# PERCEIVING DATA DISPLAYED THROUGH OSCILLATORY MOTION

**by**

**Colin Ware and Serge Limoges**

**TR94-089 November 1994**

Faculty of Computer Science
University of New Brunswick
P.O. Box 4400
Fredericton, N.B.
Canada E3B 5A3

Phone: (506) 453-4566
Fax: (506) 453-3566

# Perceiving Data Displayed Through Oscillatory Motion

COLIN WARE[1] and SERGE LIMOGES[2],
Faculty of Computer Science, University of New Brunswick, New Brunswick, Canada.

The idea of using simple sinusoidal motion of data points in a multidimensional scatter plot is explored both through a constructive analysis and by means of three experimental studies. In the system described, data values can be mapped to any combination of frequency, relative phase and amplitude as well as point size, point gray value and the center of motion. Three experiments assess the efficiency of the three motion parameters and the other display parameters in displaying correlations. The results of the first experiment suggest that relative phase is the most useful of the motion parameters, it compares favorably with gray value and point size, although it is not as effective as position on the vertical axis (the conventional scatter plot). The second and third experiments measure interference between display variables. The results show that motion does interfere with data represented by position and gray value but a phase representation is better in this respect than a frequency representation. It is argued that many natural phenomena result in related objects, or parts of objects moving in phase and this may be why relative phase is an effective display method.

**Running Title:** Motion for Information Display

**Keywords:** Statistical information display, Animation.

---

1 Requests for reprints should be sent to Colin Ware, Faculty of Computer Science, Box 4400, University of New Brunswick, New Brunswick, Canada, E3B 5A3.

2 Now at IBM Toronto Labs, Don Mills, Ontario, Canada.

6/2/94

## INTRODUCTION

In recent years there has been considerable attention given to the problems of displaying multivariate discrete data using a number of novel display methods such as Star plots, Chernoff faces, plots using color and shape and generalized draftsman's plots (Chambers, et al, 1983; Ware and Beatty, 1986). With advances in computer graphics technology it is possible to manipulate many more display options than in the era of paper based display. For example, color and visual texture are far more controllable on a CRT display than they are using print media, and the use of motion to display data values becomes feasible.

There is a body of work from a number of diverse areas which bears on the subject of motion for information display. Disney style character animators understand through their craft how to make motion convey information In particular, their work with exaggerated squash and stretch effects may be useful in illustrating the effects of forces on inanimate objects ( Thomas and Johnston 1981).

More formal studies of the simple motion of a small number graphic objects shows that such things as causality, pushing, pulling can be conveyed reliably with a few motion parameters (Michotte, 1963; Lethbridge and Ware, 1989). Michotte in particular has done extensive psychophysical experiments concerning the one dimensional motion parameters of two objects required for one graphic object to appear to *cause* the motion of another. This work has obvious application in graphical user interfaces because the perception of causality is impossible, or at least very difficult to induce with a static display.

There is a developing literature on the representation of algorithms using animation, to show such things as the sequencing of events, the positioning of ordered data objects, flow of control, and the transmission of information (Stasko, 1990; Brown and Sedgewick, 1988, Bennett, 1993). These studies demonstrate the advantages of animation for visual information display, but say little or nothing about the relative information content conveyed through animation. It is left to the craftsmanship of the system designers to use parameters of motion in a way that meaning is conveyed to the user.

Recent studies from the vision research literature show that motion can elicit "popout" effects. That is, moving objects can be searched in parallel by the human visual system for targets with anomalous direction of motion (Driver et al. 1992, Braddick and Holliday). For rotary motion, different rates of motion can also be discriminated on the basis of a parallel search (Julesz, 1970). These results are particularly encouraging for the use of motion in information displays because they suggest that motion is one of the "preattentively" processed attributes of vision and therefore motion, like color, size and orientation will be rapidly and efficiently interpreted.

It is abundantly clear that the visual flow of information which occurs as we move through space tells us a great deal about the 3D layout of objects in space (Gibson et al, 1959). J.J. Gibson introduced the notion of "ecological optics" whose essence is that the human visual system has evolved to help guide us through 3D space and that we are therefore tuned to detect and interpret natural "ecological" motions, such as the flow of visual information. A related phenomenon is the fact that brain tends to interpret objects in motion as if they are connected by rigid rods (Cutting, 1986). Because of this active interpretation, images which perform motions consistent with the projection of simple rotation about an axis are perceived as existing in 3D space. This phenomenon, which is sometimes called the kinetic depth effect has been exploited to allow the perception of discrete 3D scatter plots in a number of commercial data display packages (e.g. Donoho et al, 1988).

