A Note on the Probability of Recession: Can Statistics Canada's Leading Index Predict as Well as MARS

by
Peter S. Sephton

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DEPARTMENT OF ECONOMICS
THE UNIVERSITY OF NEW BRUNSWICK
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Peter S. Sephton
Professor
Department of Economics
University of New Brunswick
Fredericton, New Brunswick
Canada E3B 5A3

Sephton@UNB.CA

voice 506 453 4828 / 4666 ext 7386
fax 506 453 4514

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Abstract

In this note we estimate the probability of recession using the revised leading index published by Statistics Canada. The results are compared to probabilities derived from a new non-parametric regression routine. While the indexes provide similar information on the probability that the economy is currently in recession, the non-parametric approach appears to offer more reliable information. Two out-of-sample forecasting exercises demonstrate the potential benefits to the use of the multivariate adaptive regression spline model.

I Introduction

There have been a number of recent studies that attempt to describe the business cycle using methods that allow the duration of the state of the economy to play a role in the transition to a new state. This duration dependence literature grew out of the desire to model changes in the state of the business cycle, since leading indicators generally perform poorly in "real time". Hamilton (1989) provided a non-linear Markovian filter that was capable of capturing changes in regime: this has since been extended by others, such as Lam (1990), where the autoregressive component is not constrained to have a unit root; and Durland and McCurdy (1994), who allow the transition between states to be duration dependent. Empirical results are consistent with the view that the duration of expansions and contractions affect the transition to the next state. Not surprisingly, evidence is somewhat mixed on the degree to which duration dependence is important.[1]

The purpose of this note is to attempt to resurrect the leading index as a useful indicator of the state of the economy. Statistics Canada has recently revised their leading index to be more timely and up-to-date. Of the ten series in the index, five are now contemporaneously available, while five others are available with a one or two month lag. The new index has been published since February 1993 : the old index was discontinued in December 1992. While the new index is more timely than its predecessor, it does not attempt to capture the non-linearities that the duration-dependence literature appears to have identified. To this end, a new non-parametric regression routine will be used to estimate the probability of recession. The multivariate adaptive regression spline (henceforth MARS) model allows very general non-linear additive and multiplicative effects between a set of predictors and the dependent variable. Its extension to logit regression allows us to obtain estimates of the probability that the economy is currently in a recession as a function of the ten

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[1] For example Diebold and Rudebusch (1990) find evidence only in pre-war US cycles.

components of the new Statistics Canada leading index. On full sample estimation we find that the MARS approach generates probabilities that are coincident with the actual dates of recession and expansion. Probabilities derived from a simple logit model of the change in the Statistics Canada index appear to lag the onset of the 1980, 1981-82, and 1990-91 recessions, while they lead the associated recoveries. Two out-of-sample forecasting exercises predict the 1981-82 and 1990-91 recessions. The MARS model is consistently out-performs the Statistics Canada index vis a vis predicting a recession: the Statistics Canada index fairs better at signalling recovery. This suggests they may be used together to identify the current (and near term) state of the business cycle.

The next section describes the construction of the new Statistics Canada leading index. This is followed by a brief description of the MARS modelling approach. Logit models illustrate that the components of the leading indicator contain useful information on the last two recessions. Conclusions and suggestions for further work follow.

II The New Statistics Canada Leading Index

Roy (1993) describes the new composite leading index. It differs from the old index in that it is more timely. For example, in early June, the old leading index would have been published for March: that is, the published value would provide information on the state of the economy in March. This may be of limited value in June, since the state of the economy in June may be much different from what it was in March.

In June, five of the components of the leading index are available for May, and one is available for April. The new leading index that is published in early June is now called the composite index for May. It is based on five May series, one April series, and four March series. Table 1 provides a summary of the ten components of the index and their dates of availability.

Roy (1993) provides information on the error rate of the new index: it has provided five false signals (of recession or recovery) since 1970, whereas the old index had over 11 false signals after 1970.

