Incorporating Bitcoin into the Canadian Investment Landscape

by

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Abstract

Within the past few years, bitcoin has attracted a lot of attention as an emerging investment asset and alternative to traditional investments. As a relatively new financial asset, bitcoin offers investors diversification opportunities and a potential hedging strategy due to its low correlation with other investments. Using daily price data between 2017 and 2022 for bitcoin and five other assets assumed to represent the Canadian investment universe, this report evaluates the role that bitcoin plays in improving portfolio efficiency under a Modern Portfolio Theory (MPT) framework. Applying the Markowitz portfolio selection criteria, this report also assesses bitcoin's impact on the efficient frontier and the composition of the most efficient and least risky portfolios accessible to Canadian investors. Finally, this study describes the risk-return relations that exist between bitcoin and the other five assets using mean-variance analysis. Findings from this report will have implications for regulators and investment professionals, especially those relating to emerging technologies.

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Chapter 1

Introduction

The 1990's were a very significant period in human history, they can be credited for the foundational development of many of the digital technologies we enjoy in our world today. The development of cryptocurrencies, for instance, started in the early nineties, even though there was no significant breakthrough in the space until 2009 when Bitcoin (BTC) was successfully introduced to the world [Hougan and Lawant, 2021]. Bitcoin, the first fully scaled cryptocurrency, was presented as a solution to the problem of transferring value between individuals without the need for a central intermediary. At its inception, bitcoin was sold for less than one U.S. cent, however due to its increasing popularity, it is valued at thousands of dollars today and this enormous rise in value has placed it among the most controversial financial instruments of the 21st century. Bitcoin and other cryptocurrencies are becoming an increasingly popular asset class as they offer an alternative to traditional investment assets in terms of value and use. Some bitcoin proponents even claim that excluding it from an investment portfolio might be one of the huge errors a 21st century investor can make.

According to the Bank of Canada's Bitcoin Omnibus Surveys (BTCOS), between 2018 and 2020, nearly 90% of the Canadian population were aware of bitcoin but only 5% owned it [Balutel et al., 2022]. The most common reasons cited for owning bitcoin were related to its use for investments or based on interest in the technology. Bitcoin owners were susceptible to certain risks evidenced by the fact that about half of current and past owners stated that they had been affected by events such as price crashes, losing access to funds, scams, or data breaches.

There is ever growing interest in cryptocurrencies and plenty of investors are increasing their exposure to this emerging asset class. One of the most important factors to consider when dealing with cryptocurrencies is the relatively high volatility of prices. Due to their risky nature, moderate exposure is often the recommended approach when introducing crypto assets into a portfolio. However, a more technical question is to what degree can one's exposure be considered moderate?

The answer to this question will depend on individual-level characteristics such as an investor's expected returns on their investments, associated risk profile, ethics, and other factors in the general economy at large. The portfolio allocation decision involves ascribing weights to different assets based on pre-defined risk-return objectives and beliefs about the future performance of assets available to investors [Markowitz, 1952]. There is a very well developed background of research on strategies and tools for the portfolio allocation process. In truth, there cannot be a single best strategy for all Canadian investors, however, this report provides a general framework for incorporating bitcoin into a portfolio using concepts from Modern Portfolio Theory (MPT).

The goal of this report is to develop a framework for incorporating bitcoin into an investment portfolio while taking expected return, risk, and any other key factors into consideration. Once developed, such a framework could be extended to other cryptocurrencies although a different set of factors will need to be taken into consideration. The sub-objectives of this report include the following:

1. Develop a review of the relevant literature on key developments in the cryp-

tocurrency space.

2. Outline the procedure for retail and institutional investors to include bitcoin into their investment portfolios.

3. Outline the process for extending the framework developed to other cryptocurrencies and assets.

The rest of the paper is organized as follows. Section 2 presents a review of the relevant literature concerning bitcoin and the Markowitz portfolio optimization process. Section 3 describes the methodology used for the empirical process along with the data series analyzed. Section 4 discusses the estimation results and section 5 concludes the report.

Chapter 2

Literature Review

Bitcoin is a cryptographic currency which aims to be completely distributed, free from central authorities, and anonymous. The cryptocurrency comes at the end of a long and rich history of digital currency efforts going back over 30 years. It was invented in 2009 by a programmer pseudonym known as Satoshi Nakamoto and its creation follows precise rules derived from the gold market. Miners[1] competitively use sophisticated computer resources to solve cryptographic problems and verify the validity of transactions. Their success is rewarded by newly issued bitcoins [Briere et al., 2015]. A brief analysis of the community-funded bitcoin.org project gives the sense that at some point, the cryptocurrency was envisioned as a utopian monetary system through which individuals and businesses would be able to process transactions without many of the regulatory difficulties attached to the traditional financial system.

[Chen, 2021] envisions bitcoin in a cryptoeconomy where bitcoins are adopted as the means of exchange in a manner similar to the U.S. dollar. Today, however, the cryptocurrency is yet to live up to this reality. Bitcoin is now viewed by many individuals as an investment asset with individuals and organizations alike placing large amounts of their wealth into the cryptocurrency due to the potential short and

[1] Bitcoin mining refers to the computational work undertaken by players to run the bitcoin network in hopes of earning new bitcoins. Miners refer to the individuals who partake in this mining process.

long term benefits associated with it.

The major factor which distinguishes bitcoin from other currencies is the presence of an algorithm which must function correctly for its operation. The operation of this algorithm relies on three forms of consensus for its maintenance. Participants on the bitcoin network must maintain consensus on the rules to determine the validity of transactions, on which transactions have occurred in the system, and on a non-trivial value assigned to the currency. These consensus are connected and the failure of any one could lead to the unravelling of the other two. The consensus also relies on certain assumptions that participants in the system will cooperate in certain ways [Kroll et al., 2013].

A bitcoin is a fixed-value cryptographic object represented as a chain of digital signatures over the transactions in which the bitcoin was used. Each bitcoin is owned by a bitcoin address, which consists of a public key. The owner of a bitcoin can create a transaction by signing an assertion that bitcoins are transferred from one address to another. A transaction may involve many input and output identities. If the total value of the input bitcoins exceeds the value of the output bitcoins, the difference is interpreted as a transaction fee which is paid to the miner who successfully appends that transaction to the blockchain [Kroll et al., 2013].