Although the above studies suggest that motion has some interesting potential for displaying data, they say very little about the relative efficiency of motion compared to more common display parameters used for multidimensional discrete data display. The fundamental data display problem is to determining which graphical attributes are effective in assisting the interpretation of data. This is a problem of great complexity, because the display dimensions, such as size, color, orientation are not independent from one another and the perception of them depends on the task (e.g. Ballesteros, 1989). In the present study we consider the simple sinusoidal motion of data points as a display option.

There are many unanswered questions. For example, can object motion display as many gradations in value (just noticeable differences, or *ands*) as say, gray values? Are these differences perceived as orthogonal to other display parameters such as object shape and size. Does the fact that an object is moving make it harder to perceive its shape, or is that perceive shape distorted in some way? In this paper we present a series of studies designed to investigate the utility of simple sinusoidal motion of data point for the purpose of representing real valued attributes of multidimensional discrete points. In practical terms we extend the dimensionality of the scatter plot by adding motion in the x and y directions and control the frequency, amplitude and phase of the motion. We also change the data point sizes and gray values.

Our research strategy has been two pronged, first we designed and implemented a package which allows us to explore the possibilities of multidimensional data display, capable of displaying up to 11 data parameters from 200 discrete data points. These display parameters are listed below.

- X position $X$
- Y position $Y$
- X amplitude $\alpha_x$
- Y amplitude $\alpha_y$
- X frequency $\phi_x$
- Y frequency $\phi_y$
- X phase $\theta_x$
- Y phase $\theta_y$
- X point size $S_x$
- Y point size $S_y$
- Gray Value $\lambda$

Motion is controlled using the following equations

$$P_{xt} = X + \alpha x \sin(\phi x (t + \theta x)) \quad (1)$$
$$P_{yt} = Y + \alpha y \sin(\phi y (t + \theta y))$$

Where $P_t$ represents the coordinates of the display point at time $t$, $(X,Y)$ represent the center of motion, $\alpha$ $\phi$ and $\theta$ define the amplitude, frequency and phase respectively. Data points are plotted as rectangles which can vary independently in width and height. The gray value of each data point is determined by a linear mapping between the monitor black and the monitor white, or it can be set to black. The time variable t is updated with the display update rate of the graphics subsystem, usually 30 Hz.

Note that although there are eleven items on the above list of display parameters. Only nine are actually available simultaneously for the following reason: varying the relative phase between different data points in motion is only meaningful if they are oscillating at the same frequency. Thus only two of the three motion parameters can be used at any one time in each of the X and Y directions.

The display package allows us to display a discrete multivariate data sets in different ways and to gain intuitions about the relative usefulness of display parameters. In one data set with 5 variables we are able to show the clear perception of clusters in a five dimensional space. Recently we have been successfully using the technique to explore multidimensional data representing measurements on geological core samples. This allows us to visually cluster large numbers of data attributes.

One of the difficulties in studying multidimensional display methods is the combinatorial explosion of display options as the dimensionality increase. For example given only three data dimension and 9 possible display dimensions there are (9 choose 3) = 84 possibilities, far to many for a manageable experiment. For this reason the experiments described here are narrow in focus, looking at what we thought to be the most interesting questions with two variables displayed in a variety of ways.

## EXPERIMENTS

As a task we selected the perception of correlations between variables. The first experiment was designed to determine how effective various display techniques are in discriminating high from a low correlations. The second and third experiments were designed to address the related problem of visual orthogonality, the extent to which noise on one display variable can interfere with the perception of structure displayed using other variables.

## GENERAL METHOD

We measure an observer's ability to discriminate a high correlation ( $r^2 > 0.5$) from a low correlation ( $r^2 < 0.5$) where r refers to the product moment correlation. We use $r^2$ as our metric because of studies suggesting that perceived correlation is more closely related to $r^2$ than to r (Pollack, 1960). Our method is to determine the discrimination threshold separation between high and the low correlation using a binary forced choice decision task. The subjects are given a sequence of scatterplots in two or more dimensions, using two or more of the display methods. The subject is expected only to respond with a "high" or a "low" on each trial. This allows us to generate a probability distribution curve for each of the data parameters; That is, we determine the probability of perceiving a high correlation with a set of values of $r^2$ ranging between 0 and 1.

The creation of the artificial data sets is done using the following algorithm

1. Supply a correlation matrix R, where R is a symmetric matrix containing the desired correlation values

$$R = \begin{bmatrix} 1.0 & r_{12} & \dots & r_{1p} \\ r_{21} & & \dots & \cdot \\ \cdot & & \dots & \cdot \\ \cdot & & \dots & \cdot \\ \cdot & & \dots & \cdot \\ r_{p1} & & \dots & 1.0 \end{bmatrix}$$

where p is the dimension of the data space and the $r_{ij}$ denote the Pearson's product moment correlation between the ith and jth dimensions. The values vary between -1.0 and 1.0.