From the viewpoint of constructing the probability of recession using the MARS routine, the information set used to make judgements about the state of the economy at time t contains five series dated at time t, one series dated at time t-1, and four series dated at time t-2. In what follows we will assume that the object of the leading index is to provide information on the state of the economy (recession=1,expansion=0) using the available information contained in the ten components of the leading index. Since information on real output (monthly GDP) appears with a two month lag, the leading index is providing information that we will not be able to confirm for (at least) two months. In this sense it is truly a leading economic indicator.

III The MARS Model

There are a number of attractive non-parametric regression models, the most recent having been introduced by Friedman (1991a, 1991b). For an exhaustive description of the algorithm the reader is referred to those papers. What follows is an attempt to provide an intuitive review of the modelling strategy.

The multivariate adaptive regression splines model uses an extension of recursive partitioning regression to construct a set of truncated power basis functions using cubic spline approximations to the data. For example, when modelling the relationship between a predictor x_t and the dependent variable y_t, a general model might take the form

$$y_t = a_1 + a_2 x_t + a_3 x^2_t + \ldots + a_M x^{M-1}_t + \epsilon_t \qquad (1)$$

Here, y_t is a polynomial in x_t. This can be written in a more general form, as

$$y_t = \sum_{k=1}^{M} a_k \; B_k(x_t) \qquad (2)$$

where $B_k(x_t)$ is the k-th basis function of x_t. These basis functions can be highly non-linear transformations of x_t, but note that y_t is a linear (in the parameters) function of the basis functions. The parameters a_k can be chosen on the basis of a sum of squared residuals metric. The advantages to MARS are in its ability to estimate the basis functions so that both the additive and the interactive affects of the predictor space are allowed to determine the response variable.

With piecewise polynomial approximation, the predictor space is broken into sub-regions with knots separating each region. A cubic polynomial spline function is fit to the data within that region. Summing over the predictor space yields a set of spline functions that form the fitted model. Johnston (1984, p. 392) provides a pedagogic example of the application of spline functions in economics.

MARS recognizes that spline fitting routines are sensitive to knot location, and employs a truncated power basis function to fit a cubic polynomial to the series.

Knots are chosen on the basis of a multiple regression that is analogous to retaining knots which reduce the sum of squared residuals. For example, with a single predictor the sum of squared residuals would be

$$\sum_{i=1}^{N} \{ y_t - \sum_{j=0}^{q} b_j x^j - \sum_{k=1}^{K} a_k (x_t - t_k)_+^q \}^2 \qquad (3)$$

where b_j and a_k are multiple regression coefficients on cubic (q=3) splines of x_t, and x_t relative to knot location t_k. Note that the notation $(x_t - t_k)_+^q$ includes the cubic spline of x_t relative to knot location t_k if the difference is positive, and otherwise zero.

From (3) it is clear that the addition of a knot can be viewed as adding the corresponding variable $(x_t - t_k)_+^q$. A forward and backward stepwise search is incorporated in the algorithm, with the forward step purposely overfitting the data. Insignificant terms are deleted on the backward step of the routine. Another benefit to the MARS algorithm is that the parent region is retained after it has been split into its sibling regions. This has the advantage of producing overlapping regions, without gaps, should a sibling be excluded after it has been determined to be insignificant. Many other non-parametric routines are constrained so as to disallow overlapping regions.

Model selection can be based on N-fold cross-validation, or alternatively, the generalized cross-validation (GCV) criterion of Craven and Wahba (1979). The GCV can be expressed as

$$GCV = (1/N) \sum_{t=1}^{N} \{ [y_t - f_M(x_t)]^2 / [1 - C(M)/N]^2 \} \quad (4)$$

where there are N observations, and the numerator measures the lack of fit on the M basis function model $f_M(x_t)$, and denominator contains a penalty for model complexity, C(M). The MARS algorithm has been extended to incorporate logit regression. In this case model selection is based on the modified GCV (GCV_M) criterion

$$GCV_M = [y_t - 1/(1 + e^{-fm(x)})]^2 \quad (5)$$

where fm(x) denotes the corresponding MARS estimate of the log-odds ratio. Friedman (1991a, 1991b) reports that the GCV criterion behaves well relative to N-fold cross-validation, but for present purposes ten fold cross-validation will be employed in model selection.

