mathematics classroom viewed through the lenses of SCT and the mathematics education register. Although this study lends support to several arguments from the literature, it also contributes new perspectives and coding frameworks given its novel methodological approach (discourse analysis of classroom-based audio data, triangulated with written work and post hoc interviews) to the little-researched context of

secondary FI mathematics. Consequently, there are a number of implications that stem from the key findings of the current research study.

5.2 Implications

Based on the results of this study, there are several implications for theory and practice in secondary FI mathematics. Many of these implications could be extended to other immersion content classrooms and other mathematics classrooms in which students are learning in an L2.

This study has shown that FI mathematics students at the secondary level do engage in collaborative dialogue when given the opportunity to do so. However, the type of task is of utmost importance if teachers' aim is to encourage communication that is complex and meaningful, both linguistically and mathematically. While procedural knowledge and tasks certainly have value and should be given a place in FI mathematics, these types of tasks will likely not generate much in the way of in-depth, sustained, collaborative dialogue among students. Teachers need to choose true problem-solving tasks that adhere to at least some of the fundamental characteristics of such tasks, as described in seminal literature and in more recent documents such as the NCTM *Principles and Standards* (2000). Unfortunately, the assigned textbook may not always be a sufficient resource in which teachers can find such problems. Consequently, FI mathematics teachers may need to look elsewhere in order to find good problem-solving tasks. This can arguably pose a challenge, as there is less material available for FI mathematics teachers (i.e., resources written in French) at the secondary level in particular. The result, then, is an implication also for educators and researchers regarding the need to produce more resources for secondary FI teachers that contain good problem-

solving tasks that are linguistically and cognitively (content) appropriate. Moreover, the specific task used in this study was fairly multimodal in nature, using reading, writing, kinaesthetic, and visual elements. The importance of using these types of multimodal (e.g., Jewitt, 2008) materials cannot be underestimated, as they can allow students increased access to the task (mathematically and linguistically) and can allow them to use mathematics and language in more creative ways. This can be of particular value for FI students doing mathematics in their L2 and while this might be more recognized at the elementary level, the results of this study indicate that it is true at the secondary level as well. Moreover, while the problem-solving task in this study was focused on eliciting oral communication, the point at which students were required to write was one where many valuable LREs and MREs were produced. Consequently, having a final product attached to the task, whether it is written or oral production, is a key component of success. In sum, some of the key characteristics of the problem used in this study that contributed to success (defined as having engaged students in collaborative dialogue about mathematics) included: (a) the problem connected to the SCOs and the seven general mathematical processes described in the provincial curriculum document, as well as to the NCTM (2000) principles and standards for school mathematics; (b) the problem addressed several of the criteria outlined by the NCTM (2010) as necessary for worthwhile tasks—for example, the task had important mathematics embedded in it, required problem solving, contributed to conceptual development (rather than strictly procedural development), could be approached in multiple ways, provided an opportunity for students to practice important skills, allowed different positions to be taken and defended, and (of utmost important for providing opportunity for L2 use) encouraged

student engagement and discourse; (c) the problem allowed for increased opportunities for access to the task due to its use of hands-on modelling, scale diagram drawing, and/or calculation work; (d) the problem was set in a "real life" scenario, which contributed not only to the mathematical value (e.g., the way that the "real life" element of fitting playground equipment into a space changes the purely mathematical result of the maximum area calculation) of the task but also provided opportunity and need for talking through the results and the possible solutions; (e) the problem allowed for various opportunities for mathematical success (e.g., even though some students did not ultimately find the 52 x 48 dimensions as their solution, they made the important discovery that changing the dimensions of a given rectangular perimeter changes the enclosed area); and (f) the problem, while aiming to elicit oral communication, also had a written follow-up component that served to focus students' mathematical and linguistic attention. Teachers and resource developers would do well to attend to these elements when thinking about what types of problems are needed in order to present tasks that will challenge students cognitively and linguistically, while still allowing access on multiple levels. Despite the value of these communicative problem-solving tasks that aim to have students engage in collaborative dialogue, this type of activity does not appeal to all learners. As with any classroom, a variety of teaching approaches is key—while it would not be effective to engage only in communication-based student-led mathematics, nor would it be effective to never do so.

Allowing students, particularly in L2 programs like immersion where teachers have a complex dual role of teaching both language and content simultaneously, to have students work independently can be intimidating since L2 teachers have long been