2. Generate a temporary multivariate normal space T with elements $t_{ij} = N(0,1)$, $1 \le i \le m$, $1 \le j \le n$, where m is the number of data points and n is the number of dimensions required.

3. Orthonormalize T so that the mean vector is zero and the covariance matrix is equal to the identity matrix, using the Gram-Schmidt algorithm (Anton, 1984).

4. Calculate matrix C where $R = CC^t$

Matrix C is calculated using the following algorithm:

**for** i = 1,2, ..., m
   **for** j = 1,2, ..., i
      **if** (i=j)

$$C_{ii} = \sqrt{R_{ii} - \sum_{k=1}^{i} C^2_{ik}}$$

      **else**

$$C_{ij} = \frac{R_{ij} - \sum_{k=1}^{j} C_{jk}C_{ik}}{C_{jj}}$$

5. Calculate $A = \sum_{j=1}^{i} C_{ij}\, t_{kj}$ for $1 \le k \le m$, $1 \le i \le n$

The resulting multivariate space has mean vector zero and a correlation matrix R.

## EXPERIMENT 1

The purpose of the first experiment was to measure the sensitivity of observers to correlations displayed using each of the different display methods. For this experiment different degrees of correlation between two variables were synthesized and one of the variables was always mapped to horizontal position. The other variable was mapped to one of the following:

1 Vertical position (this creates a conventional scatter plot)
2 Vertical amplitude
3 Vertical frequency
4 Vertical phase
5 Height of data points (bar chart)
6 Gray value

Display parameters

In addition to the display parameters which we explicitly manipulated, there are a number of other display parameters which may affect the efficiency of a given display technique. To give an example, the amplitude of motion can affect discrimination through frequency, if the points are only moving a small amount it is hard to perceive how rapidly they are oscillating. Thus it is necessary to provide fixed default values for display parameters that are not being explicitly evaluated. In order to give each display method the best chance, we adjusted the default fixed values for each of the experimental conditions so that they appeared to be reasonably close to optimal. The results of this exercise are reflected in the ranges given Table 1.

---

**Insert Table 1 about here**

---

Recalling that one of the data values was always mapped to horizontal position the values shown in boxes drawn with a double border around them illustrate the range of the second display variable in a given condition. Each experimental condition is represented by a row of this table. For example if we examine row 4 we find that the display window was 13 cm wide and 12.5 cm high. The frequency of vertical oscillation was 0.4 Hz and its amplitude was the full vertical extent of the window, namely 12.5 cm. The data points themselves were black and they were 1 cm wide and 1.5 cm high. The motion of the individual point varied between in phase and 180 degrees out of phase with one another. What the observer would see would be points normally distributed horizontally, moving vertically at 0.4 Hz. If there were a high correlation being displayed then the points to the right would be moving out of phase with those on the left. Points in the middle would have an intermediate phase value. It there were a low correlation then there would be no perceived pattern to the motion. There were 30 data points generated in each synthetic correlation. The background was always white at about 50 candelas/ $m^2$.

Trials

Subjects were tested in four separate sessions on different days. In each session the observer was tested in each of the six conditions, with the order of conditions randomized. Each condition involved eighty five trials consisting of five representations of each of seventeen degrees of correlation using a single display method, in a random order. On each trial, one of seventeen different $r^2$ values, 0.1, 0.15, ..., 0.85, 0.9 was synthesized in the data and displayed in the form of a scatter plot. The subject's only task was to decide whether the correlation was high or low - a forced choice binary decision.

For each of the six conditions, each subject was first shown an example of a low, middle and highly correlated chart. The subject was also told what characteristics to look for in deciding if the chart had low or high correlation value. No time constraint was put on the subjects responses.

The ten subjects used in the experiment were graduate and undergraduate computer science students. All had completed an elementary statistics course and were familiar with the concept of correlation. They were paid to participate.

Results from Experiment 1

The results summarized in Figures 1 and 2 and Table 2. Figure 1 presents a set of plots showing the probability of a "Low" response to each of the $r^2$ values for each of the six experimental conditions averaged across all ten subjects and four sessions. The X against Y position plot is a conventional scatter plot and this provides a reasonable *de facto* norm against which other techniques can be measured.

In order to derive efficiency measures from this data we fit a cubic curve through each of the plots and read off values at p=0.25 and p=0.75. The difference between these readings represents the change in $r^2$ required to change the probability of a "Low" response from 0.25 to 0.75 and this is our measure of efficiency.