Interpretation

MARS estimates can most readily be interpreted from the ANOVA representation of the model (6), where the fitted function is a linear combination of additive basis functions in single variables denoted by $f_i(x_i)$ and interactions between variables, denoted by $f_{ij}(x_{it}, x_{jt})$, $f_{ijk}(x_{it}, x_{jt}, x_{kt})$, and so on.

$$y_t = f(x_t) + \epsilon_t \qquad (6)$$

where

$$f(x_t) = a_o + \sum_{K_M=1} f_i(x_{it}) + \sum_{K_M=2} f_{ij}(x_{it}, x_{jt}) + \sum_{K_M=3} f_{ijk}(x_{it}, x_{jt}, x_{kt}) + \ldots.$$

Each $f_i(x_{it})$ is a spline representation of a univariate function. A plot of $f_i(x_{it})$ against x_{it} would demonstrate the optimal transformation of the series x_{it}. This graph illustrates the degree to which to optimal transformation is non-linear, a major benefit to non-parametric regression.

For interactions involving two variables a surface can be derived to illustrate the joint effects of the series on the endogenous variable. Beyond bivariate designs the models can be "sliced" to provide a set of surfaces that show the contribution of the two series to a smooth of the dependent variable on all variables in the system.

The choice of interaction terms can be made through a comparison of the low order ($K_M = 1$) and high ($K_M \geq 2$) order models. Friedman suggests a comparison of the generalized cross validation scores, with a model involving interaction terms chosen over an additive model only if its GCV score is lower. As part of the MARS output, the relative contribution of each variable is determined, as are estimates of the model's GCV given that a particular ANOVA function (variable) has been omitted from the model. This assists in interpreting the significance of each ANOVA function.

MARS has been extended to incorporate categorical variables and missing data: while these extensions might have important applications in other fields of economics (transportation, consumer theory, industrial organization, natural resources)[2] , the next section demonstrates how MARS can be used to predict the probability of recession.

IV The MARS Leading Index and its Relative Performance

Data on the seasonally adjusted components of the leading index was obtained from the CANSIM CD-ROM database released early in 1993. Figures were updated to August 1993 using the December 1993 issue of the *Canadian Economic Observer* (11-010). Estimation spanned March 1972 to August 1993, inclusive. Cross and Roy (1989) provided reference dates for Canadian expansions and contractions.[3]

In-Sample Analysis

The series were lagged to relate the current state of the economy to the information set available when the index was constructed. A MARS model that allowed up to ten variable interactions and up to 35 different basis functions (a relatively unrestricted model) was estimated over the full sample. Table 2 presents summary information on the estimated MARS index. Surprisingly, the MARS model does not choose to include information on new orders for durable goods or retail sales in furniture and

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[2] See Lewis and Stevens (1991) and Sephton (1992, 1993, 1994) for examples using MARS.

[3] While Statistics Canada has not released official dates for the last recession, an examination of real GDP suggests that it may have begun in January 1990 and lasted until September 1991. These dates are used in conjunction with Cross and Roy (1989).

appliances. The most important variables appear to be the average work week, retail sales of durable goods, and the TSE index. Figure 1 plots the full-sample probabilities from the MARS model against the dates of recession. It is clear that the MARS model does very well at fitting the data. Indeed, this is what we should expect, given the flexibility of the algorithm. MARS should produce a very close fit to the data. Figure 1 demonstrates that it does.

Logit models were used to estimate the probability of recession based on a regression of the dichotomous variable (1 recession / 0 otherwise) against a constant and the change in the Statistics Canada leading index. Figure 2 provides an estimate of the probability of a recession derived from this approach. The Statistics Canada model appears to consistently lag in predicting the onset of recession. With the exception of the 1974 and 1980 recessions, the leading index appears to signal recovery before it occurs. Figures 1 and 2 demonstrate that both methods track the historical data well, with the MARS model more closely related to the actual dates of recession and expansion.

