The Optimization of Urban-Rural Demand Responsive Transportation Services in New Brunswick

by

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Abstract

With over one-third of New Brunswick's population living with a disability, accessible Demand-Response Transit (DRT) services are crucial for those that need it. There is a need for data on the operation and travel patterns associated with DRT services in New Brunswick as new regional agencies are assuming transportation planning responsibilities and looking to expand services. This research used geographically aggregated passenger data from 6 months of trips by a paratransit provider to estimate trip rates and to pilot an exact solution method of Mixed Integer Linear Programming (Christie Method) for vehicle deployment. The Christie Method was able to reasonably replicate the conditions of the existing service provider subject to their level of service requirements and was then applied to a community use-case based on extrapolated trip rates. While effective, the exact solution approach increases processing time exponentially for any additional constraints, therefore other heuristic approaches may warrant future considerations.

Dedication

This thesis is dedicated to all those living with a disability navigating a world built for others. You are seen.

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This endeavor would not have been possible without my co-supervisors Dr. Trevor Hanson and Dr. James Christie. This research was completed with financial contributions from the Natural Sciences and Engineering Research Council of Canada (NSERSC) and the Regional Development Corporation (RDC) of New Brunswick. This research could not have been completed without the help of Kathleen Leger and Ability Transit. I would like to thank my mother and father for their unwavering support throughout this research and my entire university journey. I would also like to thank Ethan Drost for his constant encouragement.

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List of Abbreviations

Abbreviation	Definition
ABM	Activity Based Modeling
ADA	Americans with Disabilities Act
CBC	COIN-OR Branch and Cut
CHASS	Computing in the Humanities and Social Sciences
CMILP	Constraint Mixed Integer Linear Programming
CPC	Canada Post Corporation
CTRL	Community Transportation Research Lab
CUTA	Canadian Urban Transit Association
DA	Dissemination Areas
DARP	Dial a Ride Problem
DOT	Department of Transportation
DRT	Demand Responsive Transport
FSA	Forward Sortation Areas
GPDP	General Pick up and Delivery Problem
GPS	Global Positioning System
ID	Identification
ITE	Institute of Transportation Engineers
LOS	Level of Service
MEP	Multiple Enhanced Postal Codes

NBRN	New Brunswick Road Network
NCHRP	National Cooperative Highway Research Program
NP-hard	Nondeterministic Polynomial Time Problem
PCCF	Postal Code Conversion File
PWD	Persons with a Disability
RSC	Regional Service Commission
RUTAC	Rural and Urban Transportation Advisory Committee
SERSC	Southeast Regional Service Commission
SMAR	Scheduling, Matching, Allocation, and Reduction
TAZ	Transportation Analysis Zone
TCRP	Transit Cooperative Research Program
TSP	Traveling Salesperson Problem
UD	Universal Design
UEP	Unique Enhanced Postal Codes
UNB	University of New Brunswick
VRP	Vehicle Routing Problem

1 INTRODUCTION

Transportation is an essential link to a fulfilling life; it enables access to opportunities. In New Brunswick, currently more than 1/3 of the population is living with a disability (StatCan, 2022) and the rate of disability in Canada is growing (S. P. Morris & Statistics Canada, 2021). Some individuals living with disabilities may be unable to drive themselves, relying on family and community organization to make important trips (Lin & Cui, 2021). According to Statistics Canada, among those with disabilities aged 15 and older, just over one-sixth (17.8%) were unable to complete a trip due to the unavailability of accessible transportation (StatsCan, 2017). There is also a correlation between an aging population and increase in disability, where the proportion of the New Brunswick population aged 65 years and older was 22.8% in 2021 (compared to 18.8% nationally). Individuals who are unable to access a private vehicle and cannot use available public transportation services rely on Demand Responsive Transit (DRT) or paratransit services to fulfill important trips.

There is pressure for efficient paratransit services to accommodate the reduction in independent private vehicle drivers, as the New Brunswick population continues to age, this problem becomes more chronic. Given the typically high per person cost of on-demand specialized transport, communities seeking to develop new services can be faced with the dual challenge of balancing service provision with fiscal constraints (Sultana et al., 2018). DRT can fulfill smaller volume trips with longer durations in rural areas (Sultana et al., 2018), though this presents challenges to service delivery organizations, as rural areas can have low demand for transit and dispersed demand (Itani et al., 2024). Fewer riders and longer distances mean allocating trip expenses over a smaller client base, requiring higher subsidies to operate at an affordable cost to the client (Börjesson et al., 2020). As communities have increasing numbers of individuals who cannot drive or rely on a private vehicle (including from an aging population), there will be increasing pressure for efficient alternatives. This represents an opportunity for the transportation engineering profession to help communities better plan and forecast for transportation alternatives such as DRT.

1.1 Problem Statement

The Province of New Brunswick has a mix of urban and rural areas, with 51.2% of the population living in urban areas, and 48.8% living in rural areas (StatCan, 2022), though DRT is typically only available in its largest cities. Expanding the reach of these services is being fostered by recent changes to local governance structures in New Brunswick. These changes have assigned transportation service planning responsibilities to Regional Service Commissions (RSCs) (Government of New Brunswick, 2023b), which can include the development of DRT services on a regional basis. DRT and paratransit are both flexible transportation services, but they serve different needs and populations; DRT is for the general population, while paratransit is specific subset of DRT for persons with a disability. Planning DRT/paratransit services require accurate demand modeling to operate efficiently (Sultana et al., 2018), but there is not yet a consistent demand estimation approach employed by RSCs. It is difficult for accessible demand response transit operators to plan services when there are no consistent standards for planning DRT, especially varying service area configurations (Klumpenhouwer, 2020).

The Southeast Regional Service Commission (SERSC) representing the largest RSC by population (25% of New Brunswick's population) is considering options for deploying a fleet of accessible vehicles on a regional basis. The consideration of options is complicated by limited data available on prospective ridership, travel demands of current and future riders, as well as the availability of methods to assess vehicle deployment strategies, impacts on operational costs and user travel times. While the SERSC has retained DRT consulting support, there is still the need to research the operating conditions of paratransit, to create a system adaptable to required demand. There is a need to provide analysis of the operations resulting in research that will support decision-making within the SERSC by stakeholders and service providers.

1.2 Research Questions

There are two research questions addressed by this research study:

1. What is the demand for paratransit services in Southeast Region of New Brunswick?
2. What is the optimal fleet service delivery model for paratransit services based on an estimated demand?

1.3 Research Goal and Objectives

The goal of the research is to improve the transportation planning process for the deployment of a regional on-demand accessible paratransit services by piloting a demand estimation approach paired with a novel vehicle routing and scheduling solution (the Christie Method), using the Southeast Region of New Brunswick as a test case. The objectives are:

1. Document the standard practice approaches used in industry for planning regional on demand accessible DRT services.
2. Identify the techniques used to estimate the demand for accessible on demand services at a regional level.
3. Assess service delivery models and scenarios using an operations research approach.

1.4 Geographic Scope

The geographic scope of this project is in the southeastern region of the Province of New Brunswick. The Southeast Region as defined by the SERSC encompasses 25% of the New Brunswick population within the area shown below in Figure 1 (Lapierre & Belliveau, 2017). Servicing this population can be challenging due to the varying population densities and diverse needs of different communities. These communities require additional services such as demand-responsive transit to meet the transportation needs of individuals who are unable to drive private vehicles. The map below shows the extent of the Southeast Regional Service Commission delineated with a grey broken line. The orange sections on the map characterize communities and the light purple sections represent rural areas not within a municipality.

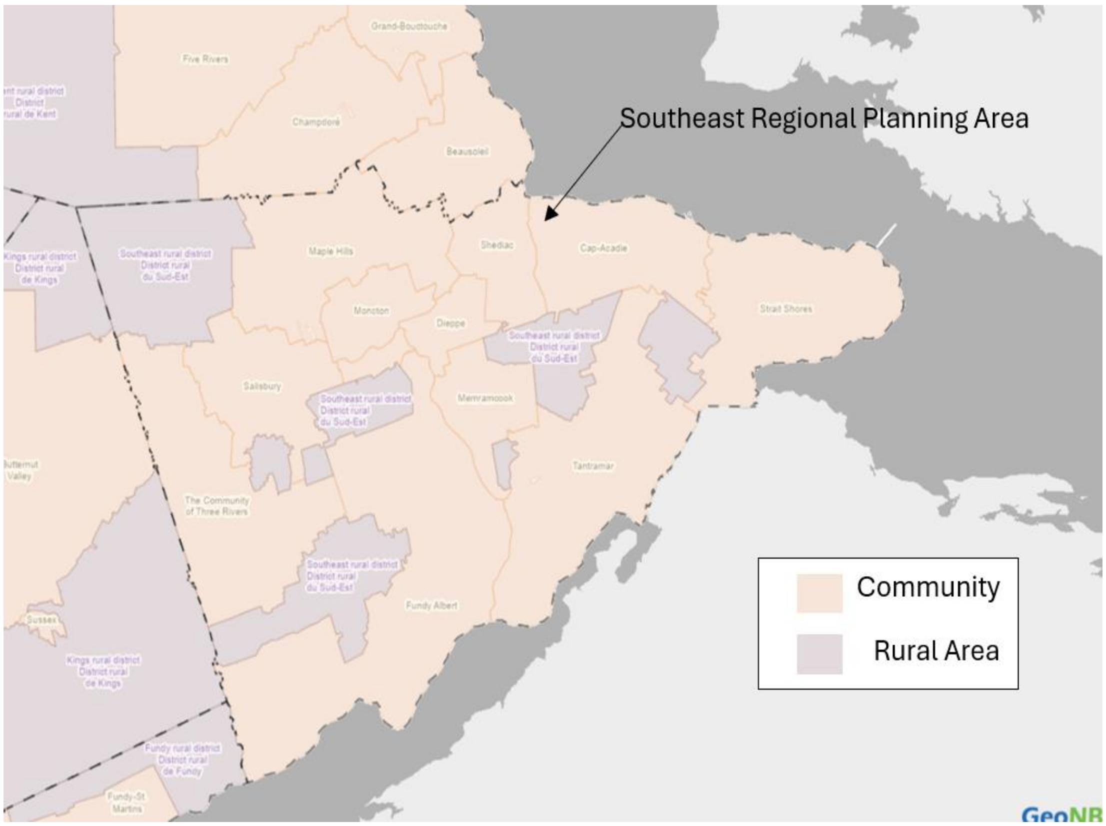

Figure 1. Extent of Southeast Regional Planning Area (GeoNB)

1.5 Thesis Organization

This thesis is organized into seven chapters, each addressing a critical component of the research process. The second chapter, the Literature Review, provides an overview of the definition and quantification of disability in New Brunswick, examines industry approaches to service provision, and outlines standard regional planning practices. It also reviews the existing literature on vehicle routing and scheduling, highlighting various methods and strategies employed in the field. The Methodology chapter (Chapter 3) details the approach for organizing, validating, and updating data to interpret travel behavior and conduct geographic analysis. It also outlines the process of estimating regional travel demand and trip generation. Chapter 4, titled Christie Method of Constraint Mixed Integer Linear Programming, introduces the Christie Method and explores its application to the Vehicle Routing Problem (VRP), explaining how this computational method addresses the complexities of route optimization of paratransit DRT. In Chapter 5, the Results are presented, focusing on the findings from the Ability Transit data analysis and the operations research conducted through the application of the proposed methods. Chapter 6, the Discussion, reflects on the implications of the research, offering insights into transportation planning practices while discussing lessons learned from both the data analysis and the modeling process. The final chapter, Conclusions and Recommendations, summarizes the high-level results and key lessons from the research, providing concluding thoughts and suggesting directions for future work in the field. Together, these chapters offer an understanding of the research, from literature review and methodology to results, discussion, and conclusions.

2 BACKGROUND AND LITERATURE REVIEW

This chapter discusses the background required for the methodology of the research completed in this thesis, as well as the literature reviewed to support the research conducted. The topics include the definition of a mobility disability, and the impact transportation has on the population of persons with a disability, approaches to offering accessible transportation, the estimation techniques for the demand of accessible transportation, the vehicle routing problem, and the Christie Method.

2.1 Understanding Mobility Disabilities and Transportation

People with disabilities comprise a sizable segment of the Canadian population representing 27% of the population (StatsCan, 2023). Of all individuals in Canada, 10.9% have a mobility disability (StatsCan, 2023). Individuals with mobility disabilities experience increased difficulties moving, which is exacerbated by lack of transportation suited to needs of persons with disabilities. Individuals with disabilities are heavily dependent on cars to fulfill transportation needs (Field & Jette, 2007). Dependence on the car is especially prevalent among older people; this is cause for concern, given that many older drivers will be unable to continue to safely drive as they age because of increasing impairments and/or disabilities (Field & Jette, 2007). When private vehicles are unavailable, people with disabilities must rely on other modes to fulfill trips. Individuals with disabilities can face barriers to travel such as transportation availability, finding an accessible parking spot, access to a service location, and transportation scheduling.

The reduction in barriers to transportation can provide more opportunities for people to travel to activities of their choosing, increasing their individual satisfaction (Yassine & Elliot Martin, 2021). Barriers to travel can cause transport-related social exclusion (Y. Zhang et al., 2022). It has been determined the result is a negative factor for overall health of individuals, especially older adults, and individuals with disabilities (Nicholson, 2012). Allowing access for vulnerable groups can provide greater benefits than just the transportation service itself. Improving transportation specifically to supplement underserved communities such as older adults and individuals with disabilities provides access to opportunities that previously had barriers.

2.1.1 Disability in New Brunswick

A survey conducted in 2018 called the General Social Survey, cycle 33: 2018: Giving, Volunteering and Participating, shows that the province of New Brunswick has one of the higher disability rates per capita in Canada. A breakdown of disability severity per province as a percentage of respondents is shown below (Figure 2).

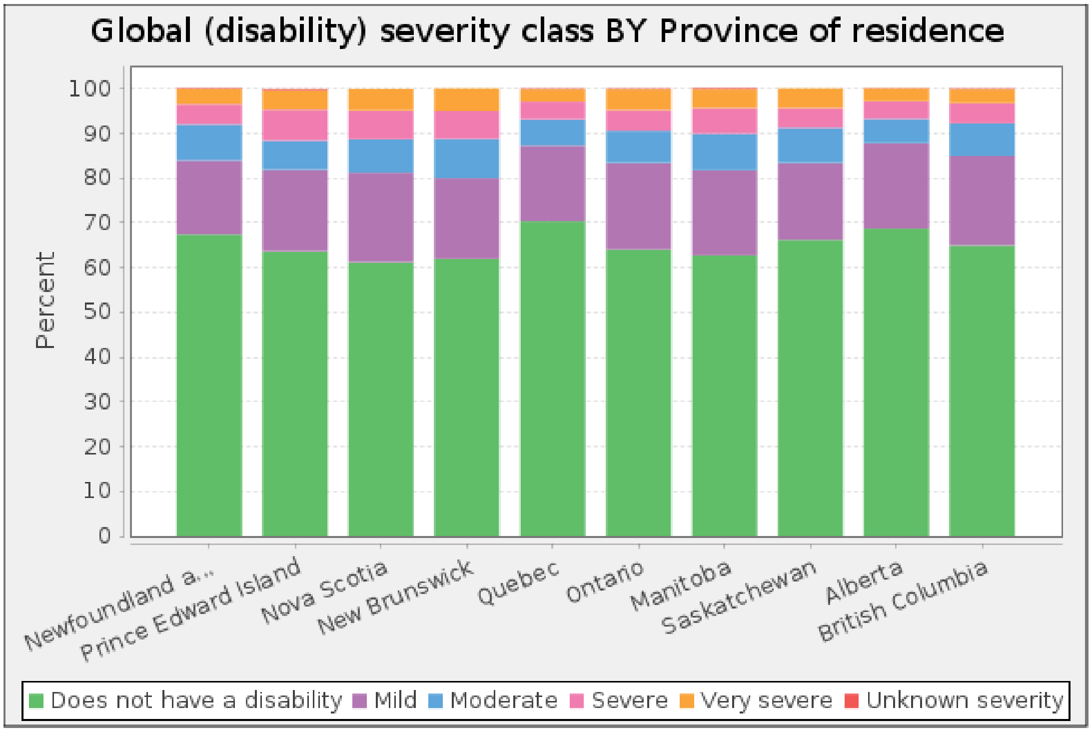

Figure 2. Disability in Canada CHASS Dataset of General Social Survey, cycle 33: 2018: Giving, Volunteering and Participating (Statistics Canada, 2021)

This indicates that the development of accessible transportation would likely address the needs of New Brunswick's population as the per capita rate of disability is the second highest in the country at 35% of the population living with a disability (Statistics Canada, 2023). There would be a subset of this population living with a disability, a value of which is unknown, that would benefit from accessible transportation. There is a correlation with disability severity and age after the age of 55. Those born in 1946 – 1964 (the Baby Boom generation), is one of the largest demographic blocks in Canada, as population are progressively aging, disability rates grow for this age group. As this demographic continues to age, the challenges associated with aging are becoming more pronounced in Canadian society, due to the proportionally large size of this group relative to other populations. To better serve the Canadian population, finding an efficient way to deliver demand responsive service is crucial as the percentage of the population with disabilities are climbing.

2.1.2 Accessible Transportation Challenges in New Brunswick

With the second largest disability percentage in Canada, New Brunswick faces a challenge of providing capacity for equitable transportation, though the demand for accessible transportation has never been quantified on a provincial scale. The document From Surfaces to Services, a document prepared by the New Brunswick Rural and Urban Transportation Advisory Committee (RUTAC), outlines requirements for an integrated transportation vision in New Brunswick (From Surfaces to Services, 2017). “The Committee recommends that governments and service providers take a holistic approach to addressing accessibility needs in transportation, including honouring existing barrier-free requirements” (From Surfaces to Services, 2017).

In Southeast Region (the largest region by population in New Brunswick, including Greater Moncton), accessible transportation services (Ability Transit) are only available within the Greater Moncton Area. In Moncton there is Ability Transit (non-profit), on-demand accessible services in Dieppe, and a non-profit group called Wheels to Wheels serving Riverview and Dieppe (Wheels on Wheels, 2011). Codiac Transpo, the fixed route transit operator for the City of Moncton, Ville de Dieppe, and Town of Riverview, offers accessible amenities on its buses, but has guidelines assistive device users must meet when using a wheelchair accessible fixed route low floor bus. For example, most electric mobility scooters, due to their overall dimensions, are not permitted on Codiac Transpo busses (Codiac Transpo, 2023). The users must be able to independently board, pay and disembark the bus (Codiac Transpo, 2023). The bus driver must secure the individual using the wheelchair after boarding. If customers do not have the ability to meet these requirements, they are not eligible to use the fixed route public transportation in the offered by Codiac Transpo.

Outside of the urban core, the only regional service is Urban-Rural Rides, which is an on-demand service using volunteer drivers and passenger vehicles to fulfil the demand. The service is not accessible, and average Urban–Rural Rides turns away 18 individuals a day looking for on demand transportation services (Urban-Rural Rides, 2023).

As urban paratransit services in the Greater Moncton Area continue to grow, there is a critical need to accurately estimate demand in order to determine the appropriate service requirements. This involves understanding travel patterns and identifying opportunities for service integration to ensure efficient and responsive delivery of paratransit services to residents. Optimizing these services requires a comprehensive approach to forecasting demand and aligning resources to meet the needs of both urban and rural populations.

2.2 Approaches to Delivering Accessible Transportation Services

Accessible transportation services are often delivered as "Paratransit" services, a supplemental (or "parallel") service offering accessible transportation on demand to persons with a disability who may not be able to use conventional transit (e.g. Ability Transit). In many cases, paratransit services may not be available to the general public (Meyer & Institute of Transportation Engineers, 2016). Paratransit falls under the umbrella of Demand-Responsive Transit services (DRT), which occupy the space between fixed-route fixed-schedule public transportation and personal taxi services (Neven et al., 2015). It differs from conventional transit in that demand-responsive transit routing and scheduling are adaptable to individual users' desires in varying degrees (Meyer & Institute of Transportation Engineers, 2016). Trip times, origins, and destinations are coordinated between the user and the providing agency, and charges are usually based on trip duration and length (Meyer & Institute of Transportation Engineers, 2016). "Dial-a-ride" was the first form of DRT, where passengers call and request in advance to specify between which two points, they wish to be served (Häll, 2011). Stated preference studies revealed that DRT would be considered as the most preferred travel mode above all by seniors with physical limitations (Nuworsoo, 2009).

2.2.1 Paratransit Services Vs. Universally Accessible Transit Services

Since paratransit services cater to clients with a disability who are unable to access conventional transit services, there are questions about efforts to make conventional transit more "universal". The Center for Excellence in Universal Design defines

Universal Design (UD) as “the design and composition of an environment so that it can be accessed, understood and used to the greatest extent possible by all people regardless of their age, size, ability or disability” (Center for Excellence in Universal Design, 2024). Universal design involves improving the standard transportation service to reduce barriers to all individuals, so that no supplemental services are required for persons with a disability. By raising the baseline for service, there is no longer a need to fill gaps by supplemental services for person with a disability. Examples of universal design principles applied to fixed route transportation services are low floor buses, auditory destination messages while on route, tactile wayfinding, and automatic door openers. While universally designed fixed-route services provide an essential and cost-effective transportation option for a broad base of users, on-demand paratransit services are indispensable for those with complex, diverse, and specific mobility needs. On-demand paratransit services offer flexibility, personal assistance, and greater accessibility for people who cannot use fixed route services, helping to bridge gaps in equity and ensuring that everyone can participate in community life.

Paratransit is a subset of DRT. Users can make a request for a ride to the organization running the paratransit service a set date and time for which they need the paratransit service. Dial a Ride was the first application of DRT, however, now Dial a Ride service no longer require you to call, they can also be booked online. The demand and service rate of paratransit services is variable, making it difficult to develop estimates (Diana et al., n.d.). Municipalities have varying ways of offering paratransit services to the community. Often these services are supplemental services to fill a gap in accessible transportation.

2.3 Characteristics of Paratransit Service Delivery

National Cooperative Highway Research Program (NCHRP) Report 832 State DOT Connecting Users and Rides for Specialized Transportation: reviews the planning efforts for paratransit services, understanding how these specialized systems are set up by regional authorities in the United States. They look at the different booking methods such as: direct trip booking, trip planning assistance, and trip booking assistance. Different regional authorities provide different planning and booking systems. Direct booking uses a software booking platform similar to Expedia to find paratransit trips for users. However, in smaller urban areas, there are few other selections for accessible trips. Jacksonville Transit Authority has their paratransit booking integrated into their route planning software where users can also book flex bus routes at no additional charge, improving linkages to the varying modes and operators (Rodman et al., 2016). Some providers report that some software does not seem to perform well in areas of low demand density (Linton Fta et al., n.d.). When accessible demand responsive transit services operate efficiently, they lower operating costs and can expand the client base (Häll, 2011). There are three models of service provisions: direct provision, contracted provision and not for profit (Regional Development Corporation Government of New Brunswick, 2023)

While there is a growing number of Canadian communities that are being served by DRT, more research is required (Klumpenhouwer, 2020). DRT offers significant potential in rural and low-density areas by providing flexible, on-demand transportation where traditional fixed routes are inefficient. However, challenges from the long-distance, low-ridership nature of rural trips, lead to high per-person costs and operational inefficiencies (Börjesson et al., 2020; Zhao et al., 2023). DRT systems servicing rural communities can be expensive due to longer trip durations and fewer riders. Despite these challenges, DRT offers key opportunities: it improves access for underserved populations, such as seniors and people with disabilities, and can be integrated with other transportation modes to optimize efficiency.

As the demand for paratransit services continues to grow, there is increasing pressure to balance service quality with cost efficiency, especially as public funding for these services declines (Park *et al*., 2012). To effectively navigate this challenge, it is crucial to accurately estimate demand for prospective services. A precise understanding of current and future demand allows transit agencies to plan and resource paratransit systems more effectively. This includes determining the optimal fleet size, scheduling strategies, and geographical coverage to meet riders' needs without overspending on unnecessary capacity. Additionally, using data-driven forecasting tools and considering factors like population trends, trip patterns, and demographic shifts can help ensure that paratransit services are appropriately scaled. With this forward-looking approach, systems can be designed to provide reliable, accessible services while minimizing operational inefficiencies.

2.4 Approaches to Estimating Accessible Transportation Travel Demand

Estimating the demand for accessible transportation services, such as paratransit, can require specialized modeling approaches that address the unique needs of individuals with mobility disabilities. Traditional models like the Four-Step Travel Demand Model, while useful for general transportation planning, often overlook the complexities of demand-responsive services and the specific challenges faced by people with disabilities, particularly in the utility of mode choice and service delivery. On-demand paratransit services, require more tailored approaches to accurately predict demand. Activity-Based Modeling (ABM) present a promising alternative, as it focuses on the activities that drive travel rather than just the trips themselves, offering a more detailed and inclusive perspective on transportation needs. Despite these advances, reliable forecasting methods for paratransit remain limited, especially in smaller cities and rural areas, pointing to the need for refined models that can better capture the demand for specialized transportation services.

2.4.1 Four-Step Travel Demand Model

The four-step model is a tool used by transportation engineers in the planning of transportation facilities and services. The four steps include trip generation, trip distribution, mode choice and trip assignment (Meyer & Institute of Transportation Engineers, 2016). This tool helps identify how and where individuals are traveling but fails to consider the impact mobility disabilities may have on the mode choice of these individuals. If paratransit is a mode within the model, challenges arise in calibrating the model to realistic service conditions (Gadepalli et al., 2024). Since on-demand paratransit has no fixed stops, a utility function for each specific service provider must be created (Gadepalli et al., 2024). The four-step model is more concerned about the trip destination class (non-home based & home based), not the activity itself that the trip generates.

2.4.2 <u>Activity Based Modeling</u>

The four-step model is a traditional trip-based model, where the basic unit of analysis is a trip (Meyer & Institute of Transportation Engineers, 2016). However, the ABM technique is derived from the patterns of people (Donnelly, 2010). Activity based modeling provides greater characteristics to non-home-based trips compared to traditional trip-based models such as the four-step model. An activity-based model considers the derived demand to travel to participate in an activity rather than enjoying the trip itself (Donnelly, 2010). An example of this would be demand for travel derived from reaching healthcare services versus enjoying a Sunday drive. The derived demand informs traveler characteristics where there is less information lost to trip classification. This is helpful because it can consider the characteristics and travel patterns of persons with a mobility disability, and their derived demand for an activity. This type of model can be helpful for paratransit tours, where the user participates in activities while they have a mode of accessible transportation available to them.

2.4.3 Paratransit Travel Demand Modeling

There is a lack of reliable methods for paratransit providers to create a demand model. The Institute of Transportation Engineers (ITE) Transportation Planning Handbook does not provide systematic guidance for modelling paratransit services. The ITE Transportation Planning Handbook simply provides a definition of paratransit and that it should be included, for municipalities to comply with Americans with Disabilities Act (ADA), an American civil rights law that prohibits discrimination based on disability. The Transit Cooperative Research Program (TCRP) provides guidance for the operation of paratransit services in TCRP Synthesis 161, based on surveyed responses from different DRT providers around the United States. There is a larger lack of Canadian studies on demand modeling for paratransit in small cities and rural locations.

TCRP Report 119 aimed to enhance the understanding of travel behavior among eligible paratransit users and included two models in the report: a sketch planning model and a regional planning model (Koffman et al., 2007) The sketch planning model allows planners to input characteristics into a spreadsheet to predict how various factors impact forecasted demand for paratransit. The model uses the population, base fare, percent of eligible riders of the population, factor if eligibility is screened, percent of the population below the poverty line, and effective on-time window for paratransit, the results are based on confidence limits of 95%-90% trips per capita are calculated, and annual ridership (Koffman et al., 2007). The regional planning model was tailored to individual metropolitan areas and accounts for demographic changes and travel variables affecting paratransit-eligible individuals. Building on TCRP Report 119, TCRP Report 158 focuses on bettering regional modeling with a disaggregate model, which has become standard practice for planning on-demand paratransit services. Disaggregate data are collected from individual persons. The data items typically represent the same variables as in aggregate data but are measured at the individual rider level (Koffman et al., 2007).

Neven *et al*., 2015 presents a method of approximating demand per year based off a percentage of the population, which then uses a synthetic group to simulate the demand for paratransit. They conducted a microscopic simulation of the demand of Persons with Disabilities (PWD) for transportation to find a detailed overview of all transportation requests that need to be processed (Neven *et al*., 2015). For this purpose, a synthetic population of all PWD was created, representing individual actors which were statistically equivalent to the real population of PWD, and their corresponding transportation requests with specific travel-related characteristics were generated (Neven et al., 2015). Each simulated transportation request was then assigned to a specific service provider. This assignment could change, depending on the scenario that is considered. Additionally, a separate vehicle routing plan was created for each provider for both a weekday and weekend day. This vehicle routing plan indicates the minimal resource requirements, in terms of vehicles, drivers and kilometres, that are required to perform all transportation requests assigned to this specific provider. This step considered heterogeneous users (persons with different severities and type of disabilities), heterogeneous vehicles (regular and wheelchair adapted vehicles) and multiple geographically distributed depots (locations where vehicles are stationed)(Neven *et al*., 2015). The majority of existing research on resource requirements for DRT services has mainly focused on finding the required number of vehicles and are based on a limited set of parameters which are assumed to be constant and applicable to all users (service area size, demand density, time window length, maximum ride time, average ride length, and others). DRT services are influenced by volatile demand, especially for non-essential trips like leisure or medical appointments, with factors such as demand density, service area, and cancellation policies impacting cost efficiency, while data-driven forecasting models can help improve logistics by predicting future demand, though issues like spontaneous ride needs and privacy concerns remain (Chandakas, 2020).

Demand is known as the input to transport modes, in the form of trips between zones, in the case of demand responsive transit, the difference is in the requests for transit. When modeling these requests each request is considered a client (Ronald et al., 2015). As stated in literature it is crucial that passengers in vehicle time is tracked (Ronald et al., 2015). There are short term models of demand responsive transit that provide insight into the travel requests, trip planning and level of service (Ronald et al., 2015). For this reason, it is important to take the real-world data from paratransit services to mirror the real work through simulation and modelling.

There is a certain amount of data needed to accurately model the demand for transportation. If the data used does not meet a certain level of accuracy the model may not be able to accurately forecast travel (National Academies of Sciences

Engineering and Medicine, 2012). The data sources used in demand modeling can be population, employment, and household size. Within the required data for modeling considers the network data for streets, which involves facility type and area type (National Academies of Sciences Engineering and Medicine , 2012).

The demand model of paratransit services involves aggregate and disaggregate components (Bradley & Koffman, 2012). The aggregate components predict how many DRT users would use the service within each Transportation Analysis Zone (TAZ). Aggregate models like regression models are estimated with single averaged variables across an entire service area (Kuo et al., 2013). Smaller sections of the service area may not respond well to the averaged variables, such as population, of the entire service area (Kuo et al., 2013). The disaggregate model components are applied to survey data, to predict the purpose, mode, and destination of trips. The model can have two scopes of modeling a regional model or a sketch model. The regional model requires detailed demographics of the area. The sketch model has more breathing room allowing changes to the on-demand transportation services and the population of the region.

Two approaches can be followed to accurately model the demand of on-demand transit services. The first approach is to create a proxy population within each TAZ of the service area (Bradley & Koffman, 2012). The second method is to use sampled data from a survey and re-expand it to each TAZ of the service area (Bradley & Koffman, 2012). This research includes both proxy and sampled demand modeling.

The sampled data will be considered the existing system data to compare the two demand modeling methods of a synthesized population.

2.4.4 On Demand Trip Generators

An American study by Deka and Gonzales published trip generators for paratransit for people with disabilities. It uses an extremely large dataset consisting of 1.91 million trips made by New Jersey Transit's Access Link clients, an on-demand service, socioeconomic data from the American Community Survey, employment data from the Longitudinal Employer-Household Dynamics, and establishment data from Dun and Bradstreet (Deka & Gonzales, 2014). The service Access Link provides curb-to-curb service to clients in 18 of New Jersey's 21 counties. Trip generators were identified at a micro level (Deka & Gonzales, 2014). Global Positioning System (GPS) data was used in the collected data to identify the generators of paratransit trips (Deka & Gonzales, 2014). Compared to most past empirical studies, this specific study focused on the home end of paratransit trips, this research also made an effort to identify the characteristics of the block groups visited by the paratransit clients and the activities in the immediate vicinity (Deka & Gonzales, 2014). It was found that requiring a reservation more than 1 day in advance is not permitted under ADA regulations in the United States; this result has no application in an American model (Bradley & Koffman, 2012). However, in New Brunswick it is often required clients make reservations over 24 hours in advance. There is a lack of data in literature determining how “reservation required" on-demand transit trips impact the demand for the service. The impact of an aging population on the paratransit has been identified; the population in all age groups over age 60 was increased by 10% to account for this in TCRP 158 (Bradley & Koffman, 2012).

2.4.5 Area Modeling

An alternative to receiving data from service providers is to model demand from an area-based approach. “A Resource Guide for Service Implementation On-Demand Transit Toolkit” (Metrolinx & Canadian Urban Transit Association, 2022) is a planning document created by Transit Integration team, within the Planning, Design, and Sponsorship team at Metrolinx along with the Research, Data and Technical Services team at the Canadian Urban Transit Association (CUTA) with support from Leading Mobility Consulting for DRT planning. Although this document is not intended to provide service delivery model decision making, it can inform general guidance about offering on demand transit in Canada.

Determining the service area size will then inform the required fleet size (Metrolinx & Canadian Urban Transit Association, 2022). The size of the DRT zone is highly dependent on the type of DRT that is proposed, the level of existing or projected demand, known or anticipated markets, desired travel times, and alignment with the transit system’s overall service goals and standards (Metrolinx & Canadian Urban Transit Association, 2022). The area modeling approach can be used to estimate the fleet required to provide a predetermined level of service (Fu, n.d.). When considering the bounds of a service area of a fixed route service, the implementation of demand responsive transit is supported in an area where the travel time between demand area and bus stops for fixed routes service is larger than a predetermined threshold value by the service provider (Zhao et al., 2023). Applications of DRT in the United States indicate that zone size can vary considerably, from 3 to 75 square kilometres (Metrolinx & Canadian Urban Transit Association, 2022). Using a heuristic algorithm, called Scheduling, Matching, Allocation, and Reduction (SMAR), is an approach to address the problem (Fu et al., 2004). This algorithm states that the types of vehicles available for selection are given; the goal is to decide the number of vehicles of each type to be used. The heuristic method is fundamentally a greedy search procedure, with the idea of using as many small vehicles as possible without loss of productivity (Fu et al., 2004). The greedy heuristic method works in stages, considering one input element at a time. At each stage, the method decides whether an input is part of an optimal solution (Metzger, 2003). The previously determined method is a heuristic method to find the approximate optimal solution not an exact solution. Often the data for demand modeling come from the national statistical agency (Dytckov et al., n.d.). The serviceable area is usually determined through costing and level of service found in the simulation stage. If the level of service is poor it is suspected that reducing the service area is a way to resolve the problem, while not increasing costs to the service provider (Metrolinx & Canadian Urban Transit Association, 2022). Fu created an analytical equation to solve for the number of vehicles required based on the characteristics of the service area. This equation is shown below.

$$FS = \beta_1 \frac{\lambda_T}{E^{\beta_4} T^{\beta_5}} \cdot \left[\tau + \frac{\beta_2}{V} \left(\frac{A}{\lambda_T \cdot T} \right)^{\beta_3} \right]$$

Equation 1

Where:

A = size of the service area (km^2);

T = a quality-of-service constraint defined as the trip service (pick up/delivery) time window (h);

E = quality-of-service measure defined as the maximum allowable ratio of excess ride time to direct driving time; the excess ride time of a given user is defined as the difference between the total ride time and the direct driving time;

λ_T = peak trip rate defined as the equivalent hourly rate at which trips need to be serviced by the system within the evaluation time interval T(trips/h) (the term λ_T T represents the number of trips during the peak period of duration T);

V = average travel speed based on Manhattan distance (km/h);

τ= boarding plus alighting time (h); and

β_1, β_2, ..., β_5= model parameters to be calibrated.

This model is set up using the same principle as a VRP, where all vehicles start and end at a depot. This analytical model proves that paratransit can be modeled as a VRP with the guiding principle that they start and end at the same location, even from an area-based approach. When the service area is not a perfect circle or square, the size of the service area can impact the fleet required by an analytical model (Fu, n.d.). Although area modeling is not often referenced in literature such as the ITE transportation planning handbook, area-based modeling can be a quick way to find fleet requirements for a predetermined service area thanks in part to the research by Liping Fu.

2.5 Vehicle Routing and Scheduling

Vehicle routing and scheduling is the process of planning the most efficient routes and schedules for vehicles to follow when delivering goods or transporting people. The goal is to make sure that vehicles travel the shortest possible distances, avoid unnecessary delays, and meet the needs of all the people or locations they are serving, all while minimizing costs and saving time. The following section presents the vehicle routing problem definition, the short term Dial a Ride Problem (DARP), the Vehicle Routing Problem (VRP), the Traveling Salesperson Problem (TSP), and the different methods used to solve vehicle routing and scheduling problems.