---

**Insert Table 2 about here**

---

The sign test is used to in order to obtain a statistical ordering of the six techniques (Siegel, 1956) This is found to be more sensitive than Scheffé's multiple orthogonal comparison technique (Limoges, Ware and Knight, 1989). We use the sign test on the data plotted in Figure 1. A display technique is judged more efficient than another if correlation values < 0.5 are more likely to be correctly judged low using that technique, and values of r > 0.5 are more likely to be correctly judged high using that technique. Thus we perform a sign test on the differences obtained between pairs of methods to obtain the ranking given in Figure 2. In this figure more efficient methods are higher on the page. If method A is significantly more efficient than method B ($p<.05$) then an arrow is drawn from A to B. This diagram is transitive, that is, if A is more efficient than B and B is more sensitive than C then A is more efficient than C.

The results from Experiment 1 show that the conventional scatter plot is the most efficient display technique. Plotting phase against X position is the second most efficient technique, and is not significantly worse than the conventional scatter plot. The most notable result here is that motion phase is actually more efficient in displaying correlation information than is gray value or point size, two commonly used display techniques. The other two motion display parameters were not as effective: Amplitude of motion appears to be no more efficient than point size or gray value, and frequency was the least efficient of all the display techniques by a wide margin.

## EXPERIMENTS 2 AND 3

It is possible that a display technique which is highly effective may still be undesirable because it interferes with the perception of other patterns which are simultaneously being represented. For example color might distort perception of size and motion seems likely to distort the perception of position (given by the center of motion). This problem of the lack of perceptual independence is likely to get worse as the number of data dimensions (and their corresponding display dimensions) increases. Experiments two and three address different aspects of this problem. Experiment 2 addresses the question of the efficiency of a scatter plot in the presence of noise on one of the other display dimensions. Experiment 3 addresses the question of the relative efficiency of the various display methods in the presence of position noise.

To address these questions we constructed displays from three dimensional discrete data, with two of the dimensions are correlated and the third containing uncorrelated, normally distributed noise.

<u>Conditions for Experiment 2</u>

In this experiment a convention scatter plot is always presented with a correlation between variable mapped to X and Y axes. Noise is added in the form of a random normal variable mapped one of

1 Y amplitude
2 Y frequency
3 Y phase
4 Y point size
5 Gray Value

The purpose being to find out how motion, gray value and size interfered with the perception of a correlation conventionally displayed. Specifically, there is always a zero correlation between each of the above dimensions and both position with respect to the X axis and position with respect to the Y axis, while there is a positive correlation between position with respect to the X axis and position with respect to the Y axis.

Conditions for Experiment 3

Experiment 3 is very similar except there is always zero correlation (noise) between X and Y positions, and there is a positive correlation between X position and each of the display parameters listed above. The idea is to find the effect of position noise on patterns represented using motion, gray value, etc.

The default values for the different display attributes are given in Table 3.

-----------------------------------------------

**Insert Table 3 about here**

-----------------------------------------------

Trials and Subjects

The method used for both Experiment 2 and 3 was identical in most respects to that used for Experiment 1. The same 10 subjects were used. Subjects were tested in four separate sessions on different days. All ten experimental conditions were tested in each session with their order randomized. For every experimental condition, eighty five trials consisting of five representations of each of the seventeen degrees of correlations were tested.

Results from Experiment 2

The results of the visual efficiency analysis are summarized in Table 4 and the results from the sign test analysis are summarized in Figure 3 (this can be interpreted in the same way as Figure 2). The point size and the gray value interfered least with the visual perception of correlation in the conventional scatter plot. Within the motion parameters, phase and amplitude interfered less than frequency. It is not surprising that changing the color and size of points interferes less with the perception of position information than does making them move. The fact that phase coding of information interfered the least supports the notion that this is the most useful of the motion parameters.

-----------------------------------------------

**Insert Table 4 about here**

---

Results from Experiment 3

The results of the sign test analysis are summarized in Figure 4. Gray value and phase showed the best resolution of correlations in the presence of vertical scatter. This suggests that both color and motion can be readily integrated by the visual system over separated data points. This is often a useful property, especially in visual cluster analysis. In this condition subjects were required to integrate over data points.

---

**Insert Table 5 about here**

---

## DISCUSSION

The results suggest that mapping display parameters to the relative phase angle of oscillating data points can be an efficient way of displaying patterns in data. Experiments 1 and 2 showed phase to be much better than using frequency and significantly better than using amplitude. Phase even compares favorably with gray value and point size, both commonly used display techniques. The main drawback with using motion as a visual display parameter is the obvious one that when points are in motion it is difficult to perceive exactly where they are. This was demonstrated empirically in Experiment 2; however, even here the effects of phase were less disruptive than either varying the amplitude or the frequency of motion. In informal studies we have found that phase coded information can be understood when the amount of motion is quite small. The large amplitude motions used in this study may not be required and small amounts of motion will clearly make the center of motion problem less severe.