Out-of-Sample Analysis

Non-parametric regression routines typically require large datasets so that the algorithms can "train" the data. For this reason we focus on two out-of-sample experiments. The first is based on estimating the MARS model and the Statistics Canada logit regression over the first 106 observations: April 1972 to December 1980 (inclusive). The estimated models are then used to generate forecasts for the probability of recession given new information on the ten components of the Statistics Canada leading index. Note that the official date for the onset of the 1981 recession is July 1981. This implies that we are assuming that the specification of the MARS model has not been updated for a full six months before the recession.

Table 3 contains information on the fitted MARS model. The only components of the leading index that are retained by the algorithm are new orders for durables, the ratio of shipments to inventories, and the TSE 300 index. The other seven series do not appear to have predictive power, since MARS has determined that the modified cross-validation score is lowest when these series are excluded. Figure 3 contains a plot of the estimated probability of recession from the MARS model and the logit Statistics Canada regression (estimated over the same time period). MARS appears to signal the recession and recovery in advance of the Statistics Canada index. The duration of the recession appears to be underpredicted by the MARS model.

If we take the view that we are most interested in how the economy will behave in the next quarter (or two), the MARS model clearly out-performs the Statistics Canada index. This is particularly true if we consider that the model could be reestimated after official recognition of the recession. Updated figures could be used to predict the onset of recovery.

The second out-of-sample exercise involves the prediction of the 1990-91 recession: the sample used for estimation spanned April 1972 to August 1989, with prediction beginning four months before the actual onset of the recession. Table 3 presents summary information on the estimated MARS index. Comparison to Tables 2 and 3 indicates that the MARS probability model is a very flexible approach to gleaning information from the ten components of the leading index: the series and their relative importance change over time. Housing starts and the TSE 300 index are relatively important determinants of the probability of recession, followed by the real money supply and retail sales of durable goods. Figure 4 plots the fitted MARS probabilities and those of the Statistics Canada logit model (again, estimated over the same time period). MARS anticipates the recession with certainty, while the Statistics Canada index lags in capturing the onset of recession. The Statistics

Canada index appears to dominate MARS in anticipating recovery. The results suggest that Statistics Canada's new leading index does provide useful information on the state of the business cycle (as proxied by the probability of recession), but that it might be usefully combined with predictions using MARS.

V Final Remarks

The object of this note was to determine whether a new non-parametric modelling routine would provide information that out-performed Statistics Canada's revised leading index. Using relatively elementary analysis with respect to estimating the probability of recession, we found that the Statistics Canada index performs very well in an out-of-sample forecasting exercise. It appears to lag in predicting recession while it leads in predicting recovery. The MARS approach appears to be capable of tracking the onset of recession with great precision, yet it is not as reliable as the Statistics Canada index in signalling recovery. This suggests there may be an opportunity to improve our understanding of the business cycle by combining the new index with information from MARS.

There are several extensions that appear to be worthwhile. The first is to examine other leading indexes for which longer periods of data over all of their components is available. This might assist in training the MARS algorithm to the data. A second area of future work might involve an analysis of series that are not components of the Statistics Canada index to determine whether there are non-linear relationships between these series and the probability of recession. Duration dependence models on Canadian data may also provide additional information on how the longevity of the stage of the cycle affects the transition to the new state.

Table 1: Components of the Leading Index

Component	Reference Month	Data Availability
Furniture and Appliance Sales	November	January
Sales of Other Durable Goods	November	January
Housing Starts	January	February
New Orders for Durable Goods	November	January
Ratio of Shipments to Stocks	November	January
Average Workweek	January	February
Employment	January	February
US Leading Index	December	February
TSE 300 Index	January	February
Money Supply	January	February

Source: Roy (1993, p.3.2)

Table 2: Full Sample Estimates

	Relative Importance %
Furniture and Appliance Sales	0
Sales of Other Durable Goods	89.12
Housing Starts	21.40
New Orders for Durable Goods	0
Ratio of Shipments to Stocks	46.97
Average Workweek	100
Employment	21.0
US Leading Index	43.0
TSE 300 Index	71.98
Money Supply	39.79

Table 3: Sub-Sample Estimates: 1972-1980

	Relative Importance %
Furniture and Appliance Sales	0
Sales of Other Durable Goods	0
Housing Starts	0
New Orders for Durable Goods	83.69
Ratio of Shipments to Stocks	100
Average Workweek	0
Employment	0
US Leading Index	0
TSE 300 Index	100
Money Supply	0

