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Facing the Mathematics: Students' Critical Awareness of the Elusiveness of Mathematical Objects

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submitted version of article

Abstract:
What happens when mathematics students are invited to ask questions and make observations about the way discourse works in their classroom? This article describes my extended conversation with a high school mathematics class about a phenomenon that caught the attention of two girls in the class. Together we observed and reflected on the direction of one's gaze during mathematics conversations. This discussion came about while I co-taught this class, prompting the students daily to become more aware of their language practices in class. This 'critical language awareness' effort aimed to address inequities in power relations within the classroom discourse by inviting student perspectives and resulted in drawing two students' attention to the power of symbols.

*Keywords***:** mathematics education, critical discourse analysis, semiotics, equity, paralinguistic, critical language awareness

Pointing at a mess on the floor, a man says to his dog, "Okay, Ginger. I've had it! You stay out of the garbage! Understand, Ginger? Stay out of the garbage, or else!" The dog looks at his master's finger and hears, "blah blah GINGER blah blah blah blah blah blah blah blah GINGER blah blah blah blah blah …" (Larson, 1983/1989, p. 230). I used this cartoon in a high school mathematics classroom to model for students the nature of our semester-long conversation in which I prompted the students daily to become more aware of their classroom language practices.

One of the more generative parts of the ongoing conversation in this research began when a pair of students made connections between the cartoon described above and their experiences learning mathematics. As I will show, their interest in the way they dealt with the elusiveness of mathematical objects exemplifies what can happen when students shift their attention from their words' referents to the words themselves, and when their views are actively sought by teachers or researchers. These conversations show how and why it is important to listen to students' voices. First, the conversations show the value in directing mathematics students to critical language awareness. In addition, the conversations reported here illuminate these students' unique perspective on their own discourse. These perspectives centre on their positions as humans, as interactive agents in mathematics discourse. Significantly, these students' perspectives emerged from a situation where they were expected to act as interactive agents who would have something important to say about mathematics discourse.

The initial question that structured my participation in this classroom and drew out student perspectives in this way was: What happens when the attention of mathematics students is drawn to the discourse in their classroom? Further questions emerged from the classroom conversations. For example, two students became interested in how they directed their gaze when communicating mathematics, and I (and some of their classmates) became interested in their question as well.

Although it might not at first sight be obvious, there is a connection between this conversation and equity. This conversation about gaze related to two of the four aspects of equity identified by Gutierrez (2011), namely *power* and *identity*, which she described as being the critical axis of equity. An invitation to contribute their perspectives gave students the *power* to exercise their voice. And, their observations in the emergent conversation highlighted human interaction, in which their *identity* became significant.

This article is structured around my account of what happened as a result of this pair of students becoming interested in the form of communication in mathematics class. The structure of the article follows the story. It begins with my observations about language in mathematics and then a description of my plan to draw out student perspectives on the language in their mathematics classroom. These two articulations of theory represent the perspective from which I listened to the students. Next, embedded in the story of two students' conversations about symbols in mathematics, I make connections between scholarship and these students' observations. Introducing this scholarship at this point of the story honours the reality that this scholarship did not figure into the structuring of the research but rather became significant because of the students' contributions[1]. Finally, from this account of the initial conversation relating to the direction of gaze, I reflect on what the students helped me to see.

Attending to Language in School Mathematics

Language can be characterized as a form of pointing. In typical conversation, we use words and groupings of words to direct attention, and we expect the people who listen to direct their attention to the things we are talking about rather than to the way we do the talking. Part of the humour in Larson's (1983/1989) cartoon depends on this natural understanding about language: the dog is supposed to look at the garbage, not at the master's hand.

However, the idea that language is merely pointing has been problematized. Just as it is natural for the dog to look at the pointing hand, it can be useful to look at the pointing language used in classroom discourse. The pointing is the location of action. For the dog, the mess on the floor is not garbage at all. It is playthings – toys. Because the master calls the dog's playthings

"garbage" and deems them a mess, he and his pointing hand are central to the situation. The master's words form the locus of action. The master made the stuff on the floor garbage by naming it so, by declaring it a mess. In this way, language here defines experience; it does not merely represent experience.