concerned, rightly, with providing enough opportunities for comprehensible input and output, which are key to language learning. When students are working together in pairs or small groups, dispersed throughout the classroom, there is no way for the teacher to know what is being said, and why and how it is being said, at all times. However, this study suggests that, when given the opportunity and when given an appropriate problem-solving task, students engage in collaborative dialogue in which they attend to both language (LREs) and content (MREs). Through this collaborative dialogue, students are able to construct knowledge and make discoveries regarding the mathematical content that they may not have made on their own, or, at least, the discoveries they make seem more meaningful (e.g., in this study, the mathematical discovery that changing the dimensions of a given total perimeter of a figure affects its area). Because the students are working on the mathematical content through collaborative dialogue, they are necessarily using, thus learning, the L2 as well. Thus, an implication for teachers of secondary FI mathematics is to trust in students' ability to work independently, and to allow them opportunities to work through problems with guidance rather than direct teacher instruction. Teachers should work toward embracing that, even though it is impossible to know everything that is said during the task, that there is (more often than not) useful attention being paid to the mathematics at hand and to the language needed to do it. Teachers may find it difficult to not know what students are saying or doing during their partner work. As a researcher, I had the advantage of listening to the entire audio recordings of the students' work and thus was able to gain insights and understandings as to how they engaged with the task. I could assess how they did based on listening in on their discussions, which was a great advantage since their final written products did not

always showcase what they had discussed in their groups. Teachers cannot likely record all of their students when assigning collaborative tasks and subsequently listen to their talk. However, it would be advantageous to do this from time to time if possible. Particularly for L2 mathematics teachers, this exercise would provide valuable insights to both the language and the mathematics students are using. One idea would be for students to record themselves using iPhones, iPads, or other technology: Students could listen back on their own communication as could the teacher, and subsequent discussions could take place based on what emerged from the recordings. Alternatively, the follow-up interviews, used in this study as a source of data triangulation, could have great pedagogical value. When using student-based collaborative problem-solving tasks in FI mathematics, following up with a short interview with the pairs would be another effective way of determining how students did, both mathematically and linguistically, with the task. With either method, the assessment should be guided by a resource-based approach to mathematics and language. In other words, students will need support, particularly if this is a new practice, with self-assessing their mathematical communication. Teachers could prepare checklists of "look fors" that students could use to assess their own communication and this could then be followed by assessment in collaboration with the teacher. As students become more familiar with this kind of exercise, responsibility could be increasingly released to them (e.g., they could create the checklist themselves, or they could analyze a piece of recording in an inductive way). Overall, the key idea would be to celebrate what students have achieved, both mathematically and linguistically, rather than focus on what they did not do. This is important in order to build the trust required for students to feel compelled and

comfortable enough to take on the risk associated with mathematical and linguistic problem solving, and, in addition to that, the risk of being recorded or interviewed. In a conversation building from that foundation, students and teachers could then proceed to discussing the challenges students might have faced (ideally, these would be brought to the forefront of the discussion by the students themselves), and how these might be addressed.

Secondary FI mathematics teachers must pay attention to, address, and support the language that students need in order to work collaboratively and problem solve. In this study, academic mathematical vocabulary was of key importance—however, it was not the only kind of vocabulary that proved essential. For L2 learners doing mathematics in the L2, academic vocabulary and everyday vocabulary (of a mathematical and non-mathematical nature) become essential parts of the communication. Thus, teachers should draw explicit attention to the lexis, forms, and lexicogrammar needed to communicate mathematically in meaningful, sustained ways. Teachers of FI mathematics should value and recognize these so-called "everyday" ways of communicating mathematically, while moving students increasingly toward academic language. Mathematics teachers in FI would do well to incorporate best practices for supporting literacy into their content area (e.g., Kartchner Clark & Brummer, 2009; Macceca & Brummer, 2009).

Providing this type of language support may help mitigate a key concern on the part of FI teachers, including those that teach mathematics: the fear of students using their L1 when working independently in pairs or small groups. This concern stems from all of the reasons discussed in Chapter 2, relating to pedagogy and policy. This study, however, showed that students' dialogue occurred mainly in the L2, although they did

use their L1 to varying degrees and for varying purposes. Often, the L1 was used for lexical searches, or as a repeat or restate to clarify meaning. In these and many other cases of LREs and MREs, the L1 served to further the dialogue and thus the language and/or mathematics at hand. Therefore, FI mathematics teachers should move toward acceptance of some use of the L1 as a resource employed by students to further their understanding of the language and the content. In this way, recent calls for judicious use of the L1 in various L2 contexts, including immersion, seem to be supported by the findings of this study. If students are permitted some overt use of the L1, teachers have the potential to know and therefore address the salient needs that emerge through the various classroom activities (e.g., how to express *fit*, in this study). Even when the L1 is used, as it often is, to engage in interpersonal interactions of various types, teachers should recognize that the students are bilingual individuals and, as such, use language(s) in different ways and for different purposes, including those that have interpersonal and emotional connections. That being said, teachers, even in FI mathematics, could harness these instances as opportunities for providing students with the linguistic know-how (e.g., lexis, forms) to use this kind of interpersonal language—these storylines are present in the mathematics classroom. "Judicious use" has been recognized as difficult to define: How much L1 is too much? Swain and Lapkin (2013) recently made some recommendations that I would argue are supported by the findings of this study and have direct implications for FI teachers (including mathematics teachers). In the younger grades, FI teachers should use the target language (French) almost exclusively; students, for their part, will need to use some L1 as they build vocabulary and proficiency in the L2 particularly at the beginning of their program. As students advance through the grades,

