

Dynamics of Coherent Structures in Turbulent Flow Over a Backwards-Facing Step

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Abstract

A long run-time ($t = 157sec$) Large Eddy Simulation is performed to elucidate the role of several key features of the turbulent flow over a Backwards-Facing Step. Preservation of the instantaneous fluctuating pressure field was central to the simulations and allowed the calculation of both field and wall-pressure statistics. Multi-point spectral results showed that the majority of the field pressure fluctuations were well represented by wall-pressure measurements at all but the highest frequencies examined. The peak spectral frequency was found to increase with streamwise distance from the step. It was shown that the increase in peak frequency was much lower than the increase in convective velocity of the structures, which indicates that mean flow acceleration is not solely responsible for this behavior.

Low-pass filtering of the instantaneous velocity fields was employed to highlight the mechanisms responsible for the peak frequencies associated with so-called shear-layer flapping near $St \approx 0.01$, which has been heavily debated in the literature. The low-frequency behavior was found to determine the instantaneous streamwise reattachment point as well as the area of the pri-

mary recirculation zone. The results indicate that flow impingement on the vertical step face results in pairs of highly three-dimensional counter-rotating vortices, with an approximate spanwise wavelength of $\lambda_z \approx 2h$, that wander back and forth along the step face. This results in a side-to-side sweeping of the flow downstream of these features, particularly near the reattachment location.

The dynamics of large-scale vortex-shedding were then isolated from the low-frequency behaviors using band-pass filtering of both the pressure and velocity fields. Examining the evolution of the structures showed that the regions of band-pass filtered fluctuating pressure increased in size as they convected downstream, and their wall pressure signature increased accordingly. The combination of this elongation and the increasing mean velocity causes the discovered increase in peak spectral frequency. The instantaneous fluctuating wall-pressures were found to be well-representative of the fluctuations above the wall, particularly for the largest regions of organized pressure fluctuations, owing to the fact that smaller-scale fluctuations that occurred farther from the wall were obscured by the larger fluctuations closer to the wall.

Dedication

To all those who supported me, especially my parents; Alanna and Jack, and my loving wife, Corinna.

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Nomenclature and Abbreviations

t	Time, s
f	Frequency, Hz
ω	Frequency, $Radians/s$
ρ	Fluid Density, kg/m^3
ν	Dynamic Viscosity, kg/ms
x, y, z	Streamwise, Wall-Normal, Spanwise Coordinates, m
dx, dy, dz	Streamwise, Wall-Normal, Spanwise Spacing, m
h	Step Height, m
ζ	Probe Spacing, m/s
ER	Expansion Ratio
TI	Turbulence Intensity, %
δ	Boundary Layer Thickness, m
θ	Boundary Layer Momentum Thickness, m
y^+	Viscous Wall Unit
x_r	Streamwise Reattachment Point, m
A_r	Recirculation Area, m^2
τ_w	Wall Shear Stress, Pa
C_f	Skin Friction Coefficient
u, v, w	Streamwise, Wall-Normal, Spanwise Velocity Component, m/s
$\overline{U}, \overline{V}, \overline{W}$	Mean Streamwise, Wall-Normal, Spanwise Velocity Component, m/s
u', v', w'	Fluctuating Streamwise, Wall-Normal, Spanwise Velocity Component, m/s

U_∞	Freestream Streamwise Velocity, m/s
U_c	Convective Velocity, m/s
p	Pressure, Pa
$\bar{p}$	Mean Pressure, Pa
p'	Fluctuating Pressure, Pa
P_0	Reference Pressure, Pa
C_P	Pressure Coefficient
q	Dynamic Head, Pa
St	Strouhal Number
$G_{nm}(f)$	Cross-Spectral Magnitude between Probes n and m
$\phi(\zeta, \omega)$	Cross-Spectral Phase Angle, <i>Radians</i>
γ^2	Two-Point Coherence
ρ_{nm}	Two-Point, Two-Time Correlation Coefficient
Re	Reynolds Number
C_o	Courant Number
LES	Large Eddy Simulation
DNS	Direct Numerical Simulation
PIV	Particle Image Velocimetry
LDV	Laser Doppler Velocimetry
LSE	Linear Stochastic Estimation
POD	Proper Orthogonal Decomposition

Chapter 1

Introduction

1.1 Background and Motivation

The desire to accurately control turbulent flow is of great importance to the fluid mechanics and fluids engineering communities, but it requires greater knowledge of the organized structures within the turbulence. With the ongoing development and improvement in experimental and numerical techniques the massive amounts of data being captured allows for potentially greater insight into the key underlying mechanisms of fluid flow. However, many of these data sets have not been used to their full potential. There remains much to be understood about the organized motions hidden within the turbulence, let alone the potential for altering the flow for our benefit. In order to accurately control the resulting flow, the understanding of the dominant flow characteristics and features must be obtained. Once the flow physics

have been well established, by what means can the results can be improved. For example, what methods can be designed to delay separation from an air-foil at high angles of attack, or Can the structures within jets and wall-jets be controlled to reduce the noise or increase the heat transfer from the wall?

The simplest form of flow-control arises from a *passive* scheme, where physical features are added to the existing geometry to either disrupt or enhance the existing features of the flow. Tabs and rivulets that generate vortices or cause boundary-layer transition to helical strakes that reduce vortex-shedding are examples of passive flow-control mechanisms. Although relatively common, the passive designs cannot accommodate much variation from the case for which they were designed. If the flow-field is varying significantly with time (such as in many high Reynolds number turbulent flows), the effectiveness of passive control means could be negatively impacted. In contrast, an *active* flow-control scheme is such that the state of the flow is measured via a sensor with a high sampling rate, and those readings are analyzed and passed onto devices that can make real-time adjustments to the flow (minus any system delay). Obviously, the crucial factor in this regard is the speed at which the flow-field results measured by the sensors can be interpreted and then relayed to the actuators. If the delay is significant, then the actuators may be altering a flow that is at a significantly different state from which the data was taken. Ideally the information must be recorded, analyzed and employed nearly instantaneously for the flow to be controlled as effectively as possible. As such, the processing time must be made to be

as short as possible, and techniques that seek to sort out and decipher the major features of the flow in real-time are becoming extremely important. The question arises: how can relevant information from the entire area of interest in real-time at both high spatial and temporal resolutions be recorded and analyzed?

Point sensors such as hot-wires and pressure transducers can measure the flow with a high degree of temporal resolution but offer very limited spatial information without the use of traverses or incorporating large numbers of individual sensors. An additional benefit is that the processing time for the data can be very quick, which would be important in an active flow-control scheme. The flow-field can be measured with high spatial resolution using a technique such as Particle Image Velocimetry (PIV), but the temporal resolution is generally quite low, on the order of a few frames per second. There are systems that can offer much higher image capture rates but are limited to small areas of interest and are extremely costly. Information on the instantaneous pressure field (which is relevant to acoustics and vibration) is also extremely challenging for these optical techniques [2]. The image processing time can also be quite significant, meaning PIV is not well suited for on-line processing in an active flow-control scheme. The technique is also not suited for real-world applications; it is a laboratory technique that requires close experimental control, as the results are extremely sensitive to the positioning of the lasers and camera. Although numerical simulations of turbulent flow can offer both high temporal and spatial resolutions, they often have very short

total run-times (relative to experiments) due to the high computational cost associated with them. In order to provide proper averaging (particularly of low-frequency features) and to accurately resolve higher order statistics, the simulations should be run for a long sampling period.

A compromise between short in-line processing times and high spatial and temporal resolutions is necessary for continued development of active flow-control applications. One such data analysis technique that seeks to achieve this goal is known as *Stochastic Estimation* (SE). A key step in SE is to build correlations between the velocity field and the corresponding time-resolved point sensor measurements (typical surface pressure or hot-wire measurements) taken from a few locations. Once developed, these correlations can be used to estimate the entire velocity field of interest using only the readings from the point sensors at any given instant in time in very short in-line processing times. This effectively creates the ability to estimate a time-resolved velocity field, which is an immensely powerful development. Surface pressure measurements are ideal to estimate the velocity field from because they are largely non-intrusive and may be easily retrofitted to current existing models and designs. Stochastic estimation is frequently employed in conjunction with techniques that seek to understand the underlying major physical features of the flow such as the *Proper Orthogonal Decomposition*. When the two methods are employed in the same study, they are known as *complementary* techniques as they both make use of the fluctuating pressure and velocity measured within the flow. Appendix A provides much greater

detail of the procedure for SE and POD. It has been shown that the accuracy of these stochastic estimations are highly dependent on the sensor type, size, location and the number of sensors, which are heavily influenced by the most important flow phenomenon [3]. It is very challenging and expensive to design experimental spatially and temporally resolved measurement configurations that can be used to study the impact of these variables. Setup and data collection can be time consuming for each and every configuration to be examined. Additionally, in order to properly assess the results from these estimated flow fields, there must be a very clear and fundamental understanding of the flow targeted for the estimations. Otherwise, the results obtained from the estimations cannot be tied back to the actual flow on which they are based.

Likely, the greatest problem with both SE and POD (which are commonly coupled together) is the interpretation of the results that the investigator is required to perform. Many times the resulting POD mode shapes or estimated velocity fields appear to be abstract art, and interpretation is left at the mercy of the imagination of the person looking at them. In order to improve both of these techniques, some rigor must be applied to ensure that we are actually making progress towards understanding the fundamental underlying physical features within turbulent flows. In order to properly evaluate the performance of any unbiased filtering or estimation techniques, we must further our understanding of key features hidden within the turbulent flow field. Ideally the features we are studying would be candidates for

flow-control as well.

The present work seeks to improve the understanding of coherent structures present in turbulent flow over a backward-facing step which would likely be the future target for active flow-control schemes. The hope is that through further understanding of these key features, proper guidelines may be developed to achieve high SE accuracy. The highly accurate flow estimations can then allow for the development and improvement of active flow-control mechanisms to alter the behavior of the coherent structures studied in this work.

1.2 Organization and Contributions

This thesis is formatted in an articles based format and is organized as follows: Chapter 2 provides a brief overall review of the relevant literature, with more detailed literature reviews included in each of the results chapters. Chapter 3 provides an overview of the signal processing techniques employed within this thesis.

Chapter 4 is a manuscript published in the AIAA Journal and provides ample validation for the simulation database and presents wall and field pressure spectral measurements of turbulent flow over a Backwards-Facing Step (BFS). The work showed how the spectral results were dependent on the streamwise measurement location, which is a critical piece of knowledge in terms of sensor placement for SE. Wall-pressure measurements were also

found to capture the pressure fluctuations within the domain as well, demonstrating the effectiveness of commonly employed, non-intrusive wall-pressure measurements, which had long been assumed, but difficult to confirm experimentally. My contribution to this work included selection of the geometry, working on the setup for the simulations themselves, the data analysis and the writing of the manuscript as the principal author.

An article submitted to the *Journal of Fluid Mechanics* is presented in Chapter 5 studying the low frequency dynamics downstream of the step by low-pass Fourier filtering ($St < 0.016$) of the velocity fields. The motivation behind this work was to further the understand of a key feature of the turbulent flow over a BFS; a low-frequency “flapping” that has been known about for 40 years, but is still heavily debated within the literature. Clear understanding of this mechanism is critical in assessing the ability of SE or POD reconstructions to capture this phenomena, as it is a primary target for flow-control applications due to its contribution to drag and noise on vehicles. It was found that the low-frequency dynamics are significantly more complicated than a simple unsteady vertical motion of a shear-layer. The recirculation zone was found to consist of several unsteady regions of counter-rotating flow (both ω_z and ω_y), with an approximate spanwise wavelength of $\lambda_z \approx 2h$. My contribution to this work included selection of the geometry, working on the setup for the simulations themselves, the data analysis and the writing of the manuscript as the principal author.

Chapter 6 is a paper that will soon be submitted to the AIAA Journal

and employs band-pass Fourier filtering of the pressure and velocity fields to further our understanding of the vortex-shedding phenomenon occurring within the turbulent flow field. Using band-pass filtering allows investigation of the vortex-shedding behavior independent of the low-frequency dynamics for the first time. The temporal evolution of the vortex-shedding was examined and it demonstrated the combined effect of structure elongation and acceleration of the convective velocity. The vortices were found to grow significantly as they convect downstream, and these large-scale structures can induce the formation of smaller-scale vortices very close to the wall. The structures demonstrated a large spanwise organization of several step heights. The unsteadiness of the vortex-shedding behavior, combined with the spanwise organization indicates that large-scale vortex-shedding would be a key contributor to aero-acoustic noise generation within the domain. Improving the understanding of this important flow feature is paramount in developing effective estimation and low-order modeling (such as POD) strategies to accurately capture the vortex-shedding occurring downstream of the step. My contribution to this work included selection of the geometry, working on the setup for the simulations themselves, the data analysis and the writing of the manuscript as the principal author.

Finally, a summary of the important contributions from each results chapter, as well as the future work that would continue on from this line of research are presented in Chapter 7.

Chapter 2

Selection of Geometry and Literature Review

In order to improve our understanding of the coherent structures and associated pressure fluctuations in turbulent flow over a BFS, the development of a comprehensive numerical database is necessary. It is desirable to employ a geometry that lends itself to ease of study and has been well documented in the literature. Instantaneous wall and field pressure (which are not normally examined) within the domain is necessary to mimic the microphone measurements that are commonly employed in experiments, and the corresponding velocity fields can be used to link the pressure fluctuations to organized motions within the flow. It would also serve the database well to have a region of strong shear that would provide a strong turbulent feature within the flow-field, ideally even a separated flow. Finally, a region of reattaching or

impinging flow can provide another region of interest for the analysis.

After careful consideration of several different geometries, a backward facing step was selected due to the presence of a strong, separated shear-layer originating from the corner of the step, a region downstream where the shear-layer reattaches to the wall, and finally the fact that two perpendicular flat surfaces makes instrumenting the geometry with pressure taps reasonably straightforward. Figure 2.1 is a simple illustration of the geometry and nomenclature for flow over a backward facing step. The backward-facing step is a common geometry and is frequently studied by computer simulations [4, 5] and by numerous experimental investigations including wall-pressure measurements [6, 7, 8], hot-wire measurements [9], velocity field measurement by PIV [1, 10, 11] and even several studies using stochastic estimation techniques [1, 10].

The flow over and downstream of a backward-facing step is controlled by a number of factors including Reynolds number, upstream turbulence intensity, upstream boundary layer thickness, step size (h) or expansion ratio $(Y_{WT} + h)/Y_{WT}$, and the aspect ratio of the test section ($AR = h/W_{WT}$) [12, 13]. The reattachment point can vary from approximately $x_r/h = 4.94$ to $x_r/h = 7$ based on the parameters mentioned above [13].

Research into flow over backward-facing steps has primarily occurred in four types of geometry, illustrated in Figures 2.2 and 2.3 for planar results, and Figures 2.4 and 2.5 for axisymmetric results. The simplest case is that of the simple planar two-dimensional BFS shown in Figure 2.2. The

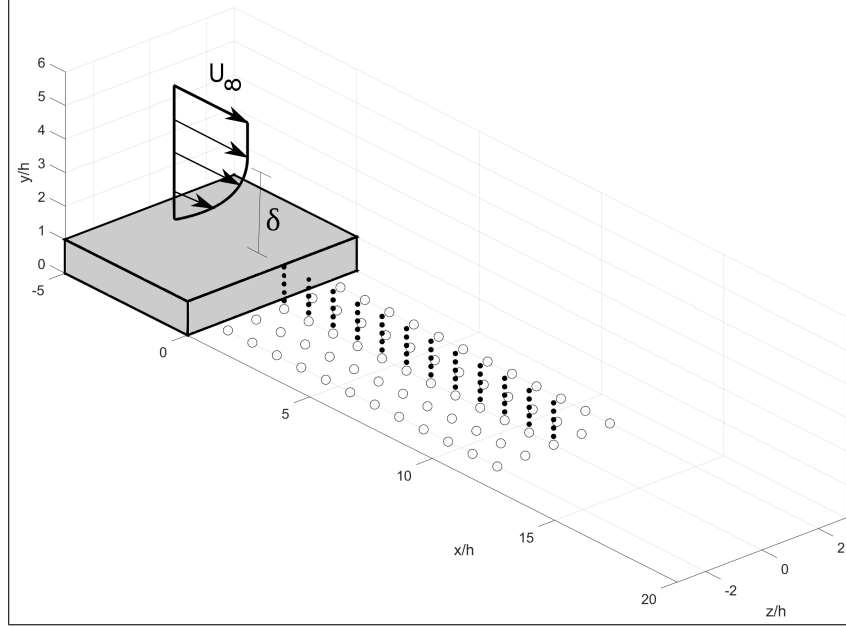


Figure 2.1: Flow Over a Backward-Facing Step

flow in a planar double-sided sudden expansion (Figure 2.3) is complicated by the presence of a second separation bubble, as mentioned in Eaton and Johnston [12].