Early structuralist approaches to language viewed it as a form of pointing with a direct relationship between object and language (Piaget, 1971). For example, the number 7, for structuralists, signified a group of seven similar items. However, saying or writing the number 7 is a human act; it involves categorizing and grouping. It can be difficult to overcome the structuralist view of the meaning of language because communication relies on using language as though it is transparent.

Post-structuralist scholarship has shown how the act of languaging has primacy over the ideas and things being represented. For example, Brown (2001) described how Jacques Lacan suggested "that notation as printed on a page, or held in a spoken word, or even held as notation in the mind, has more stability than that to which it refers" (p. 67). The way we name and talk about things is more real to us than the things themselves.

The recent shift of attention to language as a shaping force (in addition to being a descriptive tool) has made its way into mathematics education scholarship. For example, Rowland (2000), in his description of vagueness in mathematics discourse placed his analysis in the tradition of pragmatics, which "considers language from the point of view of the user – choices, constraints, purposes and so on" (p. 66). Language choices, such as the ones analyzed by Rowland, have effects in particular situations. The structuring effect of language is especially prominent in work that uses systemic functional linguistics – for example, my work with Herbel-Eisenmann identifying and interpreting pervasive word patterns in mathematics classrooms (Herbel-Eisenmann and Wagner, 2010). Brown (2001) has pointed to language as an active force saying, "Mathematical notation does not simply describe mathematical phenomena, it activates it. Language does not just describe action, it is part of it" (p. 197).

When the participants in a discipline make language-related choices, these structural choices relate to their understanding of the structure of the discipline and their position in it. Further, these choices affect others' views of the discipline and their position in it. Though most language choices are made subconsciously, it is useful for analysis to recognize that they are nevertheless choices that reflect the person's intentions in a particular context. Morgan (1998), in her analysis of mathematical text, and Kress (1993), in his more general consideration of critical discourse analysis, both argued for the appropriateness of such an analytical approach.

As we analyze discursive practice, which is an artefact of classroom culture and of individuals' participation in that culture, we are afforded insight into the participants' sense of their own relationship with the discursive system and with others in it. Linguists Chouliaraki and Fairclough (1999) have constructed a framework for analyzing discourse for critical purposes. They encouraged the use of discourse analysis to identify "the range of what people can do in given structural conditions" (p. 65). Attention to language in this way can give new insight into familiar socio-cultural contexts by making common discourse practices seem strange and no longer innocent. Kress (1990) called this the 'denaturalization' of language. Most importantly, the kind of critical attention to language described by Chouliaraki and Fairclough focuses on the choices available to discourse participants, and thus can have an emancipatory effect.

The power of critical discourse analysis for supporting alternative forms of participation in a particular discourse may be most beneficial for people learning the discourse. When critical discourse analysis is introduced in an educational setting, Fairclough (1992) calls it "critical

language awareness" (CLA). In such settings the emancipatory effect is twofold: students are empowered to understand the discourses in which they participate, and they discuss possible alternative ways of participating. While attempts to include CLA in school curricula typically involve language arts and language-acquisition classes, this article reports on CLA in a mathematics classroom, showing how CLA is also relevant to mathematics and how such critical language awareness can have an emancipatory effect for mathematics students.

Reflecting on her critical discourse analysis of mathematical writing, Morgan (1998) suggested the application of CLA in mathematics learning. The conversations reported here show mathematics students can benefit from exploring various ways of participating within the discourse space they encounter daily and how their teachers can benefit from hearing how students experience the discourse. Researchers, too, can benefit from listening in or participating in CLA conversations as students' perspectives, which are relatively unrepresented in research, are elucidated.

Raising student awareness of language and accessing student perspectives on their discourse – underpin the research question that structured my participation in the classroom situation described in this article: What happens when mathematics students are led in CLA? My effort to focus student attention on language was underpinned by my respect for students' knowledge and abilities to engage with challenging questions that relate to complex social issues. (This respect developed during my experiences teaching school mathematics prior to turning my attention to research and teaching mathematics teachers.)