Table 4: Sub-Sample Estimates: 1972-1989

	Relative Importance %
Furniture and Appliance Sales	0
Sales of Other Durable Goods	23.59
Housing Starts	100
New Orders for Durable Goods	0.107
Ratio of Shipments to Stocks	0.194
Average Workweek	0
Employment	0.316
US Leading Index	0
TSE 300 Index	81.66
Money Supply	33.34

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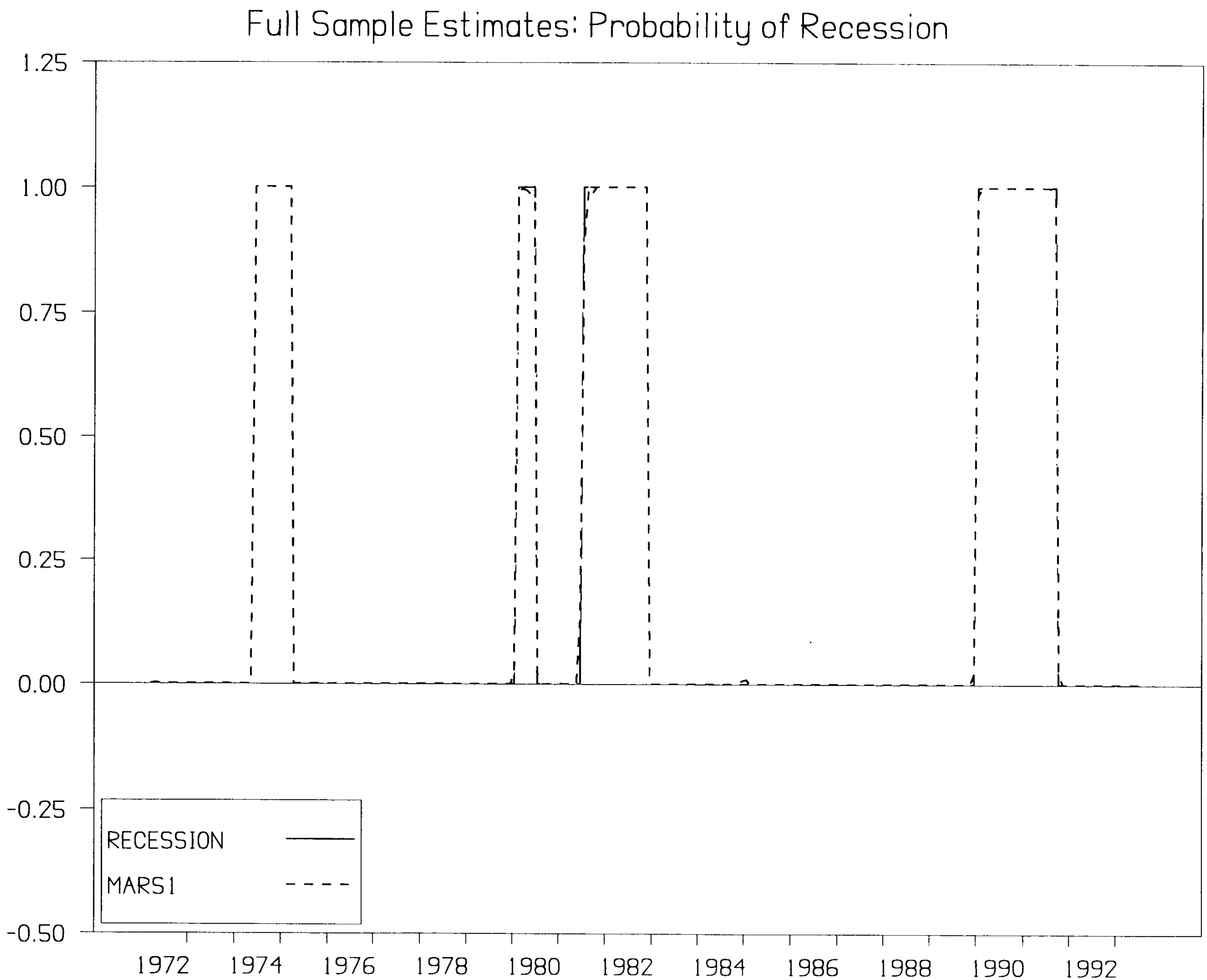
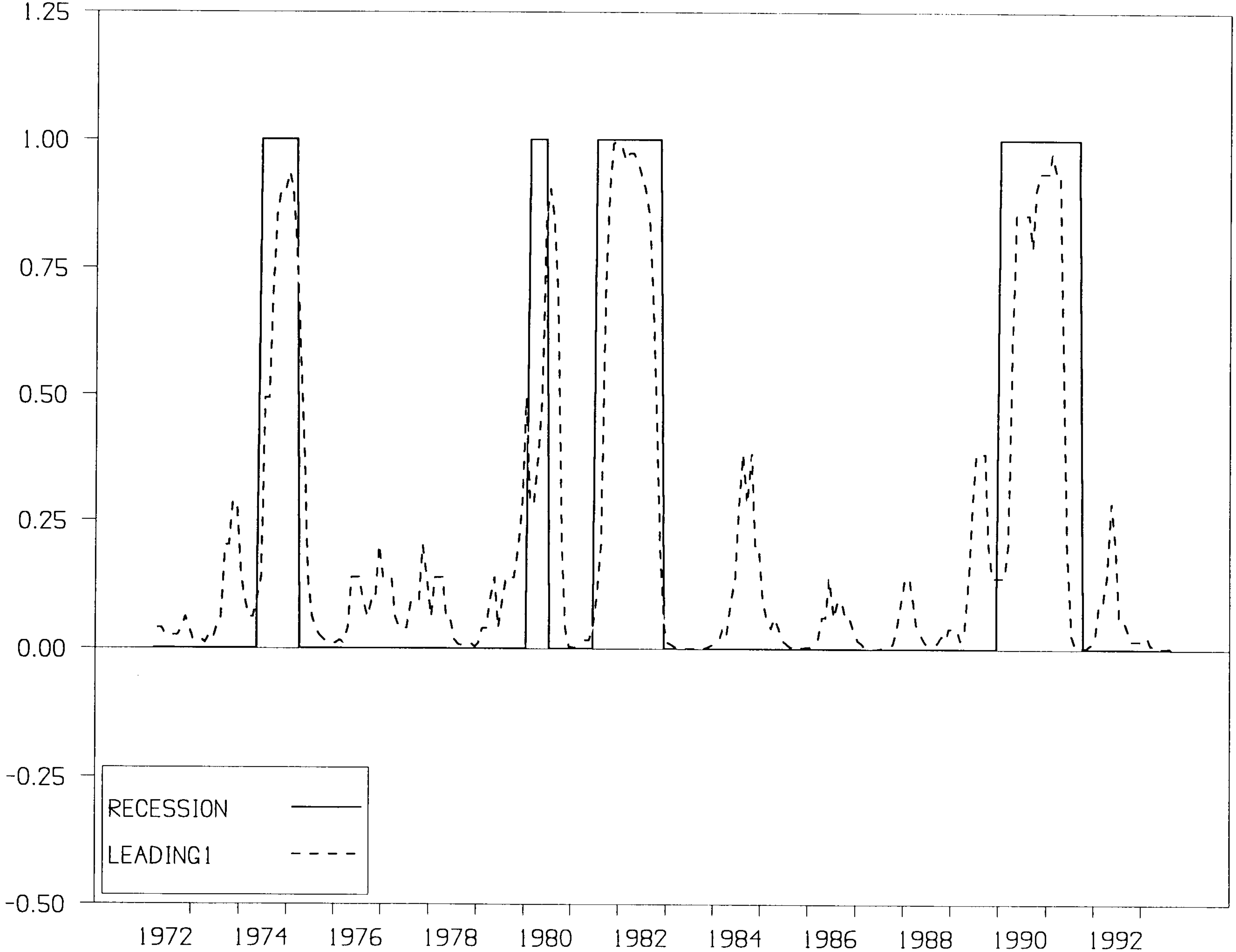
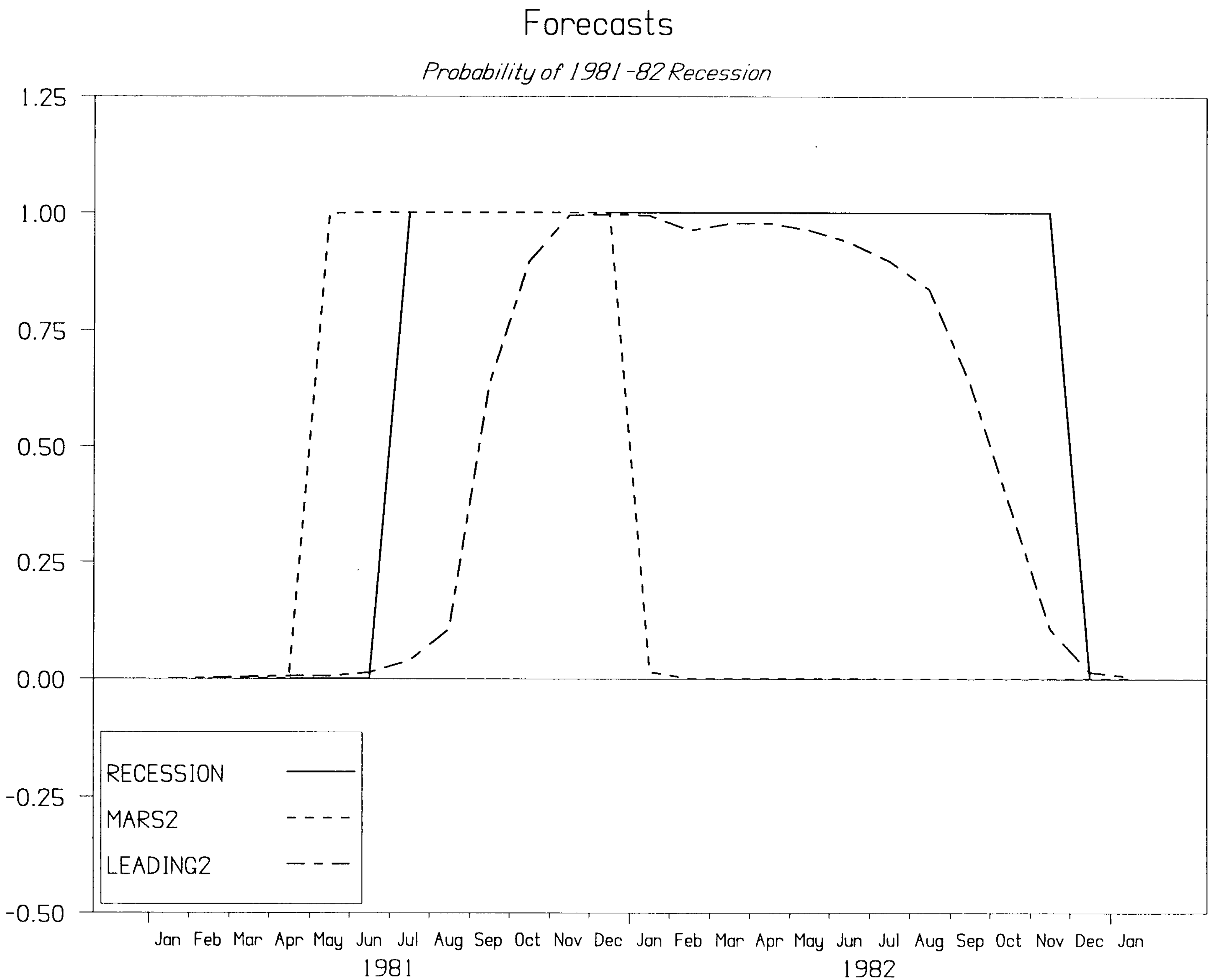
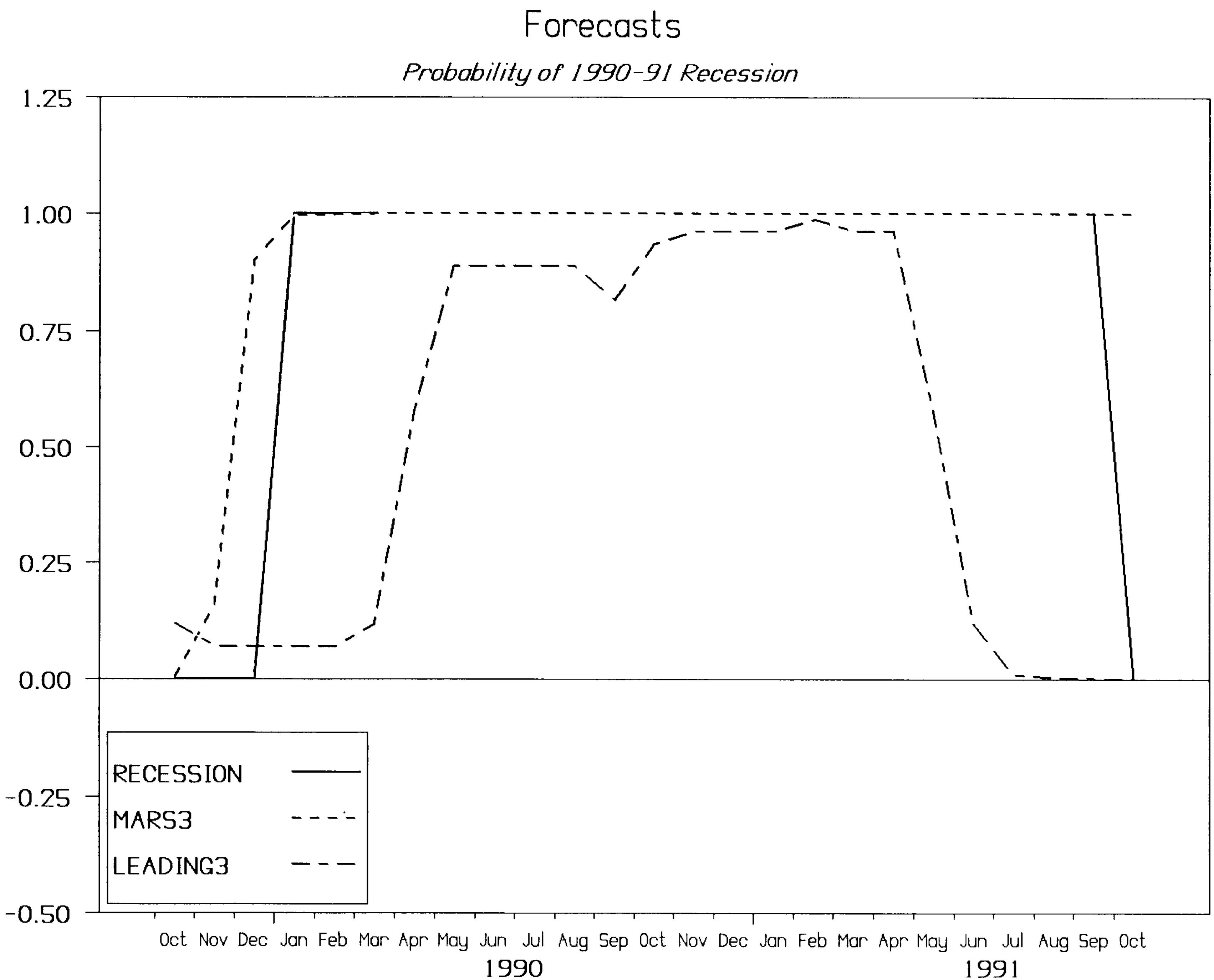

Full Sample Estimates: Probability of Recession
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
RECESSION
MARS1
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992

Full Sample Estimates: Probability of Recession




Forecasts
Probability of 1981-82 Recession
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
RECESSION
MARS2
LEADING2
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan
1981
1982


Forecasts
Probability of 1990-91 Recession
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
RECESSION
MARS3
LEADING3
Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct
1990
1991