It cannot be claimed that these studies have established any absolute value for motion as a display method, in all experiments of this type there are far too many unaccounted for variables to make such a claim. It is obvious that salience of a variable such as amplitude will depend on such things as the frequency, constant and the size of the data points that are moving. Indeed in a practical data display system, use can be made of these tradeoffs. If the parameters displayed using color are critical the plotted points should be large. If the parameters displayed using phase are critical the amplitude of motion should be large (although our experience suggests that very high amplitudes are not necessary). If the parameters displayed using point shape are critical then the motion should probably be minimal and variation in color should also be reduced. What we have done with the experiments described here is to suggest that motion has intriguing possibilities for the display of information, and that mapping data values to the relative phase of moving objects may be the most useful motion parameter. Now that animated displays are becoming a low cost option through computer graphics, the possibilities of using motion to display information deserve further study. We have only begun to scratch the surface here. however, having objects in motion need not necessarily decrease our ability to discern other perceptual attributes.

The reason why phase is an effective display method may very well have a Gibsonian ecological interpretation (Gibson et al, 1959). When phase is manipulated, data points which have the same value move together as a group, this naturally leads them to be interpreted as comprising a single visual object and to differentiate them from other objects. Many natural phenomena, such as the motion of tree branches in the wind, the schooling behavior of fish, and the flocking behavior of birds result in objects which are related moving in phase with one another, while unrelated objects move out of phase.

In our more informal experiments with the data display package we have found it useful to map variables to phase in the horizontal direction as well as phase in the vertical direction (with other variables used to determine the center of motion for each data point). What is seen is that related points follow elliptical paths, with points that are clustered in the four dimensional space moving together. Figure 5. is intended to illustrate how a cluster of points may be discriminable because of the phase of motion in a package that employs size, gray value and postion as well as motion.

## Acknowledgments

This work was funded by a grant from the National Science and Engineering Research Council of Canada. Thank are due to William Knight for advice on statistical matters.

## References

Anton, H. (1984). Elementary Linear Algebra. Wiley.

Ballesteros, S. (1989). Some determinants of perceived structure: Effects of stimulus task. In B.E. Shepp and S. Ballesteros, (Eds), Object Perception, Structure and Process. Lawrence Erlbaum: New Jersey.

Bennett, K.B. (1993). Encoding apparent motion in mimic displays, Human Factors, 35(4) 673-692.

Braddick, O.J. and Holliday, I.E. (1991). Serial search for targest defined by divergence or deformation of optic flow. Perception, , 20, 345-354.

Brown, M.H. and Sedgewick, R. (1985). Techniques for Algorithm animation. IEEE Software, 2(1): 28-39.

Chambers, J.M., Cleveland, W.S., Kleine, B. and Tukey, P.A. (1983). Graphical methods for data analysis. Boston, Duxbury Press.

Cutting, J. E. (1986). Perception with an eye for motion. MIT Press, Cambridge Mass.

Donoho, A.W., Donoho, D.L., Gasko, M., (1988). MacSpin: Dynamic Graphics on a desktop Computer, IEEE Computer Graphics and Applications, July, 51-58.

Driver, J. and McLeod, P. and Dienes, Z. (1992). Motion coherence and conjunction search: Implications for guided search theory. Perception and Psychophysics, 51(1) 79-85.

Gibson, J.J., Gibson, E.J. Flock, H. and Smith., O. (1959). Motion parallax as a determinant of perceived depth. Journal of Experimental Psychology, 58: 40-51.

Julesz, B., and Hess, R.I. (1970). Inability to perceive the direction of rotation movement of line segments. Nature, 225, 243-244.

Lethbridge, T.C. and Ware, C. (1990) Animation using behaviour functions, In Tadeo Ichikarva, Erland Jungert and Robert Korfhage, (Eds), Visual Languages and Applications. Plenum. 237-252.

Limoges, S., Ware, C. and Knight, W. (1989). Displaying correlations using position, motion, point size or point colour. Proceedings of Graphics Interface '89, 262-265. Morgan Kaufman.

Michotte, A, (1963). The perception of causality, Methuen.

Stasko, J.T. (1990). Simplifying Algorithm Animation with TANGO, Proceedings of the 1990 IEEE Workshop on Visual Languages, October, 4-6 Skokie, Illinois, IEEE Computer Soc. Press, 1-6.

Thomas, F and Johnston, O. (1981). Disney Animation: The illusion of life. Abbeville Press, New York.

Ware, C., Beatty, J.C. (1988). Using color dimensions to display data dimensions, Human Factors , 30(2), 127-142.