My trust in students' abilities and interests outweighed my worries about the challenges in bringing CLA into mathematics teaching. I thought that attention to the discourse in their classroom would help students understand what mathematics is because language is central to mathematics – mathematics can be described as a discourse, a particular way of talking about experience and ideas. Furthermore, I thought that CLA would rectify some classroom inequities. In particular, I thought students should be given more power to decide how they participate in mathematics class, and that others would benefit from hearing or reading about students' decisions about participation.

Students are likely to have a significantly different perspective on any classroom event as compared to the perspectives of teachers or researchers because of the differences in their power. Thus, students' participation in critical analysis contributes their unique voice to our understanding of what happens in mathematics classrooms. Drawing students into critical reflection on their discourse practices invites their typically-ignored voices, and positions a teacher–researcher as responsive to student concerns for learning mathematics and for mediating students' insights into their unique positions as learners.

"Critical Language Awareness" as Pedagogical and Research Method

To bring CLA into a mathematics classroom, I spent a 19-week semester with a grade 11 mathematics class of students aiming for postsecondary education, co-teaching the course with the paid teacher and collecting video and audio records of classroom discourse every day. While the focus of our work in the classroom was generally to help students develop their mathematical understanding and procedural fluency (the focus prescribed by curriculum and expected by parents and students), I tried daily to engage the students in some discussion about our language practices in the class by directing their attention to their own utterances. The form of my prompts varied, as I was continually responding to the participants. In addition to our classroom

interactions about language, I interviewed participant students and asked them to write accounts of their experiences with language in their mathematics learning.

I saw this research as an investigation of possibility, guided by Skovsmose and Borba's (2000) methodology for critical mathematics education research: "[I]t is by no means a simple truth that research should deal with what is.. . .[D]oing critical research means (among other things) to research what is not there and what is not actual" (p. 5, emphasis in original). The methodology they described aligns well with the critical discourse analysis described by Chouliaraki and Fairclough (1999), which underpins CLA. CLA too is an investigation of possibility within a discourse. While I engaged students in CLA to explore the range of possible ways for them to participate in classroom discourse, I myself was exploring the range of possibilities for me, as a teacher, to engage the class in discourse.

Teaching with differing but simultaneous purposes is not unique to CLA. Mason (1998) characterized the role of the teacher as creating conditions in which students can learn to shift their attention within a discipline. He described how it is possible for students, guided by their teachers, to develop an "inner witness" (p. 251) who extends attention beyond the performance of tasks to notice how they operate on these tasks and how these tasks relate to larger structures of practice. Such awareness of action (which Mason called *awareness-in-action*) gives rise to awareness of the discipline (*awareness-in-discipline*) with attention to the language moves. Mason's theorizing draws attention to the intimate connection between my research motivations and my pedagogical motivations. I attended to my actions as I drew students' attention to their actions both to forward their development of understanding of the discipline and to give me insight into students' experiences of learning the discipline.

The data comprised transcripts of interviews and whole-class conversations about our classroom mathematics discourse and selected student writing. I often showed students selected transcripts to prompt them to articulate their perspectives on language. The conversation described in this article began about two-thirds of the way through the course, while the class was focused on circle geometry (using properties of circles, inscribed angles and cyclic quadrilaterals in geometric proofs). For the interview that started this conversation, I had invited students to have short conversations outside the classroom during class time. I asked them about their goals for the course and the relationship between these goals and our ongoing classroom conversations about language in mathematics[2].

Critical Language Awareness in Action: An Account of Student Insights

This article focuses on interactions that began when two students who were close friends, Arwa and Tharshini (participant names are pseudonyms), initiated a conversation about their eye-movements in mathematics class, which was not the kind of focus I had in mind when I introduced the topic of language awareness. The departure of the conversation from my plans made it clear that I was hearing the students' unique perspective. The conversation started by Arwa and Tharshini was a departure from the kind of critical discourse analysis that I had prompted in our language-awareness conversations in this class. The focus of their attention was on paralinguistic features of our communication, rather than on lexico-grammatical features, which was my focus. Whether eye and head movements are a part of language is debatable. Lyons (1977) noted the wide range of meanings associated with the term *language*. He referred to worded expressions as "normal language-behavior" and classified other language behaviors as paralinguistic, though he noted within linguistic scholarship diverse use of this term.