however, they should work together with the teacher to establish classroom goals for L2 use. Students may use their L1 in private speech or to work through cognitively demanding tasks, gradually moving to the L2 as the mediating language as their proficiency and understanding increase. Teachers should continue to mainly use the L2; however, occasional, deliberate use of the L1 in order to address L2 or content learning goals (e.g., to make cross-linguistic comparisons, or to explain an abstract mathematical concept) should be an acceptable practice. Certainly, it is important that FI mathematics teachers not deny students the opportunity to engage in collaborative tasks for the fear that they might use the L1 in order to communicate. It is also imperative that FI teachers be supported in these endeavours by administration and policymakers, who likewise must recognize the potentially beneficial role that the L1 can play in the L2 mathematics classroom. Educators, parents, the public, and even students themselves would benefit from viewing FI students as bilinguals who bring unique resources to the table, rather than poor imitators of "native French speakers." When one knows two (or more) languages, being able to draw on lexical/lexicogrammatical items from the L1, such as *fit*, can signify a resource, rather than a deficit.

5.3 Future Research

This study represents the only one if its kind, to my knowledge, exploring students' collaborative dialogue in the particular context of a high school FI mathematics classroom. Although the results contribute new understandings of how students communicate mathematically in secondary FI mathematics, the study nonetheless has limitations with regard to the depth and breadth of understanding it can provide. While this study has provided answers to the research questions it set out to address, it was

impossible to analyze every aspect of the data. More research is needed in order to fully explore FI students' mathematical discourses, which would address some of the questions that remain to be investigated.

The coding frameworks used in this study, particularly those for LREs and MREs, should be tested for usefulness in different mathematics classrooms, and when using different kinds of mathematics tasks. More research is needed in order to know whether the same types of LREs and MREs emerge using other kinds of problem-solving tasks. The frameworks could also be adapted to other content areas (e.g., science) and/or other L2 contexts (e.g., learners of English as an additional language). Moreover, a finer-grained analysis of question types used during the task (e.g., on the written worksheet portion) and the kinds of LREs and MREs they elicit would help researchers and educators better understand whether certain types of tasks and questions can be used to address specific language and/or mathematical goals.

Coding and categorizing students' L1 use proved to be an extremely complex undertaking and the analysis in this study was one way in which this could potentially be done. The instances of L1 use could be analyzed from a wide variety of angles—sociolinguistics, identity, willingness to communicate/motivation, cognitive linguistics, and more. A full linguistic analysis could be undertaken in terms of L1/L2 discourse markers alone. French immersion students in Canada represent a unique example of a particular bilingual context, influenced by all of the politics and policies that go along with it. Exploring these students' L1 uses/codeswitching practices as bilinguals in this particular sociocultural context could provide insights into how FI teachers and students can best negotiate practices for effective and acceptable language use in the classroom.

Moreover, adopting a translanguaging perspective in future research could lead to broader but, also, deeper and more complex understandings of bilingual students' classroom communication and consequently have important implications for pedagogy (e.g., Creese & Blackledge, 2010; Garcia, 2009, Garcia & Wei, 2014). Specifically, studies like this one, in which students used multiple languages (i.e., French, English); various registers (e.g., "everyday," mathematical, academic); and multiple modes of representations (e.g., graphs, drawings, mathematical notations, writing, hands-on modelling); could use a translanguaging perspective to explore how all of these bilingual practices come together in a non-segmented way. Indeed, the difficulty in separating various discourses—L1/L2, everyday/academic mathematical, on-task/off-task, speaking/writing, and so on—was an important finding in this study and could be further explored from a translanguaging standpoint. Garcia (2009) noted that, "the complexities of bilingualism and the resulting translanguaging are seldom acknowledged in schools, even in bilingual education programs" (p. 37). Thus, classroom-based translanguaging research involving students and teachers would be an important avenue to pursue. Research of that nature would serve to have schools and policymakers begin to acknowledge "the complexities of bilingualism and the resulting translanguaging" and therefore open an important line of communication regarding the efficacy of bilingual programs, and what expectations might be for students in these programs regarding how they learn and how they communicate using all of their available resources.

Relatedly, this study examined students' little "d" discourses; however, there was evidence that big "D" discourses influenced what students said, how they said it, and why they said it. As pointed out in Chapter 2 of this dissertation, mathematical Discourse

includes (among other things) ways of talking, acting, interacting, thinking, and believing, and also mathematical values and beliefs (Moschkovich, 2003). Moreover, there are other Discourses at work in the context of this study (and every context) such as the Discourses of schooling, of FI, of gender, and so on. More discourse analysis research, for example, using some of Gee's (2011, 2014) tools of inquiry not used in this study, such as the big "D" Discourse tool and the figured worlds tool, would shed light on how these mathematical and other Discourses are enacted in the context of secondary FI mathematics. These types of analyses would help build an understanding of how these students view themselves as bilinguals/speakers of French and as mathematicians, which would have implications for teachers aiming to foster positive learning environments in which students are motivated and willing to learn and use language and mathematics.