Transactions are organized into blocks which contain a sequence number, a timestamp, the cryptographic hash[2] of the previous block, some metadata, a nonce[3], and a set of valid transactions. The blocks form a hash-chain (blockchain) with each new block containing the cryptographic hash of its predecessor allowing anyone to verify that no preceding block has been modified. The blockchain contains backward links but no forward links so there is a unique path backwards from each block to the beginning of the log. The forward path from a block might not be unique, hence the

[2]A hash is a function that is used to map data of arbitrary size to a fixed-size value.
[3]A nonce is an abbreviation for "number only used once."

bitcoin log has the form of a tree whose branches fork as it grows[4].

Blocks are mined through a proof-of-work computation that is thought to be difficult to perform but easy to verify. The specific computation used in the bitcoin network is taken from Hashcash, a cryptocurrency project that predates bitcoin. The difficulty of the computation is adjusted periodically by an adaptive algorithm based on recent blockchain history which aims to maintain the long term invariant that one new block be mined every ten minutes on average. This adaptive feature allows the network to compensate for increasing hardware speed and varying interests in running the network over time [Kroll et al., 2013].

The bitcoin network is free from any central bank-like intervention due to the fixed scheme money creation process pre-established by its inventor [Briere et al., 2015]. Total bitcoin supply is capped at 21 million units and new bitcoins can only be created via the mining process. Anyone with the interest and resources can choose to become a miner and mine new blocks that add transactions to the blockchain. Today, all one would need to participate as a miner is a cryptocurrency wallet, mining software, and hardware which are readily available on the market, albeit relatively expensive.

The main incentive from mining is that successful miners are rewarded new bitcoins for mining a new block. Each miner adds a special transaction to their prospective block assigning a predetermined number of reward bitcoins to anyone that they choose, typically themselves. It is important however to note that mining does not guarantee a reward since it is only the first miner who finds a solution to the proof-of-work[5] problem that claims the mining reward [Kroll et al., 2013].

The number of bitcoins created through the mining process is adjusted on a fixed schedule in which the reward is halved each time 210,000 additional blocks have been mined. The initial reward of 50 bitcoins per block was cut to 25 in November

[4] As the number of blocks increases, each new block maintains a backward link to the original block

[5] Proof of work refers to a blockchain consensus mechanism in which computing power is used to verify cryptocurrency transactions and add them to the blockchain.

2012 after the first 210,000 blocks were mined [Kroll et al., 2013] and at the time of this writing, miners were awarded 6.25 bitcoins for each block that they successfully mined.

To prevent double spending, bitcoin network participants engage in a peer-to-peer protocol that implements a distributed timestamp service providing a fully-serialized log of every bitcoin transactions ever made [Kroll et al., 2013]. The network uses a distributed ledger to publicly record all transactions on the network, hence, at any point, one can view the entire history of a bitcoin to prove that it was not already spent. Bitcoin balances are stored in a large distributed network.

The number of individuals who report using bitcoin in Canada has steadily been rising [Balutel et al., 2022] despite the fact that the Bank of Canada does not recognize it as a currency officially. The blockchain technology is also a well-advanced feature which attracts firms and governments targeting the efficiency of the payment system [Chen, 2021]. Several organizations across a wide range of industries are beginning to actively hold bitcoin on their balance sheets. Some major examples of such organizations include business intelligence firm MicroStrategy, electric car manufacturer Tesla, crypto exchange Coinbase, and financial technology firm Square.

Even governments are beginning to develop a high level of interest towards bitcoin and other digital assets. In September 2021, El Salvador became the first country to adopt bitcoin as a legal tender within the country [Arslanian et al., 2021]. The European central bank and the central banks of countries such as China, Canada, and the United States are actively evaluating a new range of digital assets referred to as central bank digital currencies (CBDC) [Soderberg et al., 2022]. These CBDCs are simply fiat currencies which share the distributed ledger and blockchain features of bitcoin. They are however more stable than bitcoin since the central banks can exert some level of control over them.

Another positive feature of bitcoin is the ease of access to the network compared to traditional financial institutions. According to the World Bank, around 1.7 billion adults, roughly a third of the world's adult population in 2017 were unbanked. Bitcoin technology could aid efforts to decrease the percentage of underbanked people across the world. The ease of access to the bitcoin network was one of the key features that led El Salvador to embrace the cryptocurrency as approximately 70 % of its population did not have a bank account at the time of this writing [Arslanian et al., 2021]. Individuals need only a mobile device with an internet connection to participate on the bitcoin network.

When used on a mobile device, bitcoin allows an individual to make payments with a simple two-step scan-and-pay process. Sending bitcoins across borders is as easy as sending them across the street allowing for quick international payments. The bitcoin network runs anytime, anywhere, allowing individuals to perform transactions during holidays or periods when banks are experiencing downtime.

Bitcoins are generally considered anonymous because bitcoin addresses are derived from public keys which could represent anyone on the internet. However, in practice, it can be rather difficult for one to maintain their anonymity. Most individuals who own or transact bitcoin in Canada do so through cryptocurrency exchanges[6] or bitcoin automated teller machines (ATM) [Balutel et al., 2022] which require some customer information by regulation. Cryptocurrency exchanges operating in Canada are held to many of the same regulatory standards as traditional financial institutions.

Also, transactions made under multiple identities, such as payments to oneself, can be linked in some circumstances and such transaction behaviour can leak identifying information. To maintain the highest level of anonymity on the bitcoin network, one would need some technical proficiency to deploy the software on their own [Kroll et al., 2013].

[6] A cryptocurrency exchange, sometimes referred to as digital currency exchange, is a business that allows customers to trade cryptocurrencies for other assets such as conventional fiat or other digital currencies.