2.5.1 Vehicle Routing Problem

The connection structure of a vehicle routing problem can be shown on a graph or map. The locations visited are classified as nodes in the problem. The number of nodes defines the sizing of the problem. Customers and the depot are represented by nodes with a specific position. There is only one depot in the VRP, and this is the start and end location of the problem. For example, a 40-node problem, the vehicle(s) visit 39 pick up/drop off locations and 1 depot. The relationship between nodes is defined as edges or links between positions. The time or distance it takes to get from each node to each other is defined as a cost represented as time or distance. There is a cost to get from each node to another node, which forms a square matrix. The cost matrix is the size of the number of nodes within the system. To simplify the vehicle routing problem a heterogeneous fleet is assumed (Munari et al., 2016). The generalised constraints of a vehicle routing problem are that each node must be serviced by an agent (vehicle). The vehicle must start and end at the depot. There can be multiple vehicles that work together to service all customers, and each customer cannot be visited more than once. Each vehicle has a limited capacity. The objective of the vehicle routing problem is to determine a set of routes that meet the needs of all customers for the least cost while meeting the previously defined constraints (Munari et al., 2016).

2.5.2 Short Term DARP

The Dial A Ride Problem (DARP) is the optimization of the DRT service provision. There are different types of demand responsive transit models such as meeting demand, real life demand, and mode choice models (Ronald et al., 2015). These models use varying algorithms and computer languages (Ronald et al., 2015). The short-term models completed in the last ten years often use C# or Delphi as the program language. The algorithm used most frequently is the Modified DARP algorithm, but insertion algorithms, successive best insertion algorithm and the integrated dial a ride problem algorithm have also been used in similar studies (Häll et al., 2009; Ronald et al., 2015).

2.5.3 Characterizing a TSP

In a Traveling Salesperson Problem (TSP), a “traveling salesperson” (or vehicle, in this case) completes a journey to multiple destinations (nodes) where all nodes within the problem must be visited and can only be visited once (as someone making sales calls), while the vehicle must start and end at the node defined as the “depot”. The depot is the home base of the service, where fleet are located. The Travelling Salesman Problem has generally been considered a NP-Hard Problem. The paratransit scheduling problem falls in the category of the General Pick up and Delivery Problem (GPDP) (Park et al., 2012). The Dial a Ride Problems (DARPs) are generalizations of the traveling salesperson problem (TSP), which has been known to be a Nondeterministic Polynomial Time Problem (NP-hard) to this point. TSP is complex and requires sophisticated algorithms to solve because the number of possible routes increases factorially. In Park et al., (2012) they focused on techniques for finding bounds for the optimal solutions and describe some heuristics to find feasible solutions. An example of the formulation of DARP is shown below (Figure 3).

$$min \sum_{\substack{i,j=1 \\ i \neq j}}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{\substack{j=1 \\ i \neq j}}^{n} x_{ij} = 1 \quad \forall i \in N \tag{1}$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} x_{ij} = 1 \quad \forall j \in N \tag{2}$$

$$u_j - u_i \geq 1 - (n-1)(1 - x_{ij}) \quad i, j \in \{2, \ldots, n\},\ i \neq j \tag{3}$$

$$1 \leq u_i \leq (n-1) \quad i, j \in \{2, \ldots, n\},\ i \neq j \tag{4}$$

Figure 3. Problem Formulation of a DARP by Miller, Tucker, and Zemlin (Park et al., 2012)

The turned A written as ∀ means “for all” in i, the ϵ means “is a member of”, this can also be represented by “s.t.” The Constraint 1 states that all nodes that exist within the network must be left once. The second constraint states that all nodes that exists within the network must be visited once. The variable “u” is a continuous decision variable tracking the location of the agent, ensuring the problem does not have any convex hulls.

2.5.4 Heuristics Analysis

Hillier and Lieberman (2015) define heuristics analysis in their book "Introduction to Operations Research" which is synthesized here. Heuristics analysis of complex problems can help find the sub optimal solution to the problem. Operations research occasionally uses only heuristic procedures to intuitively design procedures that do not always guarantee an optimal solution. The use of heuristics can be helpful to find the suboptimal solution to a large problem. There has been great progress in the field of metaheuristics. Metaheuristics provides a strategy and a general structure for designing procedures that fit particular types of problems. The use of heuristics is helpful in integer programming problems that are too large to solve for the exact solution. There are many heuristics algorithms that can be applied to large integer programing problems, to efficiently determine a sub optimal solution. A heuristics analysis method is a procedure that can find a feasible solution, but not necessarily the optimal solution for the specific problem considered. No guarantee of solution quality can be given if the solution to a problem is found using heuristics approach, although a well-designed heuristics problem can provide a nearly optimal solution.

2.5.5 Metaheuristics Analysis

A metaheuristics method is a general solution method that provides both a strategy and a general structure for developing a specific heuristic method to fit a particular kind of problem. Hillier and Lieberman (2015) define the metaheuristics method in their book "Introduction to Operations Research" which is synthesized as follows. The metaheuristics analysis method orchestrates the interaction between local improvement procedures and higher-level strategies to create a solution process. The solution process is capable of escaping from local optima solutions and can perform a large search of the feasibility of the entire region. These search procedures allow for an improved sub optimal solution compared to a simple heuristics approach. Although the metaheuristic analysis approach is helpful to find a feasible solution to large and complex problems, it does not guarantee the optimal solution to a problem.

2.6 Exact Solution Approach: Christie Method

The Christie Method, developed by Dr. Jim Christie, can be used as a short-term algorithm for the DARP offering an exact solution. The Christie Method algorithm is based in Excel allowing visualization of the steps required allowing less barriers to the use of the method (Christie, 2018). The Christie Method can use the solving power of Gurobi or COIN-OR Branch and Cut (CBC) solvers, integrated into Excel as an add-on.

Using the Christie method to solve DARPS and TSPs, can be calculated in polynomial time at small scales, differentiating from other DARP and TSP solution techniques. The Christie Method approaches the DARP as an assignment problem, including the necessary constraints and solves the problem using Constraint Mixed Integer Linear Programming (Christie, 2018). The Christie Method is described in detail in Section 4.

The Christie method differs from heuristic methods by using a constraint linear programming method with an exact linear programming solution that could be calculated in polynomial time. The other solvers use heuristic optimization techniques, constraining the problem to an NP-hard problem. The Christie Method has yet to be applied in a practical use case scenario.

2.7 Summary

There are many studies stating the challenges people with a mobility disability face while trying to access and use transportation services to reach activities (Deka & Gonzales, 2014; Field & Jette, 2007; Kuo et al., 2013; S. Morris & McDiarmid, n.d.;

Neven et al., n.d.; Petretto & Pili, 2022). This suggests demand for DRT services, but yet there is a lack of research in the demand estimation component, in particular for rural and regional services. Paratransit provides an accessible alternative to a fully accessible fixed route system. Theoretically if fixed route public transportation services used universal design techniques to service the broad spectrum of individuals who require it, fixed route paratransit would render DRT paratransit useless. No matter the accessible transportation service provision, demand modeling of these services can be classified by trip or activity (Improving ADA Complementary Paratransit Demand Estimation, 2007). Activity based modeling incorporates more traveler characteristics because it focuses on the desired demand (Donnelly, 2010). Aside from demand modeling estimating travel demand for accessible transportation services can also be done by area-based estimates (Dytckov et al., n.d.; Fu et al., 2004; Mortazavi et al., 2024). Research is required to improve the estimation techniques for the demand of paratransit service in rural areas, as detailed in section 2.4.2. Following a proper demand estimate for paratransit on demand services, varying heuristic methods were discussed, yet none in the literature for the exact optimal solutions. Further research and application of exact solving solutions are required for the vehicle routing problem, with the application to DRT paratransit. The application of the Christie Method, an exact solution method, has never been attempted before on a problem of this size. The gaps within literature prove that the work on demand responsive paratransit is still evolving.

3 METHODOLOGY

The methodology chapter describes the source data used for developing trip rates for accessible transportation users, as well as the process of validating and updating the data set for analysis. The chapter also includes the method for estimating regional travel demand and the regional trip generation of paratransit services. The data collected to be used for regional DRT demand estimation and trip generation planning are further detailed in this chapter.

3.1 Data

This research relied on secondary data provided by Ability Transit, a non-profit paratransit service, which maintains its own rider travel database in a series of Excel worksheets each representing a day of operation for a single bus, up to 6 buses a day. The data provided included the bus number, driver name, date, and rows containing unique trips including time of trip request and the location of the origin and destination in postal code form (e.g. A1B 2C3). The data provided encompassed all trips from January 2nd, 2023, to July 2nd, 2023. The table below details the extent of data provided.

Table 1. Data Provided

Number of Unique Trip Records	14,567
Number of Excel Worksheets	626
Number of Days	178
Type of Data	Trip Logs

If a trip was requested the time slot was filled in with the time requested for pick up, the location of pick up and the desired destination of the client. All personal details, including trip purposes, were redacted.

3.1.1 Organizing data for analysis

The data recorded among 626 individual Excel sheets were cleaned to take out blank cells and combined into a single worksheet to facilitate analysis. The individual sheets for each bus were combined using Excel macros. The macros used are provided in Appendix A.

3.1.2 Validating Data Inputs And Populating Empty Records

The data maintained by Ability Transit are organized to reflect their specific business processes, and as such, cannot be used directly to determine user travel behaviour due to data entry processes and a lack of trip qualifiers, such as whether multiple clients were picked up at the same time or the same place. When a client would like to book a trip, they contact Ability Transit, who has an individual assess the feasibility of being able to provide the trip, which if feasible, is then entered in the trip log for a specific day and bus. The Ability Transit data did not explicitly state which riders are picked up together or are dropped off together; their practice is to associate a single postal code with a single rider in their data, then leave the postal code blank in subsequent rows for additional riders picked up or dropped off at the same location. The researchers worked with Ability Transit to correlate their data entry practices with an interpretation of user travel behaviour (Table 2):

Table 2. Validating Data Interpretations

Data Inputs	Interpretations
Time, origin, and destination information in a single row	One rider, unique destination
Time, origin, and destination information in a single row, with date, time, origin of subsequent riders recorded in subsequent rows, and the destination cell left blank	One rider, unique origin with shared destination with other riders
Time, origin, and destination information in a single row, with subsequent rows time left blank, and origin and destination information provided	Multiple riders, same origin, unique destination
Time, origin, and destination information in a single row, with subsequent rows time left blank, and origin and destination information provided	Multiple riders from a single origin to a single destination

Given that Ability Transit's data are entered manually, it is possible there to be errors in the transcription. The data were reviewed for transcription errors by looking for a logical progression of times row by row. Often, times in the travel log data provided by Ability Transit were labeled as morning (am) trips when they were afternoon (pm) trips. This was discovered because trips were recorded in order of time requested from 7 am to 10 pm. Some trips had missing postal codes if they picked up multiple riders at one location. To save time when recording the trip requests if multiple requests were requested at one time from one location, the time was not entered. To solve this problem the dates and postal codes were filled into the blank cells, after logically checking against other trips in the travel log. In total 57 possible transcription errors were found when reviewing the data provided.

3.1.3 Updating the dataset for analysis

Each row in the data represented an individual “trip” taken by a unique rider. All trips were given sequential Trip Identification (ID) numbers to keep track of individual rider trips. The data provided in the rows of the travel logs did not provide date or bus number, important values to the descriptive statistics. Once all cells of data provided from Ability Transit were entered into the primary spreadsheet, the information in the headings of the travel logs were entered. Dates, bus numbers and driver names were taken from the headings of the travel log Excel sheets and filled into the cells in the primary Excel sheet. This was validated through discussion with Ability Transit. Two additional columns were created in the dataset to be able to identify unique “vehicle” stops for picking up and dropping off multiple passengers at the same location, with each trip record given a Pick up or Drop off ID number that increased sequentially. Unique trip records could have the same pick up or drop off ID. Origin and destination postal codes were populated in the cells where they did not explicitly exist.

Pivot tables were used to determine if there were any transcription or data entry errors in records, including postal codes, which may have had an effect on trip counts. Some postal codes were entered with no space in between the first three characters and last three characters, creating multiple groups of the same postal codes. Using the pivot value, it was possible to see errors in an easy-to-read format.

To avoid the duplication errors of postal codes when grouping the possible locations, the spaces were removed from the front and end of the postal codes. This was done using the trimming tool "TRIM" in Excel. The postal codes were trimmed to display properly in the pivot charts and tables. Following the cleaning of the data, pivot tables were created showing the count of trips to the postal codes based on the location of pick up and destination.

3.1.3.1 Introducing Derived Data Into The Dataset

Once the dataset was clear of errors, it could be used to begin analyzing the data. Because the primary dataset had 14000 rows of data and 8 columns, "IF" statements were used to automate the categorization of trips according to the interpretations in Table 2.

A pick up ID column was created by using an "IF" statement to compare the pick-up location postal code to the cell above to assign a pick up number if the cells had the same postal code and if not to add 1 to the pick up cell ID in the cell above. This distinguished the pick up locations from one another allowing an estimate of how many users were picked up at a single postal code. A binary check was created in order to not double the number of people picked up by location. In order to visualize the process defined above a flow chart was created to show the process of logic checks. The flow chart is shown below in Figure 4.

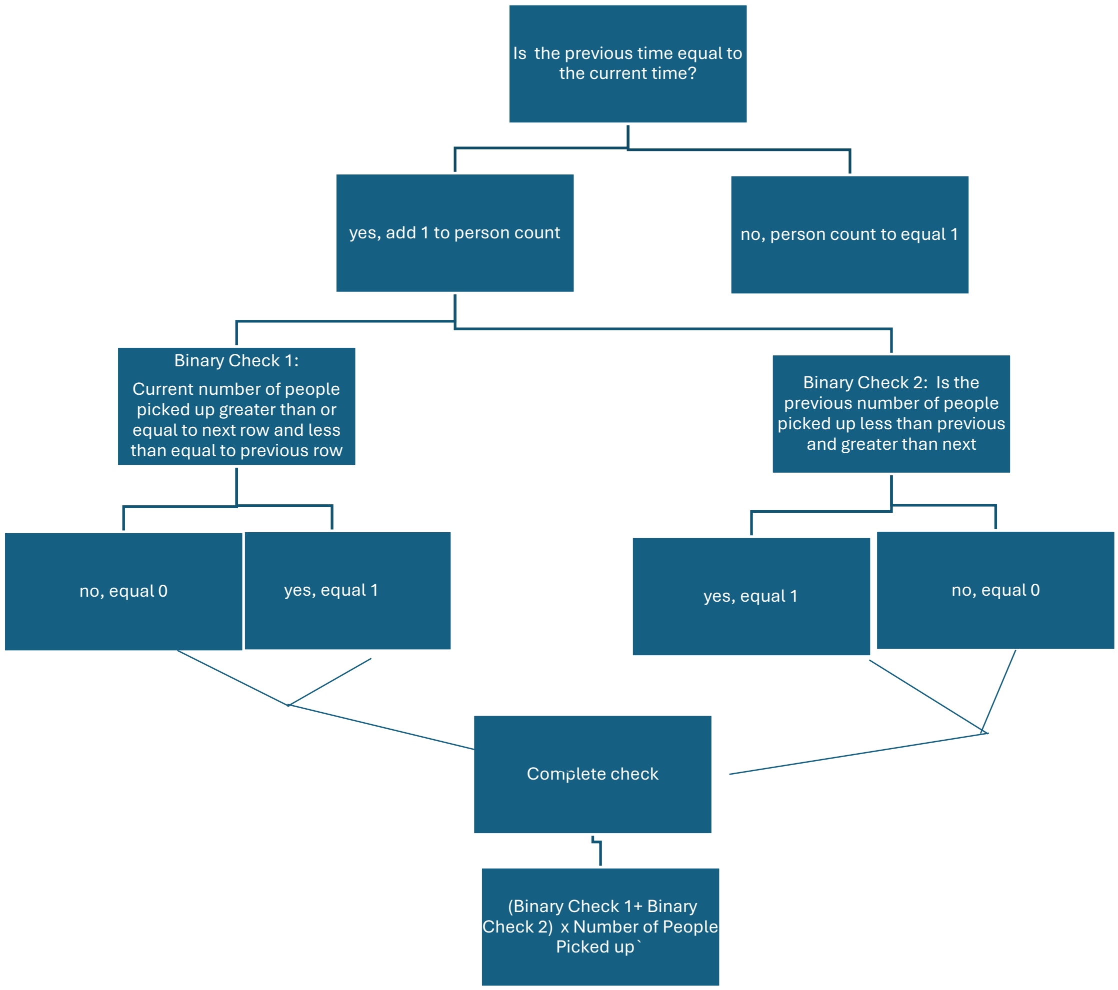

Figure 4. Binary Logic Check for Number of Riders Picked Up

The same method was applied to pick up times, as there were instances where multiple pick ups occurred at the same time from different postal codes. A logic check was necessary to determine the number of passengers on the bus at each pick up. The following logic tests are described in a flow chart above in Figure 4. Once the data cleaning and logic checks were finalized, the primary spreadsheet was ready for generating descriptive statistics. A snippet of the completed primary spreadsheet is shown in Figure 5 below, with the last three digits of the postal code redacted for privacy. The word "*complete*" in front of a column heading, means there were multiple steps before arriving at the completed result, usually involving a formula in a subsequent row to result in the value in the completed column.

Pick Up Location ID	Drop off ID	ID	Day of Week	*Date*	*Driver*	*Bus*	*Rounded Complete Time*	*COMPLETE Time*	TRIMMED Location	TRIMMED Destination	Complete Number of People picked up by Time
1	1	1	Monday	January 2, 2023	Ron	2	7:00:00 AM	6:50 AM	E1G	E1C 2	1
2	2	2	Monday	January 2, 2023	Jeff	3	7:00:00 AM	7:10 AM	E1A	E1C 2	1
3	3	3	Monday	January 2, 2023	Emile	1	7:30:00 AM	7:30 AM	E1A	E1G	1
4	3	4	Monday	January 2, 2023	Emile	1	8:30:00 AM	8:30 AM	E1E 1	E1G	1
5	4	5	Monday	January 2, 2023	Ron	2	9:30:00 AM	9:30 AM	E1C	E1C 1	1
6	5	6	Monday	January 2, 2023	Ron	2	10:00:00 AM	10:00 AM	E1C	E1B 3	1
7	6	7	Monday	January 2, 2023	Jeff	3	11:00:00 AM	11:00 AM	E1C 1	E1C 9	1
8	7	8	Monday	January 2, 2023	Ron	2	11:30:00 AM	11:30 AM	E1B	E1C 5	1
9	8	9	Monday	January 2, 2023	Jeff	3	12:00:00 PM	12:00 PM	E1C	E1A 4	1
10	9	10	Monday	January 2, 2023	Emile	1	12:30:00 PM	12:30 PM	E1C	E1C 0	1
10	10	11	Monday	January 2, 2023	Emile	1	12:30:00 PM	12:30 PM	E1C	E1A 5	1
11	11	12	Monday	January 2, 2023	Emile	1	7:00:00 AM	6:50 AM	E1C	E1C 2	0
12	11	13	Monday	January 2, 2023	Ron	2	7:00:00 AM	6:50 AM	E1C	E1C 2	2
13	11	14	Monday	January 2, 2023	Ron	2	7:30:00 AM	7:15 AM	E1C	E1C 2	1
14	12	15	Monday	January 2, 2023	Ron	2	9:00:00 AM	9:00 AM	E1G	E1A 2	1
15	13	16	Monday	January 2, 2023	Ron	2	9:30:00 AM	9:30 AM	E1A	E1E 4	1
16	14	17	Monday	January 2, 2023	Ron	2	10:30:00 AM	10:30 AM	E1A	E1C 2	1
17	15	18	Monday	January 2, 2023	Ron	2	11:30:00 AM	11:30 AM	E1C	E1C 0	0
18	16	19	Monday	January 2, 2023	Ron	2	11:30:00 AM	11:30 AM	E1G	E1E 1	2
19	17	20	Monday	January 2, 2023	Jeff	3	12:00:00 PM	12:00 PM	E1C	E1C 0	0
19	18	21	Monday	January 2, 2023	Emile	1	12:00:00 PM	12:00 PM	E1C	E1C 9	0
19	19	22	Monday	January 2, 2023	Jeff	3	12:00:00 PM	12:00 PM	E1C	E1G 2	3
20	20	23	Monday	January 2, 2023	Emile	1	1:00:00 PM	12:45 PM	E1G	E1C 2	1
21	20	24	Monday	January 2, 2023	Emile	1	1:00:00 PM	1:00 PM	E1C 6	E1C 2	1

Figure 5. Cleaned Spread Sheet of Ability Transit Data

3.1.4 Interpreting Travel Behaviour from the dataset

The dataset does not include explicit information on unique riders, the number of people picked up at each stop, the amount of time the individuals were on the bus for, the number of individuals on the bus at time, or how many individuals were dropped off at a location. Associating each trip with a unique rider was determined by applying a classification process based on an initial understanding of how Ability Transit records and inputs their data and using this to create a set of rules. Pick up location ID and drop off ID columns were created to determine how many stops the vehicle was making on their tour. These columns were created using an if statement applied to the trimmed postal code column. The "IF" statement is shown below:

$$=IF(L_c=L_n, P_p, P_p+1) \quad \text{Equation 2}$$

Where:

L_c=current location

L_n= next location

P_p=Previous pick up stop

There were five types of observations about trip characteristics from subsequent rows of data that allowed an interpretation of the most likely travel behaviour of users from incomplete data. The interpretation was validated in discussion with Ability Transit (Table 3) and used to create additional derived data such as persons per trip, the capacity of the vehicle, and how long individuals tend to stay on the vehicle before reaching their stop. These derived data, inform the creating of constraints in the model such as vehicle capacity and Level of Service (LOS) requirements. To better understand the daily operations of Ability Transit, understanding the daily trip characteristics helped to create a realistic model.

Table 3. Trip Characteristics Takeaways

Class	Observation of subsequent rows of data	Interpretation
1	Same start time and the same start location	One pick up of multiple people at one location
2	Same start time but different start locations	Multiple pick-up locations, close together. These were often representing last minute bookings squeezed in to an already assembled schedule.
3	Same start time but different start locations and same end locations	Multiple people picked up all going to the same destination
4	The destination of one is the origin of the next row	No conclusions from operation characteristics can be drawn from this.
5	Had different start times but with the same destination	This shows that not all trips are point to point and can be part of a tour but cannot be confirmed due to privacy issues. There is no limit to how frequently someone is on the bus. The longest someone would be is 30-40 minutes.

The interpretation permitted the derivation of the most likely user travel behaviour associated with the records, informing descriptive statistics such as how many people boarded the vehicle, and how many trips were made from location to location. Ability Transit also provided a constraint of the longest time an individual could be on the bus to maintain a “good” level of service. This level of service of time an individual can be in the vehicle before a decline in the overall level of service will inform a constraint in the Christie Constraint Mixed Integer Linear Programming as detailed later in Chapter 4.

3.1.5 Conducting Geographic Analysis

Ability Transit provided postal code data associated with each travel record in the log. The postal codes provide a geographic area of origin or destination, but these locations need to be converted into Statistics Canada units of geographic aggregation, such as Dissemination Areas, which would include population values and other demographic attributes from the census. This permitted the development of per capita trip rates which could be used as the basis for extrapolating sketch planning estimates elsewhere. Not only were there no demographic data associated with postal codes, but there was also no exact geographic location data in the travel log. A set of coordinates were required to create a cost matrix for use in a vehicle routing and scheduling method. The cost matrix was made of time and distance from each data point to each other. The methodology of creating and using the cost matrix is described in greater detail in Chapter 4.

The company DMTI, produces a Postal Code Conversion File (PCCF). The DMTI Postal Code data file includes six-digit postal codes for all of Canada. There are two types of postal codes provided, Unique Enhanced Postal Codes (UEP), and Multiple Enhanced Postal Codes (MEP) (Statistics Canada, 2024). UEP is used more frequently because it provides a 1:1 relationship between postal codes and Statistics Canada's standard census geography (University of Waterloo, 2024). The postal code conversion file is a text file with all 848,354 active postal code in Canada ((Statistics Canada, 2024). This data includes information copied with permission from Canada Post Corporation (CPC). Each record file contains the six character postal code, the dissemination area identifier, the dissemination block, latitude and longitude representative coordinates of the census geography, census subdivision name, code and type, geographic codes of other higher level standard geographic area, federal electoral district code, CPC information relevant to each postal code^{OM}: its birth date, retirement date, type of mail delivery, CPC community name, and various flags: single link indicator, type of representation point, and postal code^{OM} type.

The postal code conversion file included a lot of data that was not beneficial to the research. The unrelated data was to be removed, and data had to be converted to a format that could be analyzed. The .txt file was imported to excel, so the postal codes and associated data for other provinces could be removed. Each postal code in the text file was a string of characters and numbers that had no separations distinguish coordinates, from Dissemination Areas (DA) or Forward Sortation Areas (FSA). The "text to columns" feature was used in Excel to separate the values based off the size record layouts provided in the PCCF Reference Guide (Statistics Canada, 2024). After the values were separated, the irrelevant information was removed from all postal code records, this included CPC information relevant to each postal code^{OM}: its birth date, retirement date, type of mail delivery, CPC community name, and various flags: single link indicator, type of representation point, and postal code^{OM} type. Using the pivot table feature in Excel to group data by the relevant column information, all census subdivisions that were not Riverview, Salisbury, Moncton, or Dieppe region were removed from the data set. This left the appropriate data in an easy-to-understand format for the Moncton region. Next the VLOOKUP command was used to search the Moncton region PCCF data for the postal codes of the origins and destinations of the Ability Transit data. Using the VLOOKUP command returned the dissemination area, and coordinates of all the postal codes of the origins and destinations of Ability Transit trips. The new table with the coordinates, postal codes and Dissemination Areas were moved to spreadsheet for the next step of the research.

Postal codes are not the most robust spatial units of analysis available; they retain a modest degree of utility for specialized applications like transportation modeling (Grubesic, 2008). This is an example of a specialized application for the transportation model of paratransit, while protecting the privacy of the service users. Tying the postal codes to Dissemination Areas allowed trip rates to be calculated. The postal codes were tied to a dissemination area using the DMTI postal code conversion file. The DAs were the related to the population so trip rates could be calculated per capita. The population of the Dissemination Areas were found using the 2021 census information from Statistics Canada’s website. The population density of the dissemination area was used to calculate if the dissemination area was urban or rural. The population density limit for urban vs rural was based off the Statistics Canada limit of the minimum population density for an urban area is four hundred people per square kilometre (Statistics Canada, 2024). The trips rates by dissemination area are shown in Chapter 5.

Using ArcGIS, an origin-destination cost matrix was created. This matrix linked the travel times between all nodes, which were then used as a cost matrix in the Christie Method of Constraint Mixed Integer Linear Programming (CMILP), the methodology of the modeling is further discussed in Chapter 4. Prior to using ArcGIS, an origin-destination matrix based on postal code counts was created in Excel using a pivot table from the data provided by Ability Transit. The origin-destination matrix consists of 393 rows and columns. The postal codes provided are reflect trips within Ability Transit's service area.

Table 4. Summary of Origin Destination Matrix

Summary Table	
Number of OD Pairs	393
Largest Number of Trips Per OD Pair	72

Using ArcGIS Pro, a symbology was used to represent the ratio of trips that originated vs were destined for the Dissemination Areas. The percentage of trips was calculated by the number of trips originating from the dissemination area over the total number of trips to and from the dissemination area. The graduated colors symbology was used in ArcGIS to show which Dissemination Areas had more trips originating from them versus destined for them. The histogram was observed, and it showed a normal distribution, so the standard deviation method was used for the intervals. The interval sizes were one-third standard deviation.

3.2 Estimating Regional Travel Demand

The trip generation model was created and geared towards the use only for users with disabilities registered with Ability Transit. Those registered with the service must meet the requirements of the provider, such as having a mobility limiting condition. Those registered with the service can make a trip request by calling the service. Because the service operates based on a specific eligible population, the trip generating nodes were based solely on the trip request from January 2023 to July 2023 of qualifying individuals registered with Ability Transit.

The regional travel demand for paratransit services in the Southeast Regional Service Commission was estimated based on trip rates from Ability Transit. These rates were derived from a six-month period of Ability Transit trip data and are tied to the dissemination area of the postal codes where the trips originated or were destined. This trip generation data was then applied to the entire Southeast Region of New Brunswick to estimate nonprofit paratransit service demand.

Trip generation is a key component of the four-step travel demand modeling process. For demand-responsive services, such as paratransit for individuals with mobility, visual, or hearing impairments, additional variables must be considered. TCRP Report 138 broke the four-step model into a disaggregated form, focusing on user decisions related to paratransit use, such as registration for the service (Transportation Research Board, 2010). However, due to data privacy issues, a full disaggregate model was not used for this project. Instead, variables within the trip generation section were included to validate existing data sources. While a disaggregate model could improve understanding of travel behavior and the needs of individuals with disabilities, it requires significant collaboration and an ethical review to address data privacy concerns. For this project, the trip generation model focused on Ability Transit's data, including trip rates, locations, and destinations based on real trip data from January to July 2023. The model was designed for individuals registered with Ability Transit, specifically those with mobility-limiting conditions.

A disaggregate model is more than just a system-level model, it would contribute to increased understanding of travel behavior, needs of people with disabilities and has greater potential for incorporation in the next generation of regional travel demand models. Like the system-level model, a disaggregate model could be limited to ADA paratransit eligible trips and individuals (Koffman *et al.*, 2007). For this project, the generation was based on the characteristics of Ability Transit such as trip rates, trip location requests and destinations. The data was received in form of postal codes from Ability Transit. Using the total trips from the cleaned data, and the days the paratransit service was active the total trips requested per day were calculated. The number of trips per day is used to size the optimization problem. Because Ability Transit requires advanced notice for the trips, the generation was focused on one day only. The trips for one day of the model were chosen based on trip rates and patterns from a list of real trip locations and destinations in Ability Transit's dataset.

Using Ability Transit data and postal code conversion files, trips were calculated based on geographic areas defined by Dissemination Areas. The PCCF was used to associate postal codes with census subdivisions, linking origin and destination points for each trip (Statistics Canada, 2024). Population data from the 2021 Census was used to quantify trip rates for these areas, which were then applied to the Southeast Regional Commission's broader transportation planning model.

One issue that was discovered was that some postal codes that were given to University of New Brunswick (UNB) are no longer active by Canada Post. When comparing these postal codes in the PCCF file, five were invalid and therefore did not have latitude and longitudinal coordinates. These invalid postal codes could be a result of a transcription error or could be retired postal codes from clients receiving services for a long time. For this reason, the invalid postal codes were first checked to see if they had been retired and if so, deleted from the data set. After the postal codes were formatted, the associated latitude and longitude coordinates and dissemination area codes were found within the large PCCF data set.

The geographic calculations were further refined by ensuring that trips were mapped to the centroid of the most populated area within each postal code. This was crucial for accuracy, especially since some of the postal codes provided were no longer active. Invalid postal codes were checked against the PCCF data and removed, as necessary. The resulting trip generation model used demographic data (population and density) from Statistics Canada to associate trip rates with population of Moncton. These rates were calculated and presented in Chapter 5.

3.2.1 Data Visualization

Using both PowerBI and ArcGIS, Ability Transit data were analyzed to identify trip-generating patterns and destinations. All postal codes from six months of data were included. To create a heat map, pivot tables were generated to count the number of trips to each postal code, which was then imported into PowerBI, an interactive data visualization tool developed by Microsoft (Microsoft, 2024). After the heat map was created, a spot map was generated using ArcGIS. The map featured blue dots of varying sizes, with trip points indexed using a natural break method to highlight different levels of trip frequency to specific areas.

3.3 Regional Trip Generation

To calculate the demand of on demand paratransit for the Southeast Regional Service Commission, the trip rates per capita per day were found, for all Dissemination Areas within the City of Moncton. This was then applied to the different areas around the Southeast Regional Service Commission, to create a trip generation model based on the results found from the Ability Transit data. The southeast region was broken down using the zones of the Southeast Regional Service Commission. The areas for trip generation were Cap-Acadie, Shediac, Dieppe, Fundy Albert, Maple Hills, Memramcook, Moncton, Riverview, Salisbury, Strait Shores, Tantramar, and Three Rivers.

The trip rates from area to area were found in the Civilia data for the Southeast Regional Service Commission were used. The number of trips generated by on demand paratransit were found by multiplying the average trip rate, found through analyzing the Ability Transit data and multiplying the trip rates by the population of each community within the Southeast Regional Service Commission. Developing optimization scenarios

To test the effectiveness of the optimization procedures, two distinct approaches were used to create scenarios for evaluation. First, an optimization method was piloted to replicate the service parameters of a paratransit service. The goal was to simulate realistic conditions of paratransit operations, taking into account factors such as varying pick up and drop-off locations, passenger preferences, and vehicle capacity, all while maintaining the most efficient routing. The second approach applied the same optimization method to solve a small-scale community transport issue, focusing on a regional area. A model was created for Cap-Acadie, a rural area with fewer nodes, to test how well the optimization could handle service delivery in a more sparsely populated setting.

3.3.1 Piloting An Optimization Method

The process for piloting the optimization of an on-demand transport system using the Christie Method involved the need for the current data of an existing system. Using the information for an existing service the number of nodes visited during a day modeled were required. Given the existing provider has date on the nodes as they were timed and dated by a vehicle, the characteristics of the vehicle travel patterns were found with confirmation from a representative. The service model of the existing provider defined constraints for the optimization method. The constrains included the maximum number of ridders per vehicle, maximum time in vehicle per rider, and service window restrictions. Due to the size and nature of the problem, if constraints were too tight, the solver would deem the problem infeasible. As a measure of avoiding recurrent model fails, constraints were started with large windows of feasibility for the model, before narrowing in on the defined level of service. To test the method for the increase in solver and active vehicle time, the first scenario to pilot the method was a 40-node TSP. The size of the problem is 40 nodes (1 depot and 39 pick up/drop offs). The nodes were based on the spatial data discovered in the postal codes and the depot was placed in a geographic hotspot with vacant parking for fleet, in this model. This solution was followed by a 2 VRP and the vehicles were added in succession until the desired level of service was reached. An in depth methodology for the optimization is detailed in Chapter 4.

3.3.2 Applying an optimization method

To test the ability to optimize a route or service with realistic passenger volumes in a different location, a smaller scale with fewer nodes was selected. The same methodology used for the Moncton city centre 40-node VRP and TSP models was applied to the Cap-Acadie regional model using the Christie Method. This method was expanded to cover a rural area, where trips were not calculated based on received data, but rather through the trip generation procedure. This procedure utilized data from the State of the Region Transportation Report (Michaud & Hanson, 2024) to estimate trip patterns in the region.

3.4 Summary

This research utilized data from Ability Transit, a non-profit paratransit service, which provided detailed log of trips traveled from January 2, 2023, to July 2, 2023. The data, stored in Excel workbooks, consisted of daily trip logs for each bus, detailing trip times, pick-up and drop-off locations (in postal code form), and bus number. The dataset included 14,567 records across 626 sheets for 178 days. The data cleaning process addressed issues such as blank cells, missing postal codes, and transcription errors. Transcription errors, such as incorrect time stamps, were corrected by reviewing the logical sequence of trip times. Blank cells were removed, and all individual bus sheets were merged into one primary sheet using Excel macros. Data validation involved confirming trip details with Ability Transit, including scenarios like multiple riders sharing a destination. The dataset was further updated by adding missing information (e.g., bus numbers, dates) and creating unique identifiers for trips. The data were cleaned using Excel functions, and pivot tables were used to identify and resolve errors. Derived data, such as pick up locations and trip counts, were created through logical statements to analyze the number of riders per trip. Geographic analysis involved linking postal codes to geographic coordinates, and census tracts using the PCCF. This allowed the creation of an origin-destination matrix to analyze travel patterns while protecting user privacy. The incorporated demographic data from census areas was used to calculate trip rates per capita and to distinguish urban from rural zones based on population density. Finally, the data were analyzed using ArcGIS software to develop an origin-destination matrix, which was then used to optimize vehicle routing in the Christie Constraint Mixed Integer Linear Programming (CMILP) model, as to be discussed in Chapter 4 of the research.

4 CHRISTIE METHOD OF CONSTRAINT MIXED INTEGER LINEAR PROGRAMING

4.1 Introduction

The Constraint Mixed Integer Linear programing method developed by Dr. James Christie determines an exact solution to both the vehicle routing problem and Traveling Salesperson Problem (Christie, 2018). This method is helpful in discovering the optimal delivery of on demand paratransit services with fleet sizing and depot location. The Christie Method is a constraint mixed integer linear programing method, used in the optimization of TSP and VRPs (Cartwright, 2024). The Christie Method uses a system of constraints and variable matrices to model concepts such as the TSP. The methodology of the Christie Method is to set the objective function to minimize the overall cost of the system (Christie, 2018). The cost of the system can be in terms of distance, minutes, or money it takes the agent to complete all trips and arrive back at the depot. Using the Christie method to solve VRPS and TSPs, it is then no longer NP-hard, and can be calculated in polynomial time in general, differentiating it from other VRP and TSP solution techniques (Cartwright, 2024). The Christie Method approaches the VRP as an assignment problem, including the necessary constraints and solves the problem using constraint mixed integer linear programming (Christie, 2018). The Christie Method was chosen for this work, due to the previous experience with this technique, and the lack of research on this technique in the area of paratransit. The goal of using this method is to access its feasibility as a tool in the planning optimization of on demand paratransit services.