In this study, I gathered information on the students' program (EFI versus LFI) as well as their academic achievement in their FI language arts class (FILA 9) and their mathematics class (FI Math 9). These data primarily served the purpose of providing a richer description of the participants, situating each of them within the context of this classroom-based study. However, more research could be done with these kinds of data specifically, to explore whether program point of entry (i.e., EFI and LFI) has any effect on the frequency, quality, and nature of LREs and MREs produced in students' collaborative dialogue. Likewise, similar questions could be examined with regard to students' proficiency levels in French and their academic performance in standardized assessments in mathematics. In this study, it was difficult to note any clear patterns with regard to these factors; at least part of the reason for this was the small sample size. Research with a larger sample size using some standardized and calibrated assessment

measures along with possibly pretest/posttest data could shed light on these kinds of issues, and help teachers make decisions regarding how to bring the best LREs and MREs forward for students of varying academic backgrounds and proficiency levels.

Finally, one of the less anticipated but nonetheless key results to emerge from this study was the importance of choice of task in eliciting collaborative dialogue. In particular, the materials ultimately used in this study (for the Planning a Playground Activity) presented students with and required students to use multiple modes including reading, writing, listening, speaking, spoken interaction, low tech tools, hands-on models, and visuals. The nature of the mathematics problem-solving task had direct effects on students' access to the language and the content, their likes and dislikes of the task, seemingly their motivation, and the collaborative dialogue they produced. The multimodal nature of the task could have direct implications related to increased access for all students, and perhaps especially for L2 students. The participating teacher helped choose this task in particular because she wanted to incorporate these kinds of elements, which she felt she lacked in her own classroom teaching. Particularly at the secondary level, teachers sometimes struggle to integrate these types of hands-on, "real life", visual tasks, especially ones that also adhere to the key tenets of true problem solving. An action research cycle with secondary FI mathematics teachers exploring the effects of implementing problem-solving and/or multimodal tasks in the classroom would be an excellent way to investigate these issues. Particular attention could be paid to how these tasks are perceived by students, how the tasks affect their use of mathematics and language, and the ways in which these kinds of problem-solving activities might influence their identity(ies) as students/bilinguals/mathematicians/and so on. While each

of the avenues for future research I have outlined here intrigues me, this final option is one that I would be motivated to pursue most imminently.

5.4 Concluding Remarks

This study sought to answer three interrelated research questions: How do FI students at the secondary level attend to language, that is, what LREs emerge, while working collaboratively during mathematical problem solving in their L2 (French)? How do these students communicate mathematically, while working collaboratively during mathematical problem solving in their L2 (French)? When and how do these students use their L1 (English) while working collaboratively during mathematical problem solving in their L2 (French)? In order to do so, a theoretical framework was adopted that sought to bring together language and mathematics, rather than view them as separate entities. The framework combined SCT and the mathematics education register, providing a foundation upon which the entire study was based and a direction in which the literature review, methodology, and analysis would follow.

In addition to the seminal works consulted to build the theoretical framework, the literature review explored studies related to FI student achievement in French, English, and mathematics in order to provide context for the current study. Issues common to all bilingual mathematics classrooms were then investigated, such as the roles of context, policies, and polities, and the phenomenon of codeswitching. The literature review concluded with an exploration of what it means to do mathematics in an L2.

This was a classroom-based research study and as such, various preparations and challenges had to initially be met, such as university ethics procedures, and gaining access to a site and participants. The participating teacher played a key role in most steps

of the data collection process, in particular the development of the materials. The data collection took place over approximately 8 months of the school year, which included numerous site visits and interactions with student participants. The materials ultimately selected were a Planning a Playground mathematics problem-solving task, in which students were paired together and used reading, writing, oral interaction (collaborative dialogue), low-tech tools, hands-on models, and visuals/graphics to complete the activity. The main data collected were audio recordings of students' in-class oral dialogues, however these data were supplemented with researcher fieldnotes, students' written texts, and follow-up stimulated recall interviews with students as a form of triangulation.

Data were analyzed using Gee's (2011, 2014) discourse analysis theory and method. Transcripts of the collaborative dialogues were analyzed using coding frameworks for LREs, MREs, and L1 use that were based on the pertinent literature from both the L2 and mathematics education fields and then modified to better reflect emergent themes from the study at hand based on first iterations of data analysis. Several task-related findings suggested that student talk (rather than teacher talk) dominated the activity, that students mainly talked about the mathematics at hand, and that they used most of the anticipated problem-solving strategies to work through the task, although to varying degrees of thoroughness and success. Results showed that students engaged in various kinds of LREs and MREs, especially related to lexis and lexicogrammar, and also, although to a much lesser extent, form. The LREs involved non-mathematical items, non-academic-mathematical items, and academic-mathematical items. The MREs mainly involved students describing mathematical situations and expanding in order to provide explanations. Instances of L1 use emerged within the LREs (especially lexical) and the

MREs (especially expanding with repeat/restate). The L1 was also used to move the task along and for interpersonal interactions (especially vernacular and to express feelings). The LREs, MREs, and uses of the L1 were analyzed first using the coding frameworks and then the findings were further analyzed in depth using Gee's (2011) discourse tools of inquiry—in particular the "doing and not just saying tool," the "why this way and not that way tool," the "sign systems and knowledge building tool," and the "situated meaning tool"—applied to excerpts from the collaborative dialogue transcripts, and supported by the students' written work and the follow-up interviews.