Majority of the already mined bitcoins are owned by so called whales with the top 10,000 bitcoin addresses owning over 50 % of the bitcoins in circulation. The inventor, Satoshi Nakamoto, is believed to have accumulated around 1.1 million bitcoins, roughly 5 % of the total supply, over the years. An estimated 4 million bitcoins are believed to be in lost or stranded wallets due to owner's errors when storing or sending the asset [Albrecht, 2021]. Discrepancies regarding the number of bitcoins in circulation make the bitcoin network unstable, unevenly distributed, and raises serious questions regarding the feasibility of cryptocurrency as a means of exchange. Bitcoin is an open source project, and similar to other open source projects, the power of leadership is limited by the ability of anyone to fork the software by copying the current version and deploying it separately. [Kroll et al., 2013] highlight the need for a governance structure to help the system cope with long-term structural challenges. Contrary to claims that bitcoin is ungovernable and relies entirely on fixed rules laid at its founding, the rules on which the bitcoin network runs can be modified and they have been changed before by consensus amongst the network's participants. For instance, the minimum transaction amount was changed from 1 satoshi to 5430 satoshis to discourage the creation of transaction outputs of very low value. A satoshi is defined as one-hundred millionth of a bitcoin.

Another questionable issue around the bitcoin technology is the vulnerability of the network to different types of attacks. [Kroll et al., 2013] identify two types of attacks which the network might be susceptible to. The first is the 51% attack which occurs when a single miner or group of miners hold more than 50% of the network's mining capacity.

The reality of a single miner developing such capacity is highly unlikely due to the costs associated with mining. However, it is important to note that mining is generally organized into pools of coordinating miners who partition the search for the proof of work and share the mining rewards. Such pools could organize into a cartel of few powerful players who are less accountable to the much larger set of bitcoin holders. One example of such a pool is the BTC Guild which controlled over 25% of total mining power in 2013 [Kroll et al., 2013].

If a cartel is developed, it could change any of the rules enforced by the consensus and players who are not in the cartel will be obliged to follow. A cartel could choose to double spend bitcoin or ignore any transactions it does not want added to the log. In reality, if a cartel was formed, it should have no incentive to do so because people would lose confidence in the system and become unwilling to accept it as a means of exchange, hence degrading the value of the asset. As mining becomes increasingly specialized, the barriers to entry increase due to the cost associated which leads to fewer players participating in mining process, a trend observed over the years.

The second attack [Kroll et al., 2013] describe is the Goldfinger attack which is essentially a 51 % attack that aims to destroy the bitcoin economy to achieve utility outside it. There are at least three possible motivations for such an attack. Firstly, a government or institution might want to block bitcoin transactions to enforce the law, deter money laundering, or achieve some institutional goals. Second, a non state attacker might seek to gain some political or social goal, perhaps as a form of social protest. And lastly, an attacker might seek an investment gain, for instance by taking short positions in Bitcoin so as to profit if its value is diminished.

Despite significant progress in the literacy and participation of the Canadian population, bitcoin and cryptocurrencies in general are still viewed by many as an unnecessary gamble and a tool for black market transactions. Other concerns have also been raised about the energy intensive nature of bitcoin. The bitcoin network accounts for close to 1 % of global energy consumption, a feature that makes bitcoin unattractive to environmentally conscious individuals although, ideally other factors such as the energy source and potential benefits of transactions on the network should also be taken into consideration.

As a recent concept and subsequently rather unexplored financial asset, some might argue that the current bitcoin market is a bubble as history has shown that assets linked to innovation are more bubble-prone [Briere et al., 2015]. Within the past few years, bitcoin has attracted a lot of attention as an emerging asset class and alternative to traditional investment assets.

Bitcoin investments are treated by many investors similarly to financial instruments such as bonds, stocks, and commodities despite concerns that they lack government authority and intrinsic value. As a new financial asset, bitcoin offers investors diversification opportunity and potential hedging strategies due to its low correlation with other traditional and alternative investments [Chkili, 2021, Briere et al., 2015]. [Markowitz, 1952] stresses the importance of diversification both as an observable and sensible rule of investing and concludes that any rule of investment behaviour which does not highlight diversification must be rejected both as a hypothesis and a maxim. Bitcoin investments are extremely risky by nature. Their rate of return presents statistical characteristics that differ markedly from those of other assets, including gold, oil, and hedge funds. Bitcoin investments are however very attractive as they have been known to deliver exceptionally high returns in the past [Briere et al., 2015]. Since bitcoin differs from other financial assets, it might be possible to create benefits for stakeholders using portfolio analysis and risk management [Trimborn et al., 2020]. For moderately risk-averse investors, bitcoin may lead to substantial financial gains, but simultaneously to a significant increase in risk. [Briere et al., 2015] find that bitcoin-inclusive portfolios deliver superior mean-variance tradeoffs than similar bitcoin-free portfolios. This implies that including even a small proportion of bitcoin in a well-diversified portfolio may dramatically improve its risk-return characteristics.

When including bitcoin in a portfolio, a problem might arise due to the unstable nature of its market's liquidity[7]. Despite its relatively high liquidity, large bitcoin

[7] Liquidity refers to a market feature that measures how quickly an individual or firm can purchase or sell an asset without causing a drastic change in the price of the asset.

positions may take a while to clear especially during periods of crisis and panic selling. This factor should be an important consideration for institutional holders of the asset. If one wants to include bitcoin and stocks in the same portfolio, one should avoid giving bitcoin too big a weight since this could induce a liquidity problem on adjusting the position as fast as possible. Adding a liquidity constraint can improve the performance of a portfolio including bitcoin [Trimborn et al., 2020].

The goal of portfolio selection is the construction of portfolios that maximize expected returns consistent with individually acceptable levels of risk. The theory of portfolio selection together with capital asset pricing theory provides the foundation and building blocks for the management of portfolios. Modern portfolio theory (MPT) shows that it is possible to combine risky assets and produce a portfolio whose expected returns reflect its components but with considerably lower risk [Fabozzi et al., 2012]. Two questions must be raised when constructing a diversified portfolio selection: how much should be invested in each asset class and what specific securities within each class should be invested in. The process of selecting a portfolio may be divided into two stages. The first starts with observation or experience and ends with beliefs about the future performance of available securities. The second stage starts where the first ends and concludes with the choice of portfolio [Markowitz, 1952].