Due to the presence of the wind tunnel walls the planar cases can never truly be fully two-dimensional, although the work of Ötügen [14] and Papadopoulos and Ötügen [15] showed that $AR > 10$ results in approximately two-dimensional flow on average. Researchers seeking to eliminate tunnel wall effects have considered axisymmetric geometries, in which a circular geometry is two-dimensional in the $r-z$ plane due to azimuthal symmetry. Two main designs are that of internal flow of the round sudden expansion shown in Figure 2.4 employed in the studies by Devenport and Sutton [16] and Tinney

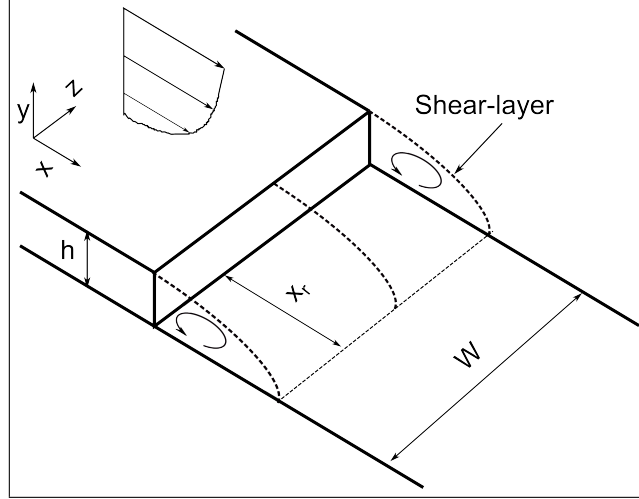


Figure 2.2: Planar Backward-Facing Step

et al. [17], and that of external flow over the axisymmetric backward-facing step (Figure 2.5) studied by Hudy et al. [1], Rajasekaran [13], and Clark et al. [10].

One of the primary features for classification of the flow over the step is the point downstream that the separated shear-layer originating from the step reattaches on the downstream wall. There are numerous methodologies to quantify this behavior, with the most common being the point at which the mean streamwise skin friction coefficient crosses from the negative values in the recirculation zone to the positive for the flow traveling downstream. Previous investigations have shown that the downstream reattachment point depends on numerous parameters including Reynolds number,

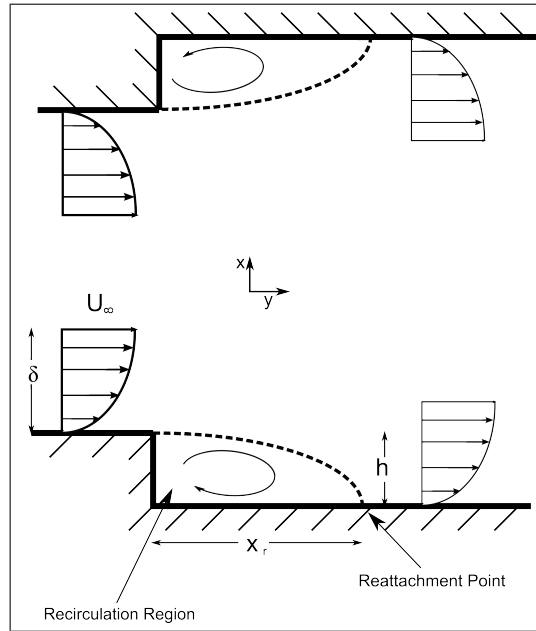


Figure 2.3: Planar Sudden Expansion

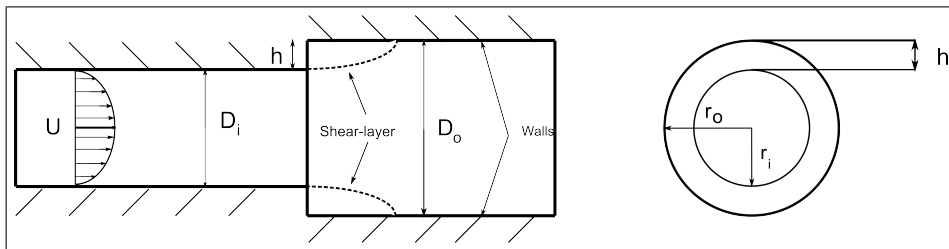


Figure 2.4: Internal Flow Through an Axisymmetric Sudden Expansion

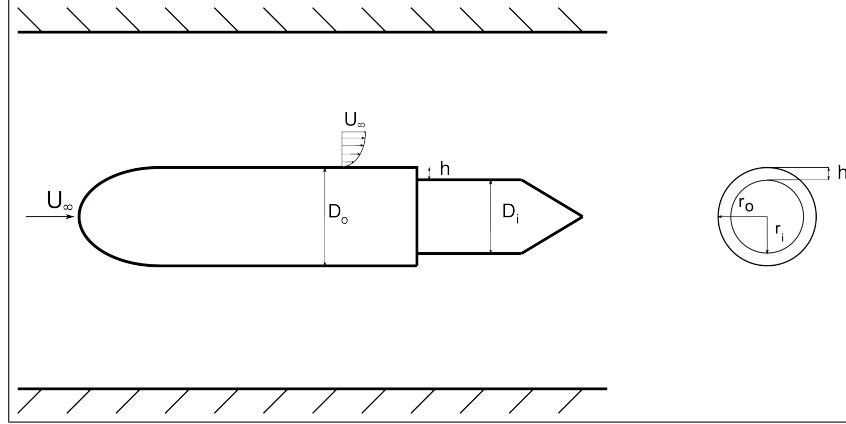


Figure 2.5: External Flow over an Axisymmetric Backward-Facing Step

upstream boundary layer thickness, upstream turbulence intensity, as well as the expansion and aspect ratios [12, 18, 14, 13].

Turbulent flow downstream of the step has been found to contain significant spectral energy peaks occurring around normalized frequencies of $St \approx 0.01$ [12, 19, 20]. The mechanisms responsible for this have been heavily debated within the literature; Eaton and Johnston [12, 19] proposed that it was a quasi-periodic bulk vertical motion of the shear-layer [12, 19] termed “flapping”, while more recent studies have offered more sophisticated explanations of a “pumping” of mass into and out of the recirculation zones [20]. Resolving these kind of low-frequency fluctuations is challenging due to equipment limitations and the slow convergence of statistics due to limited independent samples, which has lead to a great deal of debate within the literature.

Several researchers have found evidence of large-scale (on the order of

the step height) organized vortex-shedding happening downstream of the step [7, 21, 12, 1] occurring near normalized frequencies of $St \approx 0.06 - 0.09$. The strong shear downstream of the step and unsteadiness within the shear-layer can lead to the formation of rotating vortex structures. The SE work by Hudy et al. [1] on the backward-facing step was able to resolve structures that are formed in the wake of the step that had remained previously undiscovered due to the lack of time-resolved velocity field data. The results shown in Figure 2.6 support the long-standing understanding of the shear-layer originating at the corner of the step, but the proposed “*wake-mode*” [1] shown in Figure 2.7, was only able to be measured via stochastic estimation. The estimations were able to describe not only how vortices were grouping within the shear-layer as previously thought, but also that a secondary structure grows in the wake of the step before being ejected and convected downstream [1]. This provides an excellent example of why SE offers such potential for experimentalists in fluid dynamics. Until time-resolved velocity field measurements become commonplace, the ability to accurately estimate time-resolved velocity fields via stochastic means offers huge potential.

Several researchers have designed and tested methods of flow-control over backward-facing steps. Chun and Sung [22] investigated the effect of forcing the flow at the edge of the backward-facing step using a sinusoidal jet generated by a woofer for $Re_h = 13,000 - 33,000$. It was found that the reattachment length could be lengthened appreciably by selecting a forcing frequency greater than some critical value of $St = 0.8$ or reduced by high lev-

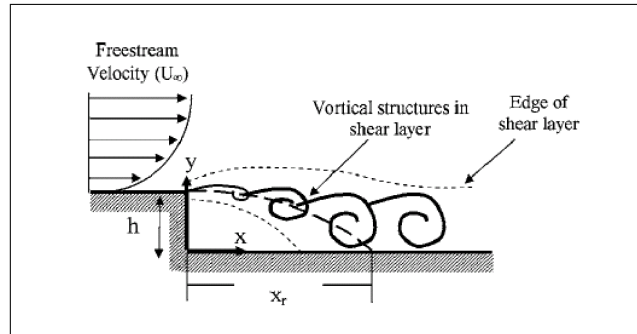


Figure 2.6: Shear-Layer Mode Proposed by Hudy et al. [1]

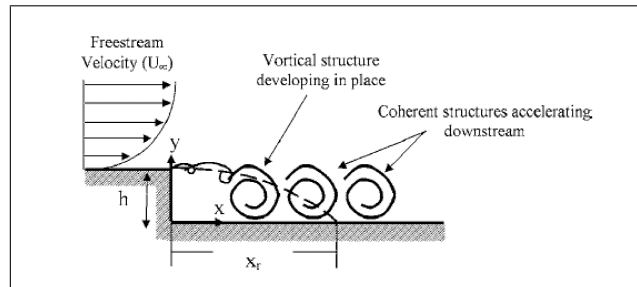


Figure 2.7: Wake Mode Proposed by Hudy et al. [1]

els of forcing near the vortex-shedding frequency from the step $St = 0.3$ [22]. The work by Gautier and Aider [23] studies the effect of moving the forcing jet from the edge of the step to a upstream position of $x/h = 2.11$ and injecting the flow normal to the wall. Studying flow over a backward-facing step in a water tunnel for $Re_h = 2070 - 2900$ with PIV showed a reduction in the reattachment length of approximately 40% [23]. The authors were able to observe the total area of the recirculation zone via their PIV measurements. They found that careful selection of jet amplitude could result in an area reduction of nearly 85%, but further increases in the jet amplitude could cause the recirculation region to nearly double in area [23].

Chapter 3

Signal Processing and Fourier Filtering

Many tools have been used to examine coherent structures in turbulent flows. Fourier analysis is one of the more established techniques for the investigation of time series data, and can be encountered in any field that has access to sufficient time-resolved data. It decomposes the signal into *basis functions*, although in Fourier space the basis functions are predetermined to be sine and cosine waves. Any signal can be reconstructed using modern Fast-Fourier Transform (*FFT*) algorithms that convert the data from a time series into the frequency domain, effectively performing the following:

$$\tilde{p}_n(f) = \int_{-T/2}^{T/2} p_n(t) e^{-j2\pi f t} dt \quad (3.1)$$

where f is the frequency, T is the sampling time for the block of data, p_n

is any time resolved signal at a location n , and t is time. The auto-spectra, G_{nn} , for the signal at position n is defined as:

$$G_{nn}(f) = \frac{\overline{\tilde{p}_n(f)\tilde{p}_n(f)^*}}{T} \quad (3.2)$$

where the asterisk denotes the complex conjugate of the signal. When considering two measurement locations for time series data, the cross-spectra, G_{nm} is calculated from:

$$G_{nm}(f) = \frac{\overline{\tilde{p}_n(f)\tilde{p}_m(f)^*}}{T} \quad (3.3)$$

The cross-spectra between two signals yields a complex result, with both the magnitude and phase angle being important. The magnitude indicates the frequencies within both signals that overlap, and the phase indicates any time lag between the two signals for a given frequency. The phase angle between the two signal is commonly used to calculate a convective velocity, U_c , which identifies the speed at which the flow features are being convected by the mean flow. The convective velocity is equal to:

$$U_c(f) = \frac{2\pi f \zeta_{nm}}{\phi(\zeta, f)} \quad (3.4)$$

where ζ_{nm} is the physical spacing between the two probes n and m , and $\phi(\zeta, f)$ is the unwrapped cross-spectral phase angle between the two probes [24].

The relationship of the two signals p_n and p_m , in time can also be examined through the use of two-point, two-time correlation, R_{nm} , defined

by:

$$R_{nm}(\tau) = \int_{-\infty}^{\infty} G_{nm}(f) e^{-j2\pi f\tau} df \quad (3.5)$$

where τ is a time-lag between the two signals. As the two signals can have vastly different magnitudes, or even different units, normalization of the correlation between the two signals is commonly done using the correlation coefficient, ρ_{nm} , found via:

$$\rho_{nm}(\tau) = \frac{R_{nm}(\tau)}{\sigma_n \sigma_m} \quad (3.6)$$

where σ_n and σ_m correspond to the standard deviation of the signals. The magnitude of the correlation coefficient is bounded between -1 and 1 . Whereas the correlation coefficient provides the relation between the two signals integrated over all frequencies, it is often insightful to examine the frequency dependence of this. This is achieved through the use of the coherence function, γ^2 , determined by:

$$\gamma^2(f) = \frac{|G_{nm}(f)|^2}{G_{nn}(f)G_{mm}(f)} \quad (3.7)$$

A common implementation of Fourier transform in practice is filtering of signals to reveal details that may be obscured by more dominant frequencies or noise. For example, frequencies of interest are selected and allowed to pass through the filter, whereas extraneous frequencies will be removed. One of the most common filters is the Butterworth [25] filter, which has a signal

gain, g , defined by:

$$g = \frac{1}{\sqrt{1 + \epsilon^2 (\frac{\omega}{\omega_c})^{2N}}} \quad (3.8)$$

where ϵ is the maximum passband gain, and N is the order of filter. This type of filter has a flat response in the pass band, which is ideal for ensuring the signal passing through the filter is undistorted [25]. Additionally, the roll-off of the filter gain can be adjusted by varying the number of poles used within the filter. By selecting a high-order filter, very sharp roll-offs can be achieved [25]. Filtering of pressure and velocity signals is an important tool in extracting the underlying coherent motions within the chaos of turbulent that obscures any organization [26, 27, 28].

Chapter 4

Wall Pressure, Field Pressure, and Velocity Field Statistics

This chapter is an article published in the AIAA Journal, co-authored by Stephen John Wilkins (Principal), Mahdi Hosseinali, and Joseph W. Hall : Stephen John Wilkins, Mahdi Hosseinali, and Joseph W. Hall, Investigation of the Unsteady Pressure Fluctuations in Flow Over a Backward-Facing Step, AIAA Journal, 57(6): 2446-2456, 2019

4.1 Introduction

The turbulent flow over a backwards facing step (BFS) is chosen as the flow of interest for a large database of numerical simulations. One of the major goals for the database is the understanding of potential impact of pressure

probe placement for the design of experimental investigations; obviously, this requires the unsteady pressure field to be correctly captured in order to study the impact of pressure sensor location. Microphones represent an effective tool of experimental flow measurements, and being able to employ this database to provide some *a priori* knowledge in regards to sensor placement is of utmost importance. Experimental investigations are primarily limited to examination of the pressures at solid boundaries where microphones and static pressure ports can be installed. Pressure measurements of the field are challenging as the number of simultaneous sensors are limited by the impact of the sensors themselves on the flow field and techniques to measure the entire pressure field simultaneously are still in their infancy [2]. Designing and setting up experiments can be extremely time consuming and very expensive, particularly when it is desirable to have flexibility in terms of sensor type, location, and quantity, as well as maintaining high spatial and temporal resolutions. These concerns are of particular importance when it comes to *stochastic estimation (SE)*, where the estimator location and numbers are critical [3].