This research study is among the few of its kind that aim to explore, to an equal extent, language and mathematics, and highlight the interrelated, intertwined nature of the two. Such work is important but messy, and the answering of certain questions leads to the posing of others. Thus, more research is needed, as suggested in the previous section of this chapter, to more fully understand FI mathematics students' classroom communication. Nonetheless, this study adds to the research conversation by providing unique and important insights into what is happening with regard to language and mathematics in the classroom, when secondary FI mathematics students are working mainly independently of their teacher on a problem-solving task in their L2.

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APPENDIX A: ETHICS APPROVAL LETTER

UNIVERSITY OF NEW BRUNSWICK
PO BOX 4400
FREDERICTON, NB
CANADA E3B 5A3
TEL 506 453-5189
FAX 506 453-3522
WWW.UNB.CA/RESEARCH
OFFICE OF THE VICE-PRESIDENT (RESEARCH)

October 15, 2015

Karla Culligan
Faculty of Education
University of New Brunswick
UNBF - Campus Mail

Dear Ms. Culligan:

RE: French immersion students' use of linguistic and mathematical resources during collaborative mathematical problem solving, **REB File # 2015-095**

The above project is approved as modified.

Approval is valid for a period of three years from the date of this letter.

Annual Reports for this project are due on the 15th January of each year, provided that this date is at least six months after the date of project approval. **Final reports** are due 90 days after project completion. Both of these reports can be found on our website at http://www.unb.ca/research/ors/forms/index.php#ethics.

Although your application was processed via Expedited Review, for your information we are providing a list of current Research Ethics Board members.

Sincerely,

R. Steven Turner

R. Steven Turner, Chair
Research Ethics Board

REB Members:
Joy Haines Bacon, Community Representative
Barbara Burnett, Community Representative
Jeff Landine, Faculty of Education
Tracey Rickards, Faculty of Nursing
Usha Kuruganti, Faculty of Kinesiology
Aloke Chatterjee, Faculty of Law
R. Steven Turner (Chair), Faculty of Arts, Department of History
Renée Audet-Martel, REB Coordinator

APPENDIX B: INFORMATION LETTERS AND INFORMED CONSENT FORMS

Student Information Letter

Dear (name of school) Student,

I am a former teacher at (name of school) and now a PhD student in the Faculty of Education at the University of New Brunswick, currently conducting a research study. This project has been reviewed by the Research Ethics Board of the University of New Brunswick and is on file as REB 2015-095. It has been approved by (name of Superintendent), Superintendent of (name of School District), and by (name of principal), Principal of (name of school). This study aims to understand how French immersion students talk about and use language and math when working together on solving math problems. This study is not about getting "right" answers to math problems. Rather, I am interested in learning more about how French immersion math students communicate when working together on problem solving tasks. I feel that learning more about this important topic will greatly benefit students and teachers, particularly as math problems and programs become much more language based. I believe that exploring these questions will help us understand how you, as students, use knowledge and skills in language and in math to problem solve. I also believe that my research will help teachers to identify different effective teaching practices.

If you agree to participate in this research study you will be audio recorded while working in small groups on math problems in the classroom. Your written work will be collected and evaluated as part of the regular math class. Information about your past academic achievement in French and math will also be collected. You will also be invited to take part in a short follow-up interview (about 20 minutes) in order to talk about your problem solving activity. The interviews may take place during your math class, or during noon hour or after school if you prefer, and they will also be audio recorded. Your teacher will also be asked to participate in separate interviews. Your participation is totally voluntary and should you decide to participate you may withdraw from the study at any time without penalty. You can agree to participate in all or part of the research project. During the interview, you may decline to answer any question, at any time. Every effort will be made to ensure that your results are made anonymous. Your real name will never be used. If you choose to participate, please indicate so by completing the informed consent form attached. You must also have your parents/guardians complete the informed consent form attached to their letter. Please return both signed forms to your classroom teacher.

Thank you for your time and consideration. If you would like further information you may contact me at 453-5136 or kculliga@unb.ca, my supervisor, Dr. Joseph Dicks, at 453-5136 or jdicks@unb.ca, or your math teacher. Should you wish to speak to someone not directly involved in this project you may contact Dr. David Wagner, Associate Dean of Graduate Programs at the Faculty of Education at 447-3294 or dwagner@unb.ca. A report of the results will be available to (name of school) once the project has ended. A copy will also be posted on our Institute website at www.unb.ca/L2.