4.2 Optimization Model

4.2.1 Objective Function

Before setting up the constraints for the optimization problem the objective cell and variable cells were determined. The objective cell of this optimization is the minimization of the total time it takes for all combined vehicles to complete their tours. A tour is the assigned route to the vehicle of nodes or drop off/ pick-ups. The objective cell is equal to the sum product of the cost matrix, that includes the length of time it takes to get from node to node, and the solution matrix. The cost matrix is described in 4.2.3. The solution matrix is a matrix comprised of all vehicle matrices. The basic objective function is shown below in Equation 5.

$$solution\ matrix\ \times cost\ matrix \qquad \text{Equation 3}$$

When time windows are applied to the TSP, the objective function is slightly different with the addition of another variable. A cost of delay variable is added to the objective function, minimizing the delay and variation from desired time window. The objective function for the traveling salesperson problem is shown below in Equation 4.

$$solution\ matrix\ \times cost\ matrix + time\ penalty \qquad \textit{Equation 4}$$

4.2.2 Variables

The variable cells for this model were the constraint service matrices and vehicle matrices. These matrices will be referred to as “variable cells” in this section and following sections. The variable cells for this model include each vehicle matrix, and service constraint matrix, a total of 31 matrices for the largest problem solved (6VRP), which all fed into the solution matrix. The variable cell matrices were required to be renamed to ensure their names were below the character limit of OpenSolver input box. The variable cells were given names of either “CM” for constraint matrices or “Vehicle” for the vehicle matrices. The size of the model needs to be identified in terms of the number of nodes to be served by the “vehicle”, though this needs to be balanced with the anticipated computational power available.

4.2.3 Cost Matrix

The cost matrix is the distance to and from all points within the model. This assigns a cost to travel from one point to another to fulfill the tour required by the TSP or VRP. The cost matrix can be in units of time, distance, or money to assign the cost of travel from one node to another. There were multiple steps required to create the cost matrix; this can be done manually.

4.2.4 Constraints

Constraints define the feasible region of the problem. All the constrains must be met to find the exact solution in the Christie Method. Based on the size of the model, the number constraints that are required vary. There are three types of constraints in this model: binary constraints, inequalities, and equalities. The constraints mold the model to fit the desired characteristics of the Christie Method and the Ability Transit paratransit service.

4.2.5 Convex Hulls

In order to minimize the computational load a of a VRP with the Christie Method, convex hulls were used. A convex hull is defined as the smallest convex polygon such that any point, within a finite collection of points, either within or at its boundary and no internal angle of a convex polygon can exceed 180 degrees (Sehta & Thakar, 2023). Reducing constraint matrices through convex hulls and variable matrices can help speed up solutions (Christie, 2018). The convex hulls are used in the algorithm to reduce the computational load because the convex hulls by definition cannot cross each other. To complete a tour starting at the depot node the vehicle must pass through the inner convex hulls to get to the outer convex hull. So, from one point within the inner convex hull to one point in another convex hull, you must pass through every other convex hull. This means that if every other convex hull within the set of points has an applied constraint, every node within the set of points has either indirect or direct constraint applied. Convex hull families are helpful for this reason to reduce the number of constraint matrices within the Christie Method of Constraint Mixed Integer Linear Programming further reducing the computational load of the optimization.

4.2.6 Time Window Constraints

To add another a layer of complexity to the model, time windows were added to the 40 node TSP and Cap-Acadie model. Time windows create a model that simulates the ordering of trips based on pick ups. The next step was to assign the time each passenger would like to be picked up by the transportation service. Applying the time windows changes the way the TSP sets the order for pick-up of individuals using the service. The simulated time for pick ups was based on the frequency and the duration determined from the descriptive statistics, with data received from Ability Transit. The constraints are applied to node j as the time window for customer. In the expression shown below it states that the time an individual is picked up by the service must be within the acceptable bounds.

$$j: [T_{j\_min}, Tj_{\_max}] \Rightarrow T_{j\_min} \leq t_j \leq T_{j\_max} \qquad \text{Equation 5}$$

The maximum time they can be picked up early is 15 minutes, and the minimum time they can be picked up late is zero. The time must be calculated as the time between the nodes of the origin and destination and the current clock time of the vehicles routing problem. This constraint involved adding an array that finds the travel time from the past node to the current node. The travel time from origin to destination can be determined by using an “IF” statement, of if node is visited then add the time from previous node to current node to time from current node to past node. The travel time of the vehicle would then have a constraint applied as shown below.

$$t_j \geq t_i + d_{ij} \times x_{ij} \qquad \text{Equation 6}$$

Using the sum product from each row of the vehicle solution matrices and the cost matrix the time from node to node was calculated if it is visited by the specific vehicle.

The addition of time windows alters the transportation planning focus of the thesis. The addition changes the model from a transportation service routing problem to an assignment problem. The order of passenger pick up is no longer the optimization process. The split of passengers per vehicles to ensure they arrive at their destination on time and efficiently is the new optimization focus. The current time of the problem is kept with a sum tracking formula. Where the time it takes to travel from previous node to current node will have a constraint applied. The equation 7 shows the time difference constraint of time desired (t_d) and time actual (t_a) must be less than 15. Using an inverse matrix of the solution matrix the previous stops were found which were then linked to t_{ia}. The fifteen in Equation 8 represents the 15-minute buffer, that the individual could be picked before their desired pick up time. The total time between the desired time and arrived time was summed and added to the objective function.

$$tja \leq tia + (tjd - tia - dij \times xij) \quad \text{Equation 7}$$

$$t_d - t_a \leq 15 \quad \text{Equation 8}$$

Using this above equation and constraint, reduces the need to have an integrated clock as this calculates the time difference between steps within the constraint.

4.3 Application of The Christie Method

The first step, to complete the Christie Method of constraint mixed integer linear programming, is to understand the size of the problem that needs to be optimized. The base problem size tested was a 40-node model. The optimization is based off one day of service of an on-demand transportation service. The size of the problem was based on the average trip characteristics found using descriptive statistics previously described in Chapter 3.3. The model was developed with synthesized travel pattern data, which was then calibrated using the data from Ability Transit. This synthesized data was to be optimized using an algorithm to determine the most efficient routing.

After the size of the problems was determined, by the Ability Transit trip characteristics, the points to be used in the Christie Method were determined by synthesizing them based on the Ability Transit data points. The model's pick up and drop off nodes' coordinates were found using the previously discussed PCCF by DMTI. Once the coordinates could be placed on a map the cost matrix could be calculated. The size of all matrices in this model for the VRP was based on one average day of travel by the on-demand vehicle service provided by Ability Transit.

4.3.1 Prerequisites for Model Setup

4.3.1.1 Route Data

Before being able to find the cost matrix the road network data for the area was required. The route data in New Brunswick is publicly available on GeoNB as a shape file (Government of New Brunswick, 2023a). Using the shape file, a GIS project was created using ArcGIS. The road data shape file was overlaid with the SERSC boundary shape file. This shape file was also found on GeoNB. The regional service commission shape file was used to split the road network shape file and find the boundary of scope in ArcGIS Pro. Once the road network was split to include only the road network within the Southeastern Regional Service Commission. The Figure 6 below shows the extent of the service area road network.

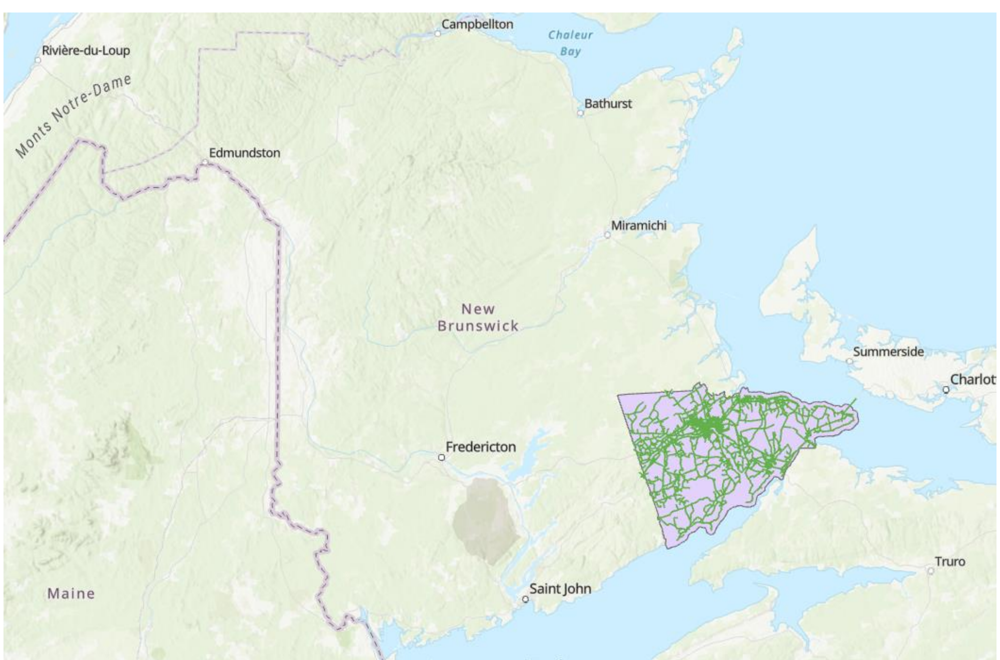

Figure 6. Scope of Research Service Area Road Network

After the road network was imported into the ArcGIS file the postal code shape file was imported. This file came from the Statistics Canada 2021 census boundary file (Government of Canada, 2021). The shape file included Forward Sortation Area data. The Forward Sortation Area is the first three characters of the postal code, designating a postal delivery area within Canada (Statistics Canada, 2021). With all the publicly available shape files in ArcGIS the necessary steps for the cost matrix could be started.

4.3.1.2 ArcGIS

The cost matrix was created using ArcGIS Pro. There were inputs required to the cost matrix such as the road files to calculated travel time, or distance and the coordinated of the postal codes to create the drop off/pick up nodes. GeoNB provides geographic files of New Brunswick such as geodatabases, layer packages, and shape files, such as the New Brunswick Road Network (NBRN). The NBRN is the official source for road data in New Brunswick. The NBRN includes road centerlines, road names, road class, surface type, address ranges and other road attributes. In order to create a network dataset, the ESRI File Geodatabase of the NBRN was required. Once the file was downloaded the file could be turned into a network dataset by using the network analyst function within ArcGIS Pro. This function helped to convert the ESRI File Geodatabase to a network dataset. This network dataset was crucial to providing the travel time and distance over the road network, for use in the cost matrix in future steps.

All postal codes used in this model were postal codes of trip start and end locations in Ability Transit's travel logs. An ArcGIS file was created with the coordinates obtained from the postal code conversion file (PCCF). Using ArcGIS Pro, the network dataset and the latitude and longitude coordinates were inserted into a map. The postal code coordinates made a point layer to act as nodes for the origins and destinations of the cost matrix.

The origin and destination cost matrix was created by running a layer analysis of a cost matrix in ArcGIS. The road connection for this cost matrix used the NBRN layer file provided by GeoNB. The NBRN layer in ArcGIS acted as the links and with the PCCF point layer acting as the nodes of origins and destinations. The origin and destination coordinates used as points for the postal codes were connected to the road layer using the nearest street to the location of the coordinate point. This is a setting that can be applied in the layer analysis in ArcGIS. The cost matrix was provided in a table of point-to-point travel time and distance. This allowed for the creation of a cost matrix. The cost matrix is the cost, distance, or time, from all nodes within the model. The Figure 23 of the point-to-point links, used for the cost matrix, is shown in Chapter 5.

The cost matrix was used to find the distance and time from each origin to destination pair in the model. This information was not only crucial for the model but was helpful to define the average trip length both in distance and in time. The cumulative distribution of the trips was calculated using a normal distribution function in Excel for the length of trips taken using the Ability Transit on demand service.

4.3.1.3 Excel Software

When setting up the constraint mixed integer programing-based Christie Method in Excel, the capacity of Excel with the number of TAZs was explored to ensure the size and number of matrices was allowed. It was discovered that Excels row capacity limit is 1,048,576 rows, and a column limit of 16,348 columns. The row limit exists because desktop Excel runs locally on a user's computer and is limited to the memory and compute resources on the computer (Microsoft, 2024). Each matrix within the Excel spread sheet will be the same size as the solution matrix, which has all the zones as both origins and destinations in order to do matrix multiplication for the Christie Method. For that reason, the Excel capacity was checked, to ensure the software did not impose another constraint on the VRP.

4.3.1.4 Solvers

The OpenSolver software was chosen because it is readily available to the public and can solve mixed integer linear programming problems. OpenSolver can solve problems with a large number of variables compared to the built in Excel solver add-in. Specifically, within OpenSolver the CBC solver was initially used in the piloting of this method with a small model. The CBC solver is an open-source linear and mixed-integer programming solver actively developed by COIN-OR (OpenSolver, 2023). The

CBC solver was initially used instead of the Gurobi solver due to downloading limitations on computers at the University of New Brunswick (UNB). OpenSolver was the add-in used on the recommendation of Dr. James Christie. OpenSolver is easily accessible to the public, so the method can be easily replicated.

In pilot testing, the CBC solver took 30 hours to complete the optimization of a 40-node TSP model (the simplest problem), therefore the Gurobi solver engine was used instead. Gurobi version 10.0.3 was used for this research for its compatibility with OpenSolver in Excel. Both the CBC and Gurobi solver engines use a Branch and Cut approach. The 'Branch and Cut' is a method used in operations research to solve integer programming problems efficiently by combining branch-and-bound with the cutting plane method to guarantee optimality (J. Zhang et al., 2023). Having the software accessible to the public was important in the early stages of this research to reduce barriers so the method could be used for the DRT community for transportation planning in the future.

4.3.2 Application of The Christie Method in OpenSolver

Excel allows visualization of the model and the constraints with the OpenSolver add-in. Building the model requires adding matrices within Excel to act as variables and constraints in the Christie method. There is no specific format matrices must be laid out within the Excel sheet, but an example of the method used for this research is attached in Appendix C. The set up of the objective, variables and constraints are described in the following sections.

4.3.2.1 Objectives

The cost matrix was added to the Excel sheet, with the cost to travel from all the respective nodes. The solution matrix was setup near the cost matrix in the Excel sheet because the sum product of the solution matrix and the cost matrix are the objective function of this model. The cell with the sum product of the total active vehicle time was set as the objective function in the OpenSolver dialogue box. There are three options for the objective cell in OpenSolver minimize, maximize, or set to an exact value. In this case the objective was to minimize the total cost of the system.

4.3.2.2 Variables

All the variables were added into the dialogue box for OpenSolver. The variables required for the Christie Method are the service constraint matrices, and vehicle solution matrices for the VRP. When applying the Christie Method to a TSP, the variables are the service constraint matrices and the solution matrix. Based on the size of the model, the number of variables shifted.

4.3.2.3 Constraints

The first constraints set in Open Solver were to ensure that all the vehicle solution matrices were binary. The matrices must be in binary because one signifies the node is visited by that vehicle, and 0 means the node was not visited by the vehicle. The binary constraint was not applied to the service constraint matrices to reduce the number of constraints, even though they need to be binary. Instead, the binary constraint was applied to the vehicle solution matrices as the service constraint matrices fed into the vehicle solution matrices, automatically requiring them to be binary to fit within the constraints. The solution matrix had a constraint that they must be greater than or equal to each applied constraint matrices A through Y.

The solution matrix had a column that sums all rows and a constraint column. A constraint was created that the sum of the solution matrix's rows must equal the constraint column. The same was done for the columns of the solution matrix, where a row was created that summed all the columns, that must be equal to the constraint row.

The highlighted node ID cells represented the depot in the solution matrices shown in Appendix C, where the constraint is equal to the number of vehicles, instead of one, like all other cells. When setting up a TSP problem, the constraint for the depot would be equal to one because there would be one vehicle servicing the area. All constraints within the open solver software are assigned a colour and are highlighted within the spreadsheet. When two matrices are connected by an expression, in this case a less than or equal to expression there are coloured connectors showing the matrices involved.

After the constraints were created for the solution matrix, similar constraints were created for the row and column constraints for each service constraint matrix. The constraints created for the service matrices include routing the vehicle from the depot to the node within the convex hull. This was done by placing a two in the row constraint for the node associated with the service matrix, and a 2 to in column constraint for the depot node. A simplified version showing the way the constraints are applied to a smaller matrix are shown below in the Figure 7. In the Figure 7 below node five is the depot and this is a service constraint matrix for node 3 in the problem. Beginning from node 5 to Node 3 to 4 to 1 to 2 and then back to 5 to ensure the vehicles follow the tour.

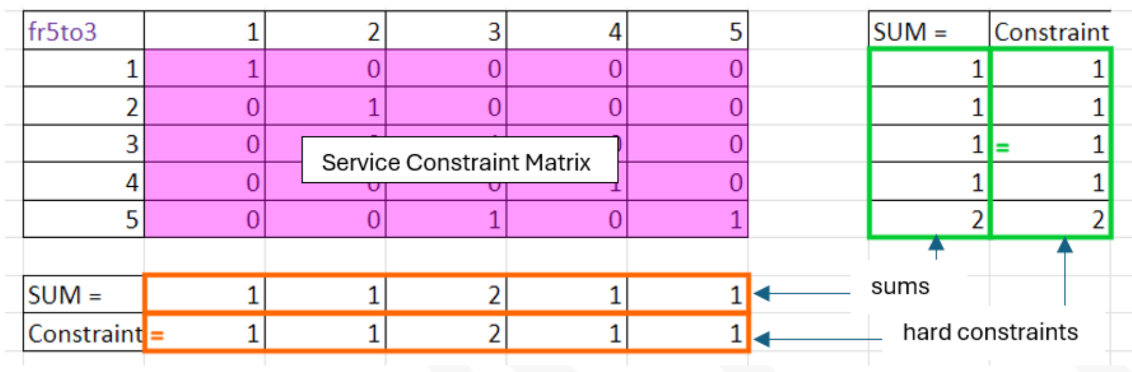

Figure 7. Service Matrix Constraints

The subtour elimination matrix, which is the addition of all vehicle matrices, had a constraint applied, that all its contents must be less than our equal to the number of vehicles. This constraint was applied because the constant is equal to the number of vehicles, and it could not be exceeded by the sum of the service constraint matrices. Subtour elimination constraints were added to both the TSP and VRP.

The connectivity matrix had a constraint applied to it of all contents must be lesser than or equal to one. The connectivity matrix is the solution matrix multiplied by the transposed solution matrix. If cells were to equal one, the route would not be connected to the tour of a vehicle.

The level of service of riders depends on the length of time they were in the vehicle; a time limit reflective of the descriptive statistics received from analyzing the Ability Transit data. The time period of 30 minutes from the pick up of the first person to their drop off was chosen as a constraint in the MILP, after consultation with an Ability Transit representative. This was applied to each vehicle solution matrix, summing the product of the vehicle solution matrix and the cost matrix.

After all the service constraint matrices listed above in Chapter 4.2.5 were created the rest of the constraint matrices were created in Excel. For each service constraint matrix created for a point in a convex hull, an applied constraint matrix must be created. An applied constraint matrix was the multiplication of the service constraint matrix by the multiplication matrix. The multiplication matrix was a matrix filled with ones except for zeros on the diagonal of the matrix. The applied constraint matrix was created to ensure the agent does not take the path of least resistance by only traveling in the diagonal of the solution matrix, which does not connect any nodes to each other. A transposed solution matrix was also created. A contact matrix was created by adding the solution matrix to the transposed solution matrix the constraints and ensuring all cells are equal or less than one. Finally, the vehicle solution matrices were created and filled with zero before the constraints were added with open solver. A portion of a vehicle matrix is shown below in Figure 8.All the solution matrices are available in Appendix C.

Vehicle B		1	2	3	4	5	6	7	8	9	10	11	12	13	14
							E1A2V1								
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	0	0	0	0	1	0	0	0	0	0	0	0
4		0	0	1	0	0	0	0	0	0	0	0	0	0	0
5		0	0	0	0	0	0	0	1	0	0	0	0	0	0
6	E1A2V1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
7		0	0	0	0	0	1	0	0	0	0	0	0	0	0
8		0	0	0	0	0	0	0	0	1	0	0	0	0	0
9		0	0	0	0	0	0	0	0	0	0	0	0	0	0
10		0	0	0	0	0	0	0	0	0	0	0	0	0	1
11		0	0	0	0	0	0	0	0	0	0	0	0	0	0
12		0	0	0	0	0	0	0	0	0	0	0	0	0	0
13		0	0	0	0	0	0	0	0	0	0	0	0	0	0
14		0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 8. Portion of Vehicle B Solution Matrix

Each vehicle solution matrix had row and column constraints applied to them, similar to the solution matrix. The vehicle solution matrices had constraints that were the sum of the rows must be less than or equal to the set column constraint. The column constraint was filled with ones. The row constraint was also an array filled with ones. The vehicle solution matrices had an additional constraint that the sum of the columns must be equal to the transposed sum of the rows. This additional constraint ensured all nodes within the individual vehicles were connected tours.

After the vehicle matrices were created and the sum of all rows and columns for each respective matrix were summed for the vehicle capacity. The spreadsheet was prepared to add the Open Solver constraints to the previously mentioned matrices.

To apply the time window constraints, new arrays were formed with constraints. Constraints mentioned in 4.2.6 were applied. In order to apply them the current, previous, and future node were required. The previous node visited, current node visited, and next node visited were found using XLOOKUP function in Excel. This allowed the order of nodes visited to be automatically updated as the solver works through solution options, creating a once dynamic process into a dynamic one. Each stop has a requested time for pick up (td), the time the previous stop + travel time from previous node to current node must be less than the time of the next stop or else it is assigned a 1. There is an additional constraint added that the sum of the column must be equal to 0, requiring OpenSolver to implement the desired time windows in the routing of a vehicle. Because the time format in Excel is nonlinear the time windows were applied in decimal formal of the 24 hours in a day. The decimal format of the 15-minute buffer was a quarter of an hour. The time windows were manual checked as well for the time in the TSP.

4.4 Adjusting the Model

The best way to start the optimization of an area is with a TSP optimization model, as it is the most basic set up. Once the TSP successfully finds the optimal solution, adjustments can be made. Once the model has been set with a TSP, changing the number of vehicles in the model the following steps need to be followed. The hard row and column constraints of the solution matrix of the depot node must be changed to be equal to the number of vehicles in the problem. An example of the hard row and column constraints for the solution matrix are shown in Figure 7. Adjusting the model with fleet meant the additional vehicle solution matrices must be added to the Excel spreadsheet. Once added to the spreadsheet, the additional matrix or matrices must be added to the list of variables and assigned a binary constraint. The new vehicle must be added to the solution matrix. If progressing from a TSP to a 2VRP, the two vehicle solution matrices must be added together to form the solution matrix.

If adding additional vehicles from a 2VRP or more the solution matrix must include the addition of all vehicle solution matrices. The constraints specific to the vehicle solution matrices must be added to the additional vehicle solution matrices.

If the location of the depot is to be changed from the existing to a new node within the model, the steps vary depending on the problem. If solving a TSP and changing the location of the depot the solution matrix stays the same, but the location of the two in column orientation constraint array for the service constraint matrices changes. If changing the location of the depot in a VRP, the number of vehicles in the row and column array constraints must move from the original depot to the new depot. The old location for the depot in the column and row constraint array for the solution matrix must be changed to a value of one. The vehicle solution matrices stay unaltered as they do not have a constraint directly applied to the matrix to define the depot, it is instead applied through the solution matrix.

4.5 Rural Application

The Cap-Acadie Region was chosen to demonstrate the Christie Method based on the demand for accessible transportation developed from Ability Transit trip rates. Previous analysis of regional travel behaviour for the SERSC by a company called "Civilia" (profiled by Hanson, et al., 2025) provided an estimate of the proportion of trips originating within Cap-Acadie and destined elsewhere. The coordinates were found for three locations within the Cap-Acadie Region, one within the Moncton

Region and one within the Shediac Region. The coordinates were then input to ArcGIS as a point layer. Because the work to create a network layer for New Brunswick was completed previously, it streamlined the process of the cost matrix in ArcGIS Pro for this problem. Once the point layer was created the origin and destination cost matrix analysis using ArcGIS Pro was run. The cost matrix was significantly smaller on a regional scale than the original Ability Transit postal code scale. The cost matrix was used to find the time from each of these points, as an initial step in the Christie Method.

In order to size model and place the nodes for the cost matrix the trip percentage was needed before creating the cost matrix. The trips allocated by percentage for origin and destinations for Cap-Acadie were estimated for the 13 areas within the SERSC boundaries (Civilia Report). The trip rates based on Ability Transit data for per capita trips estimate demand for four accessible trips per day in the Cap-Acadie Region, this is shown in Figure 34. SERSC On Demand Paratransit Origin and Destination Matrix. The locations chosen based on the trip generations by region is shown in Figure 10 below.

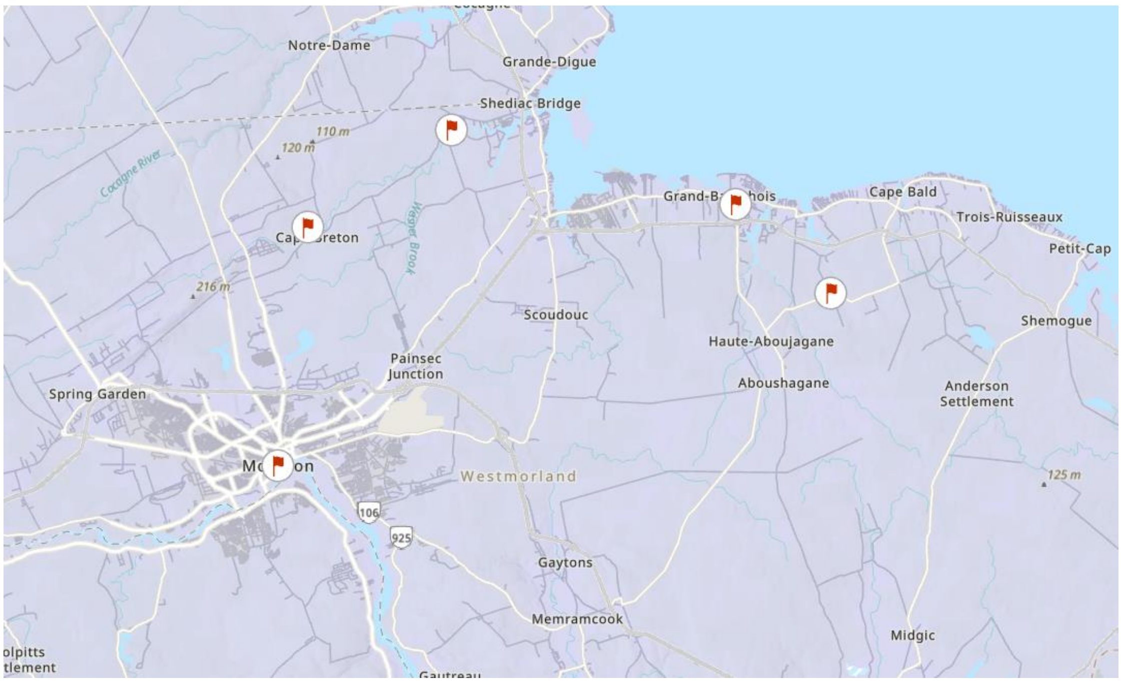

Figure 9. Cap-Acadie Model Nodes

Once the cost matrix was created, the TSP was set up within Excel using the same model set up as previously discussed in Chapter 4.2. Following the same method using the Christie Method, and the OpenSolver add-in in Excel the problem was set up in a new Excel sheet. The Cap-Acadie model is significantly smaller than the model using the Ability Transit data with only five nodes, within the area (1 depot, 4 pick up/drop offs). The depot for the problem was set in node Cap-Acadie 3. The results of using this method for regional modeling of Cap-Acadie are shown in the Results 5.3.1.

4.6 Summary

The Christie Method is a Constraint Mixed Integer Linear Programming (CMILP) technique developed to optimize solutions for the Vehicle Routing Problem (VRP) and

Traveling Salesperson Problem (TSP). It minimizes the overall system cost, whether in terms of time, distance, or money by using variable matrices and a system of constraints, transforming these typically NP-hard problems into solvable polynomial-time problems. The model's objective is to minimize travel cost for vehicles completing their routes while ensuring constraints like time windows, and vehicle capacity are met. The method employs convex hulls to reduce computational complexity by limiting the number of constraints and focusing on feasible of routes. After solving a basic TSP, the model can be adjusted to accommodate additional vehicles, modifying depot node constraints, and adding new vehicle solution matrices. Changes to the depot location also require updates to both the solution and vehicle matrices, depending on whether the model is a TSP or VRP.

The Cap-Acadie region was modeled using this method, with key locations input into ArcGIS to generate a cost matrix for regional travel. Based on population data, four trips per day were assumed, creating a smaller model compared to the Ability Transit data. The depot was placed in Cap-Acadie, and the model was solved similarly, with results applied to regional optimization. In summary, the Christie Method uses matrices, constraints, and an objective function in Excel with OpenSolver to optimize vehicle routes, and fleet for different geographic areas, such as the Moncton Region and the Cap-Acadie region.

5 RESULTS

This chapter begins by presenting summary descriptive statistics for Ability Transit, which are then used to inform the development of a service optimization model designed to replicate the existing service. Finally, the trip rates and routing methodology are applied to a use case for a community in the Southeast Regional Service Commission in New Brunswick.

5.1 Ability Transit Descriptive Statistics

Ability Transit provided data on 11,338 passenger trips during the period of January 2nd, 2023, to July 2nd, 2023 (6 months). Using the ArcGIS Pro network analyzer tool, a cost matrix was created for the 393 geographic centroids of postal codes associated with Ability Transit areas served. Of the 14,000 postal codes entries from Ability Transit, invalid postal codes account for 0.5% of the total trips over the 6-month period of data received from Ability Transit. There are 160,000 links in total so at large scale it is hard to identify link from link. The Figure 10 below, shows all the points and their links for the cost matrix.

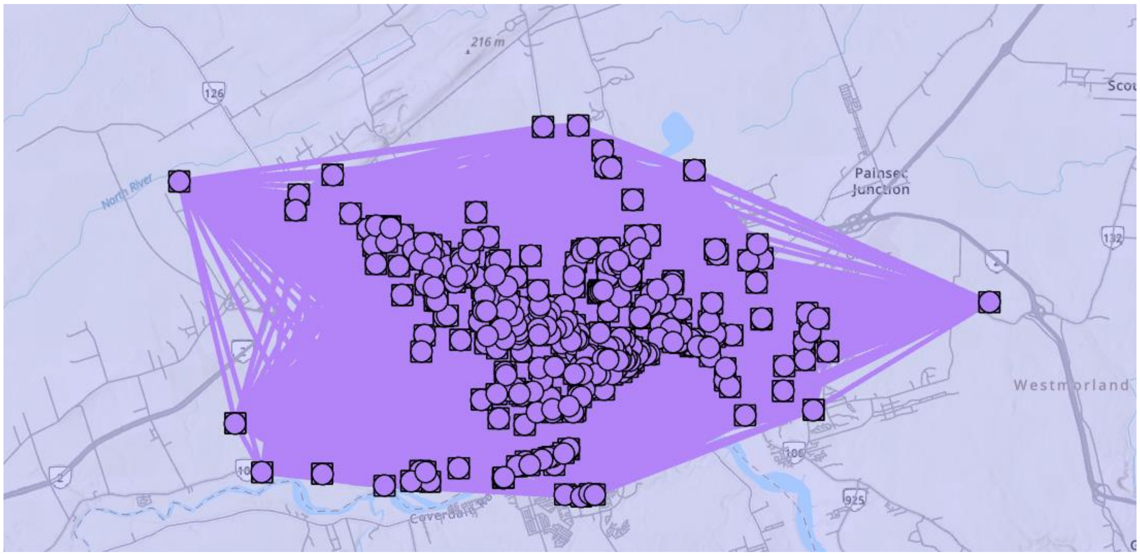

Figure 10. 400 Point Cost Matrix Layer

5.1.1 Vehicle Operational Data

The data in Table 5 summarizes vehicle operational data for Ability Transit's six vehicles over a 6-month period. Vehicle metrics include total stops, percentage of all stops, passenger trips (one-way), average daily travel time (mins), maximum daily travel time, total service time, total passenger travel time per bus, average daily km, maximum daily km, total passenger km, total vehicle.

Table 5. Vehicle Service Metrics (6-month totals and averages)

	Vehicle						
	1	2	3	4	Eve.	Eve. 2	System Total
Total stops	3,054	2,127	2,368	670	1,508	8	9,726
% of all stops	31%	22%	24%	7%	16%	0%	100%
Pax. trips (one-way)	3548	2,472	2,751	778	1,752	9	11,338
Average Daily Travel Time (mins)	175	125	136	36	87	1	558
Maximum Daily Travel Time (mins)	275	241	268	187	190	92	1,252
Total Service Time (mins)	31208	21,743	24,198	6,843	15,410	79	84,143
Total Pax Travel Time Per Bus (mins)	31072	22,166	24,178	6,454	15,441	92	99,402
Average daily distance (km)	259	181	201	53	127	1	822
Maximum daily distance (km)	366	327	368	292	293	147	1,225
Total Pax Distance (km)	46166	32,165	35,796	9,353	22,622	147	147,048
Total Vehicle Distance (Km)	39818	27,742	30,873	8,731	19,662	101	126,826

The most frequent trip-maker is Vehicle #1. Vehicle #1 is in-service everyday of operation, whereas there is more likely to be 3 vehicles fulfilling trips than 2 or 4 vehicles during the day.

5.1.2 Passenger Trip Characteristics

The data in Table 6 below details the metrics of passenger trip characteristics of Ability Transit.

Table 6. Passenger Service Metrics

Passenger Service Metrics	**Avg.**	**Max**	**Min**	**Total**
Passenger trips (one-way, per day)	62	97	7	11338
Passenger trips (weekday, one-way, per day)	72	97	40	9606
Passenger trips (weekend, one-way, per day)	34	45	7	1732
Travel distance per passenger trip (km)	13.3	27.6	1.7	147048
Travel time per passenger trip (mins)	9	57.4	2	99676
Vehicle stops	55	96	7	9726
Passenger trips per vehicle stop	1.2	5	1	

Using the cost matrix from ArcGIS Pro, the distance was calculated for each trip between origin and destination postal code and the frequency distribution of trips by travel distance is shown in Figure 11.

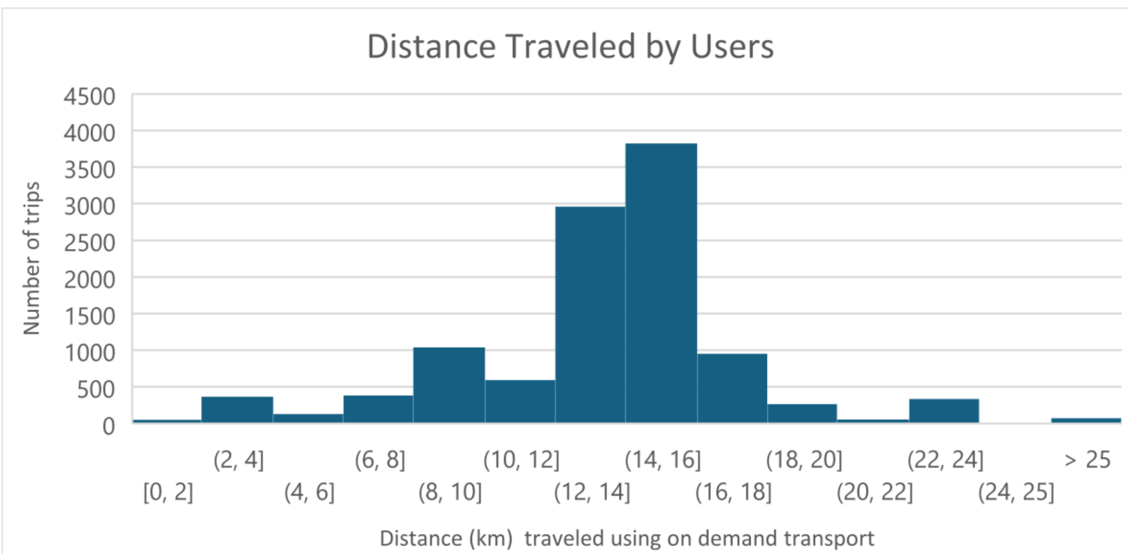

Figure 11. Frequency Distribution of Travel Distances

The Figure 11 above shows that the trips follow a normal distribution with most frequently observed trip distance of 14 to 16 kilometres. The cumulative probability of a trips distance traveled one way was also found, shown below in Figure 12. The chart shows that 99% of trips made with the service travel less than 23 km one way.