Numerical simulations, on the other hand, offer extreme versatility in terms of the flow quantities that can be extracted and measured, but they often require validation from experimental data. High spatial and temporal resolutions are possible using Large Eddy Simulations (LES) and Direct Numerical Simulations (DNS), but the required computational time is significant, particularly if long time histories are required for calculation of higher

statistical moments and correlations. Additionally, traditional synthetic eddy methods (SEM) can suffer from non-physical pressure fluctuations within the domain associated with a time-varying mass-flow rate, as well as the failure of these non-physical velocity fluctuations failing to satisfy the continuity equation at the inlet to the domain; to address this Poletto et al. [29] proposed a divergence-free SEM (DFSEM) that also ensured that a uniform mass-flow rate was present at the inlet and the boundary condition was first implemented in OpenFOAM® 1606 (June 2016). It was demonstrated that the DFSEM significantly reduced the non-physical pressure fluctuations within the domain in a channel flow [29], preserving the instantaneous pressure fields throughout the domain.

Turbulent flow over a BFS is selected for study here because it combines a turbulent boundary-layer, turbulent shear-layer, a reattachment region downstream of the step, and a recovery region [30]. It has been used as a benchmark case for most new investigative techniques, and as such and has been investigated thoroughly via a multitude of experimental techniques including microphones [7, 6, 31], hot-wire anemometers [32, 33, 34], Laser Doppler Velocimetry (LDV) [18, 35], Particle Image Velocimetry (PIV) [36, 1, 20, 37, 38] and numerical investigations using RANS [39, 40], Large Eddy Simulations [4, 41, 42] and Direct Numerical Simulations [5, 43] when developing those tools.

The flow over a BFS is known to be dominated by two major flow phenomena; a low frequency “flapping” of the shear-layer and large scale vortex-

shedding off the step [7, 21, 19, 12]. The bulk movement of the shear-layer causes very low frequency variations in the size of the recirculation zone immediately downstream of the step. The low frequency peak in the pressure and velocity spectra, often attributed to this flapping, is dominant within the recirculation zone itself, and typically occurs around Strouhal numbers of $St = fh/U_\infty = 0.01 - 0.02$ [7, 33, 21, 12, 44] for flow over BFS and splitter plates. Higher frequency peaks in the spectrum typically occurs in the range of $St = 0.06 - 0.08$ and are normally thought to correspond to large-scale (on the order of the step height) vortex-shedding off the step [7, 21, 12]. Hudy et al. [1] and Chovet et al. [45] found evidence of another type of organized shedding related to a large-scale vortex structure that grows in place near reattachment, before being ejected downstream, the so-called wake mode.

In this manuscript the mean and fluctuating velocity and pressure results from the LES examination of turbulent flow over a BFS using the DF-SEM of Poletto et al. [29] are compared against previous studies. The exact geometry and flow conditions investigated here are identical to the experimental setup of Jovic and Driver [30] and the direct numerical simulation (DNS) work by Le et al. [5], as this provided ample comparison data for this database. One major advantage of this numerical examination over experimental investigations is the ability to adequately resolve low frequency fluctuations common in most wind-tunnel experiments [7] as microphones are generally unable to accurately capture frequencies below $10 - 20Hz$. Most numerical simulations, however, usually have very short run times limiting

the number of statistically independent events that can be captured and thus the focus of this study will be on performing a “numerical experiment” where a very long run time was used ($t=157\text{sec}$, $tU_\infty/h = 1.2 \times 10^5$), which is much larger than in most simulation, including the DNS study of Le et al. [5] on the same geometry. This allows the spectra of the fluctuating wall pressure and field pressure to be examined to better understand the unsteady pressure field and improve our interpretation of unsteady wall pressure measurements. It also allows ensemble averaging to reduce the error in the spectra. This information will certainly be valuable for turbulent flow prediction techniques like stochastic estimation [1, 46, 47, 48, 49, 50, 51] and ultimately for the implementation of closed-loop flow control.

4.2 Domain Description and Boundary Conditions

An illustration of the BFS geometry in this examination is shown in Figure 4.1, with the pressure probe locations illustrated. The geometry is modeled using approximately 1.3×10^6 cells and simulates one-half a symmetric double expansion ($ER=1.2$), with a zero stress symmetry plane as the upper boundary of the domain, which is the same condition as Le et al. [5]. The lower bottom faces and vertical step are modelling as zero slip walls. In order to fully resolve the near wall regions, the grid spacing normal to the wall is selected such that the maximum value of $y^+ < 1$. Domains

using a larger number of cells were examined, but the impact of the finer grid resolution was negligible. Periodic boundary conditions are used on the side walls to preserve the mean two-dimensional flow. There are cells in the spanwise direction covering approximately $5.9h$. An advective boundary condition is applied at the exit plane ($x/h = 20$) in order to allow the passage of vortical structures out of the computational domain [5], preventing any reflected waves. As previously discussed, the inlet plane is modelled using the DFSEM approach of Poletto et al. [29]. Detailed distributions of velocity, Reynolds stresses, and turbulent length scales at the inlet are required for the DFSEM, so DNS data of a turbulent boundary layer provided by the online numerical database of Schlatter and Örlü [52] was used as the baseline data. In order to ensure ample space for the non-physical fluctuations provided from the DFSEM to stimulate and develop into a true turbulent boundary layer of the desired thickness, the domain extends 37.55 step heights upstream of the step in the streamwise direction. Investigation of the wall-shear stress indicates that physical turbulence is being excited by the synthetically generated eddies by approximately 20 step heights upstream, leaving around 17 step heights to allow the turbulent boundary layer to grow to the appropriate thickness at the upstream comparison location of $x/h = -3.1$.

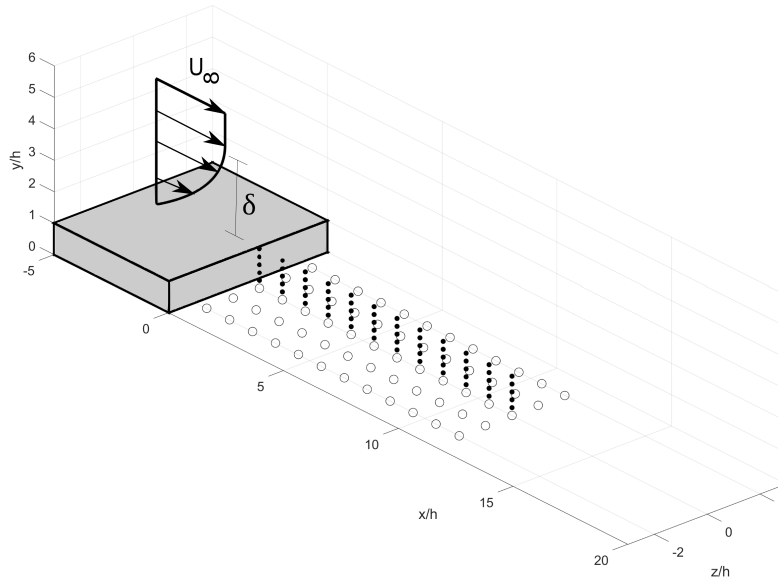


Figure 4.1: Backward-Facing Step Geometry: Open circles correspond to wall-pressure probe location and solid circles correspond to field probe locations.

4.3 Numerics

In order to provide sufficient data for a large numerical experiment database, total simulation time was $t = 157$ seconds, equivalent to approximately 1.2×10^5 eddy turnover cycles. The simulations were run using a total of 240 CPUs, and the total computation time was equivalent to 19.7 CPU years. The time step for the simulations is adjustable, fixed below $\Delta t = 2 \times 10^{-5}$ seconds ($\Delta t U_\infty / h = 0.0158$) to keep the mean Courant number less than $Co_{mean} = 0.2$. In order to ensure high enough temporal resolution to capture the expected shedding frequency occurring near $St \approx 0.06$ ($\approx 47Hz$) the data presented here is sampled at $500Hz$, which is more than 10 times the expected vortex-shedding frequency. The $500Hz$ sampling rate is approximately equal to 1 in every 100 simulation time-steps.

The LES is conducted using the pimpleFoam (merged PISO-SIMPLE) solution algorithm [53, 54] and the WALE model [55]. The filter length is set based on the cube root of the cell volume. Linear second-order discretization has been employed for the advection and diffusion terms, and a second-order backward time scheme ensured that the accuracy was kept to second order.

4.4 Results and Discussions

The first section of the results is focused on the comparison of the current study with those previously presented in the literature in terms of both velocity and pressure statistics in order to provide confidence in the results and

discussions herein. This was important as the inlet boundary condition is a relatively new addition to the software package, as well as examining the effect of employing such a long time history on this LES numerical investigation.

4.4.1 Validation of Velocity and Pressure Statistics

In this study, the boundary layer thickness was found to be $1.12h$ at $x/h = -3.1$, well within the desired matching range of $1 - 1.2h$ found in the work by Jovic and Driver [30] and Le et al. [5] at the same streamwise location. The Reynolds number based on the momentum thickness was calculated to be $Re_\theta = 610$, which was equivalent to the values obtained in those previous cases. Figure 4.2 shows the mean streamwise velocity profile of the current LES study in wall units, compared to the studies of Le et. al [5] and Jovic and Driver [30] of the flow over a BFS. Additionally the DNS results of a turbulent boundary layer from Schlatter and Örlü [52] and the expected law-of-the wall mean velocity profile are shown. The results of the present work are in very good agreement with all cases.

For further comparison, Reynolds stress profiles upstream of the step ($x/h = -3.1$) are shown in Figure 4.3, and compared with the BFS data obtained by Jovic and Driver [30] at $Re_\theta = 610$, hotwire measurements in a turbulent boundary layer by Erm and Joubert [34] at $Re_\theta = 700$, and the DNS data from Schlatter and Örlü [52] at $Re_\theta = 670$ are also provided. The general shape is very well represented. The peak values of the profiles are

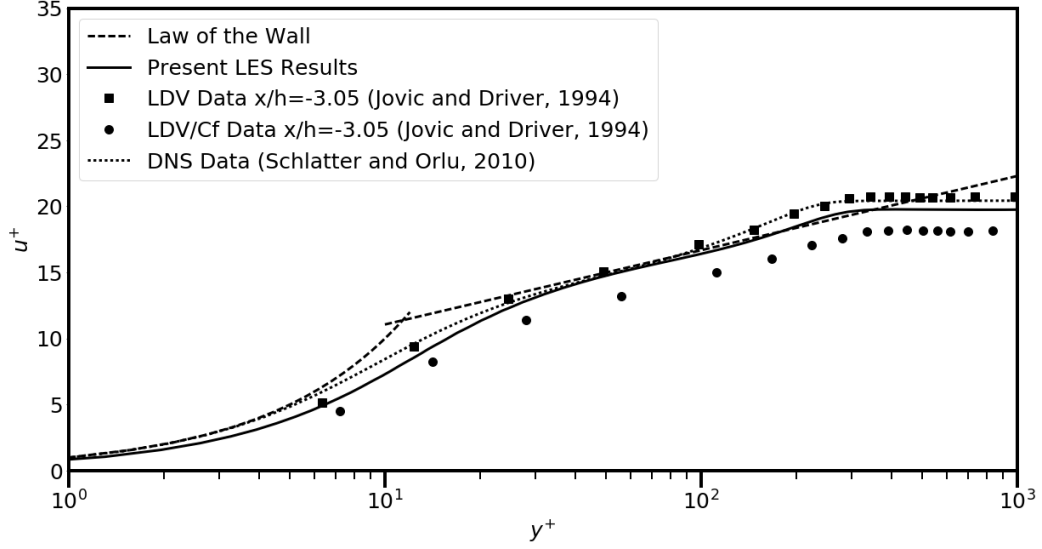
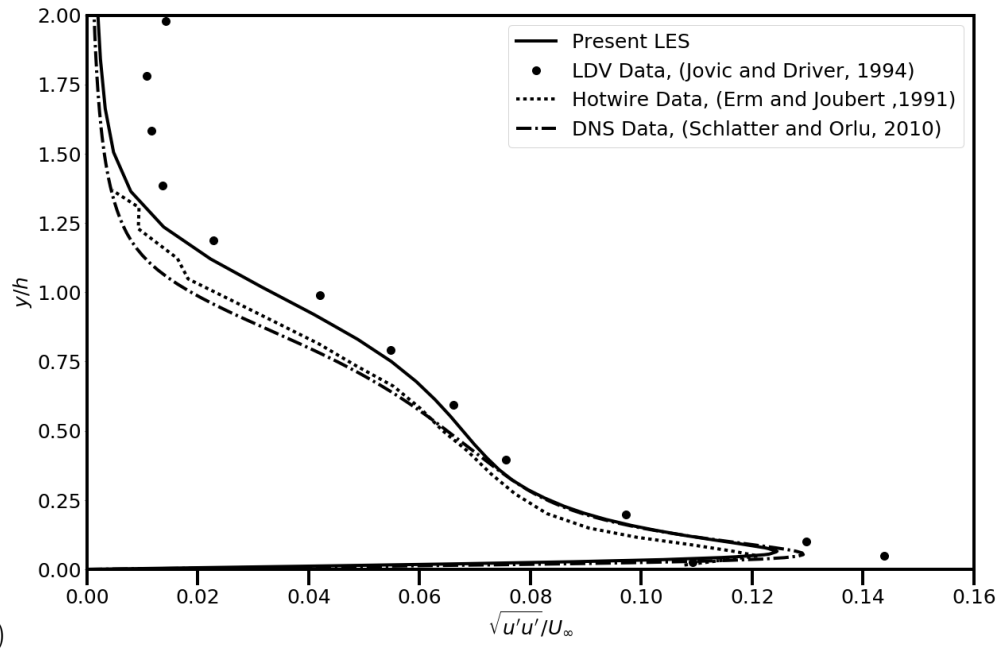


Figure 4.2: Upstream Mean Streamwise Velocity Profiles in Wall Coordinates at $x/h = -3.1$

slightly lower than the other studies, this is likely due to several factors; Jovic and Driver [30] noted that their boundary layer is over-stimulated from their trip, and unstructured solvers employed in this LES examination are limited to second-order discretization, which leads to a more diffusive simulation.

Scalar fields of both the mean normalized streamwise ($\bar{U}/U_\infty$) and mean wall-normal ($\bar{V}/U_\infty$) velocity fields are shown in Figures 4.4 and Figures 4.5. The step is illustrated by the hatched region. The flow immediately downstream and below the step is slowed down significantly resulting in the development of strong shear between a large recirculation zone ($x/h = 1 - 6$) and the largely unimpeded flow above the step as expected. There is a large downward flow downstream of the step as the flow is pulled down by the



(a)

Figure 4.3: Normalized Reynolds Stress Profile Upstream of the Step at $x/h = -3.1$

expansion of the domain downstream of the step. A smaller secondary recirculation zone ($x/h = 0 - 1$) is observed in the mean streamlines of the flow shown in Figure 4.6.