Sincerely,

Karla Culligan, BA, BEd, MEd

Student Informed Consent

I, (please print) _____, have read the description of the project entitled "French Immersion Students' Use of Linguistic and Mathematical Resources During Collaborative Mathematical Problem Solving" and understand the purpose and nature of the project as well as the procedures involved. I hereby agree to participate in the project as indicated (check all that apply):

____ I agree to participate in all aspects of the project (real-time audio recordings, collection of written work, interviews, and the collection of some personal data).
____ I agree to participate in the real-time audio recording of the problem solving activities.
____ I agree to the collection of my written work.
____ I agree to participate in follow-up interviews.
____ I agree to allow the researcher access to my grades in previous years in Mathematics and French Immersion Language Arts, as well as the results from the Grade 8 Mathematics Assessment and any standardized language assessments (e.g., OPI).

Participants are reminded that all results will be made anonymous and that you may choose to withdraw from all or part of the project at any time.

Signed: _____
Date: _____

Parents/Guardians Information Letter

Dear Parents/Guardians of (name of school) Students,

I am a former teacher at (name of school) and now a PhD student in the Faculty of Education at the University of New Brunswick, currently conducting a research study. This project has been reviewed by the Research Ethics Board of the University of New Brunswick and is on file as REB 2015-095. It has also been approved by (name of superintendent), Superintendent of (name of school district), and by (name of principal), Principal of (name of school). This study aims to understand how French immersion students talk about and use language and mathematics when working together on solving mathematics problems. This study is not about getting "right" answers to math problems. Rather, I am interested in learning more about how French immersion math students communicate when working together on problem solving tasks. I believe that learning more about this important topic will greatly benefit students and teachers, particularly as math problems and programs become much more language based. Describing and understanding the types of linguistic and mathematical resources students bring to the classroom, and how students use these resources to communicate about language and math, are important for promoting student learning of language and subject matter, and for identifying sound instructional practices for teachers.

I am requesting that students participate in this research study by agreeing to be audio recorded while working in small groups on mathematics problems in the classroom. Problems will be assigned by the classroom teacher and will be part of the regular classroom curriculum. Students' written work will be collected and evaluated as part of the regular classroom procedure. Information regarding students' academic achievement in French and math will also be collected. Students will also be invited to take part in a short follow-up interview (about 20 minutes) in order to talk about their problem solving activities. The interviews may take place during mathematics class, or during noon hour or after school if preferred, and these will also be audio recorded. The teacher will also be asked to participate in separate interviews. Students' participation is totally voluntary and should your child decide to participate he or she may withdraw from the study at any time without penalty. Students can agree to participate in all or part of the research project. During the interview, students may decline to answer any question, at any time. Your child's anonymity will be protected when results are presented by using pseudonyms. Please be assured that any data, oral or written, will be kept in a secured, locked place at the Second Language Research Institute of Canada at the University of New Brunswick. If your child chooses to participate, he or she can indicate this by completing the informed consent form attached to his or her letter. Please indicate your consent for your child to participate by completing the informed consent form attached to this letter.

If you would like further information please contact either me at 453-5136 or kculliga@unb.ca, my supervisor, Dr. Joseph Dicks, at 453-5136 or jdicks@unb.ca, or your child's math teacher, (name of teacher), at 457-6898 or (email). Should you wish to speak to someone not directly involved in this project you may contact Dr. David Wagner, Associate Dean of Graduate Programs at the Faculty of Education at 447-3294 or dwagner@unb.ca. A report of the results will be available to (name of school) once the project has ended. A summary will also be posted to our institute's website at www.unb.ca/L2.

Sincerely,

Karla Culligan, BA, BEd, MEd

Parents/Guardians Informed Consent

I, (please print) _____, have read the description of the project entitled "French Immersion Students' Use of Linguistic and Mathematical Resources During Collaborative Mathematical Problem Solving" and understand the purpose and nature of the project as well as the procedures involved. I hereby agree to have my child participate in the project as indicated (check all that apply):

____ I agree to have my child participate in all aspects of the project (real-time audio recordings, collection of written work, interviews, and the collection of some personal data).
____ I agree to have my child participate in the real-time audio recording of the problem solving activities.
____ I agree to the collection of my child's written work.
____ I agree to have my child participate in follow-up interviews.
____ I agree to allow the researcher access to my child's grades in previous years in Mathematics and French Immersion Language Arts, as well as the results from the Grade 8 Mathematics Assessment and any standardized language assessments (e.g., OPI).

Participants and their parents/guardians are reminded that all results will be made anonymous and that you and/or your child may choose to withdraw from all or part of the project at any time.

Signed: _____
Date: _____

Principal Information Letter

Dear (name of principal),

Please find attached a copy of letters to be sent to one teacher and students (and their parent/guardian) currently in Grade (9 &) 10 French immersion mathematics at (name of school). This proposed research focuses on students' linguistic and mathematical resources for problem solving. (The letters describe the project in detail.) This project has been reviewed by the Research Ethics Board of the University of New Brunswick and is on file as REB 2015-095. The Superintendent has granted approval to approach you, as school principal, to explain the project and seek your approval in moving forward. I am now seeking your permission to conduct this proposed research at (name of school). Please note that neither you nor the school staff are under any obligation to agree to participate in this research. The decision whether or not to participate remains at your discretion as school principal, and, pending your permission, at the discretion of your teachers.