The economic theory of choice uses a utility function to represent the way entities make decisions when faced with a set of choices. In portfolio theory, these entities are faced with the decision of choosing a portfolio from the set of all possible risk-return combinations [Fabozzi et al., 2012]. An underlying question is whether an investor with a particular single period utility function acting on the basis of expected return and variance (E-V) can achieve maximum expected utility [Markowitz, 1991].

Mean-variance analysis is a locally quadratic approximation of an investor's utility function and it is a practical way to maximize approximate expected utility. It is often preferred to the theoretically correct expected utility analysis because of its convenience, cost, and feasibility. The data requirements for an expected utility analysis substantially exceed those of a mean-variance analysis. There is also the issue of correctly determining the investor's utility function in expected utility analysis [Markowitz, 1991].

The rule that the investor considers expected return desirable and variance of return undesirable can be accepted as a maxim and hypothesis to guide investment behaviour over any rule which emphasizes expected returns alone. Next, consider the rule which states that an investor diversifies their funds among all those securities which give maximum expected return. Both rules imply the presence of a diversified portfolio which is preferable to all non-diversified portfolios.

This preferred portfolio is not necessarily the one with maximum expected return or minimum variance because there is a rate at which an investor will gain expected return by taking on variance, or reduce variance by giving up expected return [Markowitz, 1952]. The basic principle underlying modern portfolio theory is that for a given level of expected return, an investor would choose the portfolio with the minimum variance from among the set of all possible portfolios.

In order to use mean variance analysis in the selection of securities, one must have procedures for finding reasonable expected return and variance values that accurately represent current and future economic conditions. [Markowitz, 1952] argues that these procedures should ideally combine both statistical techniques and subjective judgement. The statistical computations to arrive at a tentative set of values and judgement to adjust these values on the basis of factors or nuances not taken into account by the formal computations.

Portfolios that provide the largest possible expected return for given levels of risk are called efficient portfolios. The efficient set of portfolios is sometimes called the efficient frontier because graphically all the efficient portfolios lie on the boundary of the set of feasible portfolios that have the maximum return for a given level of risk. Since all portfolios on the efficient frontier provide the greatest possible return at their level of risk, an investor or entity will ideally want to hold one of these portfolios. Portfolios on the efficient frontier represent trade-offs between risk and return [Fabozzi et al., 2012].

The optimal portfolio depends on an investor's preference over different risk-return trade-offs. For each level of risk, there is an efficient portfolio. The portfolio at the leftmost end of the efficient frontier is called the global minimum variance (GMV) portfolio and it is the portfolio with the smallest obtainable standard deviation [Fabozzi et al., 2012]. [Briere et al., 2015] demonstrate that bitcoin may not be added to low-risk portfolios on the frontier due to its high volatility.

While attempting to make variance small, it is not enough to just invest in many securities, it is also necessary to avoid investing in securities with high covariances among themselves. One should diversify across industries because assets across different industries, especially industries with different economic characteristics, have lower correlations than assets within an industry [Markowitz, 1952]. The principle of diversification states that as the correlation between the returns of assets combined in a portfolio decreases, so do the variance and standard deviation of the portfolio.

Today, portfolio managers are reluctant to apply quantitative risk-return optimization because of observations that it may be unreliable in practice. Mean-variance optimization is very sensitive to inputs and estimation errors in the forecasts. When used in practice, mean variance analysis has to be modified in order to achieve reliability, stability, and robustness with respect to model and estimation errors [Fabozzi et al., 2012].

Modern robust optimization techniques allow a portfolio manager to solve the robust version of the portfolio optimization problem in about the same time as needed for the traditional mean-variance portfolio optimization problem. The robust approach uses the distribution from the estimation process to find a robust portfolio in a single optimization, thereby incorporating uncertainty about inputs in the optimization process. As a result, robust portfolios are less sensitive to estimation errors than other portfolios, and often perform better than optimal portfolios determined by traditional mean-variance analysis [Fabozzi et al., 2012].

Estimating the fundamental value of bitcoin could be very challenging. [Chen, 2021] opposes the idea of a worthless bitcoin, suggesting that the marginal cost of production model supports the claim that value exists in bitcoin. Time series models such as the Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models have always been used for price forecasting [Lian et al., 2022]. The ARIMA Model combines features from the autoregressive (AR) model with the moving average (MA) model. Meanwhile, the GARCH model is used to explain the volatility clustering of financial data. [Lian et al., 2022] show that among time series models, the ARIMA-EGARCH model, an ARIMA model augmented with GARCH to express volatility, is the most accurate for predicting bitcoin prices.

Back propagation neural networks (BPNN) is an increasingly popular machine-learning based approach used as an alternative to price forecasting. [Lian et al., 2022] argues that time series models have good predictive ability considering different targets while BPNN explains non-linear data more effectively and concludes that investors can refer to BPNN models with two layers to measure the impact of news about bitcoin and price volatility on bitcoin prices [Lian et al., 2022].

[Chen, 2021] chooses the vector error correction (VEC) method as the primary method to estimate bitcoin price empirically and shows that in the short run, bitcoin price is mainly affected by medium of exchange and financial expectation forces. Blockchain technology, proxied by the hash rate, number of active addresses, and number of bitcoin wallet users daily, only has a small impact on the price of bitcoin.

To explain the dynamics of bitcoin price, one would need to consider both the transactional benefit and speculation incentive associated with bitcoin. Since no financial asset backs the value of bitcoin, its price is most likely explained by the transactional demand relative to supply implying that demand shocks better explain bitcoin price than supply shocks [Chen, 2021].

[Chen, 2021] also finds empirical evidence that market factors related to the transaction network and social media elements like the number of new members or posts on cryptocurrency-related forums could help predict future bitcoin price with high accuracy, while financial developments such as oil price and stock market index changes do not account for significant bitcoin price formation.

Currently, there is little research within existing academic literature regarding the role and influence of bitcoin as a financial asset in the Canadian investment environment. The rest of this study intends to fill this gap by highlighting bitcoin as an investment instrument from the context of a Canadian investor taking both expected return and risk as their main consideration.