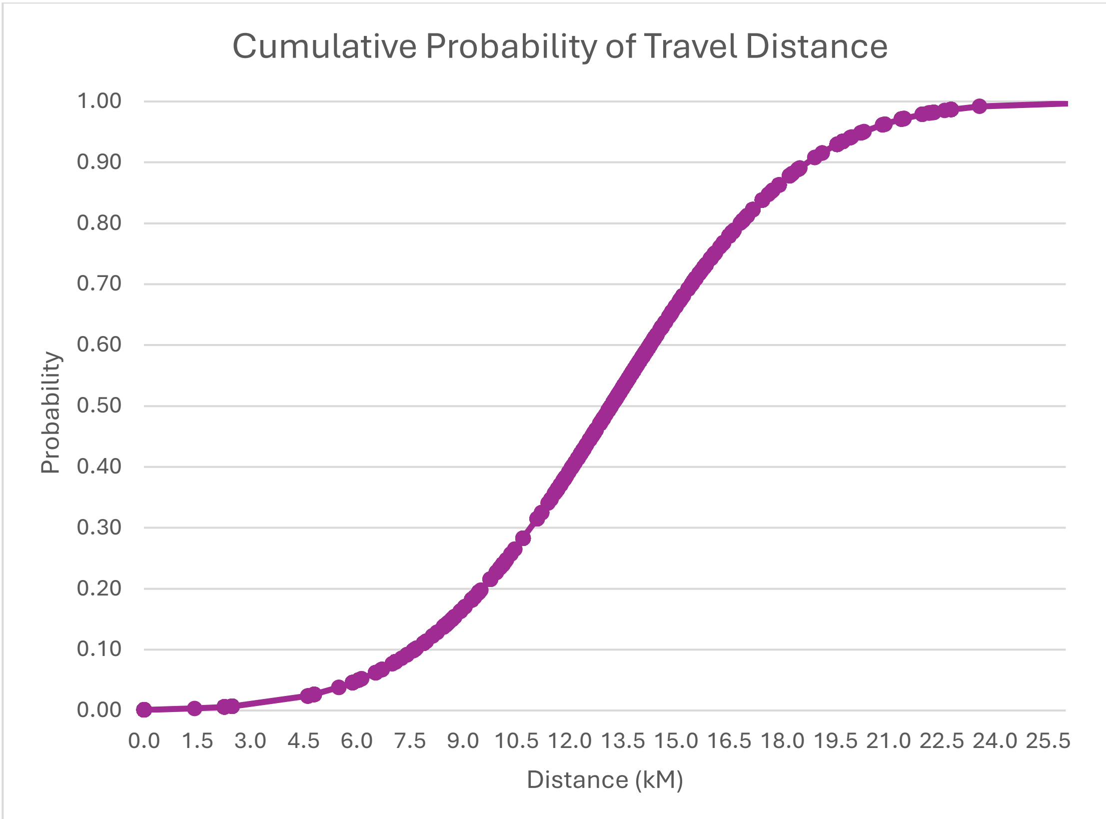

Figure 12. Cumulative Probability of a Riders Travel Distance

Table 7 shown below describes the operational behaviour of the service from Ability Transits scheduling operations. These characteristics were found using the logic check described in Chapter 3.1.2. These characteristics were taken from all trips; however, the table does not show all the trip characteristics, only unique trip characteristics. These characteristics were confirmed with an Ability Transit representative.

Table 7. Unique Passenger Trip Characteristics

Unique Characteristic	Number of Trips	Percentage of Trips
Same Time Same Start Location	1289	11%
Same Time Different Start Location	570	5%
Same Time, Different Start Location, Same End Location	448	4%
Destination Of One Stop Is the Origin of The Next Stop	411	4%
Different Start Time, Same Destination	817	7%

Although these characteristics do not represent all trips, they are helpful to classify operational behaviors of the on-demand service. For example, the “Same Time Different Start Location” shows how many times Ability Transit was able to fit someone additional into the already scheduled service in the six-month period of data. The statistics from the trip characteristics were used to inform the creation of the model and location of trip requests.

The longest trips averaged 30 mins from the start of a person’s pick up to their drop off, this was then confirmed with a representative from Ability Transit. The travel time data showed a distribution similar to the travel distance data. Figure 14 shown below shows the distribution of trip times in histogram format by bin sizes of 5 minutes, the same format as the graph above. It is important to not there are three 45–50-minute trips that are difficult to see on the graph.

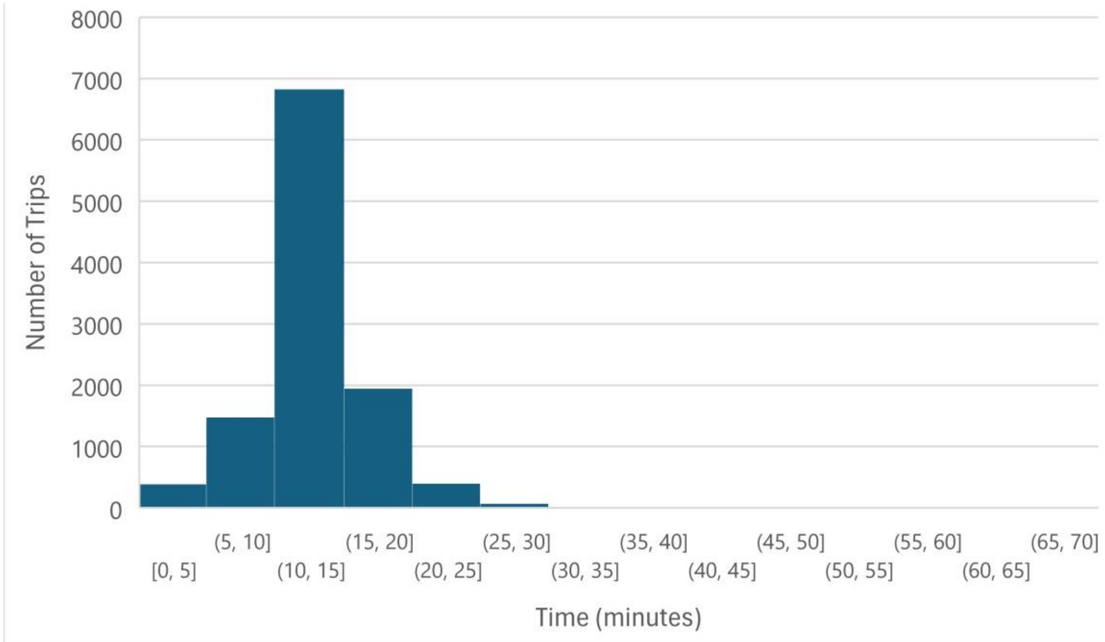

Figure 13. Frequency of Travel Time of Users by Trip

The cumulative probability of rider's trip time is shown below in Figure 15. The cumulative probability function (CPF) shows the probability of a trip being under 30 minutes for a rider is 99%.

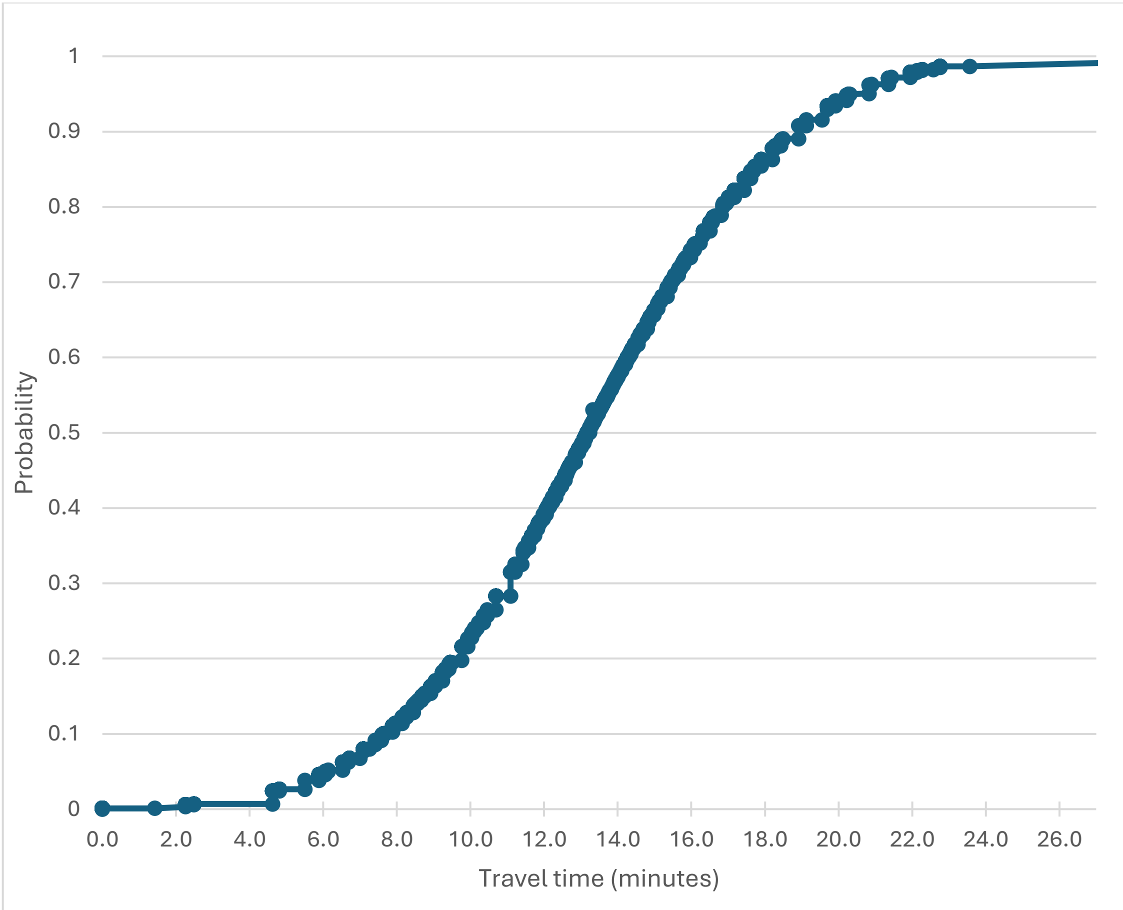

Figure 14. CPF of Travel Time

Because this paratransit service is demand responsive, the times of trips shown below are what were recorded in the Ability Transit system. The demand responsive service observed a strong peak of trips taken at both 11:00 am and 3:00 pm, a distribution of the pick up times along with the number of times it was requested and confirmed is shown below in Figure 15.

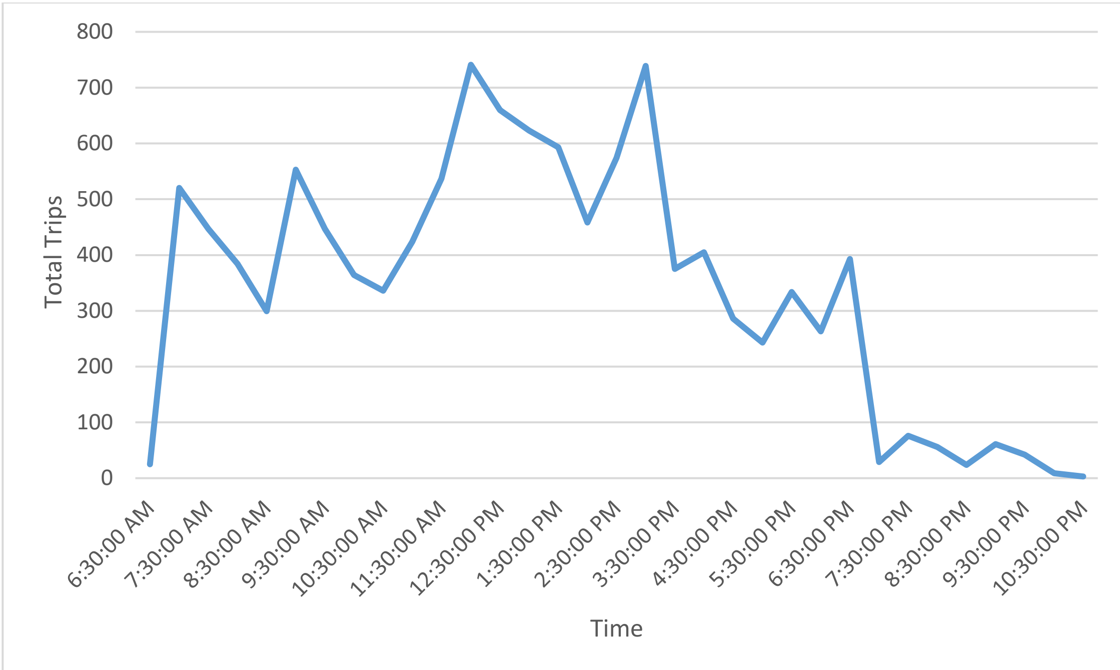

Figure 15. Trip Starts by Time of Day

The trip rates were calculated on average for the days the service was active during the six-month period of data collection. The data collected showed a large discrepancy in the hours the service provided trips. On average there were 5.6 trip requests completed per hour per day by Ability Transit. The average trip rate over the days operational in the six-month period from January 2 to July 2 was sixty-four trips per day. If the service were offered for 8 hours a day on average over the six-month period, the trip rate would be 8 trips/hour/day, however the service hours varied from 6 hours to 13 hours.

From the descriptive statistics of the cleaned data from Ability Transit, Tuesdays were the highest travel days over the six-month period and Sundays were the lowest as shown in the graph below (Figure 17).

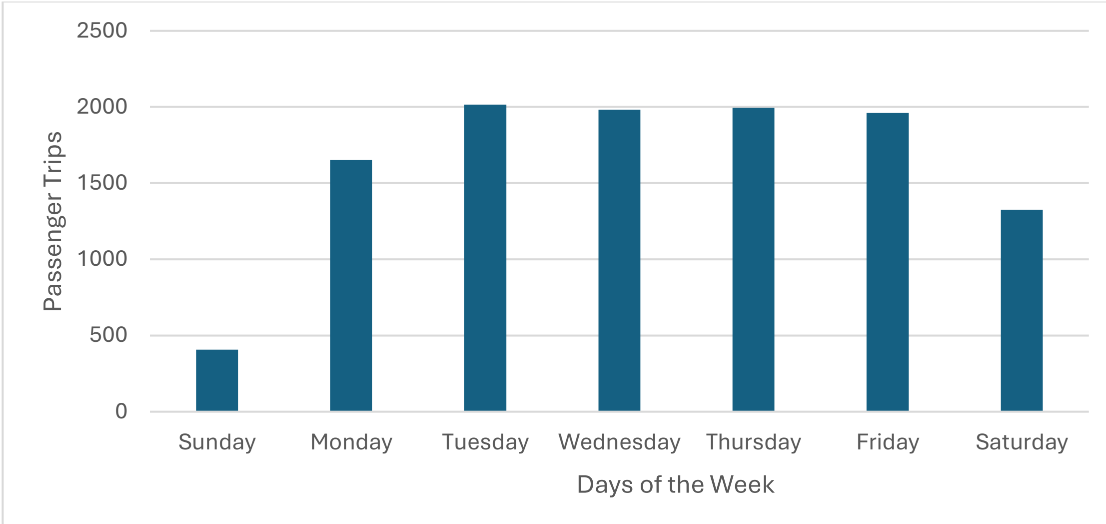

Figure 16. Total Trips by Day of Week

March 2023 was the busiest month of travel with 2084 trips taken by Ability Transit. Because the data were collected for only one day in July it is not representative of the entire month of travel, and therefore excluded from monthly totals. Figure 18 below shows the number of passenger trips per month over the 6-month period of data collected.

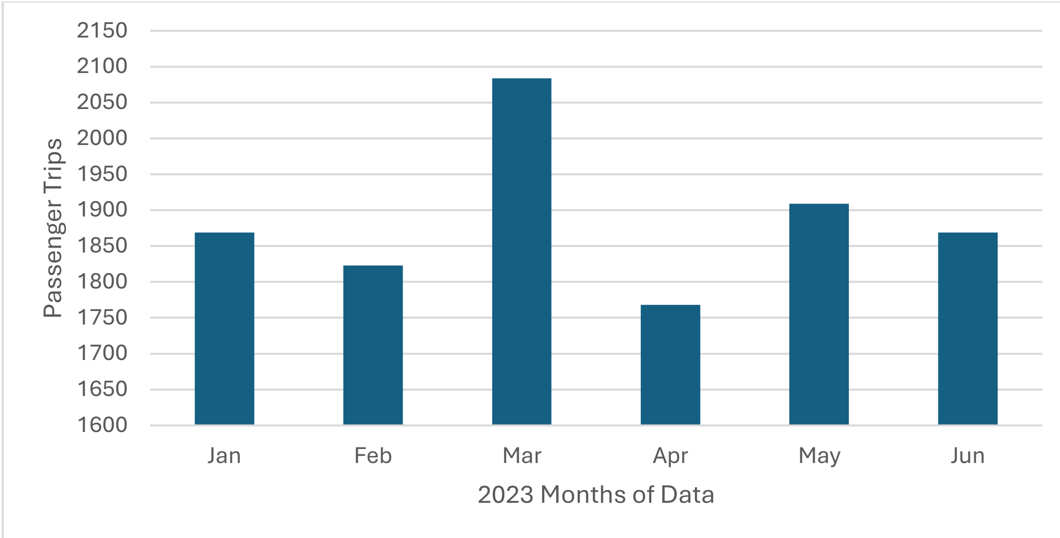

Figure 17. Passenger Trips by Month

5.1.3 <u>Geographic Analysis</u>

The use of postal codes as the location marker for trips allowed for geographic analysis while protecting the privacy of users. Using the geographic data attached to postal codes the trips per origin and destination pair were calculated and summarised in the table below. The trip origin and destination pairs fewer than six trips were suppressed for user privacy.

Table 8. Number of Trips by Origin and Destination

Pairs	6-10	11-25	26-100	100+
Number of Origin Trips by Postal Codes	90	45	68	15
Number of Destination Trips by Postal Codes	39	48	64	21

This Dissemination Areas with the most origins by postal code were 13070068, 13070044 and 13070059. The Dissemination Areas with the most destinations by postal code were also,13070068, 13070044 and 13070059 with the addition of 13070078.

The spatial differences in the trips are apparent when breaking down the location of the postal codes. The destinations of trips taken with paratransit are shown on a large scale below as well. A heat map of the locations visited by Ability Transit is shown below in Figure 18. The light blue spots show a greater intensity of trips to these locations. The light blue areas represent a higher number of trips to this location. The dark blue areas represent a lower number of trips. The legend in the top corner of the heatmap shows the scale of trips to areas.

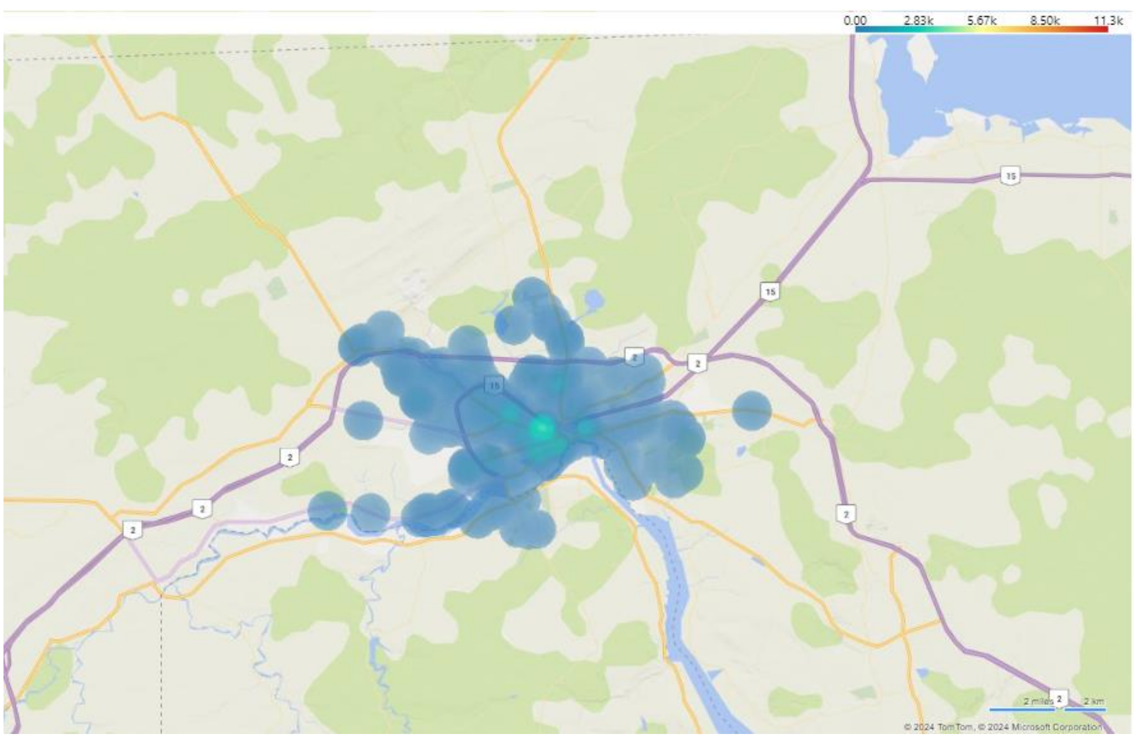

Figure 18. Heatmap of Destinations in Moncton

The trips are centered around the urban centre of Moncton, NB. A total of 86% of trips where either the origin or the destination was to an urban area over the six-month period. Ability Transit provides trips for individuals around the Moncton region, which is why the heat map shows a high density of trips in the city center.

Using Power BI and ArcGIS a map showing the areas with the most trip requests by points. The larger points shown represent recurring trips to the same postal code location. Whereas the smaller points represent the trips with the less frequent trip rates to a postal code location.

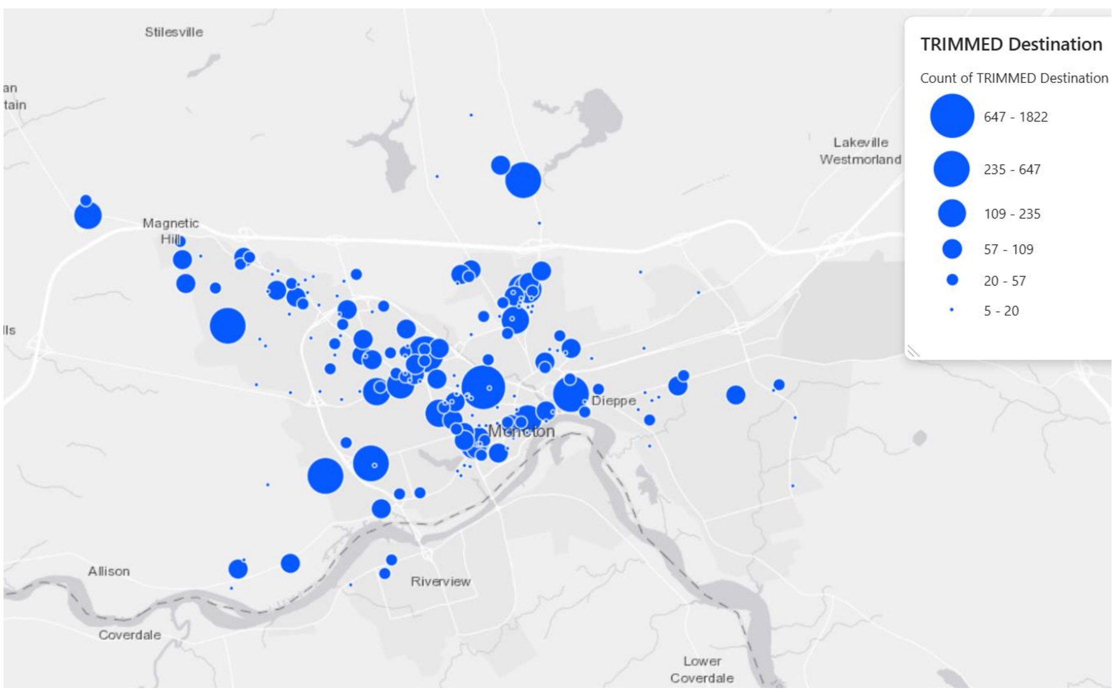

Figure 19. Trip Rates to Locations

When the trip rates are compared based on the urban Statistics Canada states that "Urban areas are those continuously built-up areas having a minimum population concentration of 1,000 persons and a population density of at least 400 persons per square kilometre based on the previous census." (Government of Canada, 2021) The extent of the urban centre by the Statistics Canada definition is shown below in Figure 20. It shows as expected that the trip rates are concentrated in the urban centre when compared to the Statistics Canada urban core delineated below.

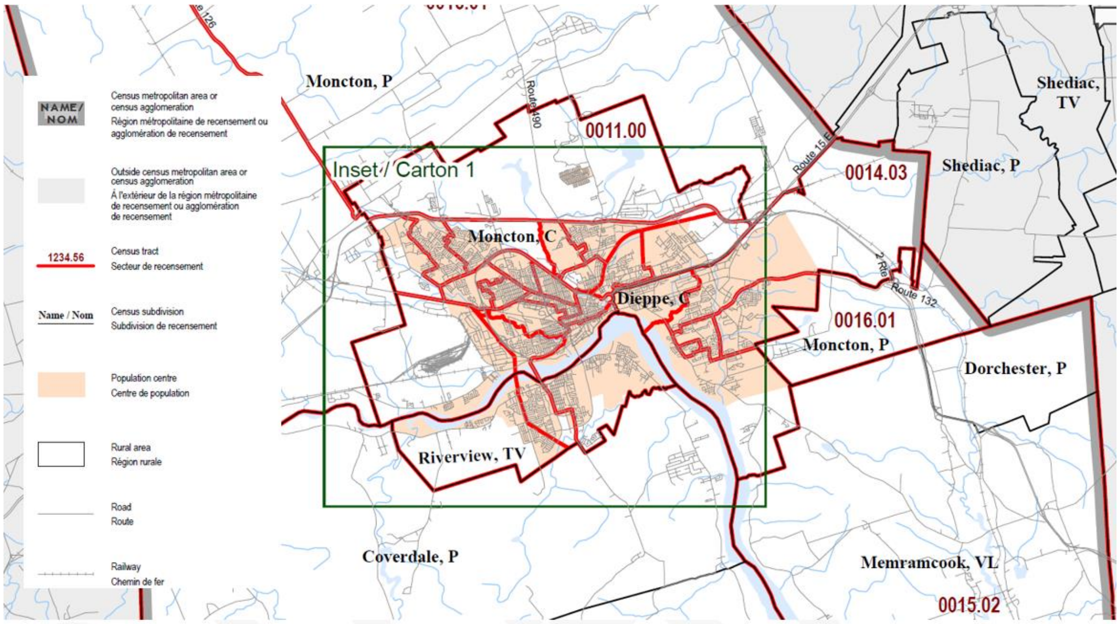

Figure 20. Statistics Canada Urban Core (Statistics Canada, 2021)

Using the PCCF file the Dissemination Areas and census tracts were linked to the postal codes obtained from ability transit. Using the University of Toronto CHASS Census Analyzer the population, and population density were linked to the Dissemination Areas where trips using Ability Transit originated from. The results of trips originating from each dissemination area per capita is shown in Appendix B. The graph below shown the percentage of trips originating from rural vs urban Dissemination Areas in the Moncton region. Out of a total of 121 Dissemination Areas from the data, 17 were rural Dissemination Areas and 104 were urban Dissemination

Areas. The dissemination areas with the highest trips were 13070068, 13070044 and 13070059, all urban Dissemination Areas.

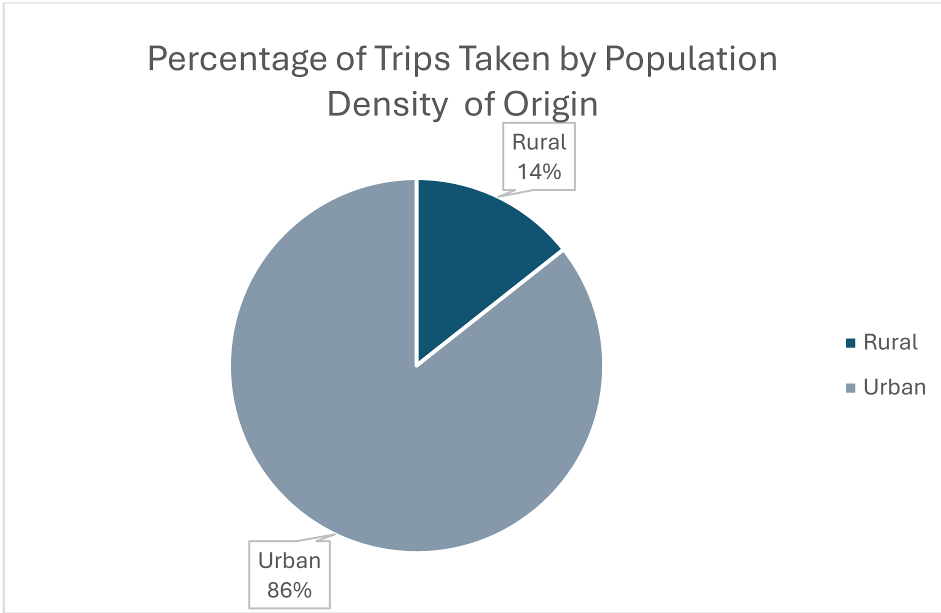

Figure 21. Percentage of Trips Originating from Urban And Rural Areas

The trips originating versus destined for the Dissemination Areas were visualised using ArcGIS. An output of the visualisation is shown in the Figure 23 below. The index attached shows the upper value of the colours visualized in the figure. A value of 0.3 would mean there are more trips to this area than originating, and a value of 0.9 would mean trips more frequently originate from this area.

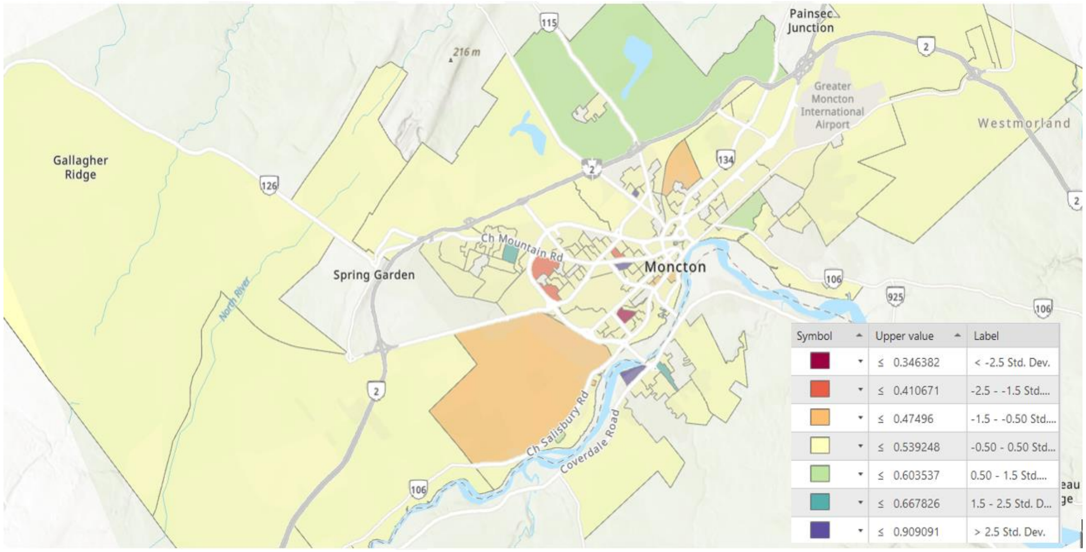

Figure 22. Trip Origins to Destinations Ratio

Most areas on the map had a yellow colour which represented the origins and destinations of trips were equal to each other. When trips had an imbalance of origins to destinations, questions could be asked, what within these areas drew attraction, such as schools, work, or medical offices.

5.1.4 <u>Trip Rates</u>

The trip rate per capita was found using the travel log data provided by Ability Transit and trip generation rates within the area from the Community Transportation Research Lab (CTRL) at the University of New Brunswick. 5.4 trips per 1000 population per week people in the Moncton area occur using as an on-demand paratransit service. This equates to 0.78 trips per 1000 population per day.

5.2 Application of Christie Method pilot (replicating existing system)

The entire service area for the study area comprises of 393 origin-destination pairs, on average 55 stops were made by the service daily, 72 stops when excluding Sunday, the least busy day. Optimizing this in an Excel-based solution is resource-intensive for a desktop computer, so strategies were employed to simplify the process while still piloting the method.

5.2.1 <u>Outlining test parameters</u>

While a 55-node network would reflect the average number of stops taken by the service, this was simplified to a 40-node matrix for the optimization of the fleet and depots (39 pick-up/drop-off nodes and 1 depot node). The model sizing was decided based on the expected exponential increase of computational load as a result of the number of vehicles stops in the problem. Therefore 40-node optimization problem was chosen as a middle ground for computational efficiency and accuracy of real-world scenarios. Figure 24 below shows their points used and their links to all other points in the model to create the cost matrix.

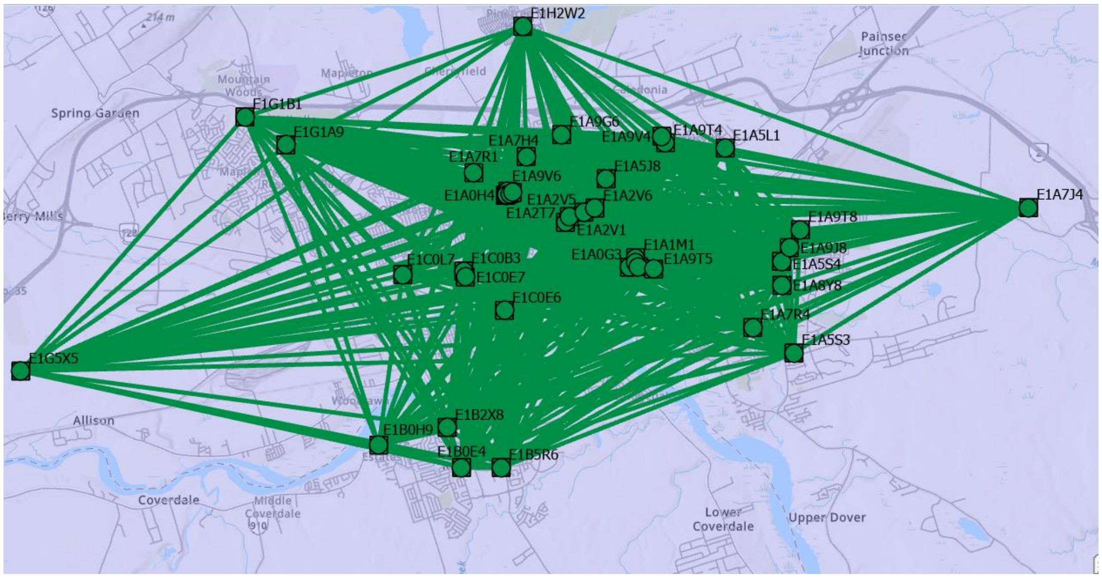

Figure 23. Forty (40) Point Scenario Cost Matrix Layer

The cost matrix output not only shows the nodes for pick up or drop off, but also the number of trip possibilities for the model. There are 1,600 possible options for the simplest form of the optimization model the TSP.

The optimization model done for the Moncton Region had 109 constraints of varying complexities. Logically applying the fewest number of constraints reduced the computational load of the problem, speeding up the solution. One method to reduce variables and constraints is using a family of convex hulls.

Once nodes are identified with their associated cost from one another in either time or distance, and a location with coordinate points, the convex hulls can be drawn. The convex hull families can be identified with the Graham algorithm, or manually (Christie, 2018). In this case given the data set was not large enough to warrant a software to automate the process and find the family of convex hulls they were found manually. The convex hull family needed to be identified before setting up the Excel spread sheet so the appropriate service constraint matrices can be created in the Excel spreadsheet. As shown in the Figure 25, below the family of convex hulls were identified manually. Every other convex hull from the first interior convex hull were highlighted in pink to identify them for a need of constraint matrices.

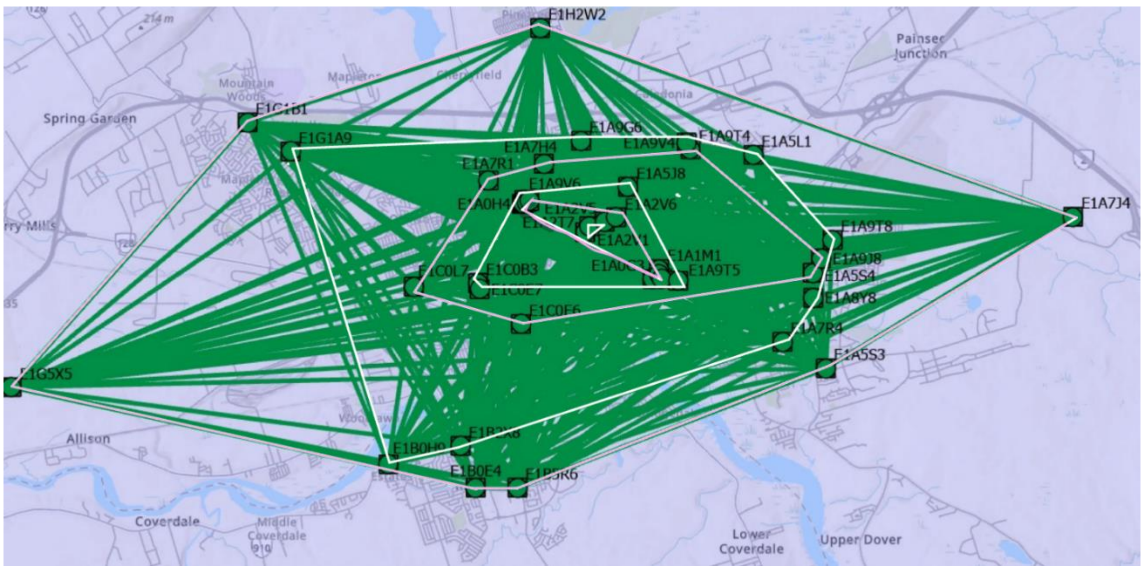

Figure 24. Forty (40) Node Family of Convex Hulls

The nodes that require constraint matrices in the setup of the Christie Method of constraint mixed integer linear programing are those that are part of the pink highlighted convex hulls. It is only required to consider convex hulls, for the constraint variables, and only the nodes associated with every other convex hull in the family. A list of the nodes requiring individual constraint matrices are listed in the table below. Each node has a letter associated which was the letter of the related constraint matrix in the Excel spreadsheet.