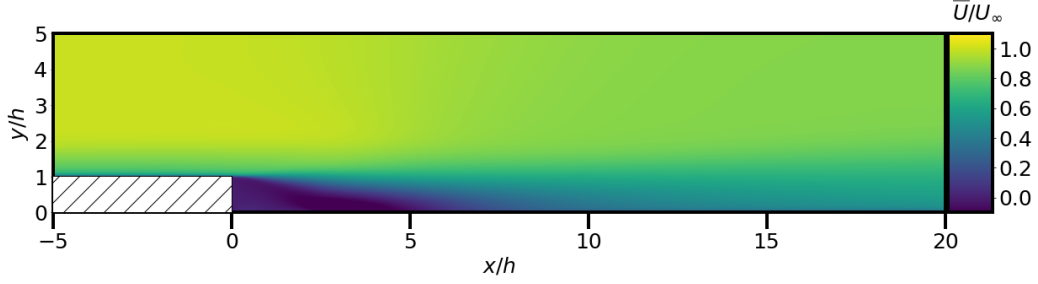


Figure 4.4: Mean Streamwise Velocity Scalar, $\bar{U}/U_\infty$

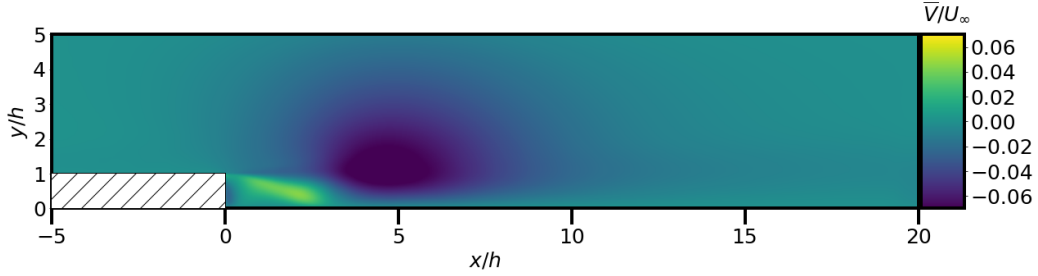


Figure 4.5: Mean Normal Velocity Scalar, $\bar{V}/V_\infty$

An important parameter in quantifying the flow over BFS is the location of the reattachment point of the shear-layer downstream of the step, x_r . There are numerous methods to determine the location, with the most reliable being the streamwise position at which the mean streamwise component of the wall shear stress is zero when moving from the negative mean shear within the recirculation zone and the positive mean shear of the flow far

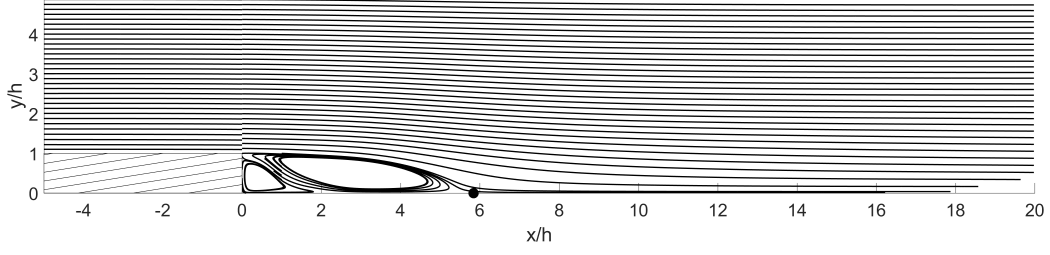


Figure 4.6: Streamlines of the Mean Flow; Dot Corresponds to Mean Reattachment Point, x_r .

downstream of the step. This is demonstrated in the plot of the skin friction coefficient, $C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$, shown in Figure 4.7. The results from the present study indicated the location for the mean reattachment point of $x_r/h = 5.84$, providing excellent agreement with both the experimental results of Jovic and Driver [30] of $x_r/h = 6 \pm 0.15$ and the reattachment length of $x_r/h = 6.28$ found the DNS results of Le et al. [5]. The maximum discrepancy between all three cases was $0.44h$ (7.4%), with the maximum discrepancy being approximately $4mm$.

Examination of the streamwise wall pressure profiles is shown in Figure 4.8, and compared with experimental microphone measurements [7, 6, 35], and the DNS data of Le and Moin [5]. In order to reduce the impact of different flow conditions and geometries, the data is normalized by the reattachment length, x_r , found in each study, based on the recommendation of Mabey [56]. This removes the differences in geometries and parameters between the various investigations. Excellent agreement is observed in both the mean pressure coefficient, $C_P = \frac{P-P_0}{\frac{1}{2}\rho U_\infty^2}$, and the *RMS* pressures (normalized

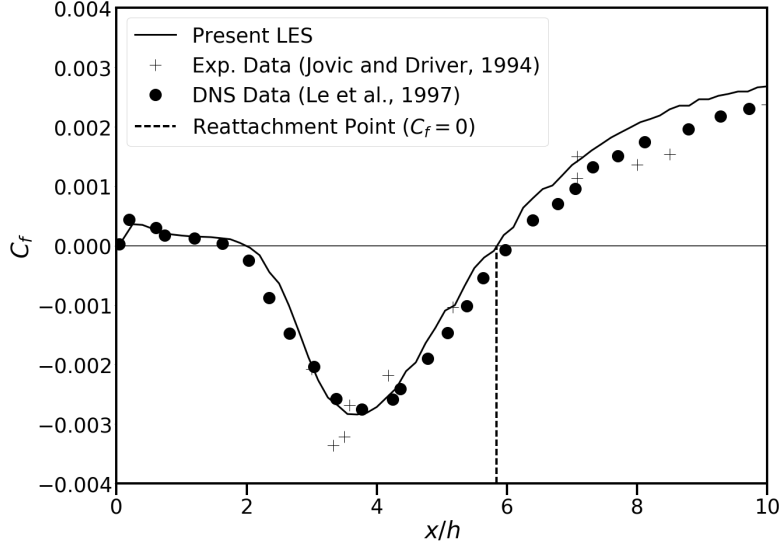


Figure 4.7: Mean Streamwise Skin Friction Coefficient.

by dynamic head, $q = \frac{1}{2}\rho U_\infty^2$) for all streamwise locations indicating that the current numerical technique is able to accurately capture the instantaneous pressure fluctuations over the BFS reliably using LES for the first time. The *RMS* pressure fluctuations increase in magnitude as the flow moves downstream of the step, reaching a maximum value just prior to the reattachment point, and decrease steadily thereafter. The excellent agreement of the fluctuating pressure could not previously be obtained using traditional SEM, as shown by Poletto et al. [29].

As the main motivation for the DFSEM was the preservation of realistic pressure fluctuations in the domain, the fluctuating pressure field averaged over the spanwise dimension and normalized by the dynamic head presented in Figure 4.9. Both the magnitude and shape of the pressure fluctuations are

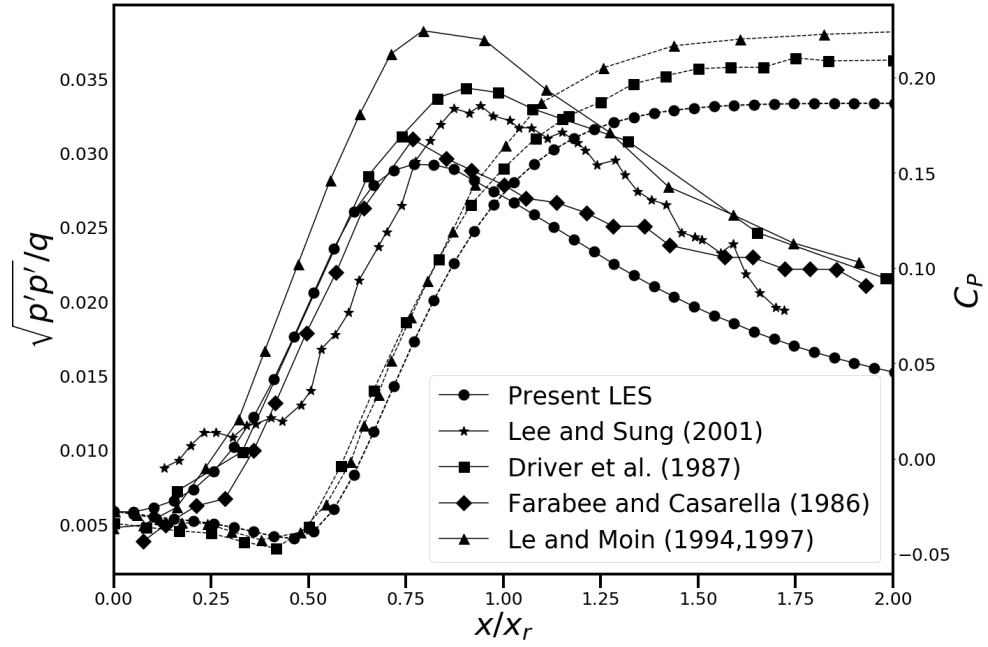


Figure 4.8: Streamwise Wall Pressure Profiles: dashed lines correspond to mean pressure coefficients, C_P (right axis), and solid lines correspond to RMS (left axis) fluctuating pressure.

in excellent agreement with the contours presented in Le et al. [5]. The peak pressure fluctuations occur just downstream of the step within the shear-layer itself. This is likely due to the unsteady passage of low-pressure cores of vortex structures within the shear-layer. Again, the preservation of realistic pressure fluctuations represents a marked improvement in the study of fluctuating pressure fields via SEM methods for LES [29].

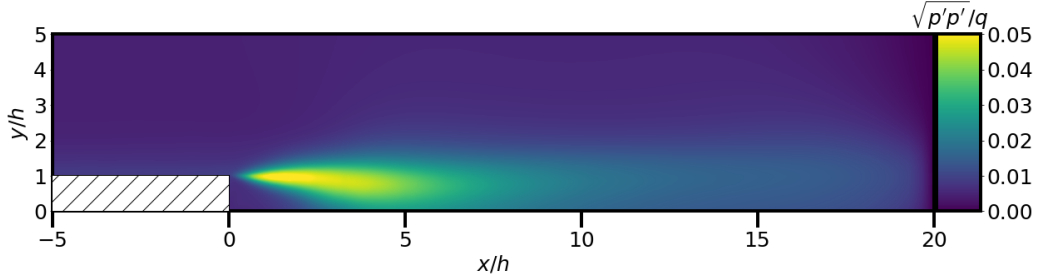


Figure 4.9: Spanwise and Temporally Averaged RMS Pressure Field

4.4.2 Wall-Pressure Spectra

Figure 4.10 presents the wall-pressure power spectra measured at several streamwise positions ranging from $x/h = 0.5$ to $x/h = 11.5$. These spectra were calculated by breaking all 157s of data into separate 1s blocks and averaging the results across those blocks, which is the standard approach when collecting data using physical experiments. Note, this is unlike most numerical simulations where very high temporal sampling rates are employed for relatively brief total sampling time. The spectra at all locations are dominated by several clear peaks. These peaks are somewhat more pronounced

than in previous experimental investigations of flow over a BFS [7, 45], which can be attributed to the lower Reynolds number used here. A significant benefit of this is that closer examination of the peaks themselves is possible. In order to make the results shown in Figure 4.10 more readable, each spectra is plotted with a fixed vertical offset, to prevent them from overlapping with one another.

Spectra very close to the step ($x/h = 0.5 - 2.5$) have very low magnitudes in general, and little evidence of any meaningful peaks as they are buried in the very slow moving fluid within the recirculation zone. The highest magnitude of the spectra just upstream of reattachment at $x/h = 4.5$ occurs at $St \approx 0.011$, highlighted by arrow (a). This peak is consistent with the low-frequency bulk movement of the shear-layer associated with flapping [7, 33, 21, 12, 44]. Spectra downstream of the mean reattachment, $x/h = 6.5 - 11.5$, show little evidence of the $St \approx 0.011$ peak, but do show a peak near $St \approx 0.025$, which increases to $St \approx 0.04$ as the streamwise position is moved further downstream, illustrated by arrow (b).

There are also prominent peaks in the spectrum at $St \approx 0.045$ at $x/D = 4.5$ which increases continually in frequency to $St \approx 0.068$ at $x/h = 7.5$, highlighted by arrow (c); this is consistent with the frequencies associated with large-scale vortex-shedding from the step [5, 7, 19] although the increase in frequency has never been addressed in the literature. A second higher frequency peak occurs at $St \approx 0.09$ and also increases continually to about $St \approx 0.11$ at $x/h = 7.5$, as highlighted by arrow (d). All of these addi-

tional higher frequency peaks are likely tied to the passage of smaller scale structures within the shear-layer or other shear-layer instabilities like those discussed by Rockwell and Naudascher [57]. A close inspection of the spectra indicates that these peaks are not harmonics of each other. As will be shown, the change in frequency of these peaks is primarily linked to an increase in convection speed of the structures coupled with a change in physical size of the structures. This information is important as it indicates that the spectral results in flows over BFS are dependent on the probe location, which can likely explain some of the variation in spectral results presented in the literature. The change in peak frequency is also critical information for SE techniques, particularly where single point estimations capture the entirety of the flow-field in a single time instant and are unlikely to capture any deformation.

The semi-logarithmic axis and vertical offsets of Figure 4.10 can make comparison of the spectral magnitudes and the rate of frequency increase challenging, so the results have been compiled into a colormap plot, shown in Figure 4.11. The horizontal axis corresponds to a linear-scale of normalized frequency, St , the vertical axis represents the streamwise location, x/h , and the colormap corresponds to the spectral magnitude at a given location and Strouhal number. As the scales are now linear, calculation of the rate of change of peak frequency with streamwise position can be determined easily. The slope of Line (b) was found to be $S_b \approx 250\text{Hz/m}$, the slope of Line (c) was calculated at $S_c \approx 833\text{Hz/m}$, and Line (d) was $S_d \approx 1224\text{Hz/m}$.

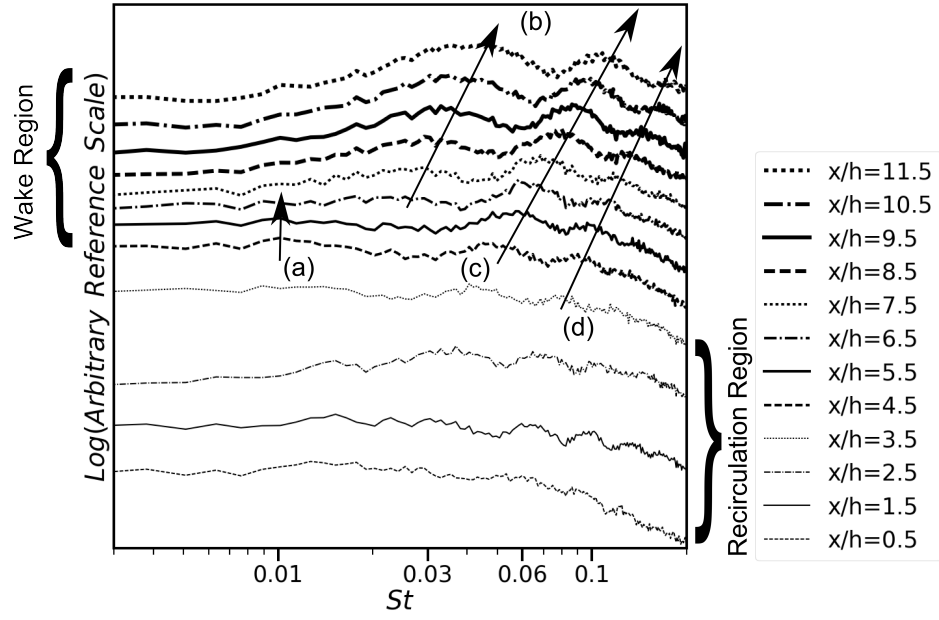


Figure 4.10: Wall Pressure Power Spectra at Different Streamwise Locations.

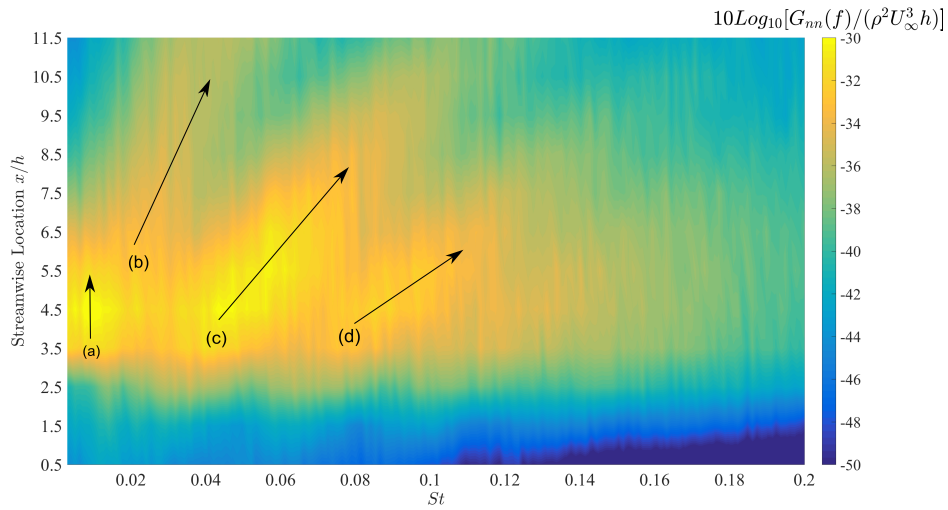


Figure 4.11: Colormap of Wall Pressure Power Spectra at Different Streamwise Locations.