Chapter 3

Methodology

The main goal of this report is to describe the risk-return relations that exist between bitcoin, an emerging asset, and other existing traditional asset classes present within the Canadian economy. The hypothesis of the study is that bitcoin improves the efficiency of an investment portfolio. In order to find the most efficient investment portfolio available to the universe of Canadian investors, this study combines concepts from mean-variance analysis with the Markowitz portfolio selection criteria to determine the mix of assets that offer the best risk-return performance. This report also analyzes how different return timelines can influence an investor's choice of portfolio. Before determining the set of efficient portfolios, certain simplifying assumptions are necessary to aid with this analysis. These assumptions include but are not limited to the following:

1. Risk averse investors.

2. The entire universe of assets available to investors includes only Canadian and U.S. stocks, Canadian bonds, Canadian real estate, gold, and bitcoin.

3. No cash holdings. Investors must spend all funds available to them. The sum of portfolio weights must be greater than one.

4. No short selling. Portfolio weights must be greater than zero[1].

5. Management and transaction fees are ignored.

Daily price data between November 1st 2017 and November 1st 2022 of 6 assets (Bitcoin (BTC-USD), the S&P/TSX Composite Index (GSPTSE), the S&P Composite 1500 Index (SP1500), the TD Canadian Aggregate Bond Index ETF (TDB.TO), the Vanguard FTSE Canadian Capped REIT Index ETF (VRE.TO), and the COMEX 30-day Gold futures contract (GC=F)) is used for this analysis. In total, 8110 observations across the five years and six assets were used to conduct the analysis.

Bitcoin, the main focus of this analysis, represented around 40 percent of the global cryptocurrency market capitalization as of November 1st 2022. Its dominance of the general cryptocurrency market, often proxied by the percentage of the total crypto market cap represented by bitcoin, has steadily been decreasing, especially within the past five years. Despite this decline, bitcoin is still viewed as a general representative of cryptocurrency market conditions as other cryptocurrencies tend to mimic its price movements.

The S&P/TSX Composite Index is the headline index and principal broad market measure for the Canadian equity market. It is the broadest in the S&P/TSX family and the basis for the multiple Canadian sub-indices issued by S&P Global. The index includes all large-cap, mid-cap, and small-cap[2] common stocks and income trust units issued on the Toronto Stock Exchange.

The S&P Composite 1500 index combines the S&P SmallCap 600, the S&P MidCap 400, and the S&P 500, which includes the 500 leading large-cap companies in the U.S. equity market. The index is designed for investors looking to replicate the performance of the U.S. equity market or benchmark against a representative universe of tradable stocks. The index represents around 90% of the U.S. equity market

[1]Assumptions 3 & 4 imply that investors are taking part in a constrained optimization problem

[2]"cap" refers to the total market capitalization of a firm traded on a stock exchange.

capitalization and selection in either of its 3 sub-indices is at the discretion of the Index Committee based on certain eligibility criteria.

The TD Canadian Aggregate Bond Index ETF seeks to track the performance of a Canadian aggregate bond index that measures the investment return on Canadian dollar-denominated investment grade publicly-issued debt. The index, managed by TD Asset Management, includes securities issued by the government and corporate entities. In order to achieve its objective, the ETF invests primarily in Canadian dollar issued bonds and debentures based on a sampling methodology that looks to closely match the aggregate investment characteristics of a broad Canadian fixed income index.

The Vanguard FTSE Canadian Capped REIT Index ETF measures the investment return of publicly traded securities in the Canadian real estate sector. The main objective of the ETF is to track the performance of a broad Canadian real estate equity index. The ETF achieves this objective by investing primarily in stocks of companies in the Canadian real estate sector.

The COMEX 30-day gold futures contract offers investors looking to gain exposure to the gold market an intangible highly liquid alternative to physical gold bullions and coins. Each contract represents 100 troy ounces of gold, however, prices are quoted in US dollars per troy ounce. COMEX is a designated contract market offering products subject to certain rules and regulations outlined by the CME Group.

The price data used below is obtained from Yahoo Finance through the *tidyquant* package available in R. The *tidyquant* package is a useful source of financial data capable of retrieving high quality data in a variety of formats from sources such as Yahoo Finance, the Federal Reserve of Economic Data (FRED), Quandl, Tiingo, Alpha Vantage, and Bloomberg. The quantitative analysis and visualizations for this paper are carried out using the *plotly, timetk, tidyverse*, and *tseries* packages in R based on a portfolio optimization framework adapted from [CodingFinance, 2018].

The daily price data available is used to calculate daily returns:

$$r_t = lnP_t - lnP_{t-1} \tag{3.1}$$

The series of daily returns are averaged geometrically to find estimates of expected daily return:

$$r_{daily} = [(1 + r_1)(1 + r_2)...(1 + r_n)]^{1/n} - 1 \tag{3.2}$$

The expected daily return estimates are then compounded to find estimates of annual[3] return used for the Markowitz portfolio optimization process:

$$r_{annual} = (1 + r_{daily})^{252} - 1 \tag{3.3}$$

The Markowitz portfolio optimization process[4], sometimes referred to as modern portfolio theory (MPT), is used to calculate the efficient frontier along with the minimum variance and optimal portfolios. To perform this procedure, portfolio weights summing up to 1 are randomly generated and assigned to the various assets under consideration. The random weights along with the estimates of annual return from equation (3) and the correlation matrix of the return series are then used to simulate 10,000 portfolios along with their expected returns and variances.

$$R_{portfolio} = w_1 r_1 + w_2 r_2 + ... + w_n r_n \tag{3.4}$$

$$var(R_{portfolio}) = \sum_{i=1}^{6} w_i var(R_i) + \sum_{i=1}^{6} \sum_{j=1}^{6} w_i w_j cov(r_i, r_j) \tag{3.5}$$

After plotting the simulated portfolios in a risk-return environment, the efficient fron-

[3]On average, there are 252 trading days in a year.