# LIST OF FIGURES

COLIN WARE and SERGE LIMOGES (Perceiving Data Displayed Through Oscillatory Motion)

COLIN WARE received a PhD. in experimental psychology from the University of Toronto, and a MMath degree in Computer science from the University of Waterloo. Since 1995 he has held a faculty position in the Faculty of Computer Science at the University of New Brunswick. His research interests include new techniques for the use of color, visual texture and motion in information display. In addition he has been studying the use of six degree-of-freedom input devices for the manipulation and exploration of 3D computer graphics environments.

SERGE LIMOGES is currently a software engineer at IBM Toronto Labs. He received BSc. and MSc. degree from the University of new Brunswick. The topic of his Master's thesis was the perception of data displayed through motion.

**TABLE 1**
Display parameters for Experiment 1

| Condition | Window Width x Height (cm) | Amplitude (% of Window) | Frequency (Hz) | Phase Angle (radians) | Point Size Width x Height (cm) | Point Gray Value (Candelas/ sq meter) |
|---|---|---|---|---|---|---|
| **1 Y pos** | 13x12.5 | 0 | 0.0 | 0.0 | 0.5x0.5 | 0.02 |
| **2 Y ampl** | 13x12.5 | 0 - 100 | 1.0 | 0.0 | 0.5x0.5 | 0.02 |
| **3 Y freq** | 6.5x12 | 100 | 0 - 0.5 | 0.0 | 0.5x0.5 | 0.02 |
| **4 Y phas** | 13x12.5 | 100 | 0.4 | 0 - $\pi$ | 1.0x1.5 | 0.02 |
| **5 Y ht** | 13x3.5 | 0 | 0.0 | 0.0 | 0.2x.1-3.5 | 0.02 |
| **6 Gray** | 10.5x7 | 0 | 0.0 | 0.0 | 0.2x2.0 | .02-55.0 |

**TABLE 2**
Summary of results from Experiment 1

| | $p = 0.25$ | $p = 0.75$ | difference in $r^2$ |
|---|---|---|---|
| **1 Y position** | 0.589 | 0.445 | 0.144 |
| **2 Y amplitude** | 0.630 | 0.355 | 0.275 |
| **3 Y frequency** | 0.696 | 0.136 | 0.560 |
| **4 Y phase** | 0.632 | 0.420 | 0.212 |
| **5 Y height** | 0.675 | 0.370 | 0.305 |
| **6 Gray value** | 0.644 | 0.319 | 0.325 |

**TABLE 3**
Default Display parameters for Experiments 2 and 3

| Condition | Window Width x Height (cm) | Amplitude (% of Window) | Frequency (Hz) | Phase Angle (radians) | Point Size Width x Height (cm) | Point Grey Value (Candelas/ sq meter) |
|---|---|---|---|---|---|---|
| **Expt 2** | | | | | | |
| **2 Y ampl** | 13x13 | 0 - 50 | 1.0 | 0.0 | 0.5x0.5 | 0.02 |
| **3 Y freq** | 13x13 | 50 | 0 - 0.5 | 0.0 | 0.5x0.5 | 0.02 |
| **4 Y phas** | 13x13 | 50 | 0.4 | 0 - π | 1.0x1.5 | 0.02 |
| **5 Y ht** | 13x13 | 0 | 0.0 | 0.0 | 0.2x.1-3.5 | 0.02 |
| **6 Gray** | 13x13 | 0 | 0.0 | 0.0 | 0.2x2.0 | .02-55.0 |
| **Expt 3** | | | | | | |
| **2 Y ampl** | 13x13 | 0 - 50 | 1.0 | 0.0 | 0.5x0.5 | 0.02 |
| **3 Y freq** | 13x13 | 50 | 0 - 0.5 | 0.0 | 0.5x0.5 | 0.02 |
| **4 Y phas** | 13x13 | 50 | 0.4 | 0 - π | 1.0x1.5 | 0.02 |
| **5 Y ht** | 13x13 | 0 | 0.0 | 0.0 | 0.2x.1-3.5 | 0.02 |
| **6 Gray** | 13x13 | 0 | 0.0 | 0.0 | 0.2x2.0 | .02-55.0 |

**TABLE 4**
Summary of results from Experiment 2

| | p = 0.25 | p = 0.75 | difference in $r^2$ |
|---|---|---|---|
| 1 Y amplitude | 0.654 | 0.276 | 0.378 |
| 2 Y frequency | 0.699 | 0.239 | 0.460 |
| 3 Y phase | 0.651 | 0.277 | 0.374 |
| 4 Y height | 0.610 | 0.415 | 0.195 |
| 5 Gray value | 0.610 | 0.402 | 0.208 |