A Fascination with Pointing and Facing Emerges

When I used the Larson (1983/1989) cartoon to talk about the difference between attending to mathematics and attending to language, I assumed the students would think of the cartoon in the same way as I had. However, in an interview two days later, Arwa told me that Tharshini's pet dog was like the dog in the cartoon. When they had been doing mathematics homework together at Tharshini's home, they had chatted about the cartoon and, as she said, "how true it was." The dog would look at Tharshini's pointing finger instead of following her gaze. This experience from Tharshini's home prompted Arwa and her to reflect on the direction of their own gazes in mathematics class. They exuded excitement whenever they talked with me about the connection between their gazes and their linguistic communication.

The interview began with me asking about their goals in this class. Arwa's goal was "to pass, and to understand why, how [mathematics] applies to life, sort of." Tharshini added, "Pass. And have enough knowledge to get the grade." I then asked them what our attention to language did for their goals. They both started talking at once and then Arwa indicated that Tharshini should answer first:

Tharshini: It doesn't really do anything for my goals but like, you know the pointing thing you were talking about?
I: Yeah.
Tharshini: From that sheet.
I: Yeah.
Tharshini: We both realize
Arwa: Yeah.

It was clear at this point in the interview that Arwa and Tharshini had already discussed their observations about pointing and had planned to tell me about them.

Tharshini: When we help each other and stuff, we're aware that we point and stuff. And I don't really understand what she's talking about. I just look at whatever she's pointing at.
Arwa: And it helps us visualize what you actually want us to know. It's like with the language talk and stuff. Because it gives us a sense about how you want us to answer the questions. And not just like, "yeah this is the answer," but explain why, and how, like the pointing picture thing. That was one of the things, that we didn't realize but when people do look at something, we actually look at it, and we're listening but we're sort of visualizing where they're pointing.
Tharshini: Yeah. You don't look at the other person.
Arwa: Yeah. It's like that cartoon.
Tharshini: It's more like imagining what they're pointing at, trying to understand what the diagram's about.
I: So, even if they're not pointing with their finger, but just talking, you just listen and imagine what it is they're talking about.
Tharshini: Yeah.
Arwa: Yeah. But if they point, you just sort of
Tharshini: If, you're more directed towards it?
Arwa: Yeah.

Tharshini noted here that a mathematical diagram is important because it directs attention to something more important: "[T]rying to understand what the diagram is about." Though their realization that the mathematics on the page refers to something else is akin to the structuralist view of language, the way they talked about this phenomenon revealed their awareness of the complexity of the relationship between symbols and referents in conversation. They realized that a person can only imagine what another person is pointing at. This relates to Radford's (2002) observation that mathematical symbols create something new because the symbols do not point to something that already exists, but rather they create the existence of the object that is pointed out. He called this phenomenon *objectifying deictics.*

The girls continued, saying that they also noticed this phenomenon in their classmates' conversations:

Tharshini: Even when Joey and Kyle argue, or Brandon, they're always pointing at the overhead [projected image]. They'd be like, "Over here, can't you see it" and stuff.
I: Yeah. And the others don't look at the person.
Tharshini: | Yeah
I: | They just look at | [*simultaneous*]
Arwa: | Yeah, they don't |
I: Yeah.
Arwa: | It's not like they
Tharshini: | Because we don't see what they | see or anything
Arwa: Yeah, because what they're seeing, we can't really see through their eyes. But, when they point we sort of visualize actually what they're looking at too, because of the focus to look at.
Tharshini: Yeah.

This tone of this conversation is lost in transcription, but that tone reflects how all three of us were very engaged and enjoyed the conversation. The numerous times that we said "Yeah" testify to our mutual sense of awe that we were recognizing the same phenomenon. Each "Yeah" felt to me like a "Yes, that's it. That's exactly it. We are seeing the exact same thing." Ironically, we seemed to be enjoying our sense that we were seeing the same thing, when in fact we were talking about the impossibility of seeing the same thing. Arwa and Tharshini each articulated this impossibility. Tharshini said, "[W]e don't see what they see," and Arwa repeated, "We can't really see through their […] eyes."