If you would like further information you may contact me at 453-5136 or kculliga@unb.ca, or my supervisor, Dr. Joseph Dicks, at 453-5136 or jdicks@unb.ca. Should you wish to speak to someone not directly involved in this project you may contact Dr. David Wagner, Associate Dean of Graduate Programs at the Faculty of Education at 447-3294 or dwagner@unb.ca. A report of the results will be available to (name of school) once the project has ended. Thank you for your consideration of this project. I look forward to your reply, which you may send via email to kculliga@unb.ca for your convenience.

Sincerely,

Karla Culligan, BA, BEd, MEd

Superintendent Information Letter

Dear (name of Superintendent),

Please find attached a copy of letters to be sent to the principal, one teacher, and students (and their parent/guardian) currently entering Grade (9 &) 10 French immersion mathematics at (name of school). This proposed research focuses on students' linguistic and mathematical resources for problem solving. (The letters describe the project in detail.) This project has been reviewed and approved by the Research Ethics Board of the University of New Brunswick and is on file as REB 2015-095. I am now seeking your permission to conduct this proposed research at (name of school).

If you would like further information you may contact me at 453-5136 or kculliga@unb.ca, or my supervisor, Dr. Joseph Dicks, at 453-5136 or jdicks@unb.ca. Should you wish to speak to someone not directly involved in this project you may contact Dr. David Wagner, Associate Dean of Graduate Programs, at the Faculty of Education at 447-3294 or dwagner@unb.ca. A report of the results will be available to (name of school) once the project has ended. I look forward to your reply and thank you for your consideration of this request.

Sincerely,

Karla Culligan, BA, BEd, MEd

Cc Dianne Kay, Director of Curriculum and Instruction

APPENDIX C: PLANNING A PLAYGROUND ACTIVITY

Garderie bout de chou NOM : _____
Membres du groupe : _____

Garderie bout de chou a récemment accepté un don d'équipement pour une cours de récréation. La garderie aimerait bâtir une nouvelle cours de récréation mais à cause de sa proximité à l'autoroute il faut que la cours de récréation soit clôturée. Le comité de collecte de fonds a collectionné assez d'argent pour acheter 200 pieds de clôture comprise de panneaux de 1 pied chacun incluant une porte d'entrée.

Vous devez trouver l'aire maximale qui peut être formée utilisant les 200 pieds de clôture et qui peut accommoder tout l'équipement. En explorant ce problème, utilisez les « dimensions nécessaires, » qui incluent l'espace requis autour de chaque objet selon les règles de sécurité. Une fois terminé, vous allez présenter vos résultats à la garderie.

Objet	Dimensions réelles	Dimensions nécessaires
La glissade	7' x 15'	19' x 27'
Le dôme	17' diamètre	20' diamètre
Le carrousel	6' diamètre	12' diamètre
Les bascules (2)	2' x 12' chacune	4' x 12' chacune 6' x 12' lorsque placées ensemble
Les balançoires	8' x 5'	32' x 36'
Les tables piquenique (2)	4' x 4' chacune	5' x 5' chacune

PARTIE A. MODÉLISATION.

1. Vous recevez 200 trombones qui représentent les 200 pieds de clôture. Chaque trombone représente 1 pied de clôture. Utilisez les trombones pour bâtir la clôture pour la cours de récréation. Essayez différentes dimensions afin de maximiser l'aire de la cours de récréation. Quelle est l'aire maximale qui peut être créée avec 200 pieds de clôture? Quelles sont les dimensions?

2. Vous recevez des modèles en papier de chacun des objets à mettre dans la cours de récréation. Une fois que vous avez trouvé l'aire maximale et que vous avez bâti la clôture avec les trombones, placez tous les objets dans la cours de récréation.

   (a) Une cours de récréation avec les dimensions trouvées en n^{o} 1 peut-elle accommoder tout l'équipement? Si oui, faites une esquisse de votre cours de récréation. Sinon, changez les dimensions de votre clôture de 200 pieds pour que l'aire soit maximisée et que tout l'équipement y entre. (b) Quelles sont les nouvelles dimensions et la nouvelle aire maximale? (c) Maintenant faites une esquisse de votre cours de récréation.

3. Que pensez-vous : pourquoi y-a-t-il une différence entre l'aire maximale d'une cours de récréation sans équipement et celle d'une cours qui inclut tout l'équipement?

PARTIE B. PRODUIT FINAL.

I. FAITES LE DIAGRAMME À L'ÉCHELLE.

1. Sur le papier quadrillé, dessinez un diagramme à l'échelle afin de modéliser votre cours de récréation. Utilisez le diagramme pendant votre présentation.