4.4.3 Two-Point Wall Pressure Statistics

The cause of the shift in peak frequency is examined further using the magnitude and phase of the cross-spectra, $G_{nm}(f)$ and $\phi_{nm}(f)$, as well as the two-point coherence, $\gamma^2(f)$. The coherence between two signals, n and m , is defined by:

$$\gamma^2(f) = \frac{|G_{nm}(f)|^2}{G_{nn}(f)G_{mm}(f)} \quad (4.1)$$

where f is the frequency in Hz, $G_{nn}(f)$ and $G_{mm}(f)$ are the auto-spectra of the two signals respectively. The coherence, cross-spectra magnitude and phase are referenced to the microphone at $x/h = 4.5$ are shown in Figure 4.12 and thus the magnitude here represents a power spectrum and there is no phase angle. Rather than plot the phase angle directly in either case, the convective velocity, $U_c(f)$, between probes at each frequency is determined from the phase angle of the cross spectra between the two signals via:

$$U_c(f) = \frac{2\pi f \zeta_{nm}}{\phi(\zeta, f)} \quad (4.2)$$

where ζ_{nm} is the physical spacing between the two probes n and m , and $\phi(\zeta, f)$ is the unwrapped cross-spectral phase angle between the two probes [24]. The angle is carefully unwrapped using an appropriate algorithm to eliminate any 2π discontinuities in the phase angle. Convective velocities computed below $St < 0.008$ are unreliable due to the extremely small phase lags between measurements at these frequencies resulting in extreme deviations in convec-

tive velocity. Additionally there is a high degree of statistical uncertainty due to the limited number of independent samples for these extremely low frequencies.

The magnitude of the cross-spectra near the low frequency peak detected in the power spectra in the recirculation region at around $St \approx 0.011$ in Figure 4.12 are apparent but decreases rapidly as the probe separation increases, more so than for the vortex-shedding peaks. The coherence is also lower at the lower frequencies, and drops significantly downstream of the mean reattachment point, indicating that the lowest frequencies are only weakly related over these spans. The normalized convection speed at this frequency is also quite low, about $U_c/U_\infty \approx 0.15$ and does not vary over the measurement span. There is a noticeable reduction in the coherence near $St \approx 0.03$ in all cases, and little to no evidence of a peak in the cross-spectral magnitude. This indicates a lack of organized shedding behavior at those frequencies. The peak observed could potentially be related to an interaction between the low-frequency flapping and the vortex-shedding from the shear-layer. As the shear-layer is moving, the vortices are not being shed from a fixed position at each instant, resulting in a peak in the spectra in between the two actual shedding mechanisms. The $St \approx 0.03$ peak is likely not a distinct flow feature, but rather a combination of the flapping of the shear-layer and vortex-shedding from the shear-layer. It could also be explained by an intermittent shedding mechanism, perhaps related to the wake-mode found by Hudy et al [1] and Chovet et al. [45].

The largest coherence in Figure 4.12 is found near the vortex-shedding peak at $St \approx 0.06$, indicating that the vortex-shedding mechanism is reasonably well organized. The cross-spectra magnitude shows a corresponding increase in frequency in this frequency range as the distance downstream is increased (highlighted by the arrows), consistent with the power spectra results. Figure 4.13 presents the same convective velocity from Figure 4.12 on linear scales in order to allow easier investigation of the changes in frequency and velocity. There is an increase in normalized convection velocity from $U_c/U_\infty \approx 0.3$ between $x/h = 4.5$ and $x/h = 5.5$ to $U_c/U_\infty \approx 0.4$ between $x/h = 4.5$ and $x/h = 7.5$, which is approximately 33% higher. This increase in convective velocity as the probe location is moved farther downstream is consistent with the results of Hudy [1], who attributed it to the increase in mean velocity outside of the recirculation zone as the flow continues downstream. However, the increase in frequency from the spectral results here was only found to be approximately 7%, whereas the convective velocity has actually increased by about 33%. If the change in peak frequency were to be solely explained by an increasing convective velocity, those rates would have to be equal; a structure passing at twice the velocity should pass fixed probe locations twice as often. As the increase in peak frequency is less than that found in the convective velocity, it is likely that the structures are undergoing an additional change that would result in a reduction in passing frequency. The most likely explanation is that the vortex structures are changing shape in addition to being convected along. Due to the region of high shear be-

tween the recirculation zone and the free-stream velocity, the structures are being stretched out and elongated in the streamwise direction in addition to being pulled along by the increasing mean flow velocity downstream of the step. The elongation of the structures would typically result in a frequency *decrease*, where the increase in mean velocity would result in a frequency *increase*. In reality we have a combined effect of these two phenomenon, and is an important development for improving SE examinations of these types of flows. Another possible explanation would be three-dimensional spanwise movement or distortions of the structures causing them to miss the probe locations. Capturing these deformations through proper sensor placement and time-lag selections would be critical to ensuring the most accurate estimations of the turbulent flow field.

An increase in peak frequency is also noted with the next highest frequency peaks at around $St \approx 0.09$ in Figures 4.12 and 4.13; the frequencies of this peak increases as the probe separation distance is increased and the normalized convection speed also increases, from $U_c/U_\infty \approx 0.35$ to $U_c/U_\infty \approx 0.45$. The change in peak frequency from $St \approx 0.09$ up to $St \approx 0.11$ represents a change in frequency of approximately 22%. Although this is closer to the percentage increase in convective velocity indicating that the smaller structures are less prone to any sort of deformation, the results still indicate that increase in convective velocity as the flow moves downstream cannot solely be explained by an increase in mean streamwise flow velocity at those locations. At even higher frequencies, the highest normalized con-

vective velocities are in the range of $U_c/U_\infty \approx 0.5$ to $U_c/U_\infty \approx 0.6$, which is consistent with the values reported by Lee and Sung [7], Hudy et al. [1], and Chouvet et al. [45]. This increase in convective velocity at higher frequencies is consistent with smaller structures more closely following the bulk flow. For frequencies less than the vortex-shedding frequency at $St \approx 0.06$, convective velocity shows peaks where the coherence is high, but is low in regions of lower coherence, where the disorganized shedding behavior results in low convective velocities on average.

In Figure 4.14 the reference probe is located at $x/h = 7.5$, which is outside the recirculation region. With this reference probe location the cross-spectral magnitudes shown at $x/h = 7.5$ correspond to a power spectra. The coherence of the furthest downstream measurements is extremely high, $\gamma^2 \approx 0.75$, indicating that there is highly coherent and organized vortex-shedding occurring around $St \approx 0.06$ outside of the recirculation region. As the probe location is past the mean reattachment point, there is little energy in the cross-spectra associated with the low frequencies, as expected as the low-frequency is associated with the reattaching phenomena which occurs upstream of this measurement location. The magnitude of the spectra and cross-spectra at the locations downstream of mean reattachment, $x/h = 6.5$ and $x/h = 7.5$, are very similar, indicating that the flow is highly related at those positions and dominated by frequencies around $St \approx 0.06$. The peak frequency value also increases as the flow moves downstream (as indicated by the arrows). The convective velocity shown in Figure 4.14 is negative because

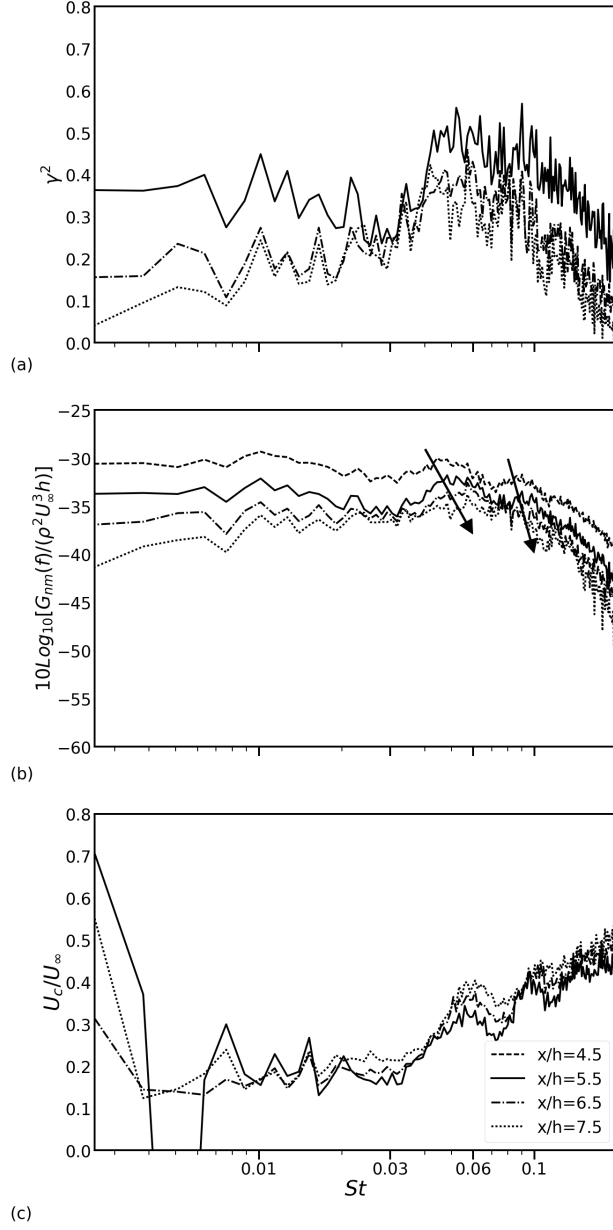


Figure 4.12: Wall Pressure Probe Two-Point (a) Coherence, γ^2 , (b) Cross-Spectral Magnitude and (c) Normalized Convective Velocity, U_c/U_∞ , with the Reference Probe at $x/h = 4.5$.

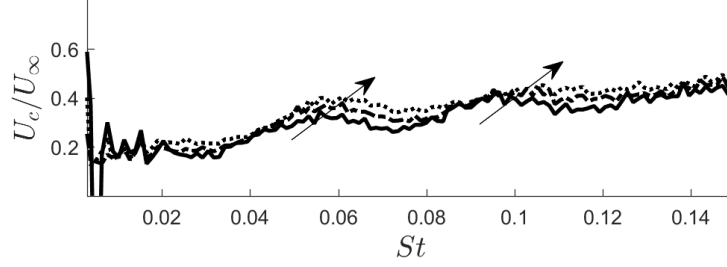


Figure 4.13: Normalized Convective Velocity, U_c/U_∞ , on Linear Scales with Reference Probe at $x/h = 4.5$. Labels are the same as Figure 4.12.

the reference probe is downstream of the other measurement locations. The largest convective velocities are found at the highest frequencies as expected. Figure 4.15 presents the convective velocity on a linear scale to highlight that there is still a discrepancy at this reference location between the increase in peak frequency ($\approx 11\%$) and increase in convective velocity ($\approx 33\%$), which is again likely tied to an elongation of the structures in the streamwise direction.

4.4.4 Field Pressure Spectra

In order to further investigate how the flow within the recirculation region and downstream of reattachment is related to a wall pressure measurement, the pressure spectra at measurement locations normal to the wall are presented at various streamwise locations is shown in Figure 4.16. Again, spectra like these are virtually impossible to obtain experimentally, especially simultaneously. The vertical dotted line corresponds to the peak frequency at each streamwise position. These results have also been compared with the wall-pressure spectra at the same streamwise position. There is generally good

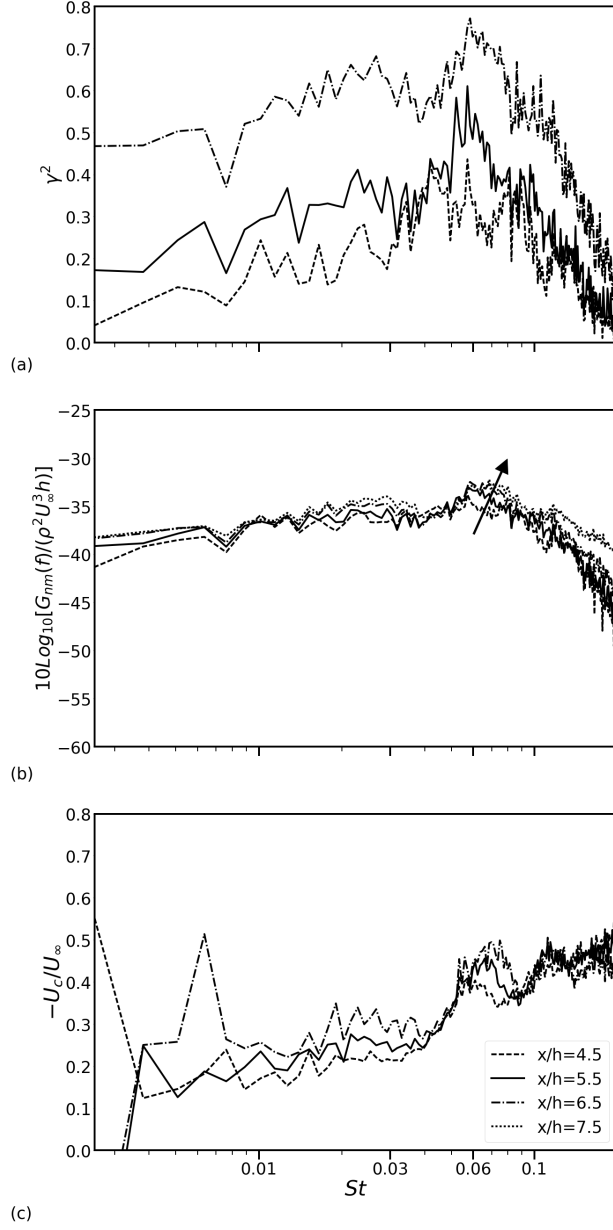


Figure 4.14: Wall Pressure Probe Two-Point (a) Coherence, γ^2 , (b) Cross-Spectral Magnitude and (c) Normalized Convective Velocity, $-U_c/U_\infty$, with the Reference Probe at $x/h = 7.5$.

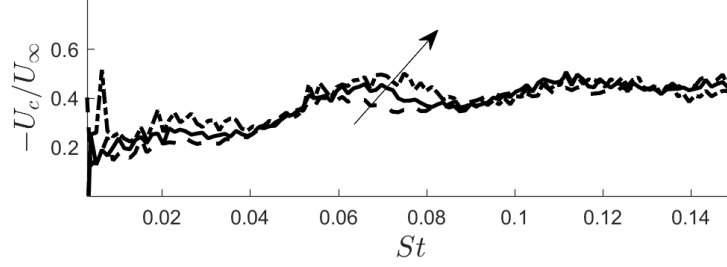


Figure 4.15: Normalized Convective Velocity, $-U_c/U_\infty$, on Linear Scales with Reference Probe at $x/h = 7.5$. Labels are the same as Figure 4.14.

agreement in both magnitude and frequency for the spectra measured at various wall normal positions, indicating that pressure fluctuations above the wall are well reflected by wall pressure measurements at those same stream-wise locations. Although, perhaps not surprising, the general collapse of the data indicates that the wall pressures are a reliable means to approximate field pressure field into the vicinity of the wall. This is not true for higher frequency fluctuations as the spectra taken at the wall tend to roll-off much more sharply at high frequencies ($St > 0.1$) than those above the wall, and this is particularly evident in the results upstream of the mean reattachment point, Figure 4.16(a). This is likely tied to the low-frequency movement of the streamwise reattachment point, which features large downward flow associated with the impingement of the shear-layer on the lower wall. Higher frequencies associated with the passage of smaller features which generally are far from the wall are obscured by the low-frequency, spatially correlated and coherent large pressure fluctuations associated with the unsteady reattaching shear-layer. Additionally, the boundary layer is much thicker in the

recirculation region which damps out the smaller structures and their pressure effect on the wall. This effect is less pronounced as the flow moves farther downstream, as the flow moves farther from the mean-reattachment point, where the magnitude of the velocity normal to the wall is greatly reduced. As shown in Figure 4.16(d), the low-frequency content decreases as the distance from the wall is increased, for all streamwise positions examined, as the low frequency movement of the shear-layer is less pronounced in the wall-normal direction compared to the movement of the streamwise reattachment point. Therefore, the faster moving smaller scale features within the shear-layer are more or less obscured by the bulk movement of the shear-layer itself. Once again, there is a noticeable increase in the peak frequency as the measurement point is moved downstream, regardless of the position normal to the wall. All wall-normal measurement locations show virtually the same peak frequency for the same streamwise position. For SE examinations this means that fluctuating pressure features within the flow field are generally well represented, particularly larger scale, low-frequency features. Luckily, those are generally the features that are most important to SE studies which generally seek to understand the largest and most important turbulent features within the flow.