Table 9. List of Constraint Matrices Nodes

Service Constraint Matrices Associated Nodes		
A	Depot Node	E1A2V1
B	Convex Hull 2	E1A2V6
C	Convex Hull 2	E1A1M1
D	Convex Hull 2	E1A9K3
E	Convex Hull 2	E1A0G3
F	Convex Hull 2	E1A9V7
G	Convex Hull 2	E1A9V8
H	Convex Hull 2	E1A9W5
I	Convex Hull 4	E1A9T4
J	Convex Hull 4	E1A9J8
K	Convex Hull 4	E1A5S4
L	Convex Hull 4	E1C0E6
M	Convex Hull 4	E1C0L7
N	Convex Hull 4	E1A7R1
O	Convex Hull 4	E1A7H4
P	Convex Hull 6	E1H2W2
Q	Convex Hull 6	E1A7J4
R	Convex Hull 6	E1A5S3
S	Convex Hull 6	E1B5R6
T	Convex Hull 6	E1B0E4
U	Convex Hull 6	E1G5X5
V	Convex Hull 6	E1G1B1
W	Convex Hull 6	E1A2V5
X	Convex Hull 6	E1A5L1
Y	Convex Hull 6	E1A2T7

The use of convex hulls reduced the number of constraint matrices for the Ability Transit model by almost 50%. Without the use of families of convex hulls in the model, this model would have required 20 additional constraint matrices. Those additional constraint matrices equate to adding 32,000 variables to the model.

In order to accurately reflect the trip travel times shown in the Ability Transit data, the percentage distribution of the trip times shown earlier in Figure 15. Trip Starts by Time of Day, above was then distributed by percentage across the trips for the Moncton region model. When applied to the 40-node model for the Moncton region the time distribution of trips is shown below in the graph in Figure 25.

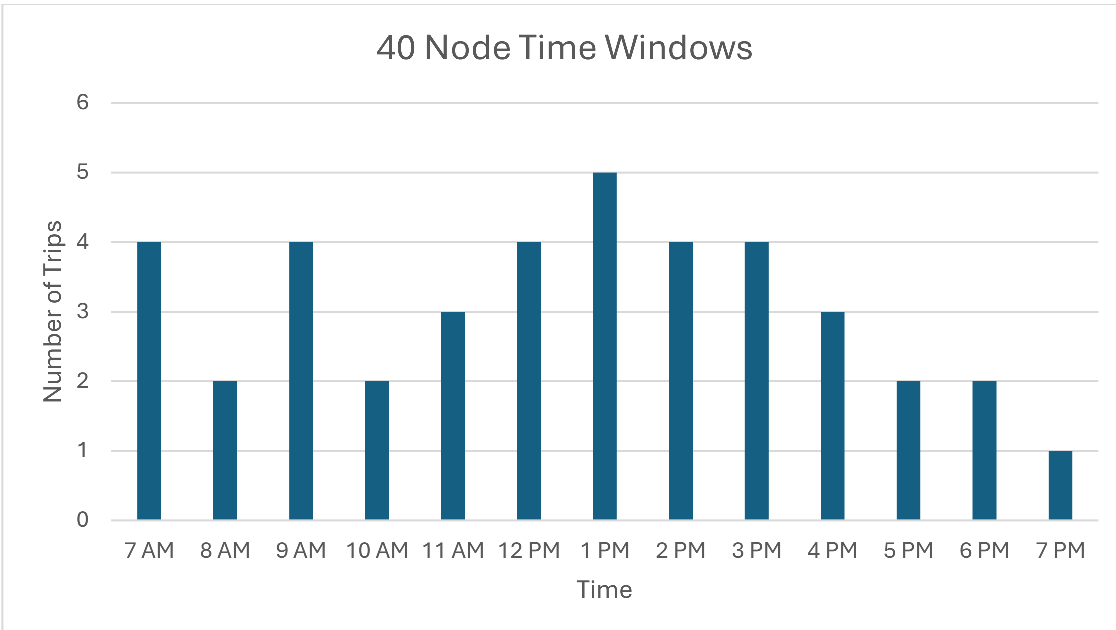

Figure 25. Trip Requests for Time Windows

5.2.2 Executing the optimization process

The paratransit organization operates with a maximum of seven vehicles (typically six vehicles), each subject to capacity constraints, and a maximum vehicle waiting time of 30 minutes. To solve the Vehicle Routing Problem (VRP), the Christie method was applied, making iterative adjustments to the number of vehicles and constraints in order to approximate the paratransit organization's operational parameters. The process involved adding one vehicle at a time to the VRP and evaluating key metrics with each iteration, including the ability of the expanded fleet to meet operational needs. Documented processing times were considered, and constraints were adjusted accordingly, while comparing the results to known fleet and vehicle characteristics. The objective was to minimize the total travel cost of the fleet, ensuring efficiency within the given limitations.

5.2.3 One-vehicle solution

The Open Solver base model in Excel took 30 hours to generate to optimal solution for the 40-node matrix TSP using the CBC engine. This model solved the traveling salesperson model (TSP) optimization of a set of 40 nodes. The TSP was used as the baseline, as it is the easiest computational problem. When switching the 40 node TSP model to the Gurobi solver engine it solved the same problem in 3 minutes. The solution to the problem 40 node TSP is shown below, in the form of a solution matrix. The total active vehicle time for the TSP is 157.1 minutes. The most efficient way to route with the least overall vehicle active time is to have one vehicle visit all the pick ups and drop offs. However, the TSP model of one vehicle does not have a level of service constraint. The level of service constraint states that the longest a client can be on the vehicle must be less than 30 minutes from pick up to drop off.

The visualization of the TSP is shown below in Figure 26. This is TSP is the baseline of the most efficient service delivery method, for an on-demand paratransit service. This is the most efficient method; however, it does not allow users the chance to request when they are pick up.

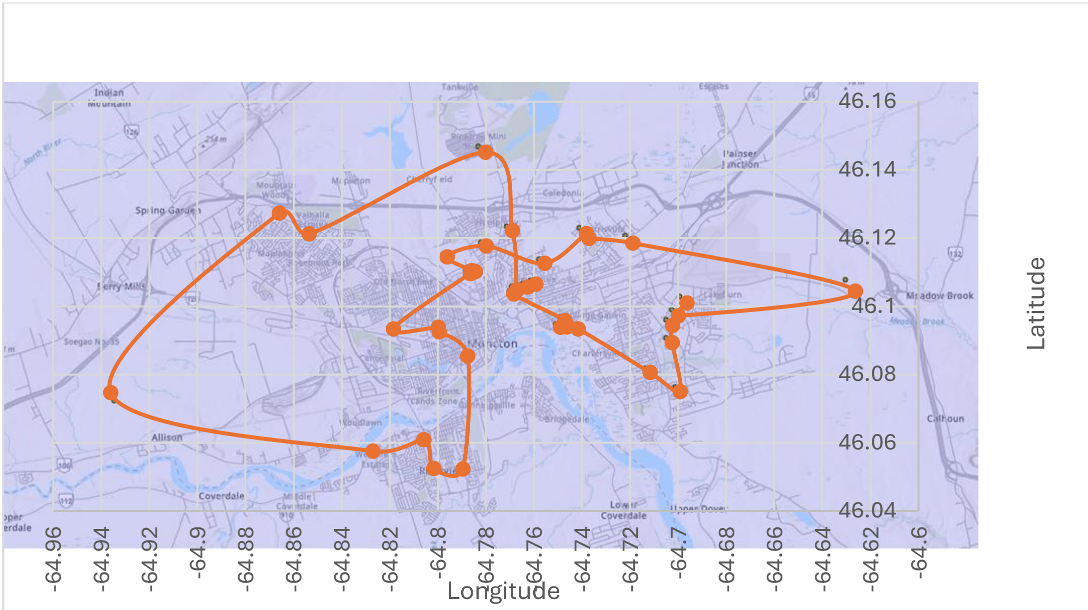

Figure 26. Route of One Vehicle (TSP)

5.2.4 Two-vehicle solution

The most efficient route, the 2 VRP is shown below. The two-vehicle routing problem was solved with an active vehicle time of 158.33 minutes. The solver took 34 seconds to solver the optimization problem. The order of the stops is shown classified by vehicles in Appendix D. The total time active vehicle time per day was 158.33 minutes. The time per vehicle ranged from 62 minutes to 96 minutes. The active time per vehicle for this model is shown in the table below. Because the time active per vehicle did not meet the level of service requirements, another vehicle must be added to the model.

Table 10. Two VRP Active Vehicle Time

	Vehicle A	**Vehicle B**
Active Time	96.4 minutes	61.8 minutes

The addition of another vehicle only added 1.6 minutes to the active vehicle time. It took the solver 104 seconds to complete the optimization of this problem, a 70 second increase from the TSP. The route of the vehicles is shown below.

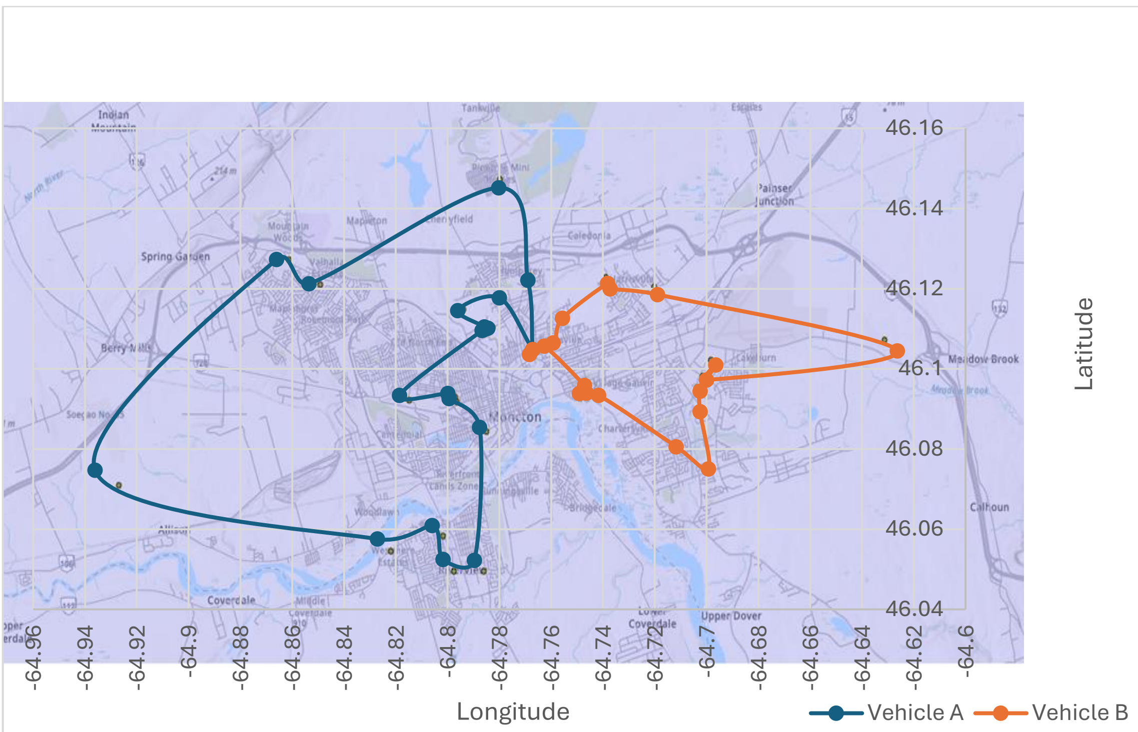

Figure 27. Solution To Two Vehicle Routing Problem

5.2.5 Three Vehicle Solution

The 3-vehicle model had an active vehicle time of 164.203 minutes. This was an increase of six minutes from the 2-vehicle routing problem. The stops are distributed amongst the vehicles is shown in Appendix D. The on road active vehicle time varied from 21 minutes to 85 minutes. The results of active vehicle time are shown below in the table 11. The results were not compliant with the level of service requirement of an active vehicle time less than 30 minutes.

Table 11. Three VRP Active Vehicle Time

	Vehicle A	**Vehicle B**	**Vehicle C**
Active Time	21.6 minutes	85.9 minutes	56.7 minutes

It took only 150 seconds to solve this problem. An increase of 50 seconds from the 2VRP. The route the vehicles take starts at the depot and travels through the nodes requesting trips. The route taken in the 3-vehicle routing problem, overlaid on a map of Moncton, is shown in the Figure 28 below. Each vehicle is represented by a colour, vehicle A is represented by dark blue, orange represents vehicle B, and green represents vehicle C.

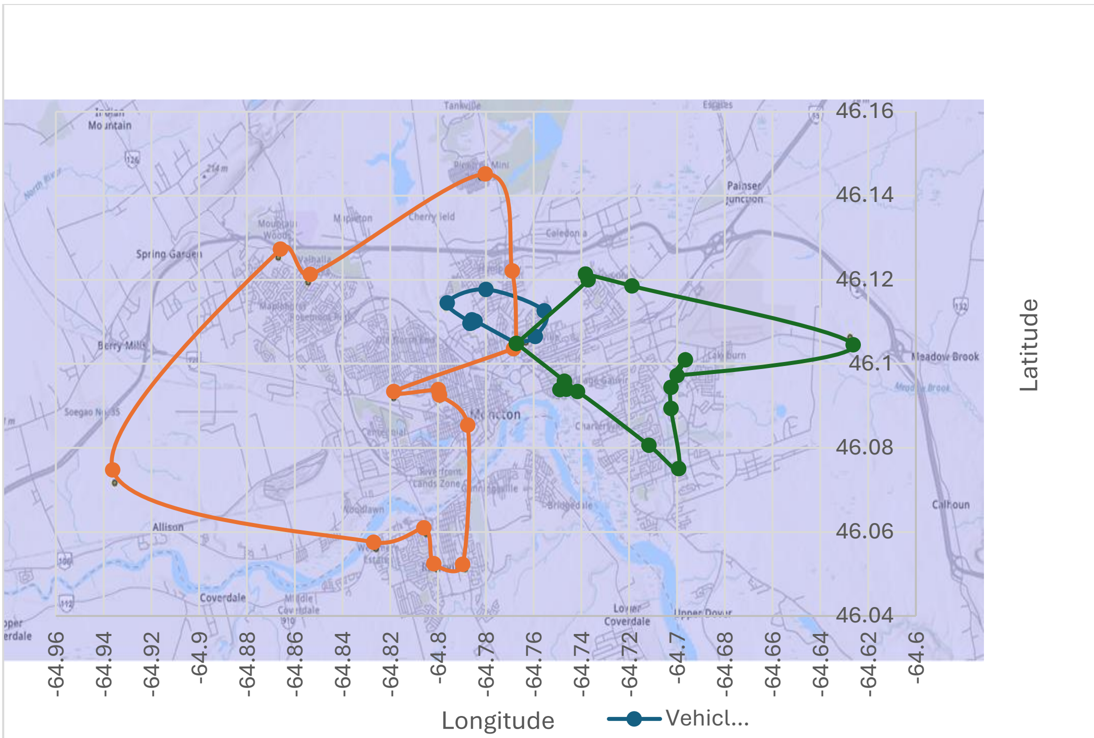

Figure 28. Solution to Three Vehicle Routing Problem

5.2.6 Four Vehicle Solution

Another vehicle was added to the model, to create a 4-vehicle routing problem. The active vehicle time was 171.539 minutes. The distribution of time by vehicle is shown below in Table 12 in minutes. The time does not meet the level of service constraint of less than 30 minutes from first rider pick up to drop off, so another vehicle must be added to the model. Vehicles A and D took longer tours to minimize the overall time of the problem, instead of distributing the time evenly amongst vehicles, which would raise the total active vehicle time. The optimization solver run time was 16491s. The order of stops is shown in Appendix D.

Table 12.Four VRP Active Vehicle Time

Vehicle A	Vehicle B	Vehicle C	Vehicle D
86 minutes	19 minutes	17 minutes	50 minutes

The route taken by the vehicles in the four-vehicle routing problem is shown below. The tours are presented for each vehicle by a colour dark blue represents vehicle A, orange represents vehicle B, green represents vehicle C, and light blue represents vehicle D. Vehicle A accounts for the largest percentage of the active vehicle time in this solution.

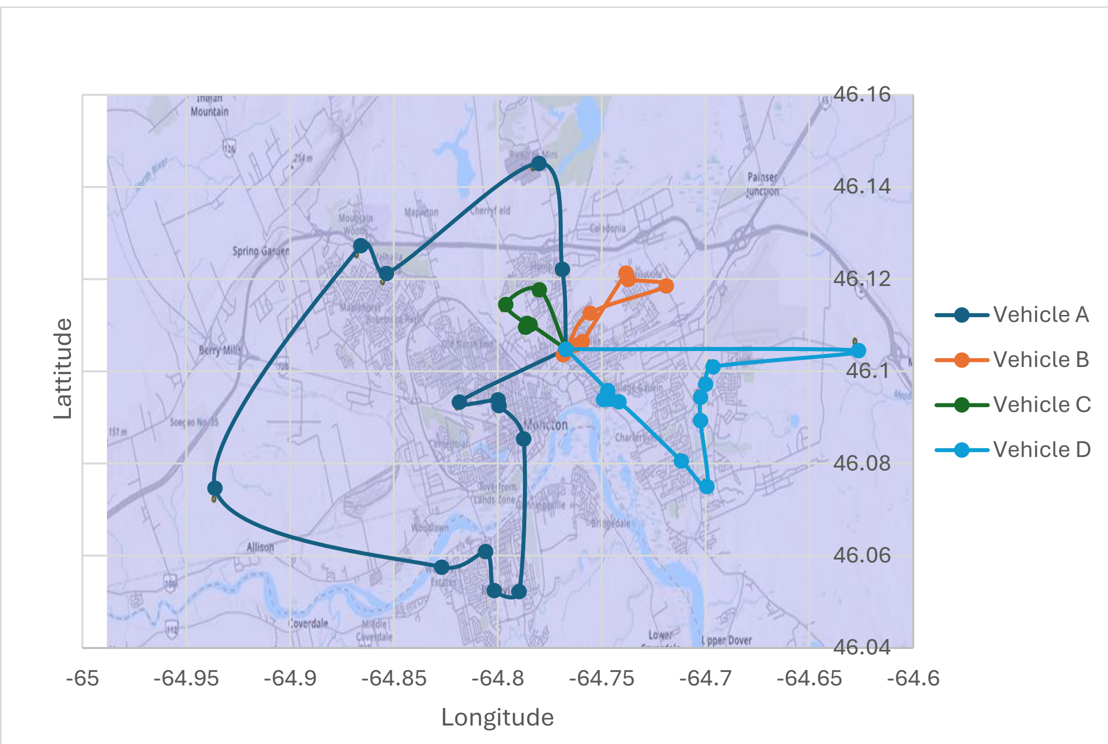

Figure 29. Solution to Four Vehicle Routing Problem

5.2.7 Five Vehicle Solution

Since the four vehicles did not meet the level of service requirements, another vehicle was added to the problem. The 5 VRP had a solver time of 19131 seconds. The total active vehicle time was 180 minutes. The order of stops is shown in Appendix D. Although it took a total time of 180 minutes for all vehicles to complete the stops the time of each vehicle is shown in the table below. Vehicle E served 6 clients and filled its capacity while only being active for 5 minutes.

Table 13. Five VRP Active Vehicle Time

Vehicle A	Vehicle B	Vehicle C	Vehicle D	Vehicle E
47 minutes	40 minutes	67 minutes	21 minutes	5 minutes

The route of the vehicles took during the 5-vehicle routing problem is shown in Figure 30 below.

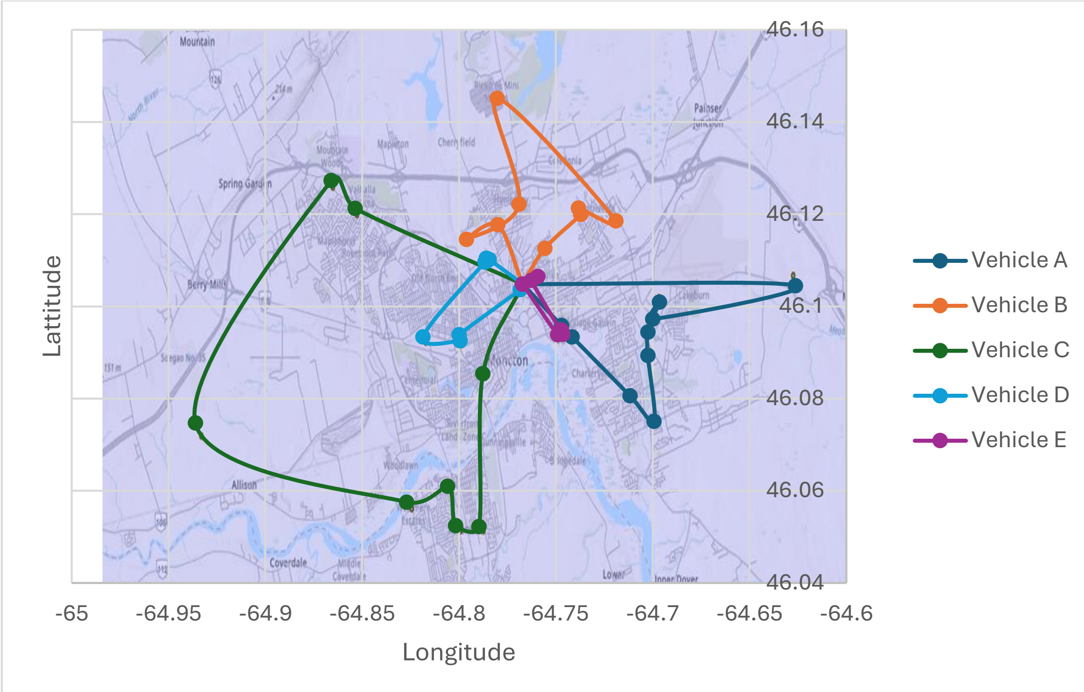

Figure 30. Solution to Five Vehicle Routing Problem

5.2.8 Six Vehicle Solution

Since the constraints for level of service were not met, another vehicle was added to the model. The solver runtime of the six-vehicle routing problem (6VRP) was 10655 seconds. The active vehicle time was 205 minutes. The time taken by each vehicle to complete the pick ups is shown below in Table 14.

Table 14. Six VRP Active Vehicle Time

Vehicle A	Vehicle B	Vehicle C	Vehicle D	Vehicle E	Vehicle F
35 minutes	44 minutes	39 minutes	37 minutes	21 minutes	28 minutes

The route the vehicles have taken is shown in the Figure 34 below overlaid on a map of Moncton. The vehicles carve out zones within the service area, they stick to in an optimal configuration of trips. As more vehicles are added to the problem such as the 6 VRP the zones become smaller and smaller.

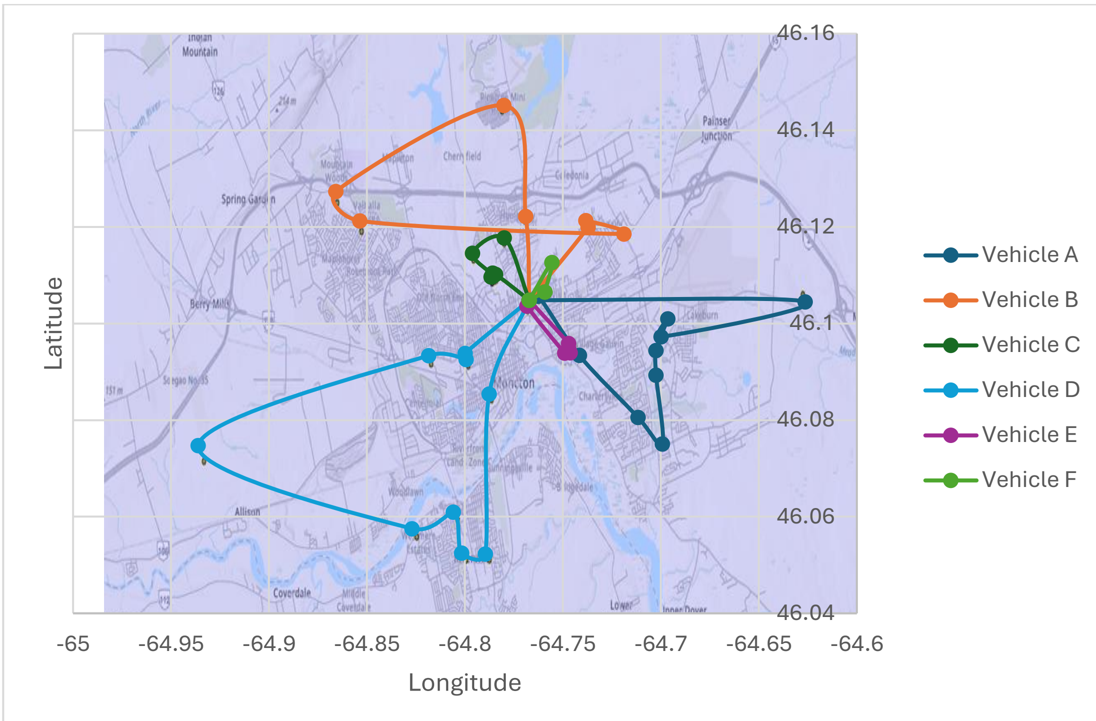

Figure 31. Solution to Six Vehicle Routing Problem Part 1

The constraint that all vehicles must be within a level of service of 30 minutes was enacted and the routing changed to show the order and routing below. With the addition of the constraint with travel time, the rural trip has a dedicated bus. The routing change is visible in Figure 32.

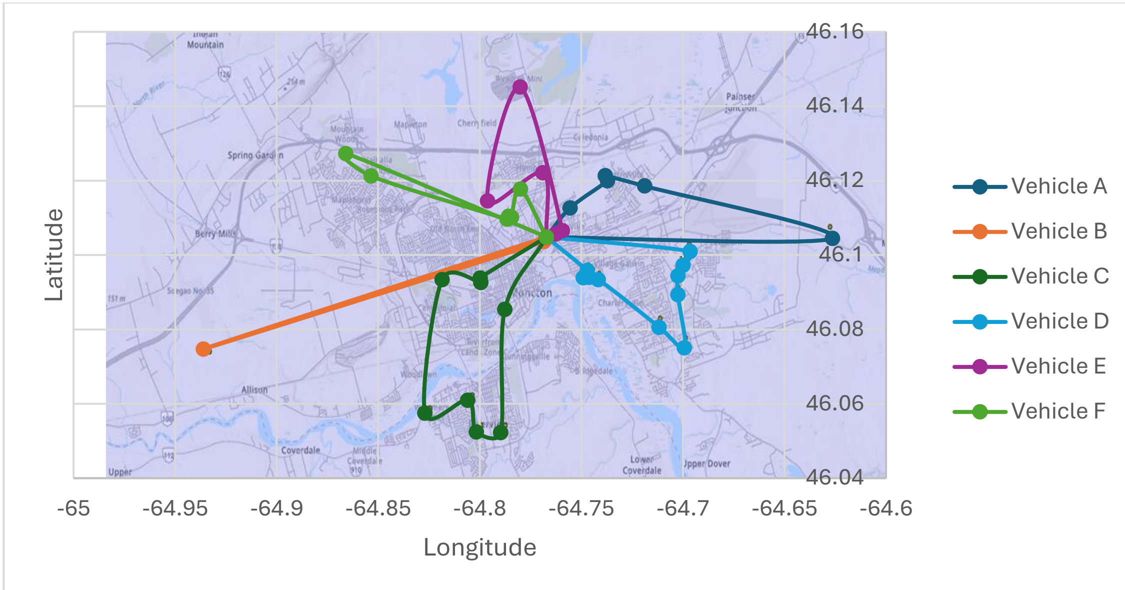

Figure 32. Solution to Six Vehicle Routing Problem with Added Constraint

In Figure 32 shown above it is clear the vehicles carved out zones within the service area, and stayed within the zones, as it had the least cost friction. The trip conducted by Vehicle B has a large travel time to one node, so only one other non-depot related stop is on that tour, whereas Vehicle D has multiple stops in close proximity, so it took advantage of the least cost route.

5.2.9 <u>Summary of application</u>

After the 40 node TSP was solved, time windows were added to reflect the known operational consideration of Ability Transit and add industry experience to the model, simulating the request time requirements. The solution to the 40 node TSP with time windows. The Christie Method in Excel proved difficult to implement time windows, so both manual checks and Excel check were done. Because the time windows change the order, showing the route does not show the optimal order just the requested order of stops. The TSP with time windows met all constraints and was solved in 1867 seconds. The overall solution time is much larger than the 6VRP because the optimal route cannot be followed.

There was no direct link to the number of vehicles and an increase in solver time, some solver times with more vehicles decreased in solver time compared to previous models. The computational time is how long it took the solver Gurobi to find an exact solution. The active vehicle time is the sum of the time it took all vehicles to complete the series of stops. The active vehicle time did not include the time of the stop. The solver time did not increase linearly with added variable.

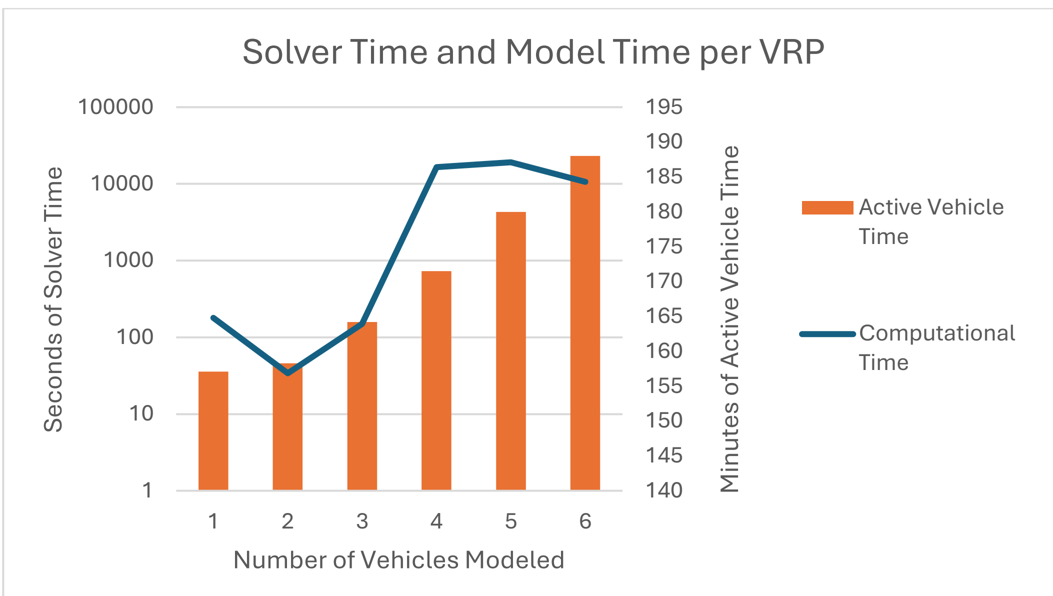

Figure 33. Solver Time and Model Time per Vehicle Routing Problem Size

It can be concluded that this model, reasonably meets the characteristics of Ability Transit. When level of service constraints were applied, they fell slightly short of the standard Ability Transit offers but the capacity of vehicle in comparison to Ability

Transit was maximized. Ability Transit uses 30 minutes from pick up to drop off at your location constraint, and this was applied to the model. The model with 6 vehicles fell slightly short of this with a few vehicles exceeding the 30-minute level of service constraint by 5 minutes. The service offers 5 vehicles during peak times to service their clients, the problem was solved with 1 through 6 vehicles to find the best fit for the data received and prove the application of this method. As the model is now calibrated to the features of an on-demand paratransit service for urban and rural clients it can be applied to new environments.

Each pilot was first applied with an open operational constraint to the on-demand service to begin to test. This method of piloting was done to test if the added constraints used for the Christie Method were correctly applied to the model. When additional vehicles were added, new matrices and constraints within the Christie Method were added and with that, the possibility of the model to fail because of a set up error. The Table 15 shown below shows the operational constraints applied to each pilot and the results and associated solver time, to show the methodical progression of pilots.

Table 15. Parameters of Models using the Christie Method

Pilot Number	Number of Vehicles	Operational Constraints	Solver Type	Result	Solver Time (sec)
1	1	Open	CBC	Feasible	108,000
2	1	Open	Gurobi	Feasible	180
3	1	Vehicle Capacity	Gurobi	Not Feasible	420
4	2	Open	Gurobi	Feasible	180
5	2	Vehicle Capacity and Service Time 20 mins	Gurobi	Not Feasible	60
6	2	Service Time 35-40 mins	Gurobi	Feasible	34
7	3	Open	Gurobi	Feasible	120
8	3	Service Time	Gurobi	Feasible	150
9	4	Open	Gurobi	Feasible	150
10	4	Service Time	Gurobi	Feasible	16,491
11	5	Open	Gurobi	Feasible	1,492
12	5	Service Time	Gurobi	Feasible	19,131
13	6	Open	Gurobi	Feasible	2,006
14	6	Service Time and Vehicle Capacity	Gurobi	Feasible	10,655

5.3 Application of Christie Method pilot (regional paratransit)

Demand patterns are important in the setup of an on-demand paratransit system. The demand patterns inform the location of paratransit hubs (Metrolinx & Canadian Urban Transit Association, 2022). There are two categories to describe the demand these services have, many to many services, and many to one service ((Metrolinx & Canadian Urban Transit Association, 2022). With the data collected from Ability Transit the demand patterns were used to create synthesized trips in the optimization model. The demand patterns allowed the model to represent the real-world conditions this service would serve, to better enable the planning of the service. Based on the trip rates from the Ability Transit data, and the trips from area to area of the Southeast Regional Service Commission, an origin destination matrix was created with the trips from region to region using the on-demand paratransit service trip rate. The percentage of trips taken in the Southeast Regional Service Commission were found from collaboration with Civilia. The Civilia cellphone data provided an estimate of the number of trips in the Southeast Regional Service Commission of New Brunswick in Winter 2023 and was then used to determine the percentage of trips in order to geographically aggregate travel demand between communities (Hanson, Whitehouse & Higdon, 2024). The percentage of trips to and from communities were found by taking the number of trips per community in the Southeast Regional Service Commission and dividing it by the total number of trips in the Southeast Regional Service Commission in New Brunswick (Hanson et al., 2025). The percentage of trips from area to area was then multiplied by the population of each area within the Southeast Regional Service Commission and the per capita trip rate was found for trips. To specifically find the number of paratransit DRT trips, the Civilia trip per community were multiplied by the paratransit DRT trip rate found from the Ability Transit data to produce the Figure 34 shown below. The Figure 34 below shows the demand of paratransit trips per day within the districts of the Southeast Regional Service Commission.

FROM/TO	Cap-Acadie	Dieppe	Fundy Albert	Maple Hills	Memramcook	Moncton	Salisbury	Shediac	SE rural district	Strait Shores	Tantramar	Three Rivers	Riverview
Cap-Acadie	2	0	0	0	0	1	0	1	0	0	0	0	0
Dieppe	0	14	0	0	0	10	0	1	0	0	0	0	1
Fundy Albert	0	0	1	0	0	0	0	0	0	0	0	0	0
Maple Hills	0	0	0	1	0	2	0	0	0	0	0	0	0
Memramcook	0	0	0	0	1	0	0	0	0	0	0	0	0
Moncton	0	9	0	2	0	63	1	1	0	0	1	0	4
Salisbury	0	0	0	0	0	1	2	0	0	0	0	0	0
Shediac	1	1	0	0	0	1	0	4	0	0	0	0	0
SE rural district	0	0	0	0	0	0	0	0	0	0	0	0	0
Strait Shores	0	0	0	0	0	0	0	0	0	0	0	0	0
Tantramar	0	0	0	0	0	0	0	0	0	0	3	0	0
Three Rivers	0	0	0	0	0	0	0	0	0	0	0	1	0
Riverview	0	1	0	0	0	5	0	0	0	0	0	0	9

Figure 34. SERSC On Demand Paratransit Origin and Destination Matrix

The majority of trips are centered around the Moncton region, with the highest origin and destinations in Moncton, Dieppe, and Riverview. The highest number of trips based on the State of the Region Transportation Report were Moncton to Moncton trips. This method estimated 63 trips per day using paratransit DRT would be within Moncton, confirming the sizing of the application using the Christie Method.. The Figure 34 shown above had conditional formatting for easy viewing, with more trips per district per day shown as a darker green. The information from area-to-area trips within Southeast Regional Service Commission allowed for the set-up of an application of the Christie Method in a district outside of the Moncton area.

5.3.1 Cap-Acadie

The cost matrix connectivity output from the Cap-Acadie example of the Christie Method is shown in the Figure 38 below. The Cap-Acadie region based on the per capita trip rates has an estimated demand of 4 paratransit trips per day. Based on the origin and destination matrix in the State of the Region Transportation Report 3 trips were within the Cap-Acadie region, one trip was to Moncton and one trip was to Shediac. The trips within the Cap-Acadie area were randomly placed in Cape Breton, Grand Barachois, and Haute-Aboujagane.