II. DÉCRIVEZ LE DIAGRAMME À L'ÉCHELLE.

2. Quelle échelle avez-vous utilisé? Pourquoi?

III. EXPLIQUEZ LA RÉSOLUTION DU PROBLÈME.

3. (a) Comment avez-vous finalement trouvé les dimensions de votre cours de récréation? (b) Pourquoi avez-vous choisi cet emplacement d'objets en particulier? (c) Pourquoi votre plan de la cours de récréation est-il le meilleur choix pour la Garderie bout de chou?

APPENDIX D: INTERVIEW GUIDE

Student Interview Protocol

1. Can you tell me a bit about yourself? How long have you been in French immersion? What do you think/feel about French? About math? How is French immersion math for you? What kinds of things stand out to you about learning math in French?
2. Can you listen to/look at (in the case of written work) this particular part of your group work? Can you stop and remember what was happening here? Can you explain/describe/elaborate on how you talked through this part?
3. Students are prompted as needed and/or asked to elaborate based on emergent ideas and points of interest.

APPENDIX E: A PRIORI (INITIAL) CODING CATEGORIES

None of the original coding categories taken from the literature is presented in table form in the body of the dissertation. Rather, they are presented as lists and/or in narrative form (this is explained in more detail in Chapter 3). Using these categories from the literature, I determined my initial "look fors" when I began coding my own data. These "look for" items or themes were as follows and were used as an a priori coding system during the initial rounds of data analysis:

- Language-related-episodes (LREs, Swain & Lapkin, 1998)
    - either (a) dialogue as an enactment of mental processes, or (b) dialogue as occasions for L2 learning (Swain & Lapkin, 1998)
        - (a) or (b) categorized into one of two groups, either "lexis-based" or "form-based" (Swain & Lapkin, 1998)
        - determine whether the LRE was related to mathematical language and/or mathematics (Barwell, 2009c, Moschkovich, 2005)
- Mathematical communication (Barwell, 2009c; Moschkovich, 2003, 2005, 2007, 2009; Pimm, 1978)
    - abstracting, generalizing, searching for certainty, use of textbook definitions/vocabulary (Moschkovich, 2007)
    - "everyday" words used in a mathematical way (meanings of utterances) [Barwell, 2009c; Moschkovich, 2007, 2009]
    - explaining, justifying, describing, elaborating, expanding, providing additional information (these sometimes, although not always, occur in bilingual students' L1) [Moschkovich, 2005]

- L1 use (Swain & Lapkin, 2000, 2013; see also Moschkovich, 2005)
    - moving the task along: sequencing (figuring the order of events); retrieving semantic information; understanding pieces of information; developing an understanding of the story; task management (Swain & Lapkin, 2000)
    - focusing attention: vocabulary search; focus on form, explanation, framing, retrieving grammatical information (Swain & Lapkin, 2000)
    - interpersonal interaction: off task (includes L1 vernacular use); disagreement (Swain & Lapkin, 2000)
    - provide a missing L2 vocabulary item; explain a concept; justify an answer; describe mathematical situations; elaborate, expand and provide additional information; reflect a level of communicative competence; reflect community norms; provide stylistic switches in the conversation; relate to memory and to routines (Moschkovich, 2005)

CURRICULUM VITAE

Karla Marie Culligan

University of New Brunswick, 2008, Master of Education

University of New Brunswick, 2002, Bachelor of Education

Mount Allison University, 2000, Bachelor of Arts

Publications:

Culligan, K. (2017). Using classroom-based research to explore mathematics and language in secondary French immersion. *Antistasis, 7*(1), 66-78. Retrieved from https://journals.lib.unb.ca/index.php/antistasis/article/view/25144/29533

Culligan, K., Le Bouthillier, J., Dicks, J., & Kristmanson, P. (2017). Les interactions orales pendant la résolution de problèmes mathématiques en immersion française : une perspective socioculturelle. In C. Dumais, R. Bergeron, M. Pellerin, & C. Lavoie (Eds.), *L'oral et son enseignement : pluralité des contextes linguistiques* (pp. 243-259). Côte Saint-Luc, QC: Éditions Peisaj.

Culligan, K. (2015). Student and teacher perceptions of first language use in second language mathematics. *Alberta Journal of Educational Research*, *61*(1), 1-19. Retrieved from http://ajer.journalhosting.ucalgary.ca/index.php/ajer/article/view/1377/pdf

Culligan, K. (2010). Pente or slope? Using student voices to explore program choice and experiences in secondary French immersion mathematics. *The Canadian Modern Language Review, 66(*3), 421-444.

Conference Presentations:

Culligan, K. (2017, February). *Using classroom-based research to explore mathematics and language learning in secondary immersion*. Invited speaker at Research Seminar: Multilingualism in Education in the High North, at UiT Arctic University of Norway, Alta, Norway.

Culligan, K., & Dicks, J. (2017, May). *Using mathematical explanation to explore secondary French immersion students' language*. Paper presented at the Canadian Association of Applied Linguistics, Ryerson University, Toronto, Ontario.

Culligan, K. (2014, July). *The purpose and implications of first language use in second language mathematics tasks*. Paper presented at the International Group for the Psychology of Mathematics Education (PME 38), Vancouver, British Columbia, Canada.