This irony also pervades my account of the conversation. In this article, I write about Arwa and Tharshini as though they were of one mind, because they regularly interrupted and continued each other's utterances. With the fluid connectivity of their utterances, it was often difficult to distinguish between their different points of view. They were good friends and reported to me a few times their fascination with the way transcripts supported their belief that they thought the same thoughts as each other. Ironically, their observations about language use and pointing reveal their awareness of the problematic nature of this belief. They spoke as though they knew what the other person was thinking even as they articulated their awareness that each could not possibly know what the other person was thinking.

Looking through Symbols to See the Inaccessible

A still photograph taken from video data shows an example of Arwa and Tharshini interacting in the way they described (Figure 1). In the image, Arwa leans across the aisle to talk mathematics with Tharshini. Arwa and Tharshini both look down at the symbols on Tharshini's paper as they talk.

Figure 1. Arwa and Tharshini facing mathematics

In my conversations with Arwa and Tharshini about the phenomenon they were describing, we did not talk about any particular examples though it was clear that their observations came from particular experiences of doing mathematics. Our conversations drew my attention to the direction of gaze in relation to drawings and other mathematical symbols, and I assumed that Arwa and Tharshini were becoming increasingly attentive as well. For me, for example, shortly after the first interview with the girls, I was in a conversation with another student, Shauna, about a particular proof. Shauna and I never looked at each other during our interaction. At her desk, she drew the diagram in Figure 2 on my scrap paper and we talked about how to establish this proof. Neither of us commented on the inaccuracies in the sketch. We saw the sketch as pointing to the idea of a circle, which we felt was a shared idea. We argued about what we could claim as known based on the information given (and, yes, we did argue). And we physically pointed with our writing instruments. Dots appear on the diagram from my pen stabs and her pencil stabs. As the argument continued further sketches were drawn, even less accurate. I was trying to help her see my way of thinking and she was trying to help me see hers.

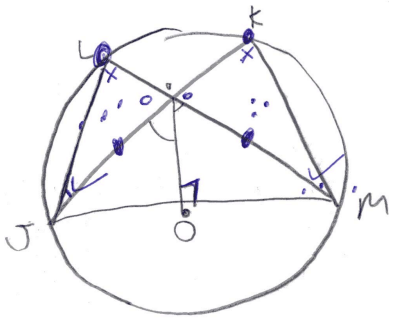

Figure 2. Proof argument jottings

In this argument, neither of us looked at each other directly. Shauna and I each wanted the other to see what we ourselves were seeing; each of us used the sketches to signify the way we saw the situation. In reflection, I recognize elements of structuralist assumptions about language during this conversation. However, I also identify elements of post-structural assumptions about language: Shauna and I both saw our symbols as creating something new for the other. Shauna even scolded me for looking at my sketch instead of hers because she wanted hers to create something for me.

Gordon Calvert's (2001) consideration of groups larger than two substantiates the idea that participants in mathematical conversations need to share their symbols:

> one person, the one sitting on the end or on the other side of the table frequently could not participate fully simply because he or she could not see the mathematical references put on paper. Communication of mathematical ideas is in large measure nonverbal in terms of its reference to symbols, diagrams, and drawings on the page, as well as the gestures which include pointing to, as well as talking about. (p. 96)

In even larger groups, such as a whole mathematics class, conversation is usually mediated through writing and other symbols on a blackboard, whiteboard or projected image. It is more difficult to see where students look if the teacher or another student stands at the board, close to the mathematical symbols. Do students look at the symbols or at the person at the blackboard? Arwa, Tharshini (and I, after they directed my attention) noticed that when students in their class argued and conversed with each other about mathematics, they did not look at each other. They continued to look at the symbols displayed at the front of the room.