**TABLE 5**
Summary of results from Experiment 3

| | p = 0.25 | p = 0.75 | difference in $r^2$ |
|---|---|---|---|
| 1 Y ampl | 0.672 | 0.194 | 0.478 |
| 2 Y freq | 0.627 | 0.130 | 0.497 |
| 3 Y phase | 0.652 | 0.339 | 0.313 |
| 4 Y height | 0.603 | 0.117 | 0.486 |
| 5 Gray value | 0.589 | 0.311 | 0.278 |

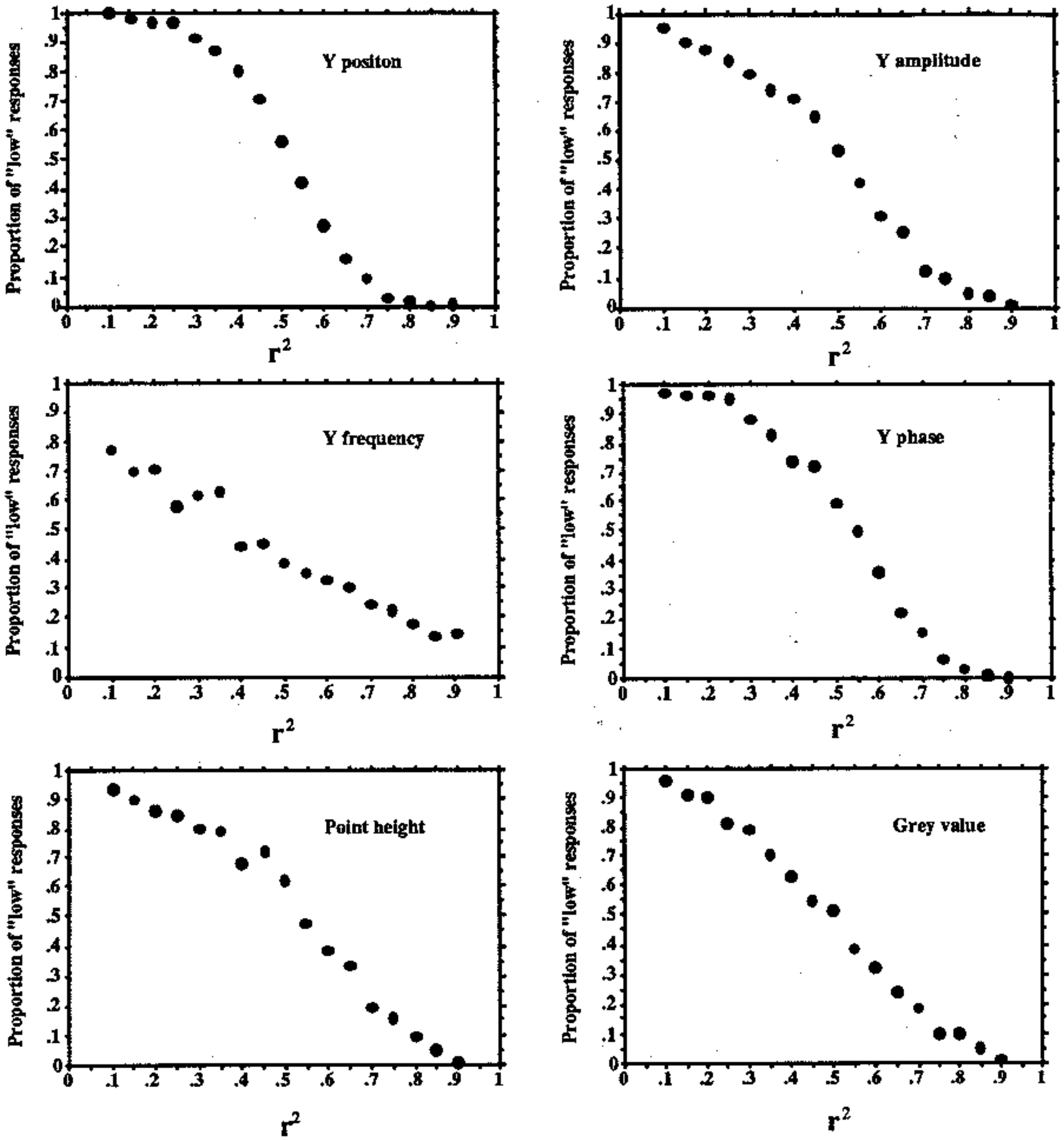


Figure 1. Proportion of "Low" responses is plotted against $r^2$ for the six different display methods used in Experiment 1.