[4]The calculations carried out in equations 1, 2, & 3 correspond to the first stage of the portfolio selection process outlined by [Markowitz, 1952].

tier [5] can be traced out along the portfolios that have the least amount of risk for a given level of return. The minimum variance and optimal portfolios can be identified within the risk-return space or from among the series of portfolio properties. The optimal portfolio is defined as the portfolio with the highest Sharpe ratio[6].

$$SharpeRatio_i = \frac{r_i - 0.0375}{\sigma_i} \tag{3.6}$$

In order to measure the impact of bitcoin on the risk-return relations of the simulated portfolios, the efficient frontier is simulated with and without bitcoin. The efficient frontier simulated without bitcoin is simply the efficient frontier simulated with bitcoin given zero portfolio weight and the weights of the other assets summing to 1. To measure the impact of an investor's time perspective on the estimated portfolios, the optimization process is also repeated across four timelines ranging from the past five years of returns to the start of this year till date. In total, 10,000 simulations were performed eight times with each set of simulations accounting for a different factors.

[5] Identifying the efficient frontier corresponds to the second stage of the portfolio selection process outlined by [Markowitz, 1952].

[6] At the time of writing, the Bank of Canada policy rate, also referred to as the risk-free rate, was 3.75%

Chapter 4

Empirical Findings

Empirical analysis begins by estimating the expected returns of the six assets under consideration using the price data described above. The expected returns are calculated using the framework described in equations 3.1, 3.2, and 3.3 above. Prior to carrying out estimations, an Augmented Dickey-Fuller (ADF) test is carried out to verify the stationarity of the return series. The test incorporates the three types of linear models generally examined; one with no drift and linear trend with respect to time, one with drift but no linear trend, and one with both drift and linear trend. Due to the frequency of the price data, the returns calculated for the six series exhibit stationarity across all timelines confirmed by the ADF test.

Table 4.1 presents the average returns of the six assets under consideration in annualized terms across the four timelines. The varying timelines capture the range of time perspectives relevant for most investment decisions that involve bitcoin. According to the results, bitcoin fails to deliver the exceptionally high returns described by [Briere et al., 2015]. In terms of returns alone, bitcoin underperforms relative to the other five assets delivering negative returns across all four timelines.

A common theme referenced in the literature is the high volatility of bitcoin relative to other traditional financial assets. An important part of this analysis involves

	YTD	1 year	2 years	5 years
Bitcoin	-0.571	-0.592	-0.048	-0.060
S&P/TSX Composite Index	-0.158	-0.138	0.081	0.006
S&P Composite 1500 Index	-0.268	-0.217	0.041	0.034
30-day Gold Futures	-0.103	-0.075	-0.073	0.052
TD Bond Index ETF	-0.139	-0.1066	-0.076	-0.006
Vanguard REIT Index ETF	-0.334	-0.279	0.028	-0.009

Table 4.1: Annualized average returns of assets over varying timelines

describing the visual and quantitative measures of volatility for bitcoin and the five other assets under consideration. Table 4.2 presents the variance-covariance matrix of the return series across the entire five years of the returns retrieved. Along the diagonal values, bitcoin exhibits a much higher variance than any of the other five assets.

Figure 4.1 further verifies the claim of bitcoin's relatively high volatility. The line graph plotting the daily returns of the six assets under consideration clearly indicates that bitcoin exhibits much larger daily highs and lows than any of the other assets.

	BTC-USD	GSPTSE	SP1500	GC=F	TDB.TO	VRE.TO
Bitcoin	0.492					
S&P/TSX	0.040	0.033				
S&P 1500	0.046	0.034	0.047			
Gold Futures	0.012	0.004	0.003	0.023		
TD ETF	0.009	0.004	0.004	0.002	0.006	
Vanguard ETF	0.026	0.028	0.029	0.002	0.046	0.045

Table 4.2: Variance-covariance matrix over 5 years of returns

Table 4.3 presents the correlation matrix obtained using the return series of the six assets under consideration. One can observe that bitcoin presents relatively low correlation with the other five assets. This finding supports the suggestion by [Chkili, 2021] & [Briere et al., 2015] that bitcoin could offer investors diversification opportunity and potential hedging benefit due to its low correlation with traditional assets.

[Markowitz, 1952] argues that in order to use mean-variance analysis for securities selection, an investor should ideally combine both statistical techniques and subjective

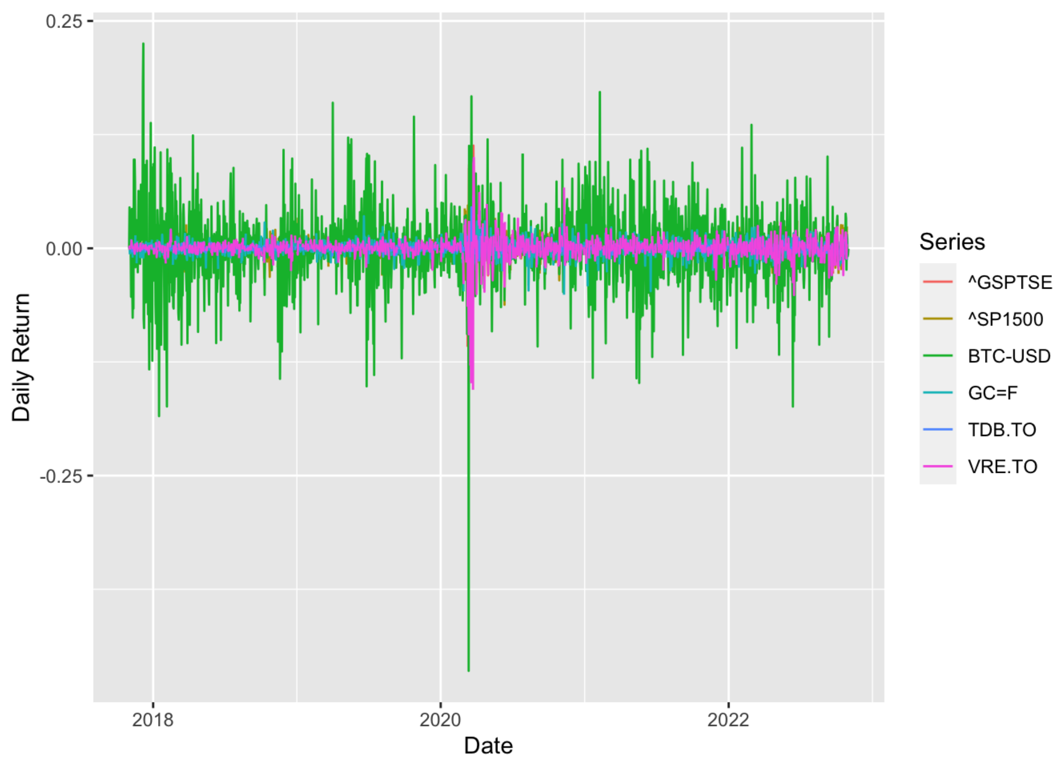

Figure 4.1: Daily returns of assets under consideration

	BTC-USD	GSPTSE	SP1500	GC=F	TDB.TO	VRE.TO
Bitcoin	1.000					
S&P/TSX	0.311	1.000				
S&P 1500	0.300	0.845	1.000			
Gold Futures	0.108	0.159	0.082	1.000		
TD ETF	0.175	0.316	0.234	0.206	1.000	
Vanguard ETF	0.175	0.731	0.625	0.065	0.289	1.000