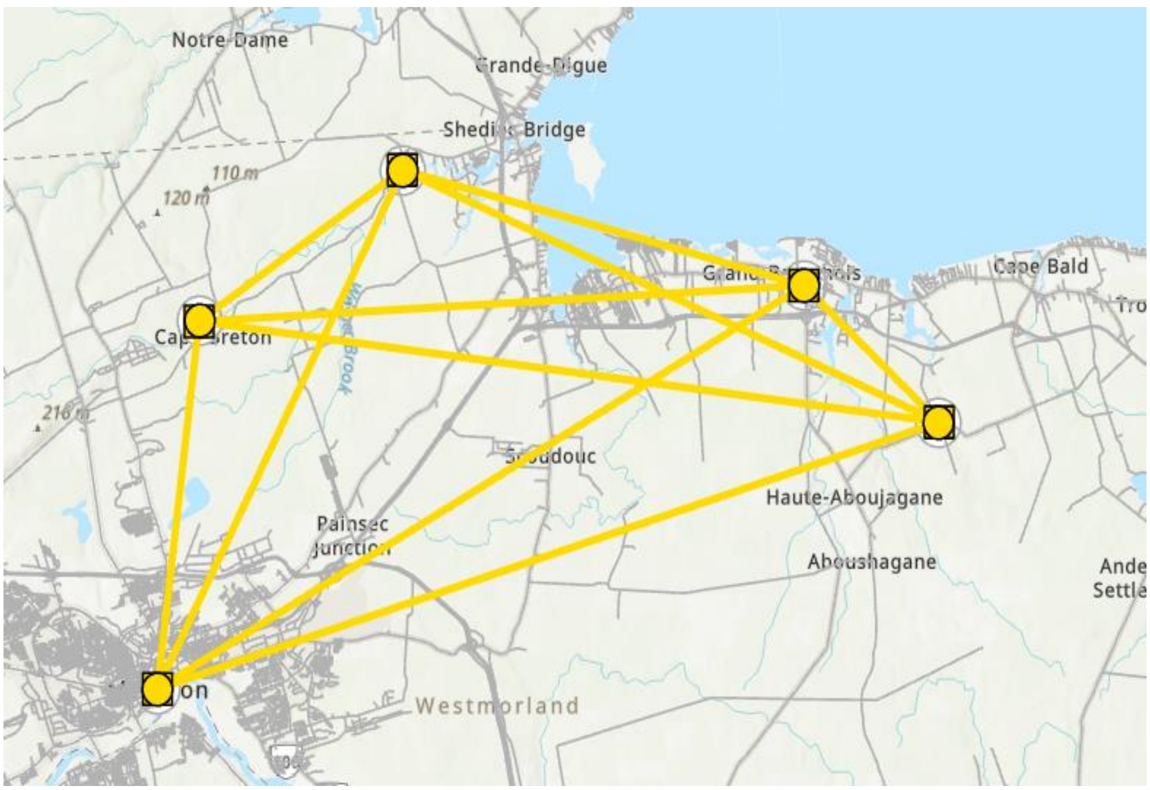

Figure 35. Cap-Acadie Origin and Destination Cost Matrix Output

The example was run with one vehicle and the active solver time was less than a second, because the problem size was so much smaller than the Moncton area

Ability Transit model. The order of stops is shown below in table 16 below. It took the vehicle 105 minutes to complete all pick ups and drop offs. The table below shows the order of the stops.

Table 16. Cap-Acadie Trip Order

Place	Order
Cap-Acadie 3	1
Cap-Acadie 2	2
Moncton	3
Cap-Acadie 1	4
Shediac	5
Cap-Acadie 3	6

The vehicle started at location Cap-Acadie 3 as it was set as the depot, this was located in Grand Barachois.

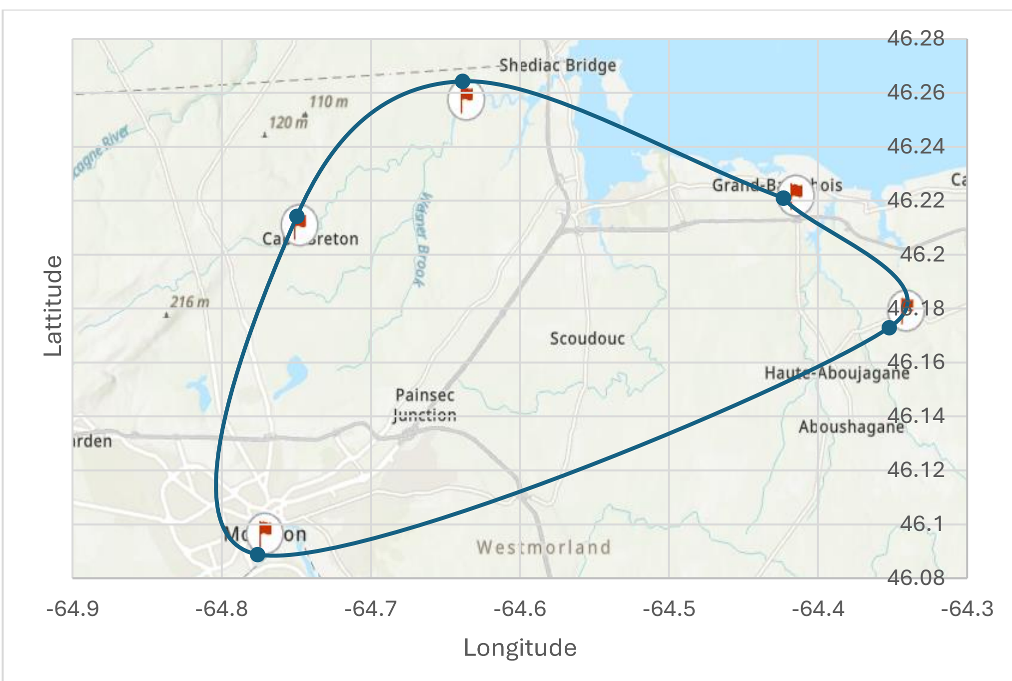

Figure 36. Solution for One Vehicle TSP for Cap Acadie

The optimal route starts in Grand Barachois and then goes to Haute- Aboujagane and continues on to Moncton. The trips are fulfilled with only one vehicle. To limit the time riders are in the vehicle not reaching their destination, the trip directly to Moncton was given an independent trip. The solution is shown with two vehicles in the Figure 37 below. The total active time for all vehicles is 135 minutes.

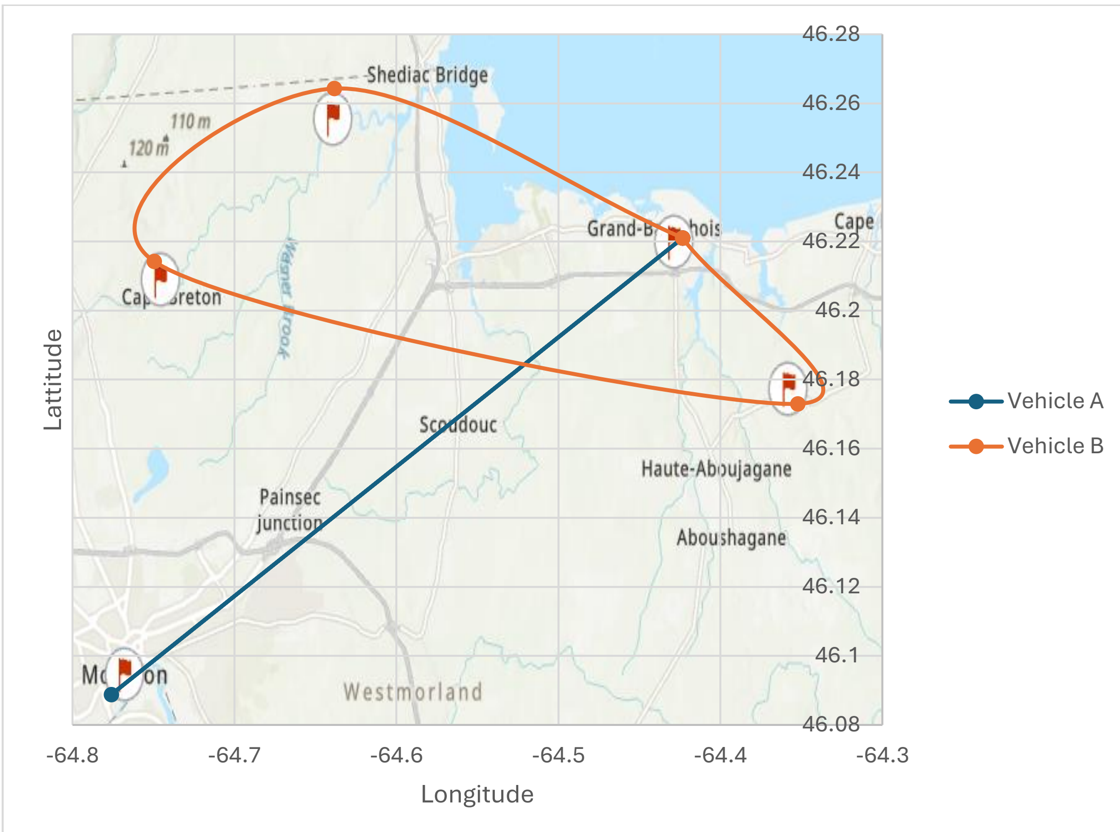

Figure 37.Cap-Acadie 2 Vehicle Routing Problem Solution

The active vehicle time for Vehicle A is 89 minutes, and Vehicle B is 47 minutes. Vehicle B starts in Grand Barachois, then moves toward Haute- Aboujagane, then Cape Breton, then towards Shediac Bridge.

Table 17. Cap-Acadie 2 Vehicle Routing Problem Active Vehicle Time

Vehicle A	Vehicle B
89 minutes	47 minutes

The increased time cost of 30 minutes of an additional vehicle does not outweigh the level of service increase for adding an additional vehicle to the fleet for the service of the Cap de Acadie area within the Southeast Regional Service Commission.

5.4 Summary

Between January 2 and July 2, 2023 (6 months), Ability Transit completed 11,338 trips, covering a total of 147,047 km. Data cleaning revealed that only 0.5% of postal codes were invalid. Using ArcGIS, a cost matrix was developed for 400 data points representing postal code centroids, which calculated travel distances and times for each trip. The average daily travel per vehicle was 175 km, with a total of 822 km traveled per day across all vehicles. The busiest vehicle (Vehicle 1) accounted for 31% of trips. The service had 62 trips on average per day, peaking on Tuesdays and lowest on Sundays. Trip distances typically ranged from 10-15 km, with the longest trips averaging 30 minutes. The trips were concentrated in the Moncton urban area, with 86% of trips either originating or ending there. Data analysis also identified that most trip origins and destinations were in urban areas, with trip rates of 5.4 per 1,000 people per week. Geographically, trip demand was highest in urban areas, particularly around Moncton's core. The analysis also revealed patterns in the scheduling, including recurring trip requests and the frequency of specific origins and destinations.

The section also discussed the application of optimization to the SERSC, specifically the Christie Method, to fleet management and routing for on-demand paratransit services. It used a 40-node matrix for a fleet and depot optimization problem, where the Traveling Salesperson Problem (TSP) served as a baseline for the active service time of the model. Solver performance was compared across different engines, with results showing significant speed improvements using the Gurobi engine over CBC. Various routing problems were solved with 1 to 6 vehicles, exploring how active vehicle times, constraints (e.g., service time), and solver times changed with each scenario. The model met most of the operational requirements for Ability Transit, although a few instances exceeded the 30-minute service constraint. Additionally, demand patterns derived from real-world data, including trip generation and origin-destination matrices, were integrated into the model for the Southeast Regional Service Commission (SERSC), with a focus on optimizing routes in Moncton and surrounding areas. The Cap-Acadie example illustrated the model's effectiveness on a smaller scale, where a two-vehicle routing problem (VRP) improved service delivery compared to a single vehicle, optimizing both active vehicle time and service efficiency.

6 DISCUSSION

The research aimed to offer valuable insights into transportation planning practices for on-demand paratransit services in both urban and rural areas. The findings related to these practices are outlined below. Collaborating with a Demand-Responsive Transport (DRT) service provider provided an opportunity to identify key data requirements for future planning. This section also explores operational research with DRT systems and presents recommendations drawn from lessons learned throughout the study.

6.1 Transportation Planning Practices

The research on established regional planning practices as a goal of this thesis found documented practices by Metrolinx. Metrolinx offers some standard practice for implementing paratransit services. Metrolinx serves the greater Toronto region in Ontario. Unfortunately, the ultimate lack of readily available planning practices for small urban centers disadvantages smaller transportation planning commissions. DRT in large urban centers act differently than smaller urban centers, when supporting transportation systems. Institutional arrangements and transportation issues can be very different in rural areas than in metropolitan areas with populations of over 50,000 (Federal Highway Administration, 2018). In metropolitan areas, the responsibility for planning lies with designated Metropolitan Planning Organizations; in small communities and rural areas of New Brunswick Regional Service Commissions are responsible for the delivery of transportation services.

The use of mixed integer linear programing can be helpful to optimize based on service constraints, transportation services within small RSCs regions. The use of the Christie Method in the Cap-Acadie example proves that it can be used in a planning practice to determine fleet sizing for an on-demand service. Due to the computational complexity of the problem breaking the modeling into reasonable pieces by the twelve areas of the SERSC could allow for the model to solve this problem in a reasonable time frame, of sub-minute in the Cap-Acadie example. This methodology struggles with multiple vehicles, so reducing the service area to a reasonable size of the governing authorities' scope of service allows the problem to help plan the implementation of paratransit fleet into the area.

6.2 State of Practice Operations

The scope of this research was to look at an on-demand paratransit system, where clients must meet specific criteria, however on demand services client bases can vary. DRT services are not always geared towards users with disabilities. When municipalities shift from fixed-route to on-demand transit services on less dense routes, area is served by DRT. On-demand transit, operated by vehicles equipped to accommodate mobility devices, serves not only individuals who need paratransit but also the general public with the same trip needs. When all user no matter age or ability, can access the same service through a single mode of transportation, it demonstrates universal design practices.

Different regional authorities provide different planning and booking systems, which impacts the accessibility of the service (Rodman et al., 2016). Direct booking uses a software platform similar to services like Expedia to find paratransit trips for users (Rodman et al., 2016). However, in smaller urban areas, such as Moncton, New Brunswick there are few other selections, if any, for accessible trips, making the booking service nonessential. In larger cities direct booking systems can help connect multiple accessible transportation services as linkages allowing continuous transport for users with a disability (Rodman et al., 2016).

The results from the ratio of trip origins versus destinations can be helpful in the planning of on demand services. When trips had an imbalance of origins to destinations, questions could be asked, what within these areas drew attraction, such as schools, works, or medical offices, this occurred with ratios less than 0.5. When Dissemination Areas had ratios greater than 0.5, they had more trips originating from them than destined for them, this could mean there was a higher volume of residences in this Dissemination Area.

6.3 Ability Transit Data

6.3.1 Trip Rates

The trip rate per capita was found using the travel log data provided by Ability Transit and trip generation rates within in the area from the Community Transportation Research Lab (CTRL) at the University of New Brunswick. A total of 5.4 paratransit trips per week per 1000 people are generated in Moncton. The latent demand for these services is not known, though historically there has been a waiting list for Ability Transit.

6.3.2 Trip Purpose

The increase in mobility and accessibility that paratransit would provide for the individual, is measured by the difference in the overall expected utility from the trip generation, distribution, and mode choice models with versus without paratransit as an alternative (Koffman *et al*., 2007). The key variables that affect someone's decision to make a trip are: age, income, employment, gender, regional effects, accessibility effects and household size (Koffman *et al*., 2007). Given the limitations that this report was published close to 20 years ago in 2007, the forecasts referenced were for 2030, which is closer to the current date than the date this source was published. The implication of mode choice is reflected in the trip rates, however the lack of personal information attributed to a trip in the Ability Transit, does not allow for inferences about trip decisions. The model could only be as good as the data from Ability Transit.

In the future setting up a model, for the optimization of paratransit, trip purpose could be helpful in deciding how to apply constraints to the model. The data received from Ability Transit consisted of a travel log with postal codes and time, which was great to offer insight into when and where trips are made in the Moncton region. The next steps to evolve the understanding of urban and rural on demand services would be to understand the "why" of a trip. Using the census data and the trip rates from

Dissemination Areas, they were broken in to urban and rural paratransit trips, however they could not be associated with any trips with a trip purpose such as home-based work (HBW) trips, home-based shop (HBS), home-based other (HBO), non-home-based work (NHW), and non-home-based other (NHO). Having trip purpose would create even more accurate model to an individuals travel behavior to help better the model constraints and could actively show the time spent using the service for home based or non-home-based trips. This trip purpose could better help us understand if riders take trip tours visiting more than one stop before returning home, and alter the model set up to handle this type of trip. In the data provided by Ability Transit it was confirmed by a representative that often individuals make one stop trips and then return home, but there were no data available to analyze that confirmed this.

6.4 Operations Research

One challenge encountered while working with OpenSolver was the character limit imposed by Excel, which restricts the variable cell input box to 255 characters. This limitation became a significant hurdle when expanding the model, as larger models were unable to function effectively because matrix names created by Excel were too long. To mitigate this issue in future projects, it is advisable to use shortened names for all matrices in Excel by creating custom names. By doing so, hitting the character limit can be avoided and facilitate the growth of the model.

6.4.1 OpenSolver

Setting up the model of a TSP, issues were presented such as slow solving speed. This prompted a new computer to be ordered. The computer was required to solve the larger 6 vehicle routing problems with 46,000 variables. Once the new computer arrived, there were barriers to using OpenSolver. The use of OpenSolver on an administrated computer requires threat exclusions to be granted by IT Services at UNB.

The OpenSolver software sets all variables equal to zero before solving the system of constraints. Because the dynamic ordering of the stops with the time windows relies on an inverse matrix, an issue arises as soon as the model starts solving. It was impossible to take the inverse of a matrix full of zeros, which snowballed through the systems of constraints and formulas requiring the inverse of the solution matrix. Also, OpenSolver repeatedly stated linear systems were nonlinear, even though the problem had been successfully run previously. The error would clear once the computer was restarted, and the model was solved again.

6.4.2 Software

During the development of a solution to address the demand for paratransit, several software options were evaluated. A key requirement was that the software needed to be compatible with the Gurobi solver engine, which was ultimately used for optimizing the demand-responsive transit model. Programming languages such as MATLAB and Python were considered for running the optimization outside of Excel.

Both MATLAB and Python were explored for implementing the Christie Method, but both presented a steep learning curve compared to using Excel with OpenSolver, which also utilizes the Gurobi solver engine. MATLAB was ultimately excluded from consideration, as its processing power when paired with Gurobi did not offer significant advantages over the OpenSolver add-in in Excel.

When trying to apply time windows to the TSP and VRP there were errors in the linearity of the problem. OpenSolver highlighted the following constraints as non-linear constraints shown in the Figure 38 below. In order to set a time window, a clock had to be used, but open solver did not like the format of time within the solver. The time clock is nonlinear, so the time was converted from the time format to a decimal format, a fraction of the 24 hours in a day. The 15-minute buffer in td was changed to 0.25 of an hour instead of 15 minutes. For this reason, in the future continuation of this research MATLAB and Python should be considered as other software for the solving of both the TSP and VRP with time windows.

ta	TIME desired	Td- Dij+15 min	tchange	dij	NEXT NODE POSTAL
0:00:00	7:00:00	≥ 6:45:00	0.291666667	0	E1A
0:00:00	7:00:00	≥ 6:45:00	0.291666667	0	E1A
0:00:00	7:30:00	≥ 7:15:00	0.3125	0	E1A
0:00:00	7:30:00	≥ 7:15:00	0.3125	0	E1A
0:00:00	8:00:00	≥ 7:45:00	0.333333333	0	E1A
0:00:00	8:30:00	≥ 8:15:00	0.354166667	0	E1A
0:00:00	9:00:00	≥ 8:45:00	0.375	0	E1A
0:00:00	9:00:00	≥ 8:45:00	0.375	0	E1A
0:00:00	9:30:00	≥ 9:15:00	0.395833333	0	E1A
0:00:00	9:30:00	≥ 9:15:00	0.395833333	0	E1A
0:00:00	10:00:00	≥ 9:45:00	0.416666667	0	E1A
0:00:00	10:30:00	≥ 10:15:00	0.4375	0	E1A
0:00:00	11:00:00	≥ 10:45:00	0.458333333	0	E1A
0:00:00	11:30:00	≥ 11:15:00	0.479166667	0	E1H
0:00:00	11:30:00	≥ 11:15:00	0.479166667	0	E1A
0:00:00	12:00:00	≥ 11:45:00	0.5	0	E1A
0:00:00	12:00:00	≥ 11:45:00	0.5	0	E1A
0:00:00	12:00:00	≥ 11:45:00	0.5	0	E1A
0:00:00	13:00:00	≥ 12:45:00	0.541666667	0	E1A
0:00:00	13:00:00	≥ 12:45:00	0.541666667	0	E1A
0:00:00	13:30:00	≥ 13:15:00	0.5625	0	E1A
0:00:00	13:30:00	≥ 13:15:00	0.5625	0	E1A
0:00:00	14:00:00	≥ 13:45:00	0.583333333	0	E1A
0:00:00	14:00:00	≥ 13:45:00	0.583333333	0	E1A
0:00:00	14:30:00	≥ 14:15:00	0.604166667	0	E1A
0:00:00	14:30:00	≥ 14:15:00	0.604166667	0	E1A
0:00:00	15:00:00	≥ 14:45:00	0.625	0	E1A
0:00:00	15:00:00	≥ 14:45:00	0.625	0	E1A
0:00:00	15:00:00	≥ 14:45:00	0.625	0	E1E
0:00:00	15:30:00	≥ 15:15:00	0.645833333	0	E1E
0:00:00	16:00:00	≥ 15:45:00	0.666666667	0	E1E
0:00:00	16:30:00	≥ 16:15:00	0.6875	0	E1C
0:00:00	17:00:00	≥ 16:45:00	0.708333333	0	E1C
0:00:00	17:30:00	≥ 17:15:00	0.729166667	0	E1C
0:00:00	18:00:00	≥ 17:45:00	0.75	0	E1C
0:00:00	18:30:00	≥ 18:15:00	0.770833333	0	E1A
0:00:00	19:30:00	≥ 19:15:00	0.8125	0	E1C
0:00:00	16:00:00	≥ 15:45:00	0.666666667	0	E1C
0:00:00	12:30:00	≥ 12:15:00	0.520833333	0	E1C
0:00:00	13:00:00	≥ 12:45:00	0.541666667	0	E1E
		Total Penalty	21.16666667		

Figure 38. Time Window Non-Linearity in Excel

To find t_{ia} of the previous stop, the array of t_{ia}, highlighted in baby blue above in Figure 38 was required to be searched. In Excel this could not be done. Excel flagged this operation as a circular reference error and would not procced. To avoid this the “XLOOKUP” function was used after the lambda function in Excel was tried, but it still failed to properly apply the time windows constraint in the early piloting stages.

6.4.3 Machine Learning

In the future, it is recommended that machine learning be incorporated into the optimization process to solve the problem more efficiently and expand the scope of the project. Following the completion of this project, a suggestion is made for someone to further explore the integration of machine learning techniques with mixed integer linear programming to identify the most effective approach. Given that a TSP 40-node model includes 30 matrices and 41,600 variables, using more advanced software could potentially simplify the process compared to Excel, especially after the method has been proven to work for demand-responsive transit optimization. Excel was chosen for this thesis due to the constraints of expanding both the problem size and the software capabilities, which would have been too large an undertaking for this particular project. However, since the methodology has been successfully applied to various locations in New Brunswick, including models with up to 49,000 variables, it is suggested that the software used for implementation could be upgraded for future work.

6.4.4 Computability

Discussion with Dr. Erik Scheme during this research revealed the lack of computational power even within MATLAB. Within his area of work this prompts him and his team to use Python for their research. Three suggested ways to improve the computational processing power were a Lambda GPU server, Google Collab accounts and ACENET, all which could not be used for this research, as they are not compatible with Excel. For this reason, if this problem is expanded in the future, the solver must be compatible with an increase in computational power. Excel does not use the graphics card within a computer, only the processing power of the CPU and the RAM. If this problem is increased in the future using a solver that can fully utilize the extents of a computer should be a considered. After the 3-vehicle routing problem the variable loading time increased exponentially. Adding a fourth vehicle to the problem, resulted in an optimization run time greater than 24 hours. Although the computer computability was increased with a better CPU and RAM, the run time decreased, on the problems with 1 vehicle, 2 vehicle and 3 vehicle problems. This research did not compare the run time of the of the heuristic solutions versus exact solutions. The impact of the additional vehicle is directly related to the increase in possibilities of the vehicle assignment options. The increase in solving time is not solely related to the number of vehicles it can be the number of vehicles related to points services. The 5 Vehicle Routing Problem ran faster than the 4 Vehicle Routing Problem due to the complexity of vehicle assignment based on the nodes.

6.4.5 Christie Method

The Christie Method of Mixed Integer Linear Programming was utilized as the optimization technique for this study, chosen for its capacity to solve the Vehicle Routing Problem (VRP), which often relies on heuristic solution technique, into one that can be solved exactly. However, additional research is needed, particularly concerning the integration of time windows. Future applications for short-term modeling of on-demand paratransit services must incorporate time windows as additional constraints to ensure accurate service simulations. The current methodology leverages Excel with the OpenSolver add-in and the Gurobi solver engine to effectively manage these constraints. While the existing approach can resolve the precise parameters of the VRP, the inclusion of time windows will require enhanced computational capabilities when applying to larger problem sets, highlighting an important avenue for future research.

When applying the level of service constraints to the problem in Excel with OpenSolver, if the constraints could not be met by the number of vehicles the solver would fail, giving no justification. For this reason, the constraints were opened to start, they sometimes had to be expanded to 35-40 minutes, to find an optimal solution when experimenting with varying fleet. The list of pilots run with operational constraints are shown in Table 15. When using the Christie Method, the level of service constraint should begin at the least constraining value and then tighten the constraints as the problem successfully solves, in an iterative practice.

6.4.6 Heuristic vs Non-Heuristic Application

It is clear through the results that the non-heuristic methods take a long time to solve. The computability of an exact solution relies heavily on more constraints and more variables. Having a model take multiple days to solve is not feasible to be applied to community partners and in industry. For this reason, it is clear to see why heuristic methods dominate industry due to the ease and efficiency of use. In the future it would be interesting to see the relationship in computation time for exact solutions versus heuristic solutions. However using a heuristic method, it could be difficult to compare to how close it was to the optimal solution. There is no measure of how close the estimates are to the optimal solution, without already knowing the optimal solution. Although the heuristic measures are quicker than the Christie Method, they do not gauge how close they are to the optimal solution. Being within the feasible region of a solution offers varying results from solution to solution, not showing the exact optimal way to solve the vehicle routing problem. Comparing the heuristics approach versus the Christie Method to a problem does not always conclude the same results, within the feasible region (Christie, 2018).

It was shown that the Christie Method is solvable in polynomial time because the time it takes to find a solution increases at a rate that is manageable as the problem size grows, as shown by Figure 33. Solver Time and Model Time per Vehicle Routing Problem Size. However, the model set up time required to load the variables into the solver engine increased in non-polynomial time. The combination of time required for loading variables and solving makes this not an efficient industry solving method.

6.5 Summary

This research explored the optimization of on-demand paratransit services in small urban and rural areas, where established planning practices are often lacking compared to larger metropolitan regions. The study utilized Mixed Integer Linear Programming (MILP) and the Christie Method to determine fleet sizing and improve the efficiency of services in areas like Cap-Acadie and Moncton, New Brunswick. Despite the limited data, including the absence of trip purpose information.

Technical challenges arose with Excel's OpenSolver due to its computational limitations, leading to suggestions for more powerful programing solutions like MATLAB or Python for future work. This research also highlighted the need for further research, including incorporating trip purpose data, and expanding the model to consider multi-depot systems or non-emergent medical transportation services. The research demonstrated the feasibility of the Christie Method for optimizing fleet management in smaller regions but pointed to areas for future improvement in both modeling and computational efficiency.

7 CONCLUSIONS AND RECOMMENDATIONS

In conclusion this research aimed at finding a method to plan for paratransit in communities in New Brunswick. The goal of this research was to establish a method for route finding and fleet calibration in the setting up of paratransit services, using optimization. Partnership with Ability Transit allowed data to be collected over a six-month period, from January 2nd, 2023, to July 2nd, 2023. The trips offered by Ability Transit in Moncton were used as the basis of geographic analysis. The location and time of trips were collected with the purpose of applying the Christie Method to optimize selection of a fleet and a depot location. The data were used to calibrate a 40-node model in Moncton with the characteristics of the Ability Transit paratransit service. Once the 40-node model was calibrated to characteristics of the on-demand paratransit service on the Southeast Regional Service Commission, the model was applied to a regional area within the Southeast Regional Service Commission to determine the desired fleet size.

The results of the Ability Transit data showed that 5.4 trips per week per 1000 population are taken with Ability Transit, based on their current service levels when extrapolated to the population of Moncton (80,800). The Ability Transit data showed characteristics of an on-demand paratransit service in New Brunswick, to build an accurate model to optimize the area with paratransit based on the demand. The Christie Method was used to optimize the fleet for the Moncton area and then applied to a test case area within the Southeast Regional Service Commission. The application of the Christie Method pilot to Cap-Acadie showed the method useful.

The Cap-Acadie test case showed that the area could be serviced with one vehicle and meet the 30 min threshold.

The Christie Method was applied to a TSP and multiple VRPs throughout the Moncton area, the model used synthesized data based on the descriptive statistics provided by Ability Transit. The model was calibrated using the information provided by Ability Transit, then applied to the region, to find the optimal service and routing of the area. Once successfully applied to the Moncton region, it was applied to one of the 12 areas within the Southeast Regional Service Commission, Cap-Acadie. The optimal routing of the vehicles was applied to the trips occurring throughout that area, based on the trip rates for on demand paratransit found through the analysis of Ability Transit's data. The trip origin and destination locations were based on State of the Region Transportation Report by & Hanson et al.

7.1 Recommendations for Continued Research

The data from Ability Transit, used to calibrate the model did not include the trip purpose. Applying the reason for the trip purpose to each trip could be helpful in deciding the tour of the trip in the model. In future research applications of trip reasoning could be helpful in the decision-making process. It would be best if future research were done using a data source that included more than time, and place for each trip could better provide statistics to calibrate the model. The lack of trip purpose reduced the ability to classify trip tours. In conversation with Ability Transit, it was confirmed that users often only make one destination per trip, for which reason tours were not considered in this model. If exploring non-emergency medical transport, investigating the relation of trip tours could be a promising area of research with this method of modeling.

The Christie method can be used in all situations where the vehicles start at an assigned depot and pick up/drop off individuals and varying nodes, all to return back to the same initial depot. In the future continued research could be done in varying languages such as Python to apply this method with more constraints. This method can be used in continued research for on demand services, delivery services, and transport services. In the future continued research into the application of the Christie Method in varying programming languages could be possible with the outlined method of this research. Possible next steps for the use of this method could include non-emergency medical transportation routing in New Brunswick, a branch of research not conducted in this thesis research.

The application of time windows within the optimization program in Excel brought challenges to the linearity of the problem setup. Exploring the use of time windows when optimizing for both the routing and scheduling of the problem could be applied in another format, other than those discussed in Chapter 4, in Excel to avoid the circular referencing error received when trying to search an array for the variable used in the time window constraints t_{ia}.

One example was provided of the application of the Christie Method for Mixed Integer Linear Programming to Cap-Acadie, an area within the Southeast Regional Service

Commission to test the method in the application of fleet sizing. This example of the method could then be applied to the 11 other areas within the Southeast Regional Service Commission.

The model appears to reasonably reflect the level of service and capacity of Ability Transit In the future it would be important to try and adapt this model to provide 100% accuracy with the real-world events and environments. In the models of the VRP, zones emerge within the model itself. In future research with the Christie Method, it would be interesting to model multiple depots interconnecting a large service area, where multiple vehicles are staged at multiple depots, and how this would impact the overall active vehicle time for better or for worse.

7.2 Impact of Research

This research aimed to enhance the transportation planning process for deploying regional on-demand accessible Demand Response Transportation (DRT) services by piloting a demand estimation approach combined with the novel Christie Method for vehicle routing and scheduling. Using the Southeast Region of New Brunswick as a test case, the study documented standard planning practices in the industry, identified techniques for estimating demand at the regional level, and assessed various service delivery models and scenarios through an operations research approach. This research contributes to the field of engineering by addressing key challenges in transportation planning, specifically in the context of regional on-demand accessible DRT services. By developing and piloting a demand estimation approach coupled with an innovative vehicle routing and fleet optimization the study provides engineers with valuable tools to optimize the deployment and efficiency of accessible transportation systems in rural and urban communities.