The convective velocity between two probes at different streamwise (x/h) positions, but identical wall normal (y/h) positions, are shown in Figure 4.17 for several streamwise locations. The convective velocities measured by probes upstream of reattachment, Figure 4.17(a) and (b), are generally

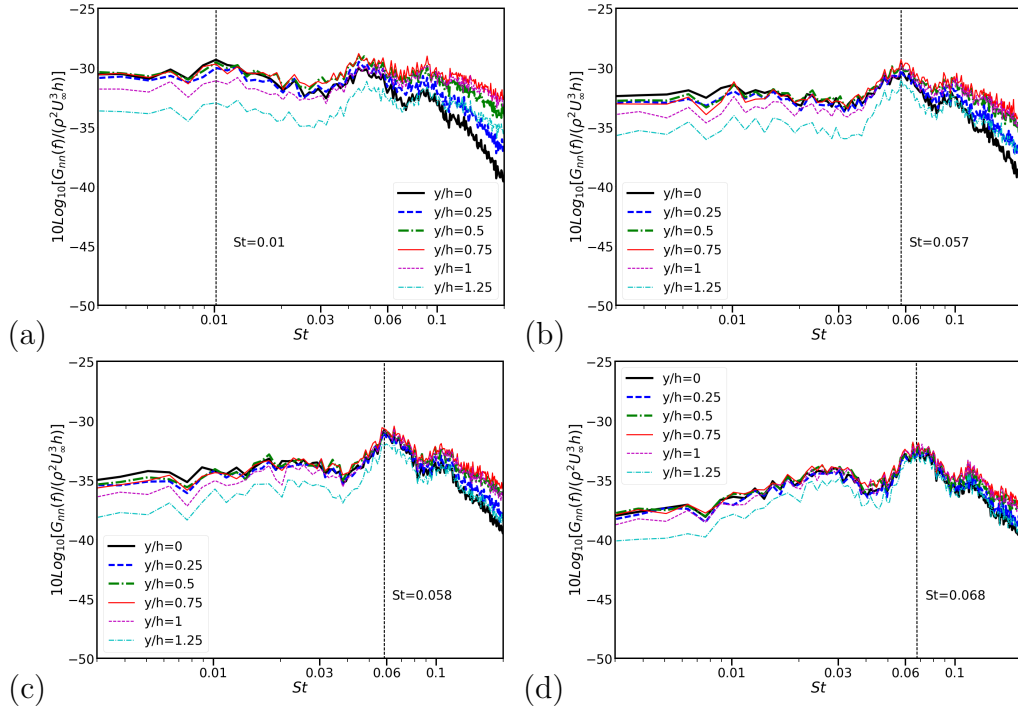


Figure 4.16: Wall and Field Pressure Spectra at Various Wall-Normal Positions and Fixed Streamwise Locations: (a) $x/h = 4.5$, (b) $x/h = 5.5$, (c) $x/h = 6.5$, and (d) $x/h = 7.5$. Dashed line corresponds to location of peak spectral frequency of the wall-pressure probe.

lower than those measured further downstream at a given frequency, likely due to flow being entrained within the slower moving flow upstream. Results near of the mean reattachment point, Figure 4.17(a) and (b), show little variation in the convection velocity as a function of wall normal position, indicating that structures away from the wall are convecting at the same speed as those near the wall, or more likely, are directly causing correlated pressure fluctuations at the wall. There are two small peaks around $St \approx 0.06$ and $St \approx 0.11$, which are consistent with the expected vortex-shedding at those frequencies. The convective velocity near $St \approx 0.06$ in Figure 4.17(c) was found to be $U_c/U_\infty \approx 0.5$, which was in agreement with the value found from the single peak time delay from the two-point, two-time correlations between probe locations, indicating that the delay in the cross-correlation peaks from different streamwise positions is likely tied to this mechanism. There was no significant change in the value of convective velocity as the distance from the wall was increased when computed from the single time delay approach. For higher frequencies there is generally a slight increase in the convective velocity at a given frequency as the distance from the wall is increased. A likely explanation is that the mean streamwise velocity farther from the wall is higher, causing structures to convect more quickly than those trapped in the slower flow nearer to the lower wall. There are several peaks observed in the convective velocities that coincide with the peaks in the power spectra, indicating that there are multiple features within the shear-layer that convect at distinct velocities. It is also worth noting once

again that the higher-frequency fluctuations are not well captured using wall pressure measurements.

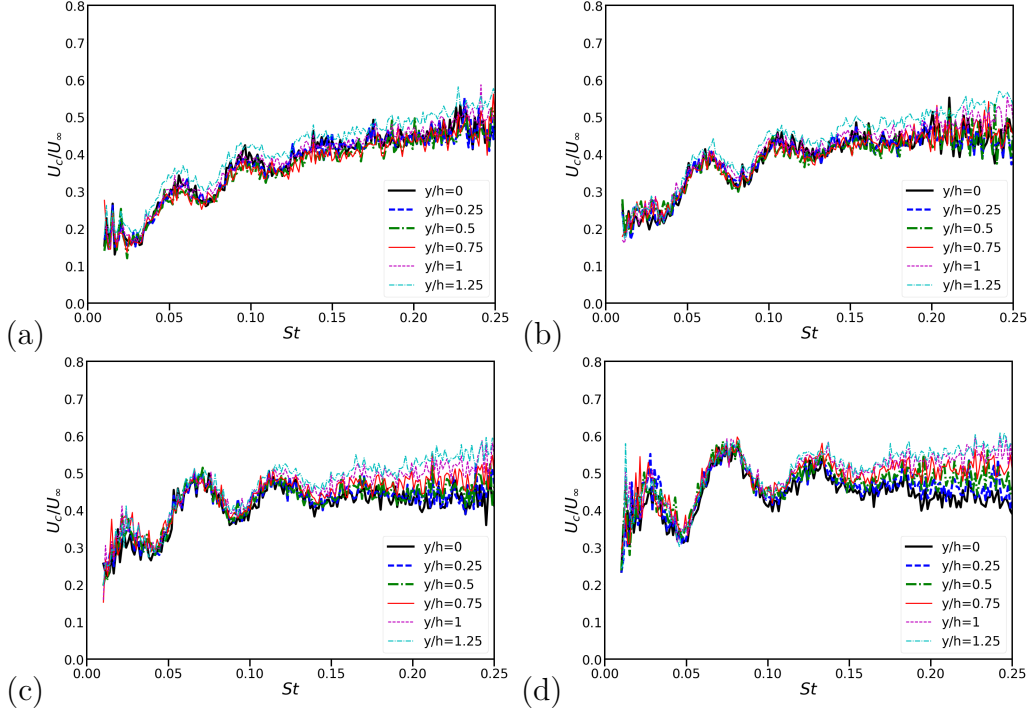


Figure 4.17: Normalized Convection Velocity, U_c/U_∞ , at Various Wall Normal Positions for Probes at: (a) $x/h = 4.5$ and $x/h = 5.5$, (b) $x/h = 5.5$ and $x/h = 6.5$, (c) $x/h = 6.5$ and $x/h = 7.5$, and (d) $x/h = 7.5$ and $x/h = 8.5$.

4.5 Conclusions

Turbulent flow over a backwards facing step at $Re_h = 5100$ has been investigated using a divergent free synthetic eddy method (DFSEM) inlet boundary condition to preserve realistic pressure fluctuations within a large eddy sim-

ulation (LES) in OpenFOAM® 1606. The geometry investigated is the same as experimentally examined by Jovic and Driver [30] and the DNS work of Le et al. [5]. Excellent agreement is found with those studies in velocity profiles, Reynolds stresses, skin friction and pressure results. The mean reattachment point was found to be $x/h = 5.84$, which is in good agreement with previous investigations. Fluctuating wall-pressure measurements taken downstream of the step have been validated with the microphone measurements of Lee and Sung [7], in terms of magnitude, distributions and spectra illustrating that the DFSEM was extremely successful in preserving the fluctuating pressure throughout the domain. This is a marked improvement over traditional SEM which cause non-physical pressure fluctuations within the domain for LES examinations.

Multiple peaks observed in the pressure spectra indicate the existence of several features of the flow, ranging from low-frequency bulk movement of the mean shear-layer, $St \approx 0.01$, higher frequency vortex-shedding, $St \approx 0.06 - 0.1$, and potential combinations of the two, $St \approx 0.03$. Wall pressure measurements were shown to be effective at capturing the field pressure fluctuations for all but the highest frequencies measured. This is likely to the smaller scale high frequency fluctuations being obscured by the organized and larger low-frequency pressure fluctuations. Convective velocities computed from correlations of the fluctuating pressures tended to increase as the flow moved downstream or away from the wall, peaking near $U_c/U_\infty = 0.5$ to 0.6 , which is in good agreement with previous wall pressure results of flow over a

BFS. Additionally, an increase in peak frequency in the wall pressure spectra as the measurement location was moved downstream was found in this study. The increase in convective velocity ($\approx 33\%$) and frequency ($\approx 7\text{--}11\%$) for the primary vortex-shedding peak, were not found to be directly proportional, indicating that the increase in peak frequency cannot be solely explained by flow acceleration as commonly accepted, but that the structures are likely undergoing additional elongation in the streamwise or spanwise directions. If the acceleration of the mean flow was solely responsible for the increase in convective velocity, then the shedding frequency would increase at an equivalent rate. Further analysis of the velocity fields is planned to investigate this hypothesis.

Chapter 5

Low-Frequency Dynamics of Flow Over a Backwards-Facing Step

This chapter is an article submitted to the Journal of Fluid Mechanics, co-authored by Stephen John Wilkins (Principal), Mahdi Hosseinali, and Joseph W. Hall.

5.1 Introduction

Turbulent flow over a backwards-facing step (BFS) is a popular case for study as it contains several differently behaving regions of varying flow; an upstream turbulent boundary layer, a separated shear-layer originating at the corner

of the step that reattaches on the lower wall, and a recovery region far downstream of the step. Significant effort has been devoted to studying this flow with an abundance of experimental methodologies, often examining the flow with high temporal resolution, but with a limited number of measurement locations using microphones [7, 6, 31, 51], hot-wires [32, 33], and Laser Doppler Velocimetry (LDV) [18, 35]. On the other hand, high spatial resolution techniques such as Particle Image Velocimetry (PIV) [51, 36, 1, 20, 37, 38, 58] are also used extensively to study the flow over the BFS. In the aforementioned experimental techniques, convergence of many higher-order statistical properties can be readily achieved by capturing large numbers of statistically independent measurements using experimental techniques by employing long total run-times for most types of experimental data collection. Equipment and techniques (such as stereoscopic time-resolved tomographic PIV) that are capable of both high-spatial and high-temporal resolutions are currently subject to significant limitations on measurement volume or Reynolds number.

Numerical simulations, on the other hand, can readily offer both high spatial and temporal resolutions, but have traditionally been used for only short simulation times (relative to experiments) due to the high computational cost associated with using a large number of elements and the small time-steps required for numerical stability of the solutions [59]. Turbulent flow over the BFS has been examined numerically using different numerical methodologies; including Reynolds Averaged Navier-Stokes Equations

(RANS) [39, 40], Large Eddy Simulations [4, 41, 42, 60] and Direct Numerical Simulations [5, 43]. In this present examination a long run-time numerical database investigating the dynamics of turbulent flow ($Re_h = 5100$) over a Backwards-Facing Step (BFS) was created using a Large Eddy Simulation (LES). The present numerical simulation is intended to get the best of both worlds, having the high resolutions possible with a numerical examination, but extending the total run-time to that typically employed when using experimental techniques.

Several important mechanisms have been identified in turbulent flow over the BFS, including a low-frequency, $St = fh/U_\infty \approx 0.01$, “flapping” or “pumping” [7, 12, 19, 20]. The significance of the low frequency ($St \approx 0.01$) behavior is still unclear. Early works on the flow over BFS by Eaton and Johnston [12, 19] and Durst and Tropea [61] hypothesized that the low-frequency energy content was due to a quasi-periodic vertical motion of the shear-layer, termed “flapping”. Researchers have also proposed that the low-frequency dynamics are governed by the unsteady behavior of the recirculation zone and the reattachment on the lower-wall [35, 62, 63], which may or may not be tied to the movement and ejection of large vortical structures from within the recirculation zone itself [63]. Statnikov et al. [20] examined the low-frequency behavior occurring at $St \approx 0.01$ using classical two-point correlation length and Dynamic Mode Decomposition (DMD), and found that the spanwise wavelength of the low-frequency “pumping” was approximately $\lambda_z \approx 2h$. A recent PIV study by Ma and Schröder [64] showed that

the flapping motion was responsible for a large portion of the turbulent kinetic energy, and was strongly related to the location of reattachment. Ma and Schröder [64] also stated that the different features proposed by the various researchers were likely describing similar phenomena, rather than being distinct mechanisms. Chapter 4 showed that the peak frequency measured in wall-pressure spectra was a function of streamwise position. The lower frequencies around $St \approx 0.01$ were found to have the largest spectral peaks in the primary recirculation zone, and the $St \approx 0.06 - 0.09$ peaks were observed to be the largest spectral magnitudes downstream of the mean reattachment point, which may be linked to the vortex-shedding off the step corner.

Large-scale organized vortex-shedding, $St \approx 0.06$ to $St \approx 0.09$ [7, 20, 12, 21], has also been found to occur in BFS flows, as well as vortex structures that grow in a stationary streamwise location before being ejected downstream [1, 45]. Higher-frequency shear-layer structures, $St \approx 0.2$ [35, 38, 65] are also present in the separated shear-layer originating from the sharp corner of the step. The relative importance of each of these flow features are highly dependent on the streamwise location within the flow, as demonstrated in Chapter 4.

The present work aims to investigate the behavior of the flow at these low frequencies and the associated spanwise wavelengths using very long runtime simulations so that the low frequencies can be resolved and examined. These low frequencies are challenging to resolve experimentally due to sen-

sor limitations, *i.e.* the ability of microphones to resolve low frequencies, and numerically, as relatively long simulation times are required. They will be filtered based on the spectral results found in Chapter 4 using the same numerical database. The time-averaged Reynolds stresses of the low frequency features are examined and discussed. The dynamics of the structures are then examined using instantaneous streamlines, velocity fields and skin friction coefficient. Proposed flow mechanisms are illustrated and discussed.

5.2 Domain Description, Boundary Conditions, and Numerics

Figure 5.1 illustrates the flow geometry used in this investigation. The domain examined is one-half a symmetric double expansion (Expansion Ratio, $ER=1.2$) and is divided into approximately 1.3×10^6 elements. Examination of the results using finer grid resolutions did not significantly alter the results or conclusions obtained. Mean two-dimensionality is maintained through the use of periodic side boundary conditions, and the domain spans $6h$. A zero-gradient outlet boundary condition is applied to the velocity at the exit plane ($x/h = 20$), and there was no evidence of any reflected waves within the domain. Both lower walls and the vertical step are modelled as zero slip walls, a zero stress symmetry plane is employed as the upper boundary condition, matching the boundary conditions employed by Le et al. [5]. In order to fully resolve the near wall regions, the grid spacing normal to the wall is selected

such that the maximum value of $y^+ < 1$ upstream of the step. The Divergence Free Synthetic Eddy Method (DFSEM) of Poletto et al. [29] was set as the inlet boundary condition as it preserved the instantaneous fluctuating pressure within the domain, which allowed both velocity and pressure fields to be captured at each instant. Full distributions of the velocity, Reynolds stresses, and turbulent length scales were needed for the DFSEM to generate the proper eddy lengths upstream of the step, data was found in the online DNS database of turbulent boundary layer flow by Schlatter and Örlü [52]. The domain extends to $x/h = -37.55$ upstream of the step to ensure that physical turbulence has been stimulated by the generated eddies and leave enough room to allow the turbulent boundary layer to develop to the proper thickness desired to match the same case as the experimental work of Jovic and Driver [30], as well as the DNS work by Le et al. [5] at the comparison location at $x/h = -3.1$.

A variable time-step is used, fixed below $\Delta t = 2 \times 10^{-5}$ seconds ($\Delta t U_\infty / h = 0.0158$), to ensure that the solution remains numerically stable by keeping the mean Courant number below a threshold of $Co < 0.2$. Data was extracted from the simulation database at $500Hz$. The statistical results presented were calculated using all $157sec$ of run-time data, while the low-frequency dynamics were examined by stepping through and filtering each individual block of $1sec$ duration and keeping running statistics.