Table 4.3: Correlation matrix over 5 years of returns

judgement to determine estimates of expected return and variance. This study focuses on the statistical portion of the estimation process by using the geometric mean of return as the estimate of expected return and the diagonal values of the variance-covariance matrix as estimates for expected variance. Any subjective judgements are left for future analysis.

Using the estimates for expected return and variance to calculate portfolio risk and return as in equation 3.4 and 3.5, the efficient frontier is plotted to visually describe the 10,000 portfolios simulated. Figure 4.2 presents the results from the portfolio simulation procedure across the four timelines. The 2-year and 5-year figures exhibit the upward-sloping curved shape of the efficient frontier described by [Fabozzi et al., 2012]. The 1-year and YTD figures, on the other hand, exhibit a downward-sloping curve as they are heavily influenced by the declining economic conditions and general depreciation of assets across board over the past year.

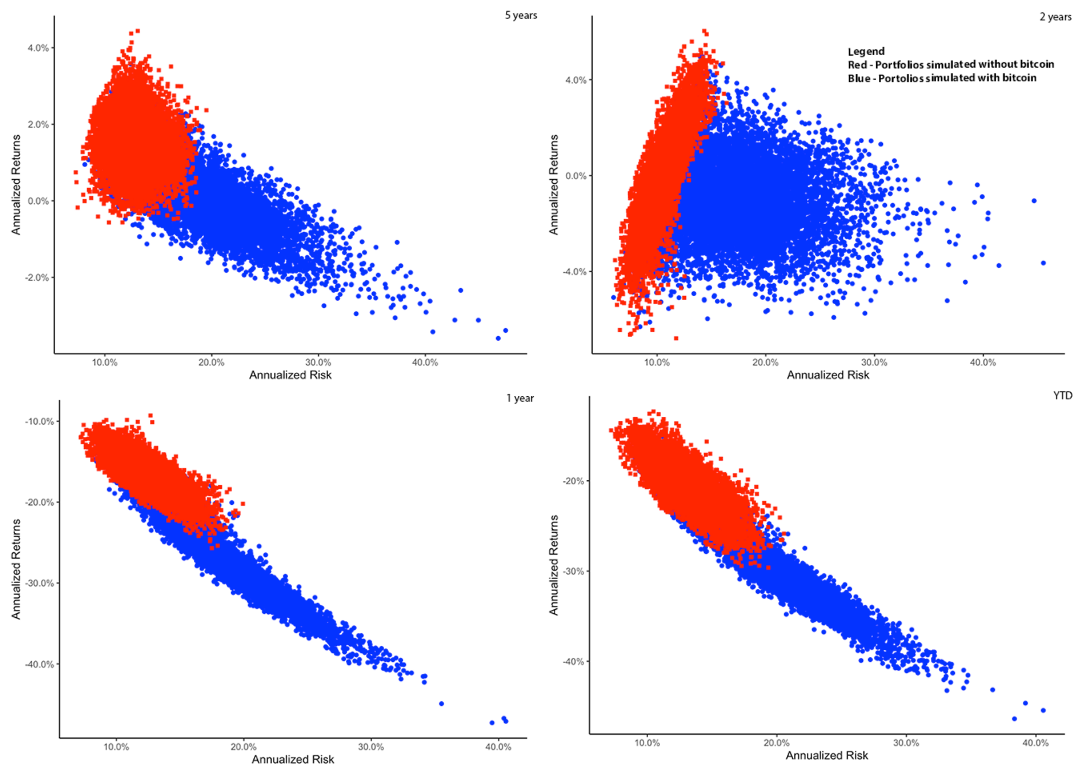

Figure 4.2: Portfolios simulated over varying timelines

The distinction between portfolios simulated with and without bitcoin shows that the portfolios which contain bitcoin are much riskier than those that do not. In figure 4.2, across all timelines, the portfolios simulated with bitcoin extend the frontier along the inefficient portion and seem to contribute very little along the efficient portion. Table 4.4 presents the role of bitcoin in the portfolios with minimum variance and maximum Sharpe ratio.

Due to the random nature of the simulation procedure, the portfolio with maximum

Sharpe ratio is used a representative of the optimal portfolio. The Sharpe ratio is generally viewed as a measure of portfolio efficiency even though the portfolio with maximum Sharpe ratio may not represent a rational choice of portfolio . As demonstrated by [Briere et al., 2015], bitcoin plays a minor role in the minimum variance portfolio, a result that remains consistent across all four timelines. In the two shorter timelines, bitcoin weighs heavily in the portfolios with maximum Sharpe ratio however, its inclusion significantly declines as the timelines are increased.

	YTD	1 year	2 years	5 years
Minimum variance portfolio	0.012	0.025	0.019	0.002
Portfolio with maximum Sharpe ratio	0.647	0.653	0.001	0.045

Table 4.4: Bitcoin weight in 2 key portfolios over varying timelines

Tables 4.5 and 4.6 present the risk, return, and Sharpe ratio of the portfolios with minimum variance and maximum Sharpe ratio respectively. The tables distinguish between the portfolios simulated with and without bitcoin. The minor inclusion of bitcoin in the minimum variance portfolios improves their risk-return relations observed by the slightly higher Sharpe ratios across all four timelines. However, bitcoin's inclusion does not have an observable impact on the returns of these minimum variance portfolios.