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APPENDIX A EXCEL MACROS CODE

```
Sub Merge_Sheets ()
Dim startRow, startCol, lastRow, lastCol As Long
Dim headers As Range

'Set Master sheet for consolidation.
Set mtr = Worksheets("Master")

Set wb = ThisWorkbook
'Get Headers
Set headers = Application.InputBox("Select the Headers", Type:=8)

'Copy Headers into master
headers.Copy mtr.Range("A1")
startRow = headers.Row + 1
startCol = headers.Column

Debug.Print startRow, startCol
'loop through all sheets
For Each ws In wb.Worksheets
   'except the master sheet from looping
   If ws.Name <> "Master" Then
     ws.Activate
     lastRow = Cells(Rows.Count, startCol).End(xlUp).Row
     lastCol = Cells(startRow, Columns.Count).End(xlToLeft).Column
     'get data from each worksheet and copy it into Master sheet
     Range(Cells(startRow, startCol), Cells(lastRow, lastCol)).Copy _
     mtr.Range("A" & mtr.Cells(Rows.Count, 1).End(xlUp).Row + 1)
```

```
        End If
Next ws

Worksheets("Master").Activate

End Sub
```

Tj Excel macros code

```
Sub TracePath()
   Dim ws As Worksheet
   Dim fromNode As String
   Dim currentRow As Integer
   Dim currentColumn As Integer
   Dim nextNode As String
   Dim path As String
   Dim lastRow As Integer
   Dim lastColumn As Integer
   Dim i As Integer

   ' Set your worksheet
   Set ws = ThisWorkbook.Sheets("Sheet1") ' Change "Sheet1" to your actual sheet
name

   ' Set the range boundaries
   lastRow = ws.Cells(ws.Rows.Count, "CJ").End(xlUp).Row ' Find the last row of
data
   lastColumn = ws.Cells(137, ws.Columns.Count).End(xlToLeft).Column ' Find the
last column
```

```
' Ask user to input the starting node
fromNode = InputBox("Enter the starting node (e.g., NodeA):")

' Find the starting row based on the fromNode
currentRow = Application.Match(fromNode, ws.Range("CJ137:CJ" & lastRow), 0) +
136 ' Add 136 because row 137 is the start
If IsError(currentRow) Then
  MsgBox "Starting node not found!", vbExclamation
  Exit Sub
End If

' Initialize path as starting point
path = fromNode & " -> "

' Loop to trace the path
Do
  ' Search for a "1" in the current row (which indicates a path)
  For currentColumn = 134 To lastColumn ' Start from column CK (column 134)
    If ws.Cells(currentRow, currentColumn).Value = 1 Then
      ' Find the corresponding "To" node
      nextNode = ws.Cells(137, currentColumn).Value ' Row 137 contains "To"
nodes
      path = path & nextNode & " -> "

      ' Move to the next "From" node (current "To" node)
      currentRow = Application.Match(nextNode, ws.Range("CJ137:CJ" &
lastRow), 0) + 136
      If IsError(currentRow) Then
        MsgBox "End of path reached or invalid next node!", vbInformation
```

```
            Exit Sub
          End If

          Exit For
        End If
      Next currentColumn

      ' If no path is found (no "1" in the current row), end the loop
      If currentColumn > lastColumn Then
        MsgBox "No more paths found!", vbInformation
        Exit Sub
      End If
    Loop

    ' Remove last arrow (" -> ")
    path = Left(path, Len(path) - 4)

    ' Output the path
    MsgBox "Path: " & path, vbInformation
End Sub
```

APPENDIX B TRIPS BY DISSEMINATION AREA

DA	Pop/ Km2	Origin/ Capita	Destination/ Capita	Urban / Rural
13060007	410.4	0.00	0.00	Urban
13060010	2398.2	0.01	0.01	Urban
13060012	1549.5	0.00	0.00	Urban
13060014	624.6	0.01	0.01	Urban
13060019	1786.4	0.05	0.05	Urban
13060028	114.8	0.01	0.01	Rural
13060030	1805.9	0.04	0.01	Urban
13060031	1505.6	0.01	0.01	Urban
13060032	916.3	0.00	0.00	Urban
13070033	255.3	0.10	0.10	Rural
13070034	729.3	0.01	0.01	Urban
13070035	334.6	0.01	0.01	Rural
13070036	499.6	0.03	0.03	Urban
13070037	1844.7	0.02	0.02	Urban
13070039	1497.58	0.03	0.02	Urban
13070040	789.8	0.06	0.06	Urban
13070041	2170.4	0.06	0.06	Urban
13070043	499.6	0.00	0.00	Urban
13070044	1340.5	0.83	0.82	Urban
13070045	1816.7	0.12	0.12	Urban
13070046	3206.8	0.04	0.05	Urban
13070047	1975.9	0.26	0.26	Urban

13070049	370.5	0.01	0.01	Rural
13070051	2893.7	0.00	0.00	Urban
13070052	1652	0.03	0.02	Urban
13070055	1762.2	0.01	0.01	Urban
13070056	2893.7	0.14	0.14	Urban
13070057	6216.2	0.02	0.02	Urban
13070058	6783.4	0.02	0.00	Urban
13070059	2845	0.34	0.36	Urban
13070066	754.7	0.23	0.20	Urban
13070067	674.8	0.10	0.10	Urban
13070068	1636.2	3.64	3.64	Urban
13070069	3169.4	0.01	0.01	Urban
13070070	5283.8	0.03	0.03	Urban
13070071	4294.5	0.02	0.02	Urban
13070072	2797.9	0.03	0.01	Urban
13070073	4745.6	0.05	0.05	Urban
13070074	2775.1	0.02	0.04	Urban
13070075	3304.7	0.02	0.02	Urban
13070077	5870.7	0.14	0.14	Urban
13070078	1723.4	0.94	0.93	Urban
13070079	3327.3	0.29	0.29	Urban
13070080	3864.7	0.07	0.07	Urban
13070081	2781.7	0.07	0.07	Urban
13070082	2934.2	0.12	0.12	Urban
13070083	4698.3	0.14	0.14	Urban

13070084	4326.8	0.00	0.00	Urban
13070088	1411.6	0.03	0.03	Urban
13070089	2873.8	0.00	0.00	Urban
13070090	2554.6	0.06	0.06	Urban
13070093	2264.8	0.44	0.44	Urban
13070094	1952.7	0.15	0.15	Urban
13070095	4698.7	0.01	0.01	Urban
13070096	1063.6	0.07	0.07	Urban
13070097	2764.8	0.22	0.22	Urban
13070099	1702.9	0.20	0.20	Urban
13070100	194.9	0.07	0.07	Rural
13070101	1438.4	0.01	0.01	Urban
13070102	2170.9	0.09	0.10	Urban
13070105	2140.6	0.04	0.03	Urban
13070106	2108.7	0.01	0.01	Urban
13070107	1772.7	0.05	0.05	Urban
13070109	1577.1	0.14	0.14	Urban
13070110	2441.5	0.06	0.06	Urban
13070113	1462.3	0.01	0.01	Urban
13070114	432.9	0.06	0.06	Urban
13070115	492.6	0.24	0.24	Urban
13070116	1898.5	0.64	0.62	Urban
13070117	2061.9	0.23	0.23	Urban
13070118	2677.7	0.13	0.13	Urban
13070119	4385.6	0.27	0.27	Urban

13070122	6473.4	1.05	1.20	Urban
13070123	4166.7	0.06	0.06	Urban
13070124	4809.6	0.04	0.04	Urban
13070125	3012.4	0.02	0.02	Urban
13070127	4886.5	0.03	0.03	Urban
13070128	5194.057	0.23	0.23	Urban
13070129	4533.5	0.02	0.02	Urban
13070130	3748.9	0.10	0.10	Urban
13070131	1211.3	0.01	0.01	Urban
13070132	2003.4	0.01	0.01	Urban
13070133	321.1273	0.85	0.86	Rural
13070134	1099.6	0.26	0.29	Urban
13070135	2562.5	0.08	0.08	Urban
13070136	3040.1	0.00	0.00	Urban
13070137	2481	0.09	0.10	Urban
13070138	2877.6	0.00	0.00	Urban
13070139	2073.7	0.03	0.13	Urban
13070140	2033.1	0.00	0.00	Urban
13070141	3032.9	0.10	0.10	Urban
13070142	630.1	0.17	0.18	Urban
13070143	4259.6	0.63	0.64	Urban
13070146	374.7	0.11	0.11	Rural
13070147	2575.8	0.01	0.01	Urban
13070175	507.9	0.01	0.00	Urban
13070180	1335.4	0.00	0.00	Urban

13070182	505.2	0.00	0.00	Urban
13070225	4703.7	0.02	0.02	Urban
13070226	48	0.38	0.47	Rural
13070227	1112.786	0.05	0.08	Urban
13070256	82.8	0.01	0.01	Rural
13070257	54.3	0.17	0.18	Rural
13070260	20.5	0.01	0.01	Rural
13070261	19	0.00	0.00	Rural
13070275	3.8	0.00	0.00	Rural
13070277	39.3	0.27	0.19	Rural
13070278	1628.9	0.08	0.08	Urban
13070321	647.3	0.23	0.23	Urban
13070322	2587	0.04	0.04	Urban
13070323	928.5	0.22	0.23	Urban
13070325	2463.8	0.01	0.01	Urban
13070326	2008.733	0.00	0.00	Urban
13070327	2902.7	0.01	0.01	Urban
13070328	3023.6	0.15	0.15	Urban
13070329	2898.6	0.10	0.07	Urban
13070330	1904.8	0.01	0.01	Urban
13070331	1329.3	0.00	0.00	Urban
13070354	30.1	0.05	0.05	Rural
13070355	25.3	0.05	0.05	Rural
13070036,	499.6	0.00	0.00	Urban

APPENDIX C EXCEL SHEETS

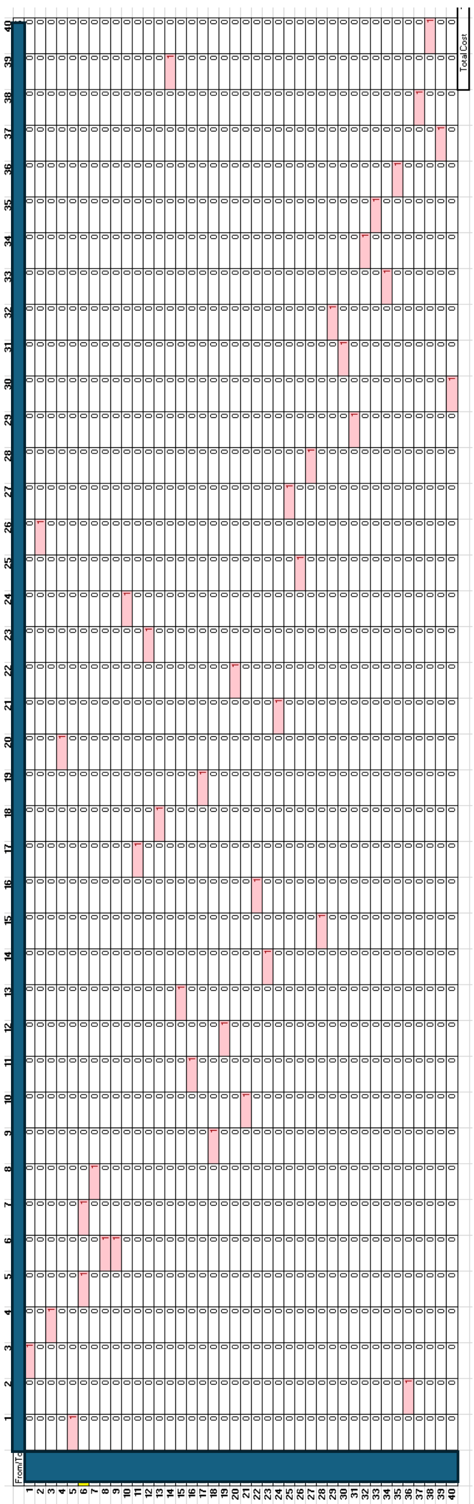

Figure 39. TSP Solution Matrix

Vehi	E1A0G3	E1A0H4	E1A1M1	E1A1M2	E1A2T7	E1A2V1	E1A2V5	E1A2V6	E1A5J8	E1A5L1	E1A5S3	E1A5S4	E1A7H4	E1A7J4	E1A7R1	E1A7R4	E1A8Y8	E1A9G6	E1A9J8	E1A9K3	E1A9T4	E1A9T5	E1A9T8	E1A9V4	E1A9V6	E1A9V7	E1A9V8	E1A9W5	E1B0E4	E1B0H9	E1B2X8	E1B5R6	E1C0B3	E1C0E6	E1C0E7	E1C0L7	E1G1A9	E1G1B1	E1H2W2	E1G5X5
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
36	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0