OpenFOAM® 1606 was used to perform the LES, with the filter length set by using the cube root of the cell volume. The solver employed was

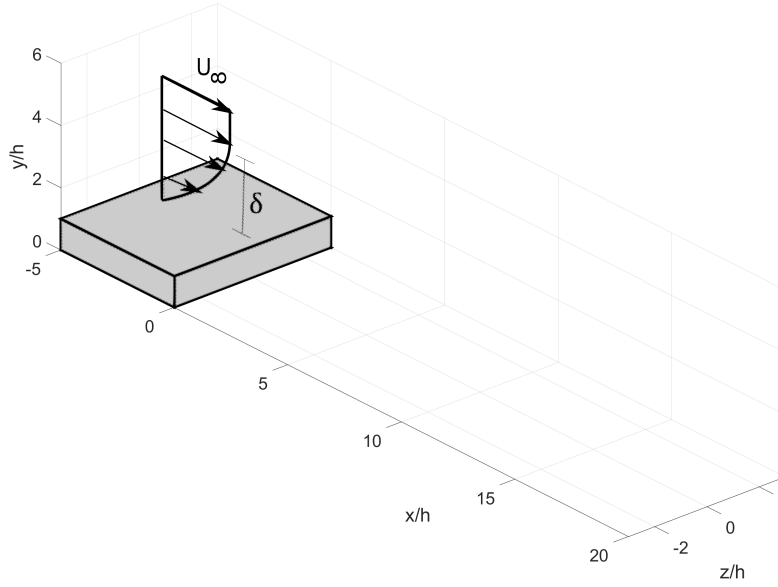


Figure 5.1: Illustration of the Backward Facing Step Geometry

the pimpleFoam (merged PISO-SIMPLE) algorithm [53, 54] and the eddy viscosity is handled using the Wall-Adapting Local Eddy Viscosity (WALE) model [55]. Second order discretization was employed for both the advection and diffusion terms, as well as a second order backward time scheme ensured that the accuracy in all cases was held to second order. Further details of the simulation and validation results can be found in Chapter 4.

5.3 Results and Discussion

Figure 5.2(a) illustrates typical wall-pressure spectra, like those shown in Chapter 4, at several streamwise locations. Significant energy is present in all cases near $St \approx 0.01$, which is consistent with the known low-frequency

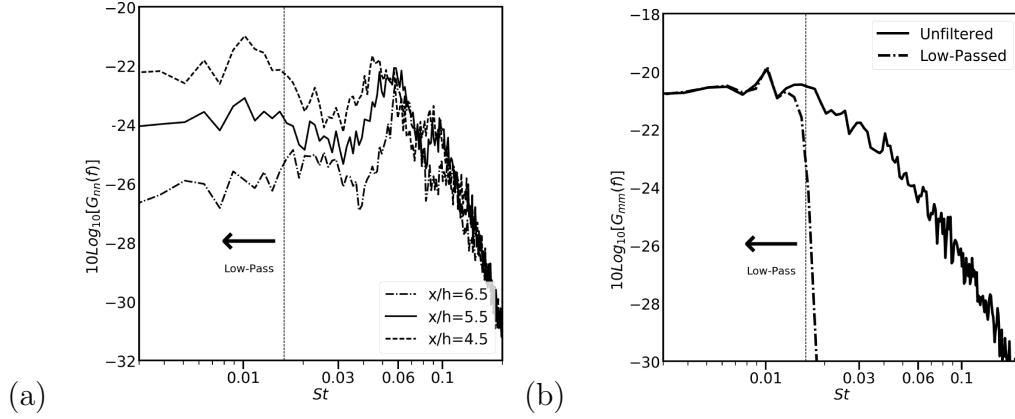


Figure 5.2: Spectra: (a) Fluctuating Wall-Pressure, p' , Spectra Measured at $z/h = 0$ and Various Streamwise Positions, and (b) Unfiltered and Low-Pass Filtered Streamwise Fluctuating Velocity, u' , Spectra Measured at $x/h = 4.5$, $y/h = 0.5$, and $z/h = 0$. Vertical dashed line illustrates low-pass cut-off frequency, $St < 0.016$.

peak found in other studies [12, 19, 7, 21]. The largest peak occurs at $St \approx 0.01$ at $x/h = 4.5$. The positions further downstream showed peaks in the pressure around $St \approx 0.03$ and $St \approx 0.06$. The velocity spectra measured at $x/h = 4.5$, $y/h = 0.25$, $z/h = 0$ is shown in Figure 5.2(b), and shows the largest spectral peak near $St \approx 0.01$, consistent with the pressure spectra.

In order to shed better light on the low-frequency dynamics at $St \approx 0.01$, the velocity fields are filtered using an 8th order low-pass Butterworth filter with a cut-off frequency of $12.5Hz$ ($St < 0.016$) to isolate the low-frequency behaviors. Butterworth filters are used because their response remains flat across the passing frequencies and a high filter order ensures a steep roll-off outside the cut-off points [25]. The filter is applied both forward and backward to prevent any phase distortion. The effect of this filter on the velocity

spectra is clearly shown in Figure 5.2(b). Running averages and standard deviations are computed as the algorithm steps through all 157 blocks of data. The normalized frequency resolution was $\Delta St = 0.0012$. Filtering of both pressure and velocity data from turbulent flows is commonly employed to extract coherent features from the flowfield using either Fourier based methods for time resolved data [26, 27, 28], Proper Orthogonal Decomposition (POD) for spatial data [66, 67, 68, 47, 69, 70, 71], or a combination of both [67, 72, 50].

The low-pass filtered velocity spectra in Figure 5.2(b) shows the steep roll-off above the cut-off frequency, and the spectra below the cut-off including the $St \approx 0.01$ peak are preserved through the filtering operation. The cut-off frequency was selected to remove the impact of higher frequency features that were notable in the pressure spectra around $St \approx 0.03$ and $St \approx 0.06$, which are particularly evident in the results from further downstream. The impact of varying the cut-off frequency was investigated up to $St < 0.24$, but the differences were negligible and did not effect the conclusions drawn.

Figure 5.3 is presented to demonstrate the impact of the filtering operation on the instantaneous streamlines, normalized streamwise velocity, u/U_∞ , skin friction coefficient, $C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$, and reattachment point, x_r , measured at $z/h = 0$ for a typical snapshot of the flow. Clearly the low-pass filtered velocity fields are more orderly. The point at which $u = 0$ ($C_f = 0$) is important as it traditionally has been employed to determine where the flow reattaches

and reverses [63], but only has a single reattachment location in the time-averaged sense. It is clear that identifying a single, instantaneous location of reattachment from the unfiltered results in Figure 5.3 is impossible, the flow can separate and reattach at several locations downstream of the step. As evident in the unfiltered C_f plot in Figure 5.3(e), the instantaneous skin friction coefficient can cross the zero threshold multiple times (hence why C_f is unreliable for instantaneous identification) indicating several regions of positive and negative streamwise velocity associated with the complex turbulent flow near the location of shear-layer impingement. The $C_f = 0$ methodology can also be problematic in PIV studies, where the velocity field is generally not fully resolved down to the wall, so no C_f information is available. In those cases, the Forward Flow Probability (FFP) [63] is commonly used to determine the reattachment point, which is also not suitable for instantaneous identification as it needs a series of instants to determine the average location of streamwise reattachment. Another methodology to delineate the recirculation zone is to bound it by using the streamline originating from the step and impacting at the reattachment position [73, 74]. Figure 5.3(a) makes it clear that instantaneous unfiltered streamlines are very challenging to use in identifying the primary recirculation region, as they can impact the lower wall at several streamwise locations. There are actually several smaller regions exhibiting flow reversals and recirculation. The difficulties in defining and identifying the streamwise position of flow attachment along the lower wall may be responsible for some disparities in the literature of

different reported reattachment lengths, which are likely very sensitive to the method of identification. The low-pass filtered results; however, almost always possess a single, easily identifiable primary reattachment point evident in Figure 5.3(d), so $C_f = 0$ can be reliably used in the low-pass filtered results obtained here to define the instantaneous filtered reattachment point. Observation of the time series showed that the low-frequency dynamics are largely responsible for setting the location of reattachment on the lower wall, which is not unusual as it dominated by the mean flow.

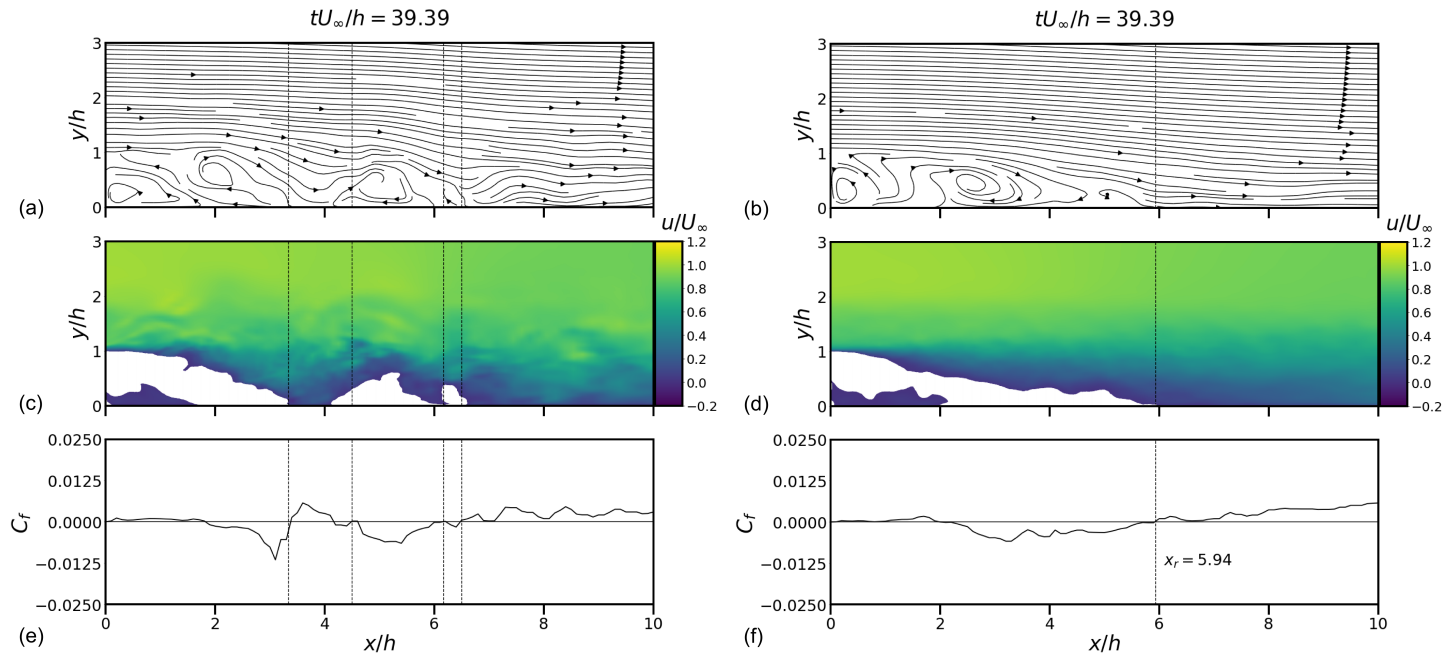


Figure 5.3: Comparison of Instantaneous Unfiltered (*Left*) and Low-Pass Filtered Results (*Right*) for (a,b) Streamlines, (c,d) Normalized Streamwise Velocity Scalar, u/U_∞ , and (e,f) Skin Friction Coefficient, C_f measured at $z/h = 0$. White areas correspond are overlaid on locations of negative streamwise velocity.

The Reynolds stresses are significantly impacted by the filtering operation; both in distribution and magnitude. The streamwise component of the Reynolds stress are shown in Figure 5.4 for both the unfiltered and low-pass filtered results. In order to better illustrate the time-averaged locations of the primary and secondary recirculation zones, the mean streamlines are overlaid in white. As expected, the removal of the higher-frequency turbulent structures has resulted in a marked reduction of the magnitude of the normalized Reynolds stress, $\overline{u'u'}/U_\infty^2$, with the magnitudes of the low-pass filtered results being approximately 30 – 35% lower than the unfiltered case. Due to the low variance within the low-pass filtered results, the uncertainty of the mean low-pass filtered streamwise velocity is less than 1%. The unfiltered results had the highest magnitude in the shear-layer whereas the low-pass filtered results are the largest in the recirculation region. Additionally, the interface region between smaller secondary recirculation zone that occurs closer to the step, $1 < x/h < 2.5$, shows magnitude of the streamwise Reynolds stresses just below the level of the primary zone indicating that this secondary region plays an important role in the low-frequency dynamics of the flow downstream of the step; the role of this region will be examined in detail later in this paper.

Figure 5.5 presents the magnitudes of the normalized wall-normal component of the Reynolds stress, $\overline{v'v'}/U_\infty^2$, for the unfiltered and low-pass filtered velocity fields. The largest magnitudes in the unfiltered velocities are found in the highly turbulent regions within the shear-layer itself. Thus the strong v fluctuations in the shear-layer are linked to structures associated with higher

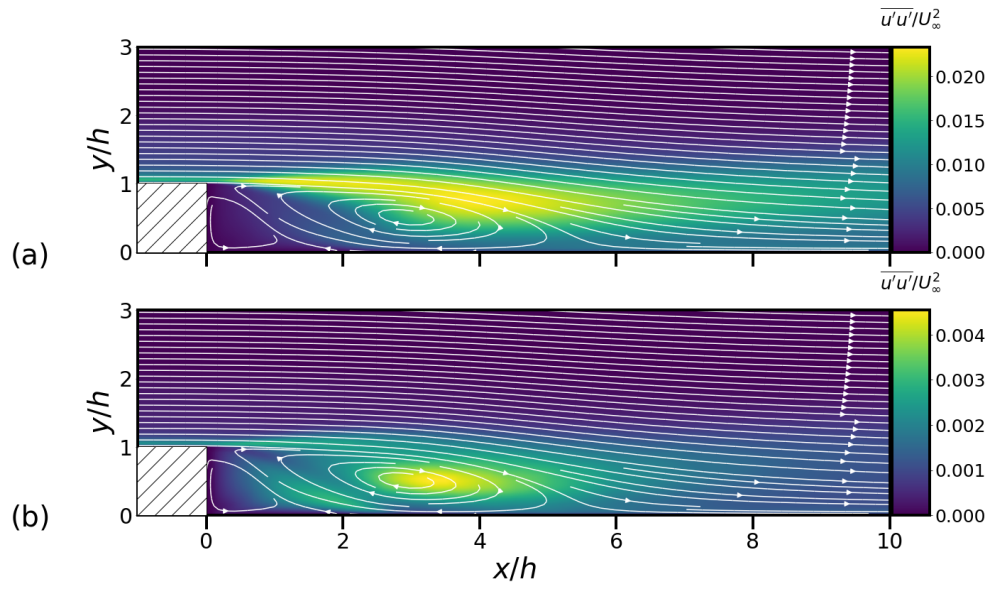


Figure 5.4: Normalized Streamwise Component of the Reynolds Stress, $\overline{u'u'}/U_\infty^2$, (a) Unfiltered and (b) Low-Pass Filtered. Mean in-plane $\overline{UV}$, streamlines are overlaid in white.

frequencies, like vortex-shedding. The largest magnitudes in the low-pass filtered results are encountered just above the wall in the region upstream of the streamwise location where the shear-layer tends to reattach, in the region of $x/h = 3$ to $x/h = 5$; here, the flow is directed strongly downwards towards the wall and combined with the streamwise movement of the reattachment point, large variations in wall-normal velocity are encountered in this vicinity. The high $\overline{v'v'}$ associated with the lower frequencies being near the wall indicate that the low frequencies are tied to the vertical “flapping” near the reattachment point, and not the turbulent structures within the shear-layer itself. This again shows the effectiveness of this low-pass filtering method to study the isolated dynamics of the recirculation region due to the removal of the smaller-scale, vortex-like structures.