It is said that the eyes are the windows to the soul. We may look into the other person's eyes to see the soul, but it appears that we need to look at symbols, including written symbolic artefacts of a person's utterances, in order to face their mathematical ideas and reasoning. Arwa and Tharshini's sense that this feature of communication was connected to the impossibility of two people actually seeing the same thing when they look at the same symbols added depth to my reflection. This nuance suggests that they were aware that the symbol is not the mathematics – it is a medium of the mathematics.

With the combination of their two observations, that mathematics is seen through symbols and that no two people can see any symbol in the same way, Arwa and Tharshini's sense of the inaccessibility of mathematics resembles observations by Sfard (2000), Duval (1999) and others. Duval wrote, "there is no direct access to mathematical objects but only to their representations" (p. 24), and Sfard noted that rational numbers, for example, "are not palpable objects that can be seen, heard, and touched. […] An implied entity remains behind the

scene and its existence can only be inferred from the processes performed on its 'representations'" (p. 38).

Sharing Mathematical Symbols

After my interview with Arwa and Tharshini, I asked the entire class where they looked when communicating mathematics. A few days later, Rory, who was not a friend to Arwa and Tharshini, reported in class discussion that she had experimented with directing her gaze. She described her motivation this way: "sometimes you try to look at the person, because you should be looking at them when they're talking." But she found that when she looked at her friend Kalli instead of Kalli's mathematical symbols, she could not understand Kalli's words. (After this comment, whole class discussion about the phenomenon was effectively terminated by another student's rude joke. However, I continued to talk with Arwa and Tharshini about the phenomenon.)

Rory found that she had trouble understanding when she was not looking at the symbols being discussed. Her reflection prompted me to think about effective communication practices for mathematics educators. For example, when speaking mathematics, teachers who expect students to write notes as well as understand should pause enough for students to write without missing out on listening – the kind of listening that requires looking at the writing and symbols being referred to by the speaker.

Furthermore, Rory's account of her experimentation drew my attention to another aspect of mathematical symbols. Rory talked about Kalli's symbols and I wondered about this sense of ownership of symbols. Ownership relates to power; whomever controls the symbols controls the discourse. When Kalli drew symbols (which in this case were probably diagrams related to circle geometry) during their mathematical conversations, were they not Rory's too? This question relates to Bakhtin's (1953/1986) observations about the complexity of utterances and to whom the words belong. He noted that many utterances can be taken as a combination of the speaker's word, another person's word and as a word that does not belong to anyone.

> [A]ny word exists for the speaker in three aspects: as a neutral word of a language, belonging to nobody; as an *other's* word, which belongs to another person and is filled with echoes of the other's utterance; and, finally, as *my* word, for, since I am dealing with it in a particular situation, with a particular speech plan, it is already imbued with my expression. (p. 88)

This question of ownership prompted for me questions about the notion that there can be taken-as-shared meanings (e.g. Cobb, Wood, Yackel and McNeal, 1992). What exactly is taken as shared – the mathematical object or the symbolic representation of the mathematics? And, if it is shared, is this an example of democracy in action, or does the normalization of symbols suggest an external power controlling all? Because participants in mathematical conversation often look through the same mathematical symbols (they share the symbols), there is a natural expectation that they all see the same thing. The expectation appears in the grammar of classroom discourse, which often suggests there is one shared way of seeing (Herbel-Eisenmann and Wagner, 2010).

Noticing the Power of Symbols

After talking with Arwa and Tharshini about the direction of students' gazes, I tried to envision ways of making the nature of mathematical symbols clear to other students. What

metaphors would be most accessible? In my last interview with these girls, they made a powerful comparison that connects with equity issues in any situation involving symbols. Their comparison also demonstrated that our conversation continued to colour their experiences just as it was coloring mine. They said that when their history teachers would point at maps to talk about Napoleon's invasions of neighbouring countries, students were supposed to be thinking about the wars – the fighting, the movement of fighting men through populated and rugged terrains, the social issues that prompted the fighting and the social issues that were caused by the fighting. Sometimes, when their teachers pointed at maps while talking about the Napoleonic wars, Arwa and Tharshini would find themselves thinking about the wars as though they were transformations of shapes on the map (as the boundaries changed with the moving fronts). They said that they were supposed to be looking through the map instead of looking at the shapes on the map. This comparison reflects their awareness that maps are representations of things that cannot be seen in a classroom.