Figure 40. Vehicle A Solution Matrix 2VRP

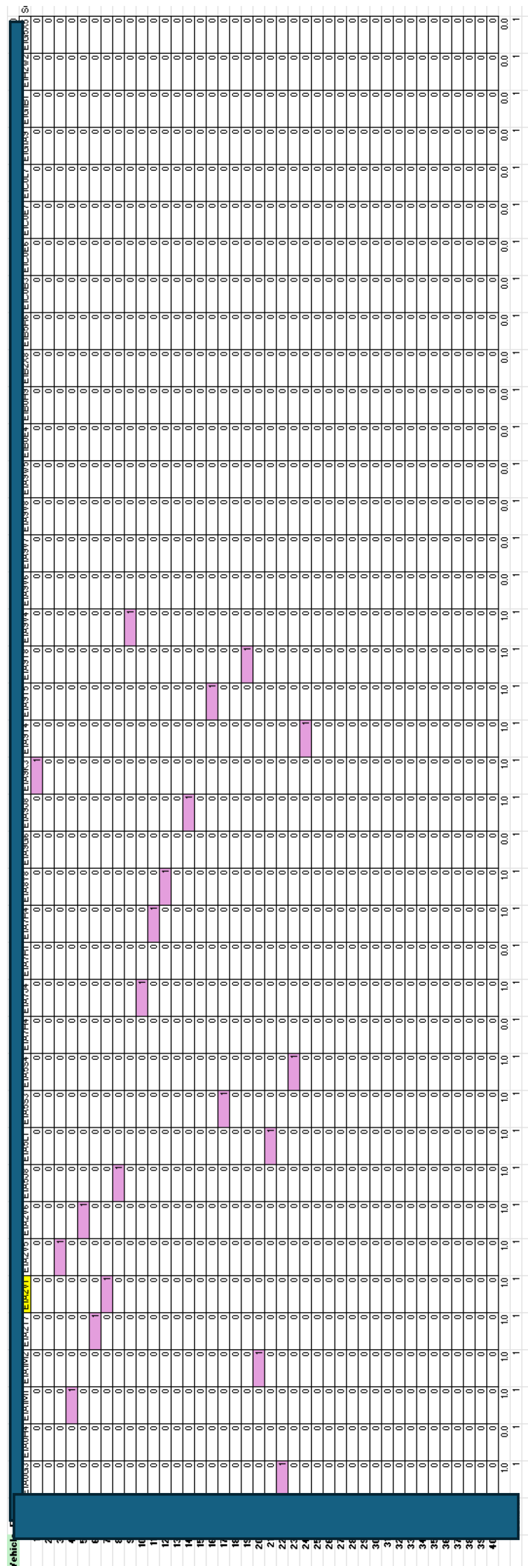

Figure 41. Vehicle B Solution Matrix 2VRP

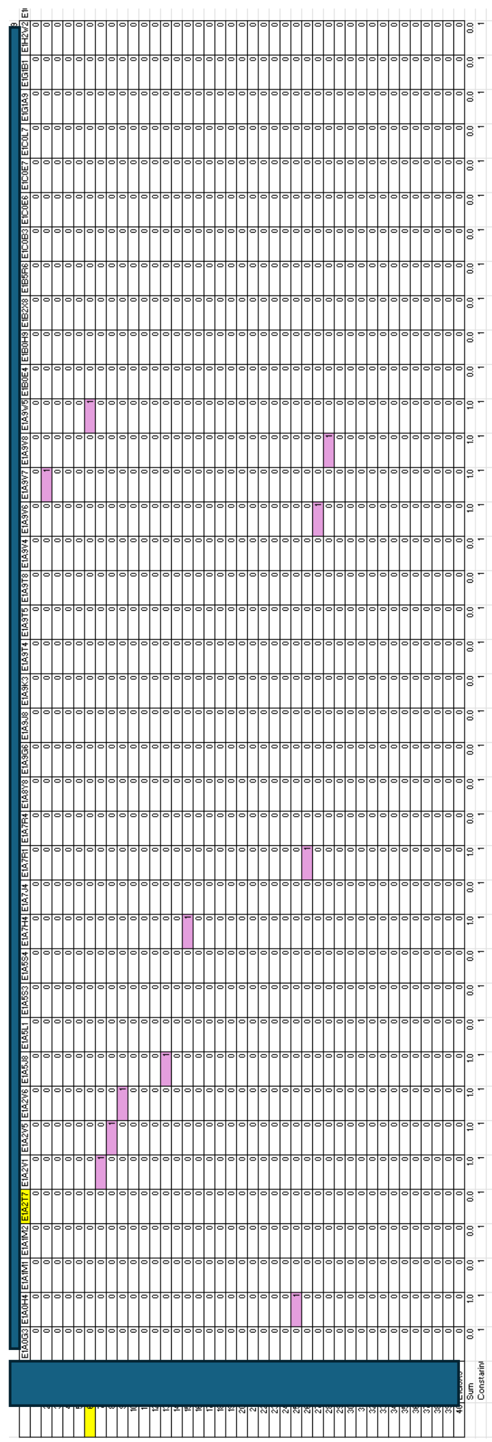

Figure 42. Vehicle A Solution Matrix 3VRP

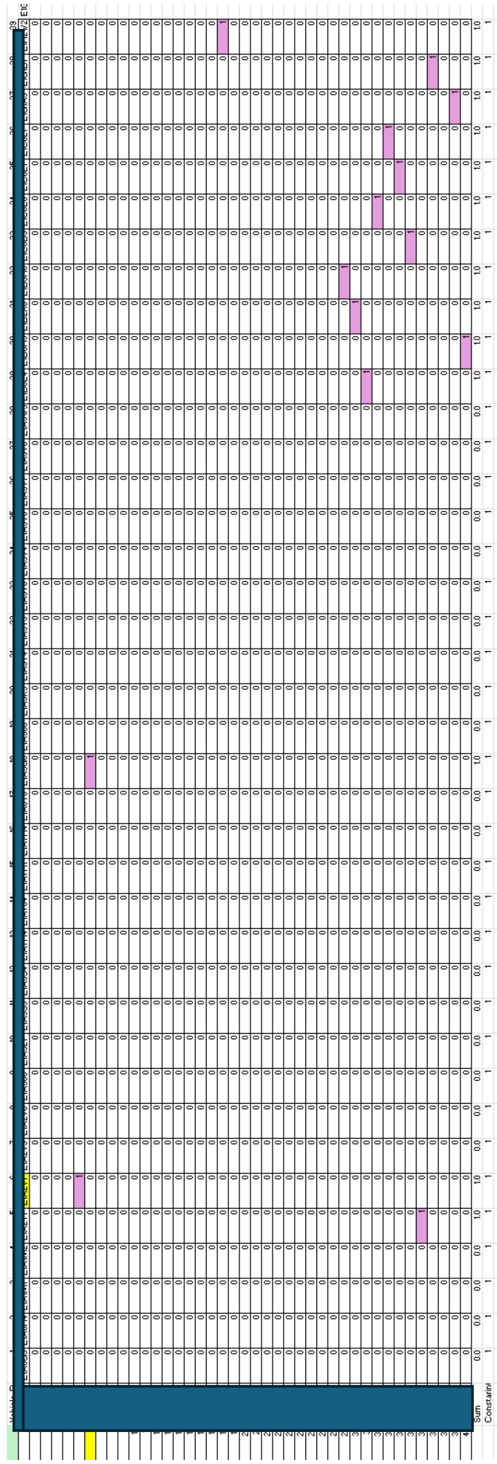

Figure 43. Vehicle B Solution Matrix 3VRP

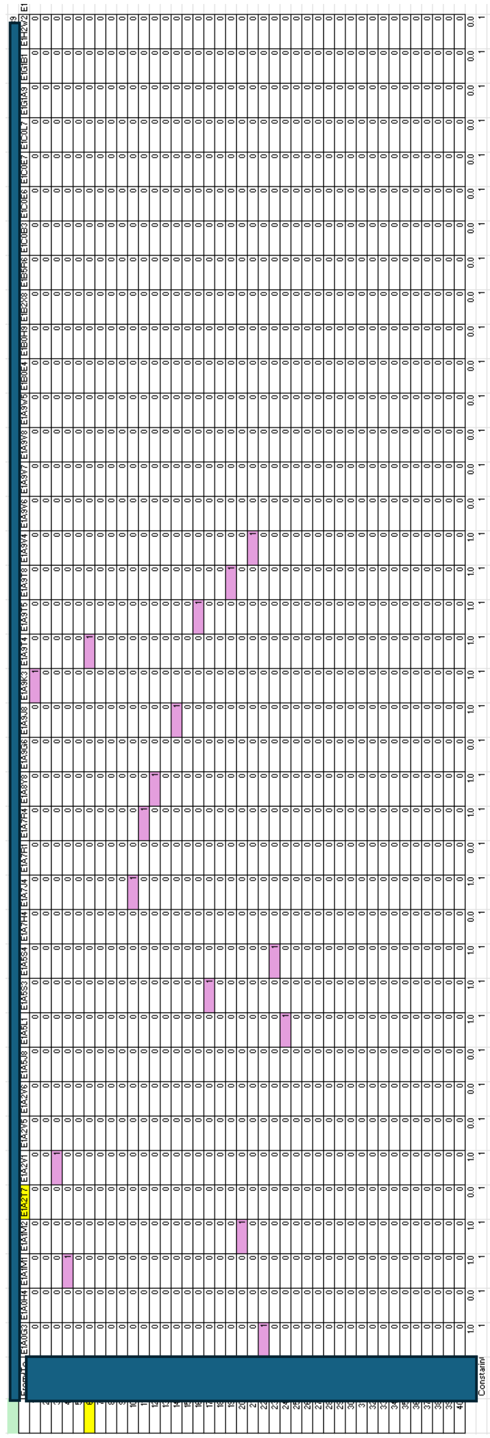

Figure 44 Vehicle C Solution Matrix 3VRP

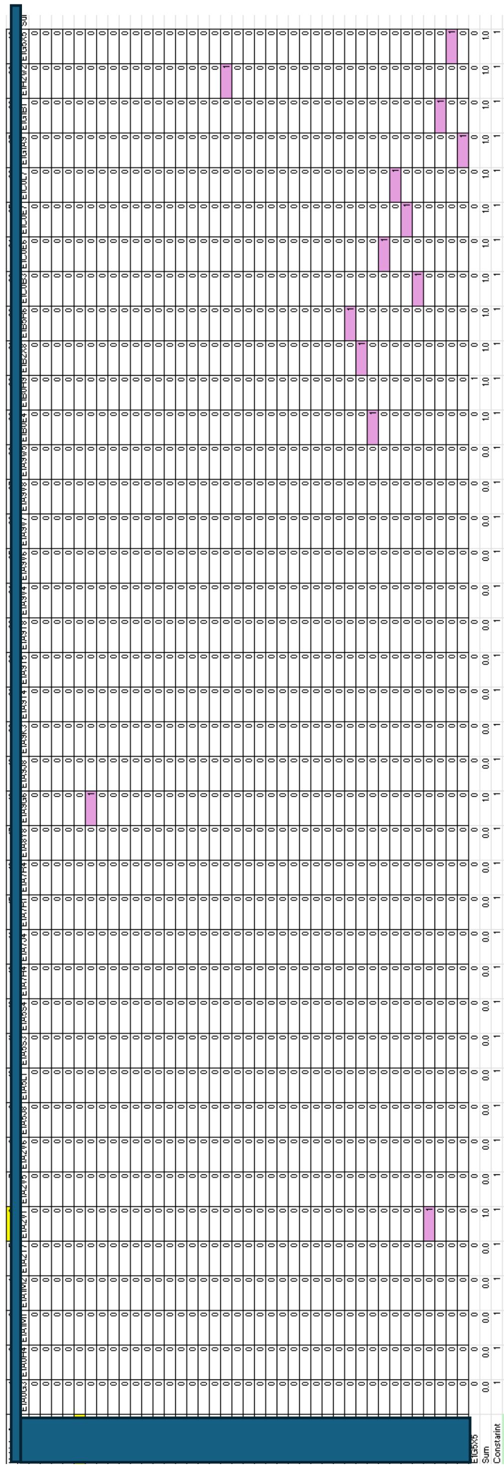

Figure 45. Vehicle A Solution Matrix 4VRP

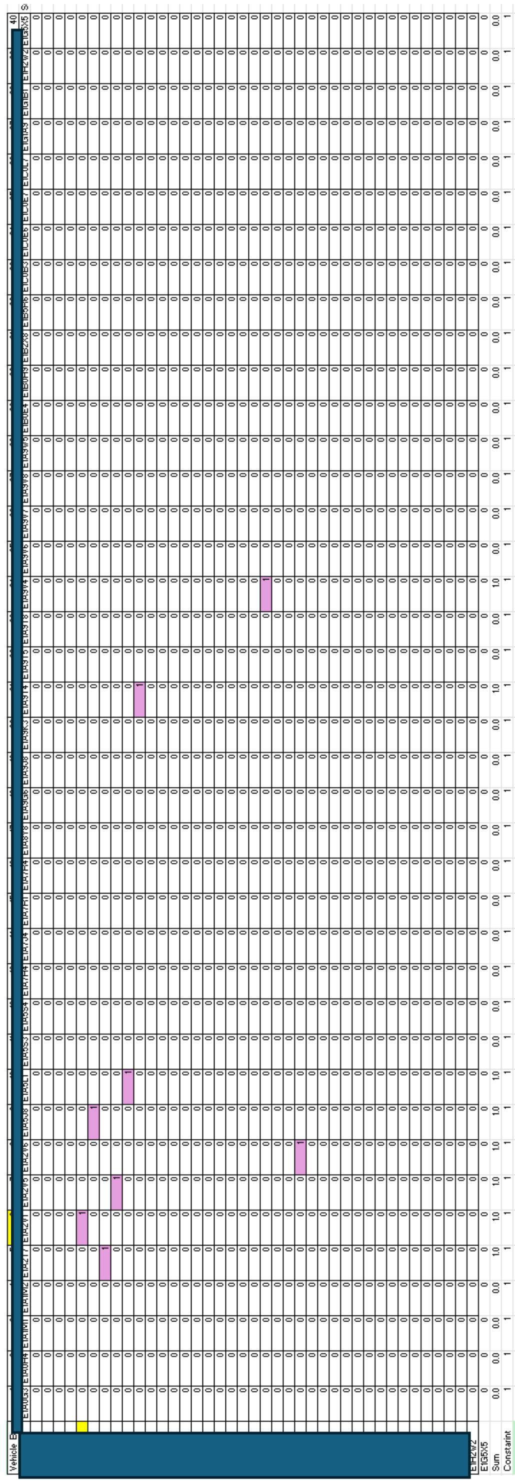

Figure 46. Vehicle B Solution Matrix 4VRP

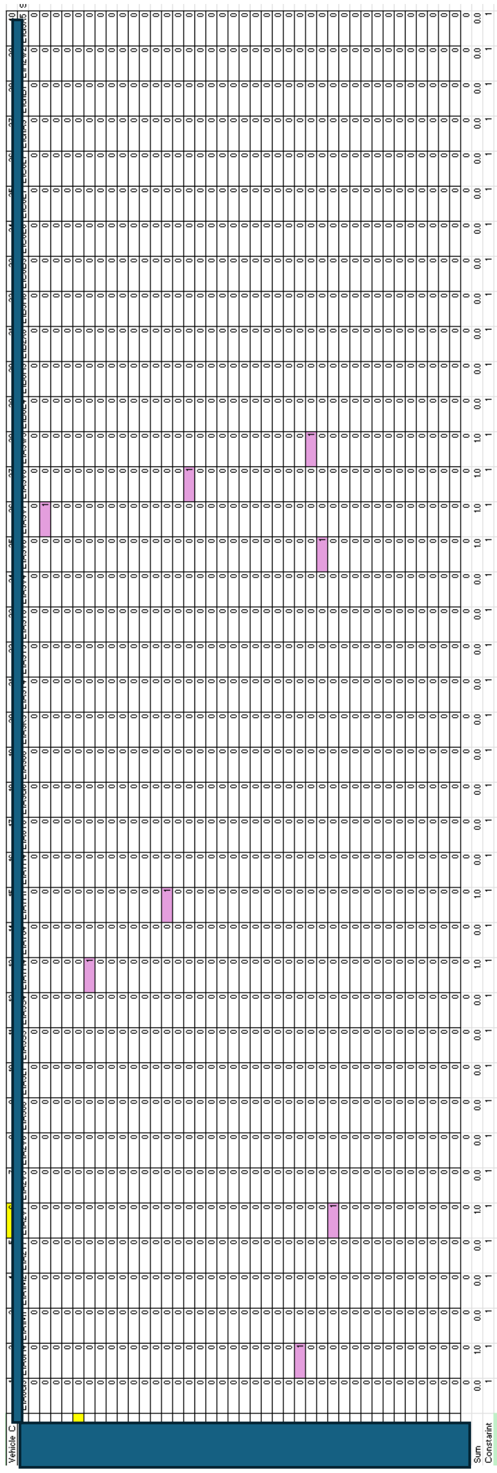

Figure 47 Vehicle C Solution Matrix 4VRP

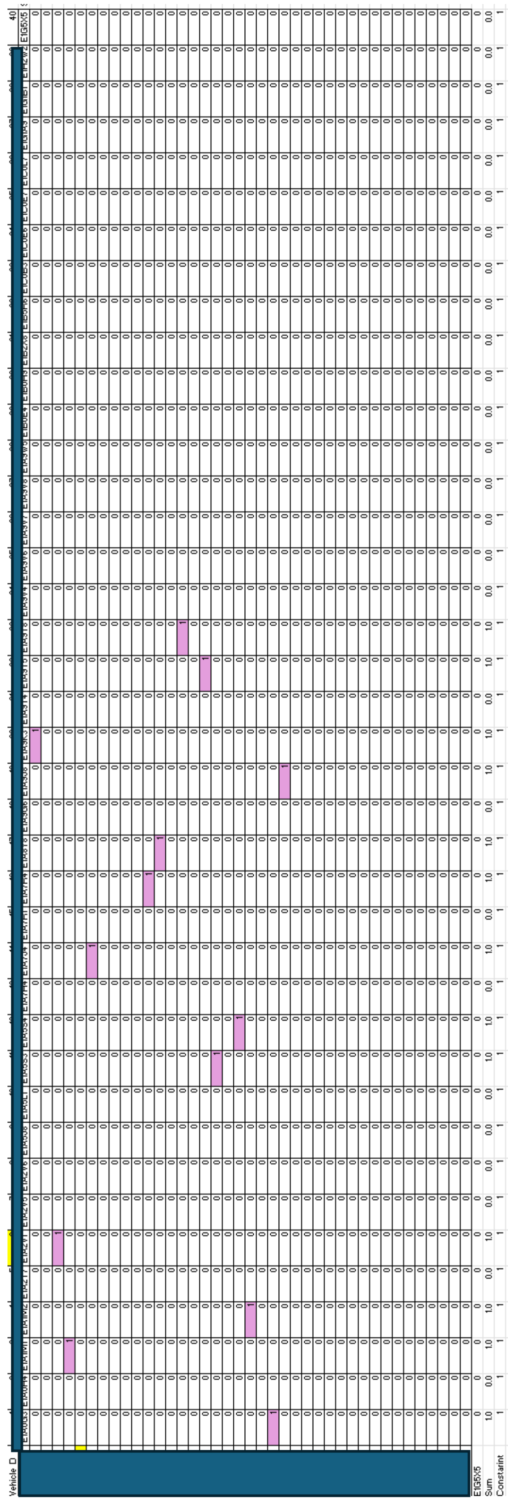

Figure 48 Vehicle D Solution Matrix 4VRP

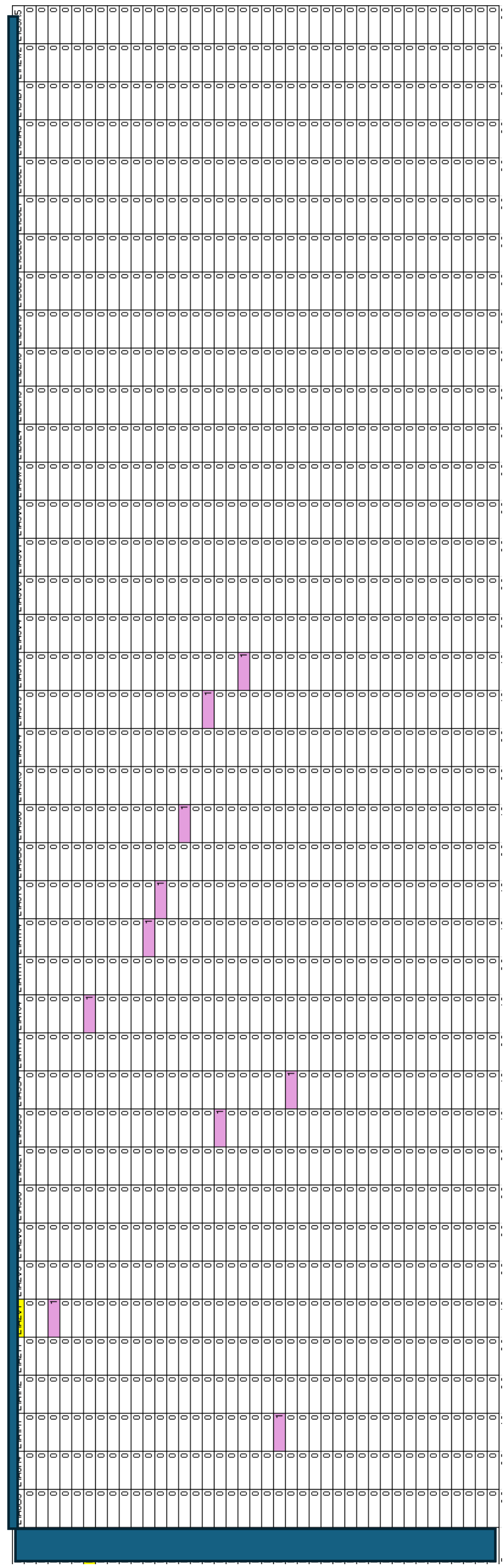

Figure 49 Vehicle A Solution Matrix 5VRP

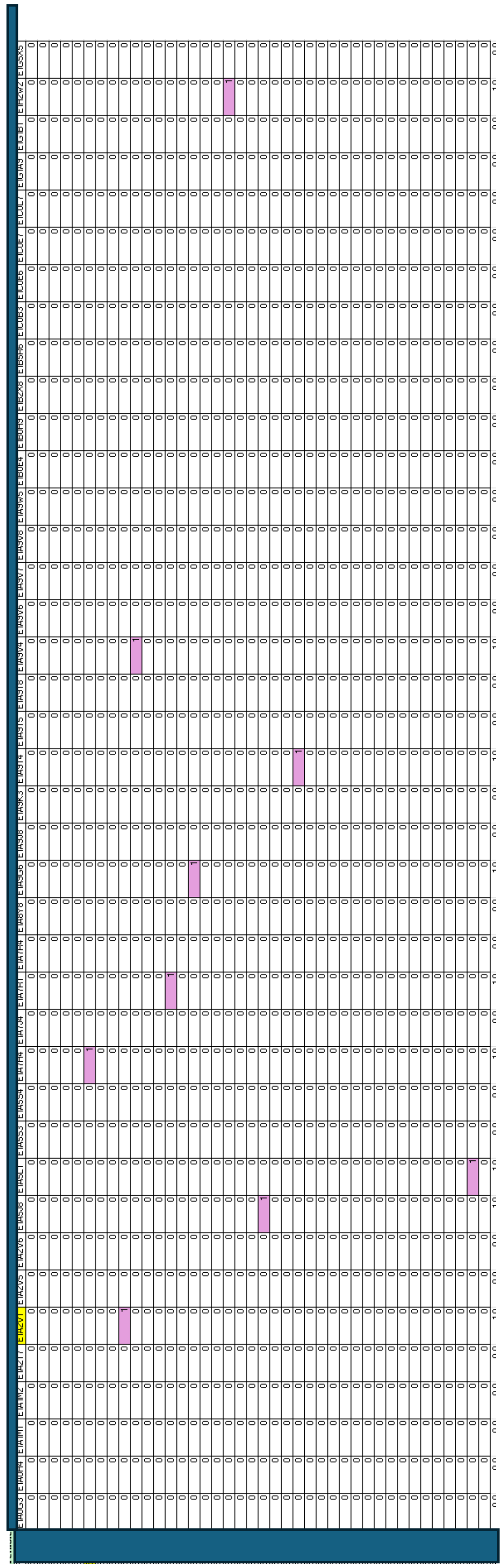

Figure 50 Vehicle B Solution Matrix 5VRP

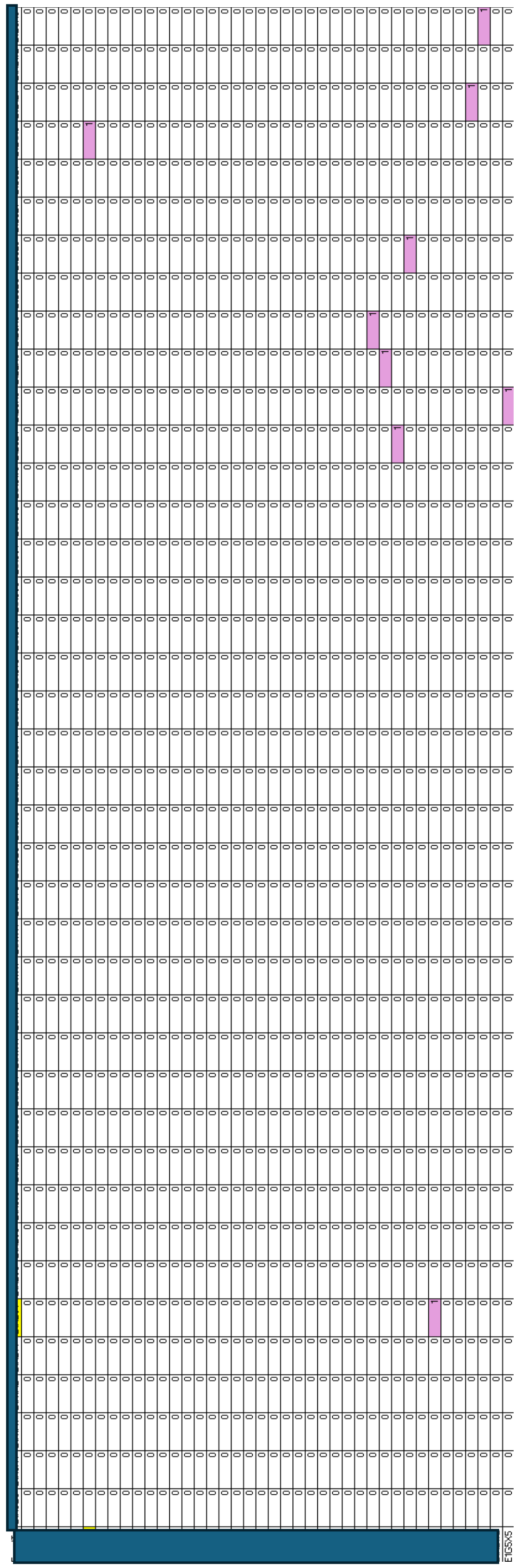

Figure 51. Vehicle C Solution Matrix 5VRP

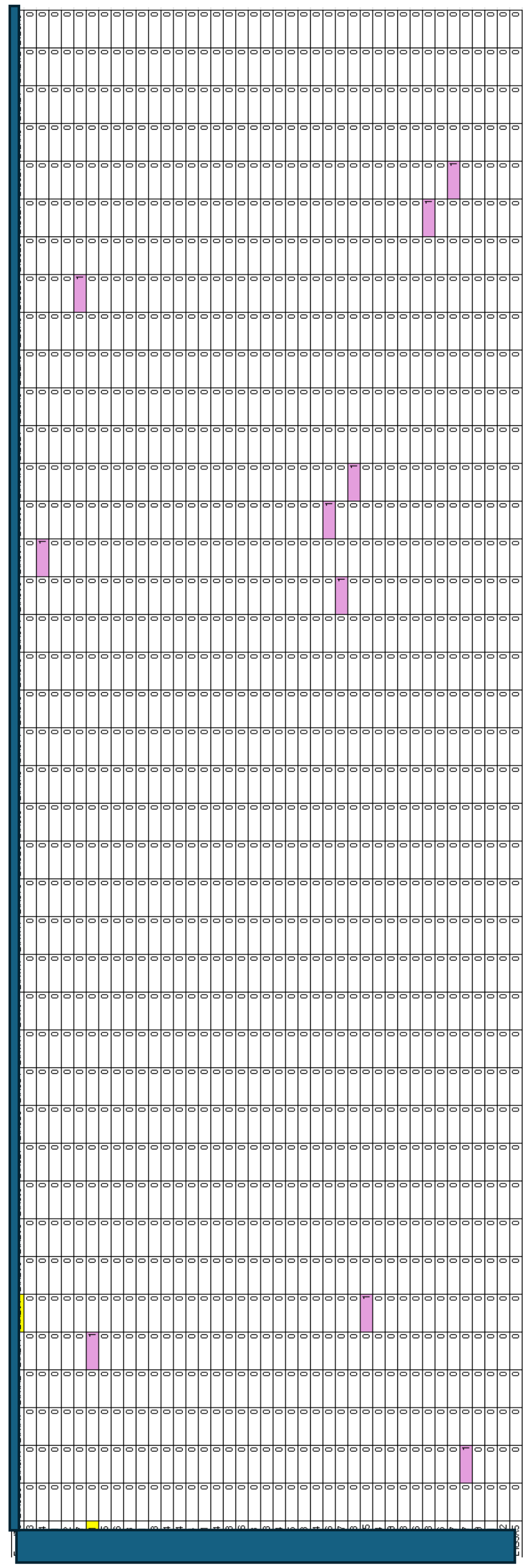

Figure 52 Vehicle D Solution Matrix 5VRP

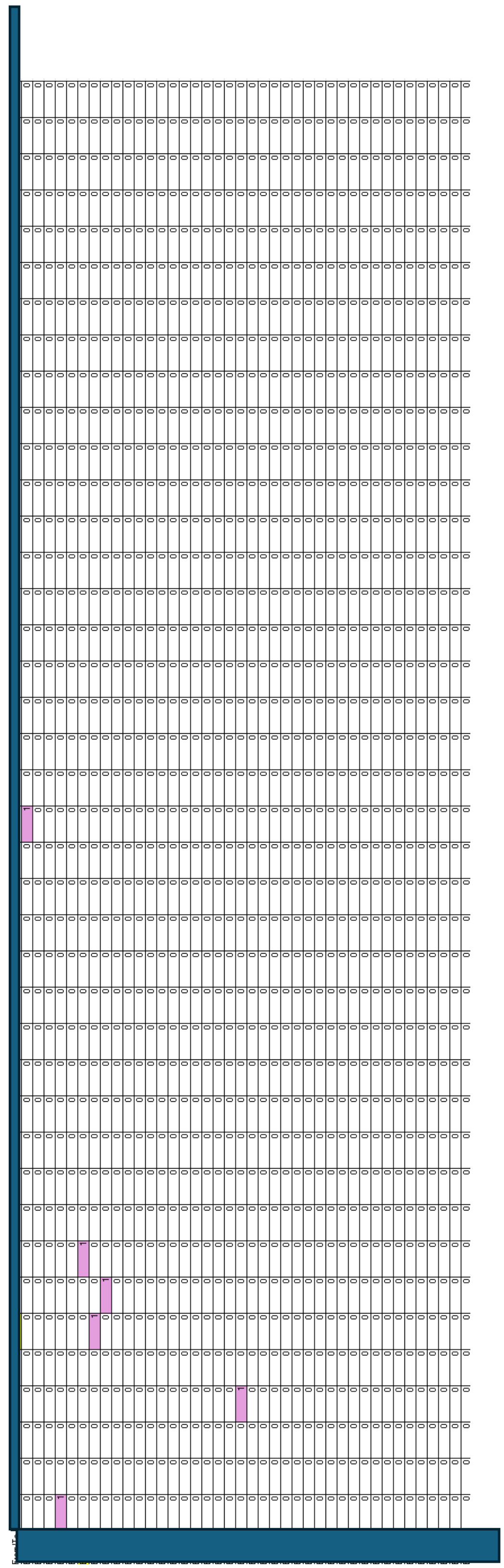

Figure 53 Vehicle E Solution Matrix 5VRP

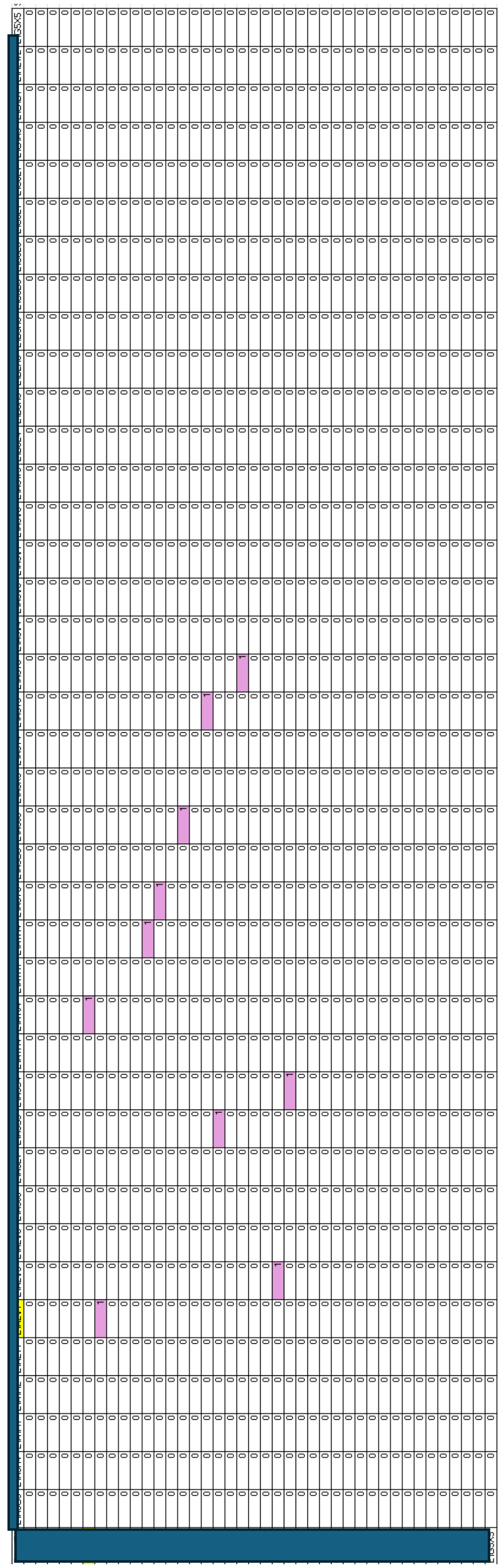

Figure 54 Vehicle A Solution Matrix 6VRP Part 1

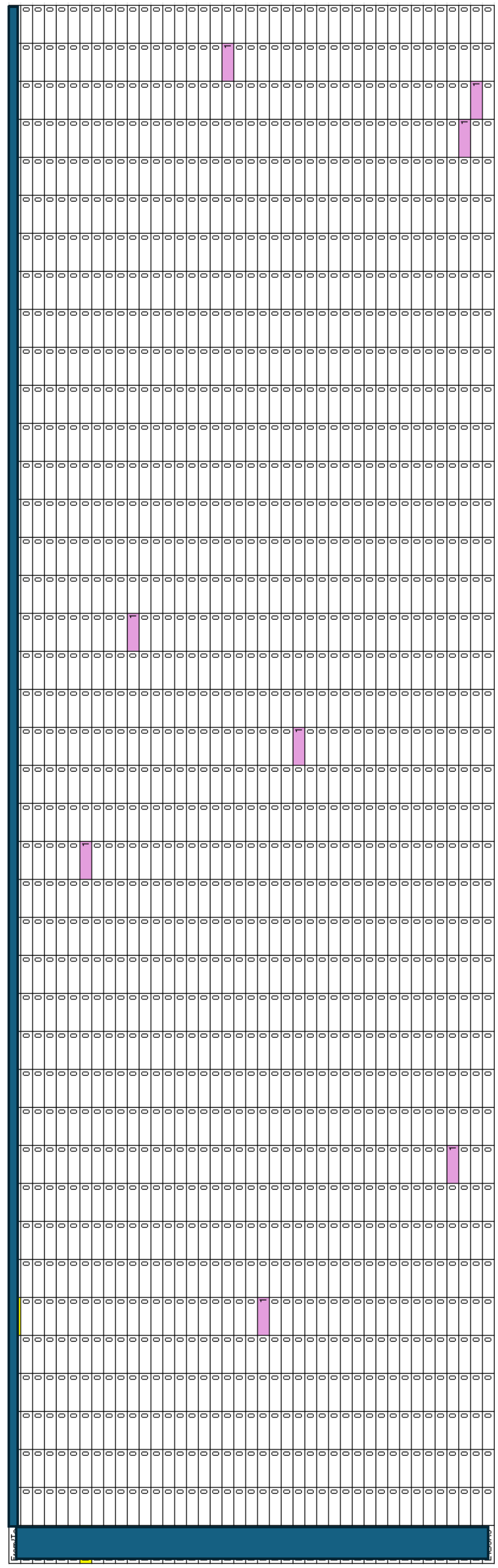

Figure 55 Vehicle B Solution Matrix 6VRP Part 1

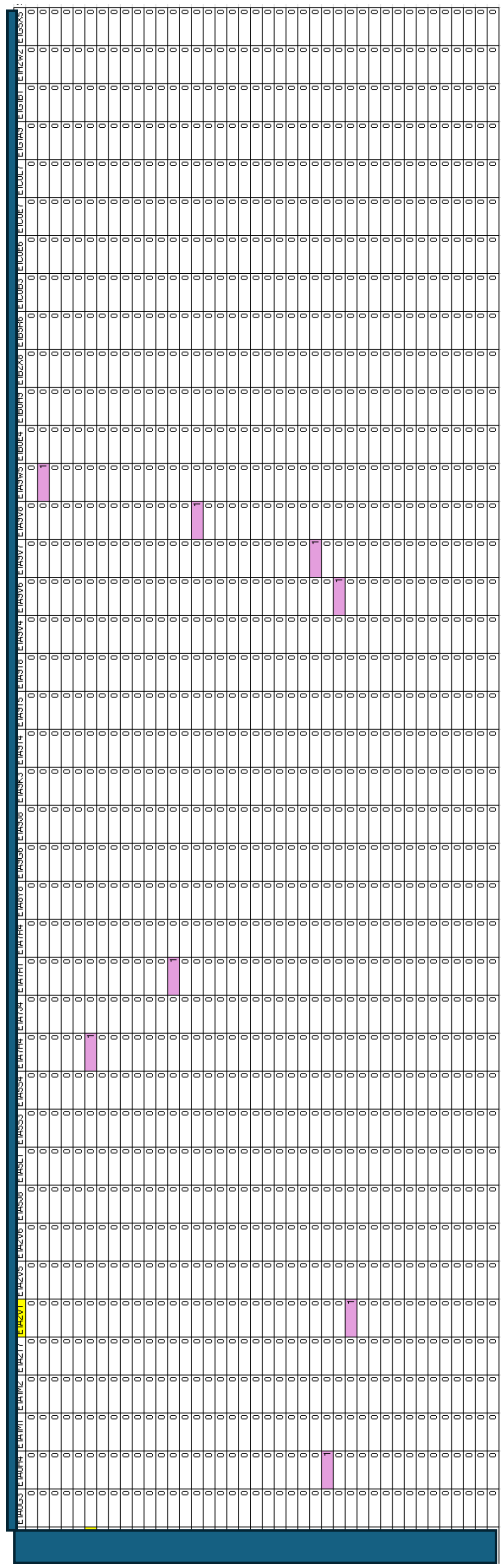

Figure 56 Vehicle C Solution Matrix 6VRP Part 1

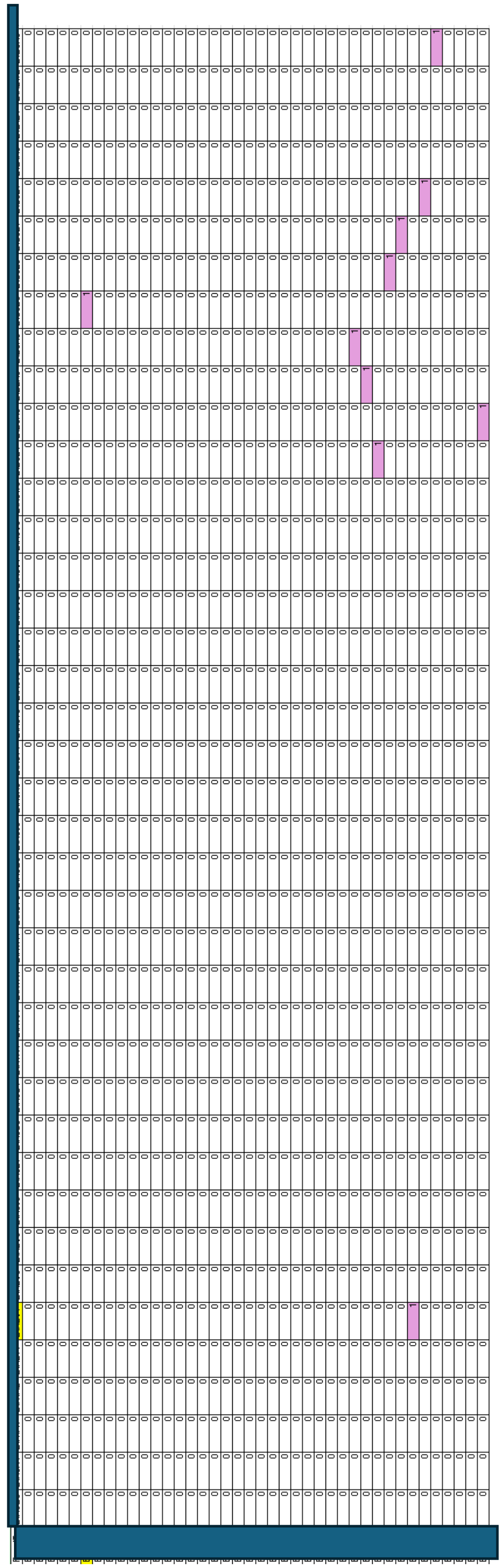

Figure 57 Vehicle D Solution Matrix 6VRP Part 1

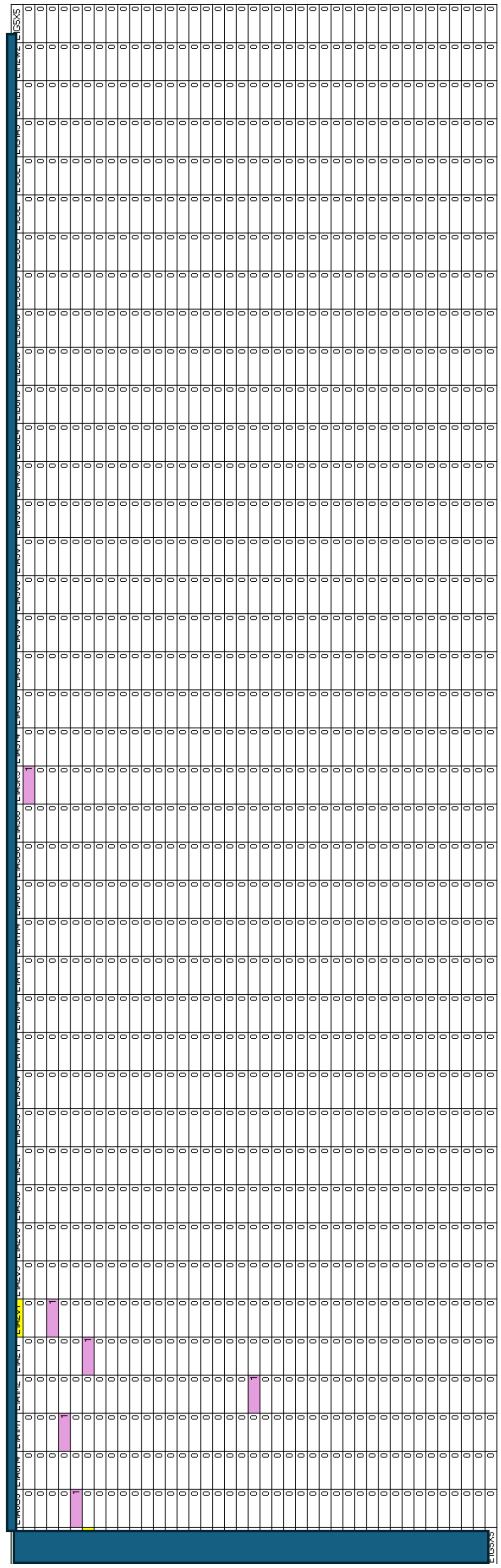

Figure 58 Vehicle E Solution Matrix 6VRP Part 1

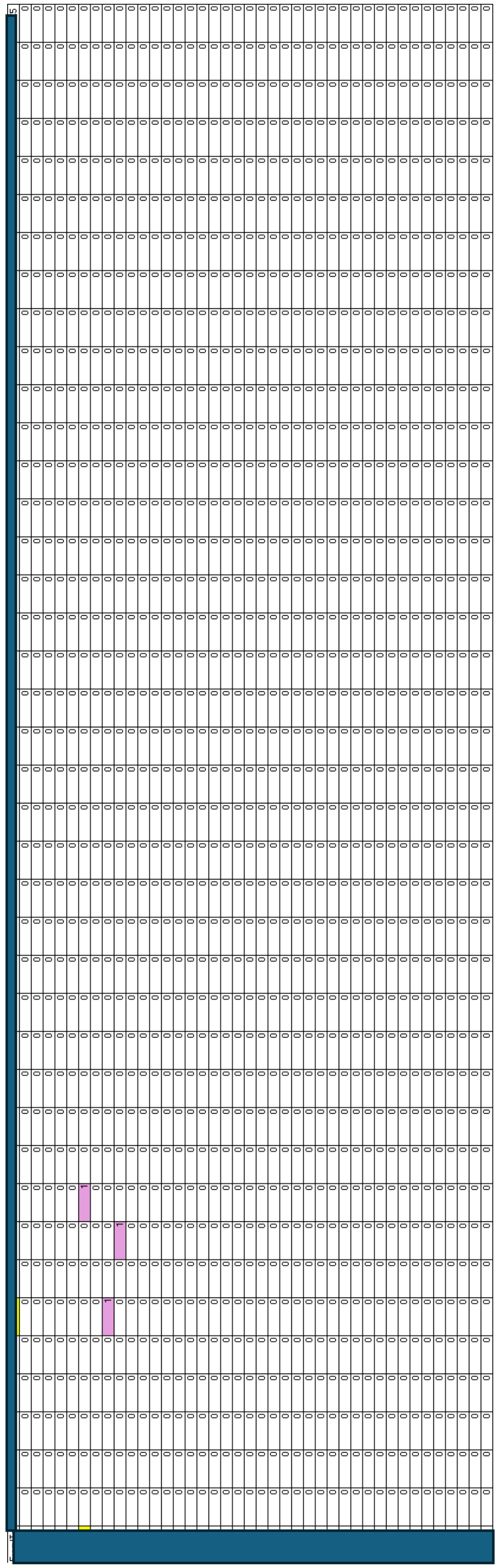

Figure 59 Vehicle F Solution Matrix 6VRP Part 1

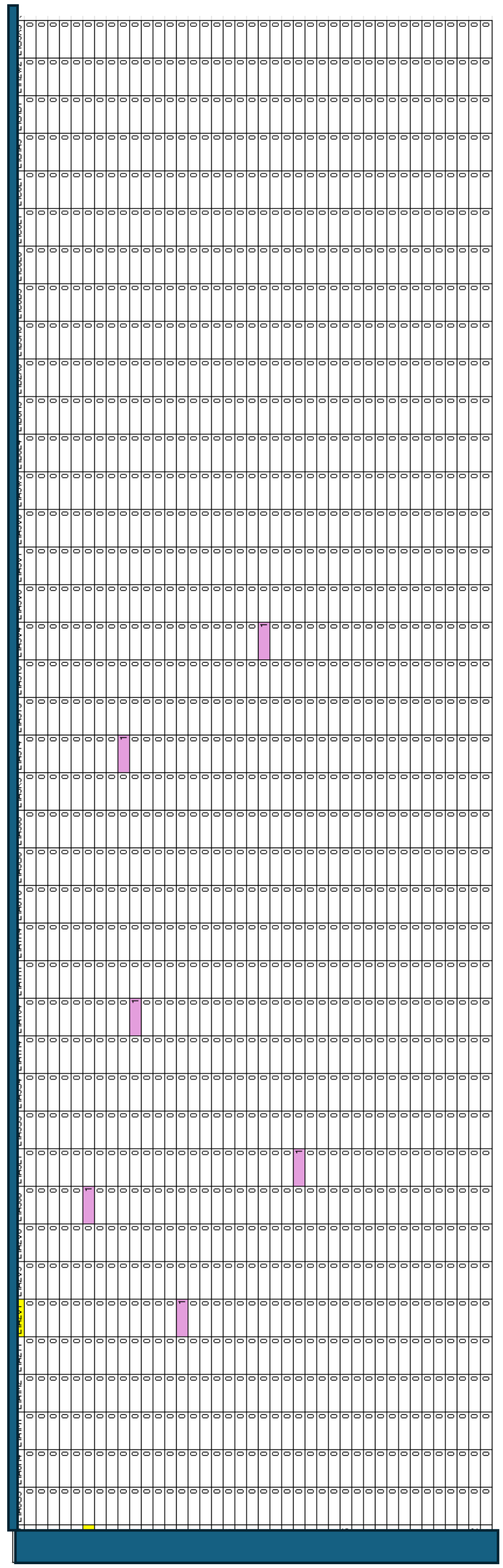

Figure 60. Vehicle A Solution Matric 6VRP Part 2

F	E1A0G3	E1A0H4	E1A1M1	E1A1M2	E1A2T7	E1A2V1	E1A2V5	E1A2V6	E1A3J8	E1A3L1	E1A3S3	E1A3S4	E1A7H4	E1A7J4	E1A7R1	E1A7R4	E1A8Y8	E1A9G6	E1A9J8	E1A9K3	E1A9T4	E1A9T5	E1A9T8	E1A9V4	E1A9V6	E1A9V7	E1A9V8	E1A9W5	E1B0E4	E1B0H5	E1B2X8	E1B5R6	E1C0B5	E1C0E6	E1C0E7	E1C0L7	E1G1A5	E1G1B1	E1H2W2	E1G5X5
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E1G5X5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 61 Vehicle B Solution Matrix 6VRP Part 2

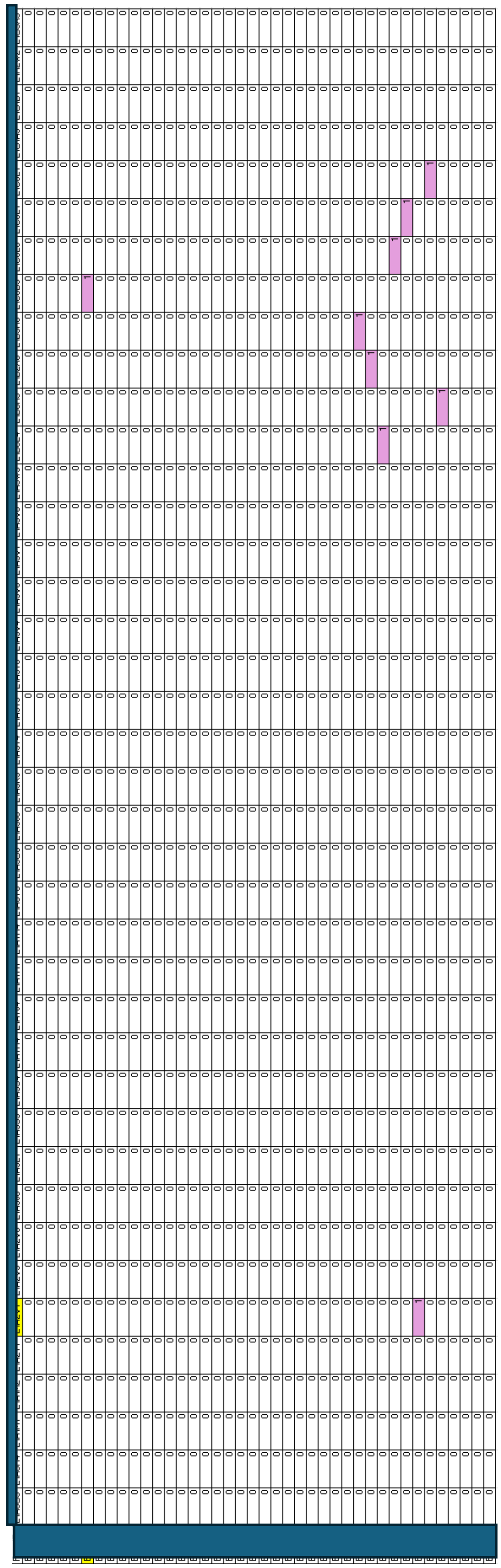

Figure 62 Vehicle C Solution Matrix 6VRP Part 2

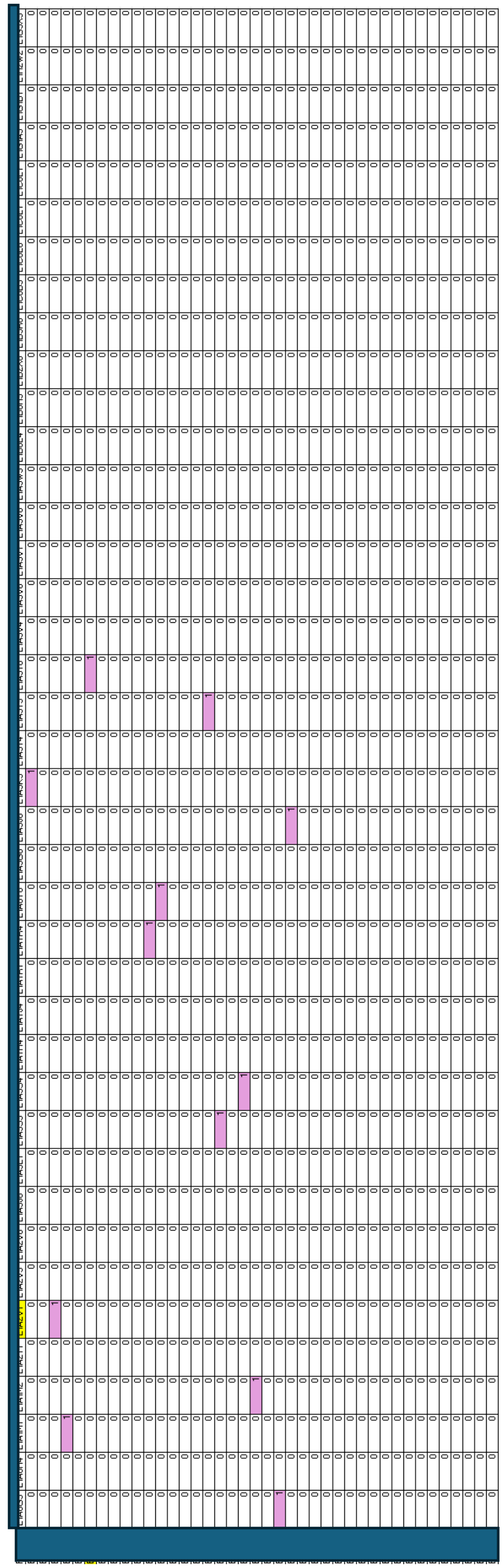

Figure 63 Vehicle D Solution Matrix 6VRP Part 2

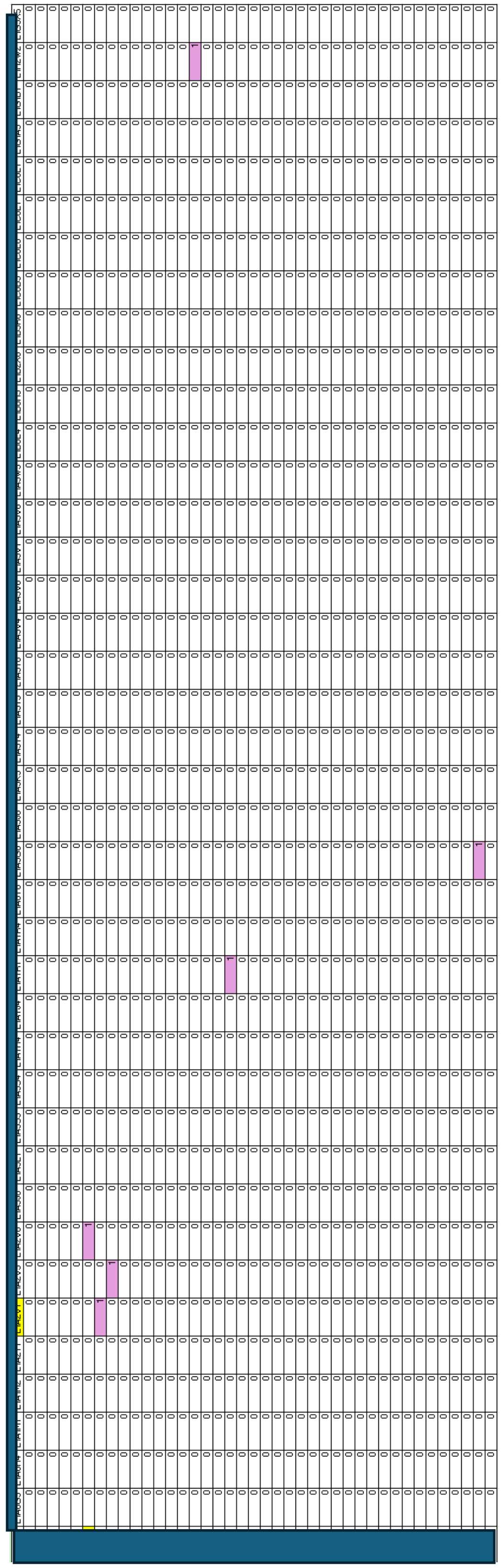

Figure 64. Vehicle E Solution Matrix 6VRP Part 2

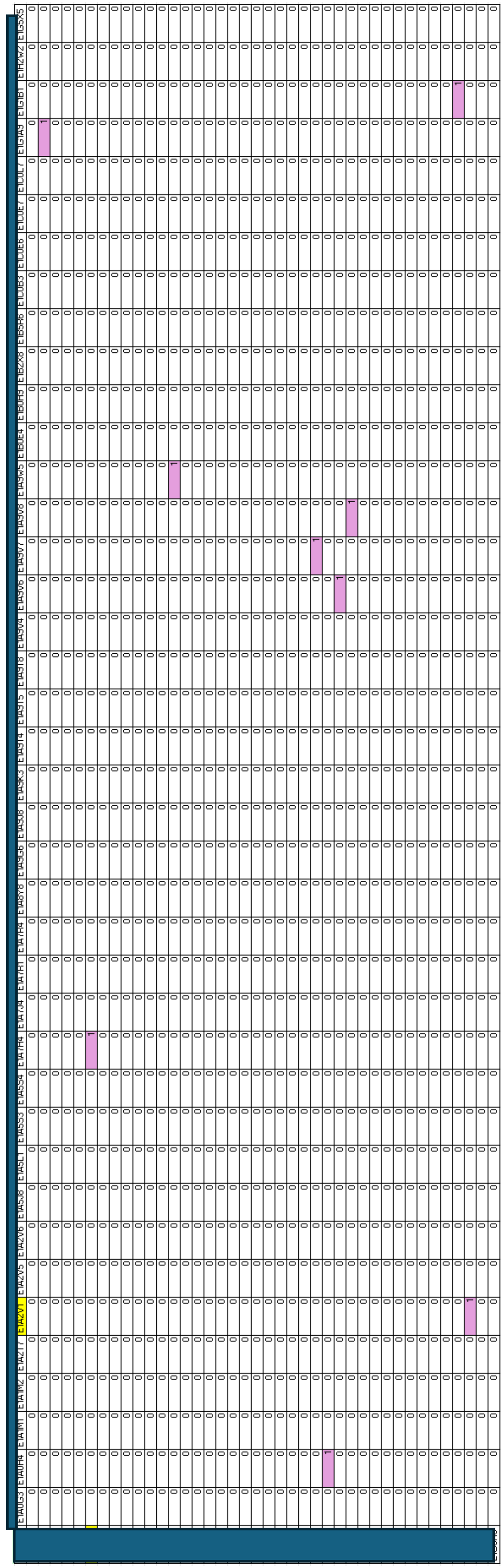

Figure 65 Vehicle F Solution Matrix 6VRP Part 2

APPENDIX D VRP ROUTES

Table 18. TSP Stop Order

Order	Stop
1	E1A[redacted]
2	E1A[redacted]
3	E1H[redacted]
4	E1G[redacted]
5	E1G[redacted]
6	E1G[redacted]
7	E1B[redacted]
8	E1B[redacted]
9	E1B[redacted]
10	E1B[redacted]
11	E1C[redacted]
12	E1C[redacted]
13	E1C[redacted]
14	E1C[redacted]
15	E1A[redacted]
16	E1A[redacted]
17	E1A[redacted]
18	E1A[redacted]
19	E1A[redacted]
20	E1A[redacted]
21	E1A[redacted]
22	E1A[redacted]
23	E1A[redacted]
24	E1A[redacted]
25	E1A[redacted]
26	E1A[redacted]
27	E1A[redacted]
28	E1A[redacted]
29	E1A[redacted]
30	E1A[redacted]
31	E1A[redacted]
32	E1A[redacted]
33	E1A[redacted]
34	E1A[redacted]
35	E1A[redacted]

36	E1A[redacted]
37	E1A[redacted]
38	E1A[redacted]
39	E1A[redacted]
40	E1A[redacted]
41	E1A[redacted]

Table 19. Two Vehicle Stop Order

Order	Vehicle A	Vehicle B
1	E1A[redacted]	E1A[redacted]
2	E1A[redacted]	E1A[redacted]
3	E1H[redacted]	E1A[redacted]
4	E1G[redacted]	E1A[redacted]
5	E1G[redacted]	E1A[redacted]
6	E1G[redacted]	E1A[redacted]
7	E1B[redacted]	E1A[redacted]
8	E1B[redacted]	E1A[redacted]
9	E1B[redacted]	E1A[redacted]
10	E1B[redacted]	E1A[redacted]
11	E1C[redacted]	E1A[redacted]
12	E1C[redacted]	E1A[redacted]
13	E1C[redacted]	E1A[redacted]
14	E1C[redacted]	E1A[redacted]
15	E1A[redacted]	E1A[redacted]
16	E1A[redacted]	E1A[redacted]
17	E1A[redacted]	E1A[redacted]
18	E1A[redacted]	E1A[redacted]
19	E1A[redacted]	E1A[redacted]
20	E1A[redacted]	E1A[redacted]
21	E1A[redacted]	E1A[redacted]
22	E1A[redacted]	

Table 20. Three Vehicle Stop Order

Order	Vehicle A	Vehicle B	Vehicle C
1	E1A[redacted]	E1A[redacted]	E1A[redacted]
2	E1A[redacted]	E1A[redacted]	E1A[redacted]
3	E1A[redacted]	E1H[redacted]	E1A[redacted]
4	E1A[redacted]	E1G[redacted]	E1A[redacted]
5	E1A[redacted]	E1G[redacted]	E1A[redacted]
6	E1A[redacted]	E1G[redacted]	E1A[redacted]
7	E1A[redacted]	E1B[redacted]	E1A[redacted]
8	E1A[redacted]	E1B[redacted]	E1A[redacted]
9	E1A[redacted]	E1B[redacted]	E1A[redacted]
10	E1A[redacted]	E1B[redacted]	E1A[redacted]
11	E1A[redacted]	E1C[redacted]	E1A[redacted]
12		E1C[redacted]	E1A[redacted]
13		E1C[redacted]	E1A[redacted]
14		E1C[redacted]	E1A[redacted]
15		E1A[redacted]	E1A[redacted]
16		E1A[redacted]	E1A[redacted]
17			E1A[redacted]

Table 21. Four Vehicle Stop Order

Order	Vehicle A	Vehicle B	Vehicle C	Vehicle D
1	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
2	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
3	E1H[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
4	E1G[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
5	E1G[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
6	E1G[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
7	E1B[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
8	E1B[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
9	E1B[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
10	E1B[redacted]			E1A[redacted]
11	E1C[redacted]			E1A[redacted]
12	E1C[redacted]			E1A[redacted]
13	E1C[redacted]			E1A[redacted]
14	E1C[redacted]			E1A[redacted]
15	E1A[redacted]			

Table 22. Five Vehicle Stop Order

Order	Vehicle A	Vehicle B	Vehicle C	Vehicle D	Vehicle E
1	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
2	E1A[redacted]	E1A[redacted]	E1G[redacted]	E1A[redacted]	E1A[redacted]
3	E1A[redacted]	E1A[redacted]	E1G[redacted]	E1C[redacted]	E1A[redacted]
4	E1A[redacted]	E1A[redacted]	E1G[redacted]	E1C[redacted]	E1A[redacted]
5	E1A[redacted]	E1H[redacted]	E1B[redacted]	E1C[redacted]	E1A[redacted]
6	E1A[redacted]	E1A[redacted]	E1B[redacted]	E1A[redacted]	E1A[redacted]
7	E1A[redacted]	E1A[redacted]	E1B[redacted]	E1A[redacted]	E1A[redacted]
9	E1A[redacted]	E1A[redacted]	E1B[redacted]	E1A[redacted]	
10	E1A[redacted]	E1A[redacted]	E1C[redacted]	E1A[redacted]	
11	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]	
12	E1A[redacted]			E1A[redacted]	

Table 23. Six Vehicle Stop Order Part 2

Order	Vehicle A	Vehicle B	Vehicle C	Vehicle D	Vehicle E	Vehicle F
1	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1A[redacted]
2	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1C[redacted]	E1A[redacted]	E1A[redacted]
3	E1A[redacted]	E1H[redacted]	E1A[redacted]	E1C[redacted]	E1A[redacted]	E1A[redacted]
4	E1A[redacted]	E1G[redacted]	E1A[redacted]	E1C[redacted]	E1A[redacted]	E1A[redacted]
5	E1A[redacted]	E1G[redacted]	E1A[redacted]	E1G[redacted]	E1A[redacted]	
6	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1B[redacted]	E1A[redacted]	
7	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1B[redacted]	E1A[redacted]	
8	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1B[redacted]		
9	E1A[redacted]	E1A[redacted]	E1A[redacted]	E1B[redacted]		
10	E1A[redacted]			E1C[redacted]		
11	E1A[redacted]			E1A[redacted]		

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