	YTD	1 year	2 years	5 years
With bitcoin				
Return	-0.154	-0.126	-0.051	0.013
Risk	0.089	0.078	0.064	0.085
Sharpe Ratio	-2.16	-2.09	-1.37	-0.294
Without bitcoin				
Return	-0.152	-0.115	-0.053	0.003
Risk	0.073	0.072	0.058	0.076
Sharpe Ratio	-2.58	-2.10	-1.56	-0.452

Table 4.5: Minimum variance portfolio over varying timelines

In terms of returns, bitcoin worsens the performance of the portfolio with maximum Sharpe ratio across all four timelines. Portfolios that include bitcoin underperform those that do not. Also, the inclusion of bitcoin does not seem to have much of an impact on the Sharpe ratios in any of the timelines. One should however note that the results in the two shorter timelines are heavily influenced by the extreme volatility experienced by all assets over the past year.

	YTD	1 year	2 years	5 years
With bitcoin				
Return	-0.463	-0.487	0.054	0.040
Risk	0.417	0.427	0.128	0.132
Sharpe Ratio	-1.200	-1.228	0.130	0.017
Without bitcoin				
Return	-0.238	-0.100	0.059	0.041
Risk	0.214	0.119	0.131	0.126
Sharpe Ratio	-1.287	-1.155	0.166	0.031

Table 4.6: Portfolio with maximum Sharpe ratio over varying timelines

Before concluding this report, it is important to describe the limitations of the estimation procedure performed and how this might impact any potential analysis that comes from it. First of all, this estimation procedure does not take into consideration the exchange rate fluctuations between the U.S. dollar (USD) and Canadian dollar (CAD). This report assumes that currency values are the same at both the time of deposit and withdrawal, however, in reality, this assumption will not tend to hold generally.

Over the 5 years of data being analyzed, the exchange rate between the U.S. and Canadian dollar ranged between 0.690 and 0.831. A brief observation of the fluctuations between both currencies shows that they tend to be negligible in the short-term, but, as the holding period of an investment increases, the fluctuations increase also. This finding implies that the current framework used in this report can only be used for short term decisions and as the length of time increases, any results will need to be further verified using a more analysis that accounts for exchange rate changes.

Also, in the two shorter timelines, bitcoin is over represented (over 65 %) in the portfolios with maximum Sharpe ratio. This exaggeration occurs because of the high volatility experienced by the asset over the past year. The Sharpe ratio is a biased measure of efficiency because it is heavily influenced by extreme values as observed in this report. In reality, it is highly unlikely that a rational investor will leave such huge amounts of their investments in bitcoin especially during the past year when the asset depreciated significantly. As suggested by [Trimborn et al., 2020], most investors tend to add constraints to their portfolios through modern day investment instruments like limit and stop orders.

Chapter 5

Conclusion

This report attempts to analyze the role of bitcoin, if any, in a well diversified portfolio and determine if the asset improves portfolio efficiency using the Markowitz portfolio optimization framework. Comparing the efficient frontier and portfolios with minimum variance and maximum Sharpe ratio among those simulated with and without bitcoin, this report finds no substantial evidence to support the claim that bitcoin improves portfolio efficiency across board.

In fact, the empirical analysis suggests that including bitcoin in a portfolio potentially diminishes its performance in comparison to similar portfolios that do not include bitcoin. However, results from the optimization procedure carried out in this report ought to be taken with a grain of salt. Aside from any issues that might arise due to the extremely high volatility of bitcoin over the past year, results from the estimations should ideally be verified by a more advanced robust optimization procedure.

Modern robust optimization techniques allow portfolio managers to find optimal portfolios that are less sensitive to estimation errors. Using the distribution from the estimation process, this approach finds a robust portfolio which often performs better than the optimal portfolio determined by traditional mean-variance analysis. The robust portfolio is obtained in a single optimization and it is less sensitive to estimation errors [Fabozzi et al., 2012].

Post modern portfolio theory (PMPT) is a burgeoning field in finance that builds on the works of modern portfolio theory (MPT) by focusing on downside risk specifically. Risk is defined as the standard deviation of negative returns instead of all returns as in standard MPT and the Sortino ratio is used as a replacement for the biased Sharpe ratio. By assuming investors have asymmetric risk caring more about downside deviation than upside, PMPT reports more reasonable investment outcomes and decisions than standard MPT [Rom and Ferguson, 1994].

The framework outlined in this report can be used by both retail and institutional investors to analyze the impact of bitcoin on their portfolios, although institutional investors will likely have more considerations. An investor will however need to make certain subjective judgements before they can further utilize the framework. A decision needs to be made on whether the expected return values obtained through the geometric average of the return series are suitable for the optimization process or if the return values will need to be adjusted in preparation for the process. The investor might also need to account for exchange rate fluctuations if they intend to use the model for foreign investments or decisions that involve lengthier holding periods.

Finally, as observed in the results across the various timelines, the economic conditions that are present during the period taken into consideration tend to influence the composition of the efficient portfolio. This report was performed after the Covid-19 pandemic at a time when most assets, bitcoin in particular, experienced significant reduction in value observed by their relatively low expected return estimates. This analysis also occurs during a period when the Bank of Canada has consistently raised the risk-free rate in attempts to curb inflation and influence economic activity.

Alternatively, suppose this report was carried out a year ago when asset prices peaked, the framework employed would deliver an entirely different set of estimates and portfolio composition. While employing this framework for analysis, it is important for a portfolio manager to account for the current and future economic conditions amongst the subjective decisions that need to be taken. Expected return and risk estimates should ideally be adjusted in anticipation of incoming economic conditions. Also, the risk-free rate used in Sharpe ratio calculations should reflect beliefs about any future activity of the central monetary authority.

This analysis can be extended to other cryptocurrencies and alternatives to bitcoin, however, a different set of factors will need to be considered. In truth, the days of excess returns in the crypto world are probably over. Regulators around the world are constantly trying their best to control the digital space and this will likely be the case of digital currencies in due time. Regardless, investors and finance professionals are on the look out for the next big thing hoping to enjoy the early adopter advantage that was present with bitcoin. Future research into the identification and analysis of such assets will provide some utility for regulators and investors alike.

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