Relating to the tension between structuralist and post-structuralist views of language, one might argue about which is more important, the map of the war or the fighting and famine in the war. For the people on the ground during the war, surely the concrete experiences of hunger and pain were paramount. However, the wars would have been planned with maps. The maps, in a sense, made the warring happen where it happened. Further, maps drawn after the war structure the image of the war for people outside the war, people like Arwa and Tharshini. Again, both the architects of a war and the interpreters of a past war would do well to attend to the concrete experiences of people affected by their maps and manipulations of maps. Maps, like language and all other symbols, are powerful. Critical attention to the way representation works allows us to use and see these symbols differently.

Adler (2001) described a similar situation, the *dilemma of transparency*. In her analysis of issues faced by mathematics teachers in multilingual environments, she drew on Lave and Wenger (1991), who had suggested that in fluent practice resources are used as though they are transparent. She extended their image by including language as a resource. Non-verbal mathematical symbols are also a resource, though not all students have the same facility with the resource. Symbols need to be used as though they are transparent. However, just as with language, there is value in occasionally looking at the symbols instead of only looking through them. Indeed, this is what Arwa and Tharshini were doing when they talked with me about the role of symbols in mathematics communication. I suggest that this time of looking at the symbols while looking through them and of watching peers look through symbols afforded these two girls an opportunity to understand their mathematical thinking and reasoning processes better.

Reflection: Locating Mathematics

The limitations of the research arrangement prevented me from following Arwa, Tharshini and their classmates more. However, I would expect our conversations in the class continued to influence their experiences, just as they have influenced mine. Our conversation sparked my interest in two somewhat related aspects of the way students turned their gazes during mathematics discussion. I have noted the connection between students turning their gazes away from their interlocutor's face and linguistic structuring that masks human agency. Now I will explore how language expresses the physical distance between our mathematics and ourselves.

It is significantly different talking about a pipe (as in Magritte's famous painting: "Ceci n'est pas une pipe"), which my conversation partners and I can all touch, than talking about a

square, a parabola or a proof, which only exist in our imaginations. Yet I note the tendency to speak of these inaccessible things as though they are present like a pipe that is in our hands. In English, mathematical utterances are often characterized by the present tense and by pointing words like *here* and *this*.

Linguists use the word *deixis* to describe the process of pointing with words. The root of the word is the Latin word for *finger*. In pragmatics, which is the field of linguistics that is interested in the effects of people's language choices, deixis has further significance. Rowland (2000) has described significant aspects of deixis in mathematics learning, but focused on vagueness in pronouns, which are pointing words. I have also considered pronoun deixis in student-teacher interactions in mathematical investigations and in interviews with the participants in the investigations (Wagner, 2003).

However, Arwa and Tharshini directed my attention to other pointers. Levinson (1983), whose work is not focused on mathematics learning, distinguished between distal pointers, such as *that* and *there,* and proximal pointers, such as *this* and *here*. The proximity we suggest in our language choices can imply our sense of intimacy, control over , or distance from the thing we are pointing at. We use words proximal pointers (*this* and *here*) to refer to things that are present and within reach, and we use distal pointers (*that* and *there*) to refer to things that are not within reach and thus not under our control.

My limited knowledge of another language helps me understand the apparent oddity of our practice of referring to distant things as though they were proximate. In siSwati, a Bantu language of southern Africa, there are three words for referring to the position of objects – *lapha* (pronounced LAH-pah*)*, *lapho* (lah-POH) and *lapha* (lah-PAH)**.** Notice the different stress in the pronunciation of this last version, despite the fact that it is spelled the same as the first one. To distinguish between these two, I will use bold font for the emphasized syllable. ***La****pha* means *here*. *Lapho* means *there*. And *la****pha*** is often translated orally as "on the other side." It refers to things beyond our view. I find it interesting that the words for *here* and *way over there* are the same word with a mere difference in emphasis – ***la****pha* and *la****pha*** – especially since the emphasis of syllables is usually unimportant in siSwati. If I say "*this* piece of paper, here in my hand," I would use ***la****pha*. If I say "*that* piece of paper on the table" and I am pointing at it so that my audience can see it, I would use the word *lapho*. If I am outdoors and talking about a piece of paper on my desk in my office, I would use the word *la****pha***. It is far away, beyond our view.