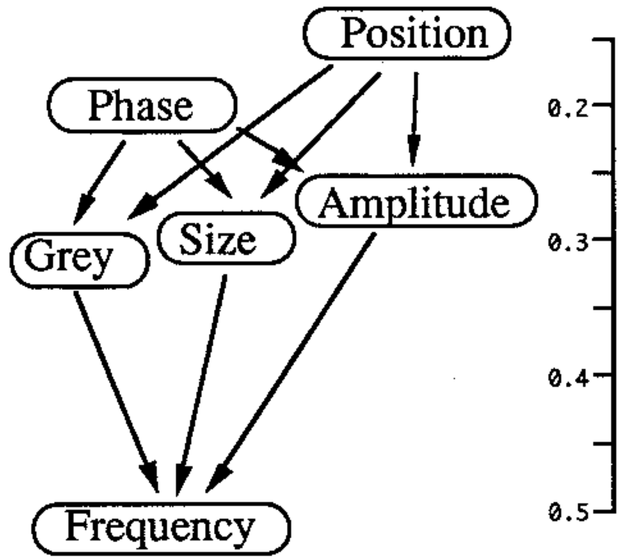


Figure 2. Summary of results from Experiment 1. Efficiency of various display techniques at conveying correlation information. The scale shows the change in $r^2$ required to change the probability of a "Low" response from 0.25 to 0.75.

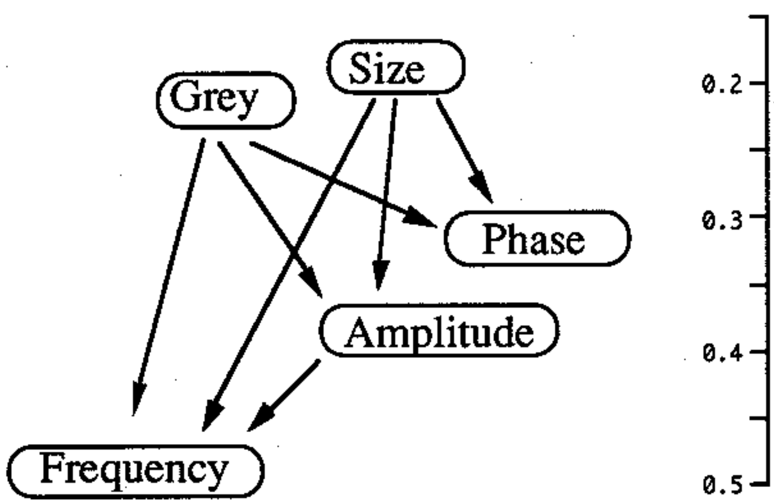


Figure 3. Summary of results from Experiment 2 . Efficiency of a scatterplot varies in the presence of interference noise from various motion parameters, height and gray values.

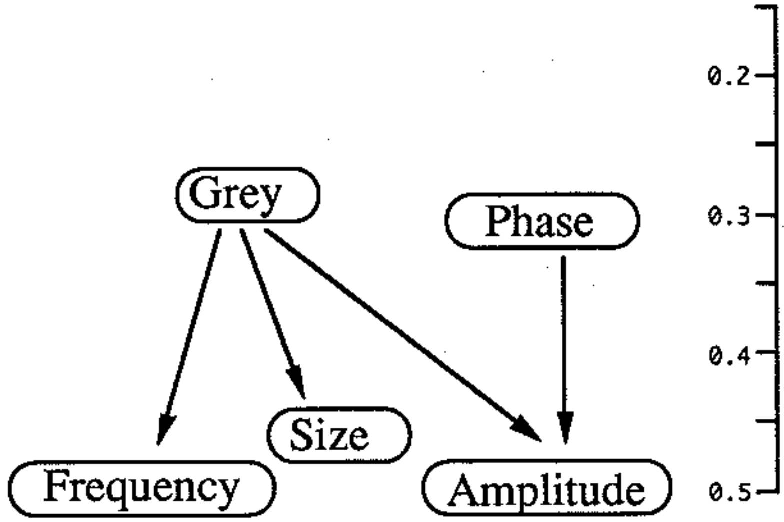


Figure 4. Summary of Results from Experiment 3. Efficiency of display methods in the presence of position noise.

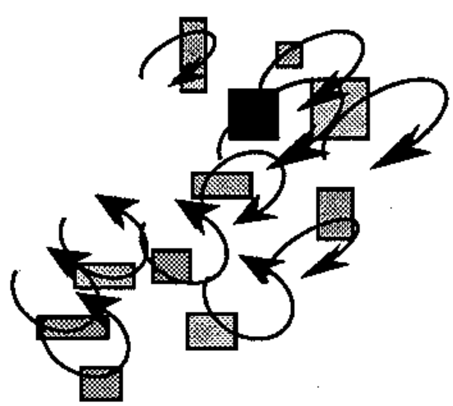

Figure 5. It is difficult to illustrate moving patterns in a static diagram but this pattern illustrates how motion phase may be used to make the it apparent that there are two distinct clusters in a multidimensional discrete data set.