In English there are similar practices. We talk about far away or abstract things as if they are proximate by using the present tense and pointing words like *this* and *here*. The alternatives – using the past tense and the words *that* and *there* – suggest distance. Levinson (1983) described how proximal pointers are used to refer to things that are physically or conceptually present. Thus even far away things have a sense of presence when they are discussed because they are conceptually present.

When I talk with a friend about a piece of paper that is far away on my desk and describe how I can fold it to achieve a certain result, I display a sense of proximity with my pointing words when I say "this paper." I also indicate proximity by using the present tense: "Fold the top right corner to the centre line." These common ways of speaking suggest that the paper is right here in my hand, where my audience can see it. (I may even gesture to suggest the folding, as if I had the paper in my hand.) Yet in reality it is far away – *la****pha***. Indeed, it might not even exist. Though it might be a hypothetical piece of paper, it does exist in my thinking and in my audience's thinking. Because we see this paper only in our imaginations, it feels even more proximate than the sidewalk we are standing on. However, we each imagine our own "piece of

paper," and we cannot see each other's papers. We talk as though we see the same paper, but our words that point at and refer to this mythical piece of paper actually refer to more than one piece of paper. Each person refers to her or his own imagined piece of paper.

In a conversation a few weeks after the interview described earlier, Tharshini gave an example of such a situation. She described conversations she had had with another friend who was also taking grade 11 mathematics but not in her class. When they talked about their mathematics learning, they spoke as though they had experienced the same lessons even though they knew that their lessons had been significantly different. Tharshini said that when she would talk about a lesson she had just experienced, her friend "wouldn't really be focusing on what I'm saying, but [...] trying to remember back whatever *they* did that day, and then try to understand."

We talk as though we experience the same thing, even if it is a piece of paper on the other side of the world. That is where mathematics is. It is *la**pha***, beyond our senses, but we talk about it as if it is here. I sense that this gulf between the *actual* location of mathematical objects (inaccessible) and their *apparent* location (proximal and tangible) suggested by the way we speak of them is one reason why communication in mathematics is complex and often frustrating.

The conversations and interviews with students reported in this article lead me to believe that if students became aware of this problem they might feel better about being confused. When I was in grade school, my teachers told me that mathematics was precise, unambiguous. I believed them. I was successful in mathematics, but now I am concerned about the students who struggle with mathematics and wonder how they experience mathematical discussions. How do they feel about not seeing inaccessible things that are supposed to be so clear? How do they feel about not seeing things that are talked about as though they are present and at hand? I think that these are all good reasons to be confused during mathematical discussions. I am not suggesting that we change the way we talk about mathematics to make mathematical objects seem less proximate. Instead, I suggest that there is value in helping students become aware of the reasons behind the semiotic tensions related to speaking mathematically.

Significantly, some students in the researched classroom I have described summed up their sense of the value of attending to language by saying that it made them more comfortable in class. The students in my classroom who noted this benefit did not seem to know why they felt this way, but they claimed a new sense of security, a feeling that they had not experienced in their previous mathematics classes. It seems ironic that their awareness of the complex and perhaps messy relationship between symbols and meaning in mathematics would make them more comfortable, perhaps because it can be comforting to know that a situation is messy and complex and that confusion is can be one appropriate response.

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[1] The focus of the article is on the students' observations and the reflections they prompted for me. Though these observations and the resultant reflections touch on ideas that have drawn significant research attention in mathematics education, namely semiotics and gesture, I do not suggest that the students' insights are building on that literature. Rather, the point is to show students' own perspective, which was relatively unmediated by the research tradition.

[2] Other streams of conversation from this class include an account of our conversations about mathematics teachers using the word *just* to suggest simplicity (Wagner, 2008), and an account of our conversations about agency (Wagner, 2007). Each of these accounts gives further insight into the application of CLA both as a research method and as an orientation for mathematics teaching.

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