Although the mean velocity in the spanwise direction was found to be zero as expected, the magnitude of the Reynolds stress, $\overline{w'w'}/U_\infty^2$, are non-zero as shown in Figure 5.6. Once again, the largest magnitudes for the unfiltered case, Figure 5.6(a), are located within the shear-layer. Interestingly, when the results are low-pass filtered, Figure 5.6(b), the largest magnitudes are encountered on the step face near $x/h = 0$ and $y/h = 0.9$. The mechanisms behind this behavior will be examined in greater detail later on in this paper when the dynamics of this region are discussed. Additionally, large values are found very close to the lower wall around the mean reattachment point in the region of $x/h = 3$ to $x/h = 6$, and are likely associated with the three-dimensional dynamics. The filtered $\overline{w'w'}/U_\infty^2$ results suggest that the

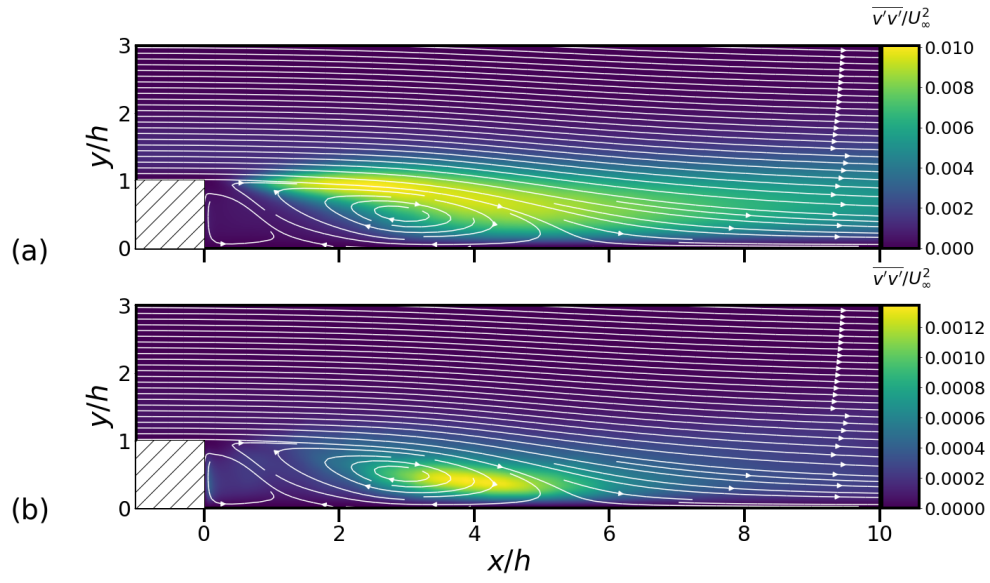


Figure 5.5: Normalized Streamwise Component of the Reynolds Stress, $\overline{v'v'}/U_\infty^2$, (a) Unfiltered and (b) Low-Pass Filtered. Mean streamlines are overlaid in white.

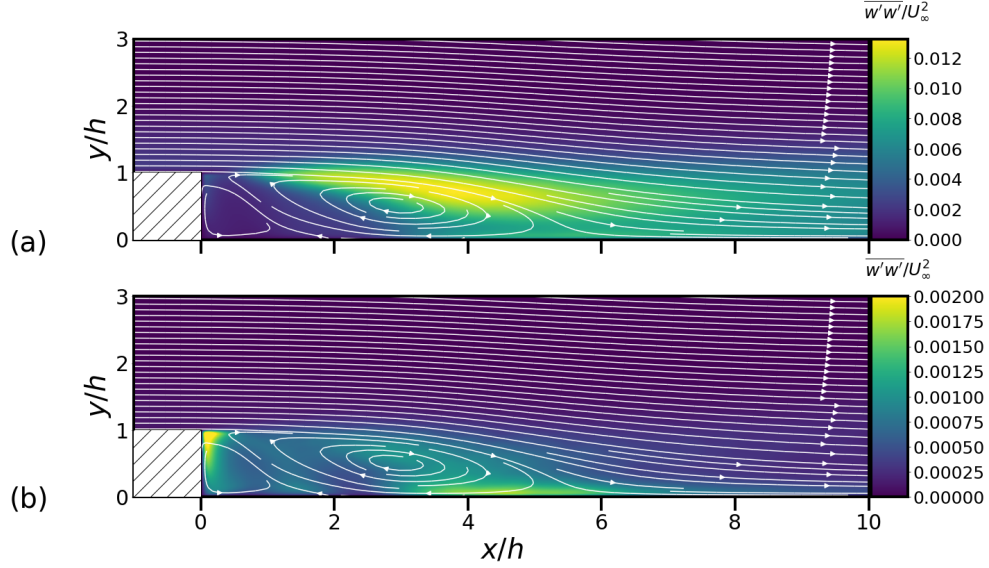


Figure 5.6: Normalized Streamwise Component of the Reynolds Stress, $\overline{w'w'}/U_\infty^2$, (a) Unfiltered and (b) Low-Pass Filtered. Mean streamlines are overlaid in white.

dynamics of the structures associated with these low frequencies are important in the near wall regions close to the reattachment point as well as along the face of the step.

The low-passed filtered and unfiltered normalized shear component of the Reynolds stresses, $-\overline{u'v'}/U_\infty^2$, are shown in Figure 5.7. The largest magnitudes in both cases are found where both u' and v' are both large, which is found just above the wall near the point of primary reattachment and recirculation. The magnitude of the shear component is approximately 50% of the streamwise normal stress, $\overline{u'u'}/U_\infty^2$, and roughly the same magnitude as the wall-normal component $\overline{v'v'}/U_\infty^2$. The $\overline{u'v'}$ terms are often associ-

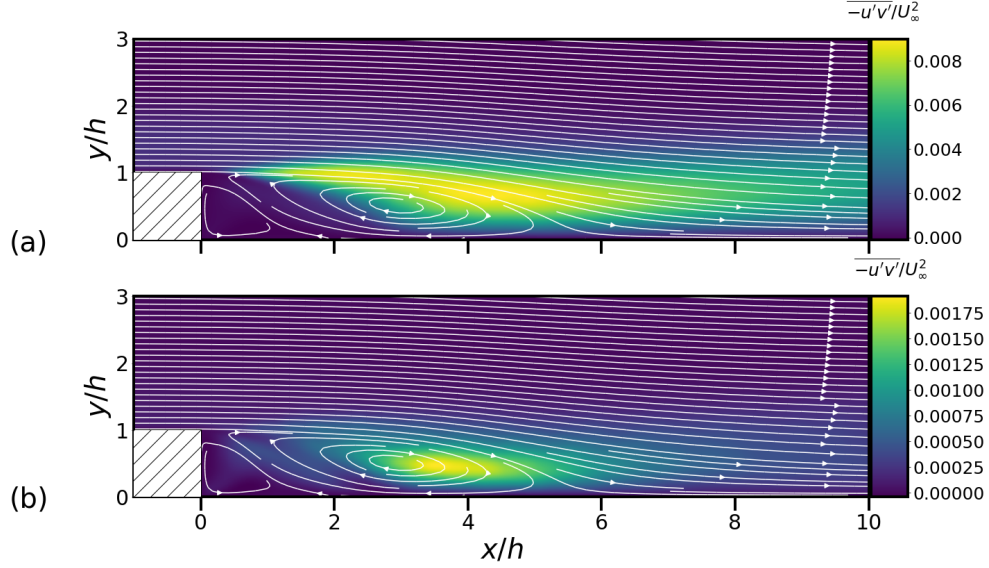


Figure 5.7: Normalized Streamwise Component of the Reynolds Stress, $-\overline{u'v'}/U_\infty^2$, (a) Unfiltered and (b) Low-Pass Filtered. Mean streamlines are overlaid in white.

ated with the presence of spanwise organized vortex structures, which is why the unfiltered results are concentrated within the shear-layer. The fact that the magnitude of the low-pass filtered results are larger near the wall once again suggests that this method is effective at removing the impact of higher frequency, turbulent features within the shear-layer.

As previously discussed, Eaton and Johnston [12, 19] hypothesized that the low-frequency energy content was due to a quasi-periodic vertical motion of the shear-layer, termed “flapping”. It is obvious from the results shown in Figures 5.3 that the low-frequency dynamics of the step flow are much more complicated than a simple uniform vertical movement of the shear layer.

Perhaps a more useful method to investigate the low-frequency behavior of the flow is to examine the area of the primary recirculation region at each instant, which can account for variations in the shear-layer distance from the wall. As the data presented has been linearly interpolated on a Cartesian grid, the area of the area of flow reversal is easily computed by counting the number of negative velocity vectors and multiplying it by the element size, $dx \times dy$, which is the same approach employed by Pearson [75] in their study of flow over a forward-facing step. Figure 5.8 clearly shows just how much of the size of the primary recirculation zone is controlled by the low-frequency dynamics of the step flow. Almost the entirety of the unfiltered recirculation zone is captured when looking at the low-pass filtered results. Higher frequencies may distort the shape of the recirculation zone slightly, but the dynamics of the primary recirculation zone are principally occurring at low frequencies.

In order to better investigate the spanwise dynamics of the flow, instantaneous streamwise velocity scalar just above the wall at $y/h = 0.25$ are examined for the unfiltered and low-pass filtered cases in Figure 5.9. Significant variations in the spanwise direction are observed. Additionally, the location of where $u = 0$ is more easily identifiable in the low-pass filtered results, and is highlighted by the solid black line. In both cases, the flow downstream of reattachment is almost entirely directed in the downstream direction, $u > 0$. The primary recirculation bubble ($u < 0$) is once again highlighted by the white color. The dark blue regions surrounded by the

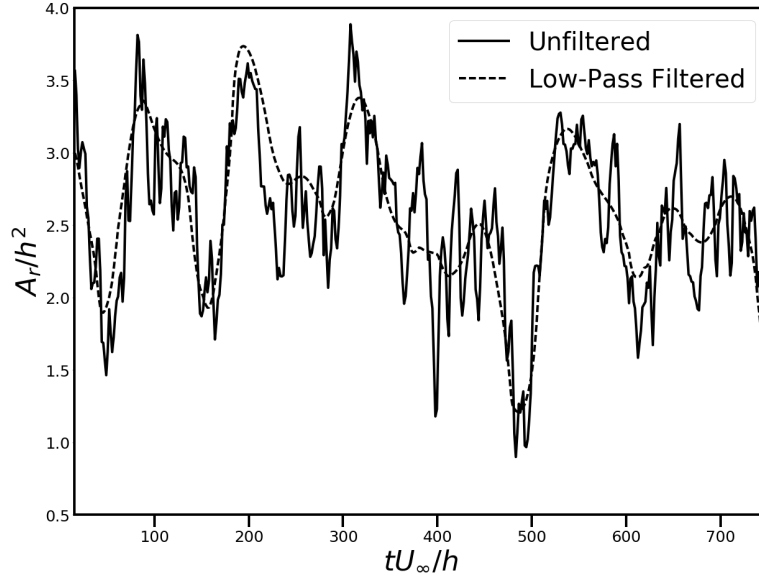


Figure 5.8: Area of the Primary Recirculation Zone, A_r/h^2 , for the xy Plane at $z/h = 0$.

white areas correspond to the secondary recirculation bubbles. Interestingly, there are large regions where there is little to no evidence of the existence of the secondary recirculation; the streamwise velocity is almost entirely negative all the way back to the step face.

Figure 5.10 presents a plot of the instantaneous uw streamlines and the instantaneous u and w velocity vectors on a plane just above the lower wall at $y/h = 0.25$ at a single instant. Interpretation of the unfiltered case in Figure 5.10(a) is challenging; however, the presence of several interesting flow features can be more easily observed in the low-pass filtered results in Figure 5.10(b). In both cases, the flow downstream of reattachment is almost entirely directed in the downstream direction, $u > 0$. At the locations

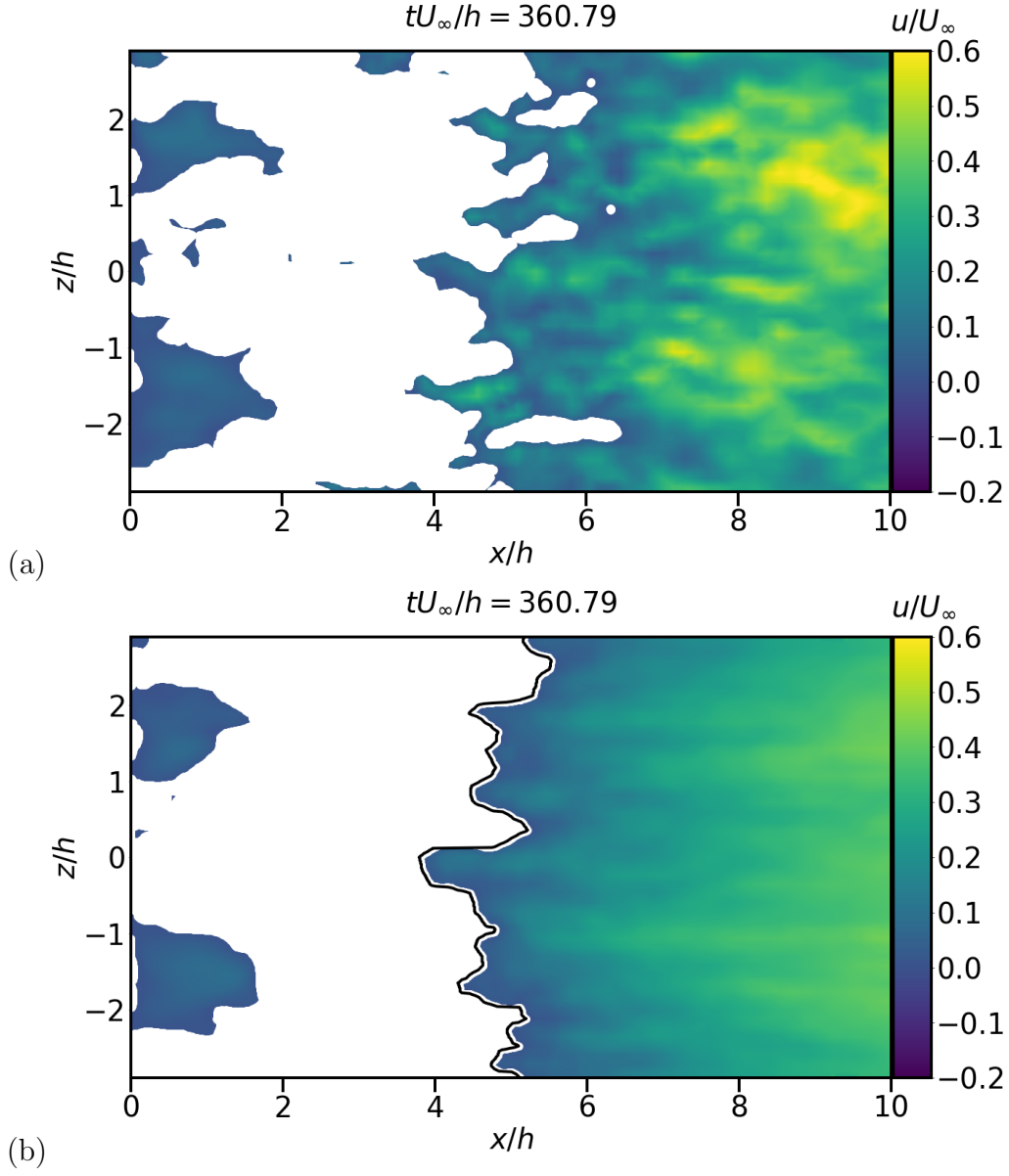


Figure 5.9: Instantaneous Spanwise Variation in Streamwise Velocity, u/U_∞ , for (a) Unfiltered and (b) Low-Pass Filtered at $y/h = 0.25$. White area correspond to locations of negative streamwise velocity. The black line corresponds to approximate location of $u = 0$ in the low-pass filtered results.

where the flow reaches and impacts upon the step face at $x/h = 0$ it splits, with some of the flow directed along the face of the step in both spanwise directions. This results in significant rotation in the wall-normal plane, as evidence by the swirling velocity vectors highlighted by the red circles. The same features can be observed in the unfiltered case, so they are physical features of the flow, not artifacts of the filter. Additionally, the resulting counter-rotating regions of flow are highlighted by the dashed blue ellipses in Figure 5.10(b). While it was clear from the mean streamlines shown in Figures 5.4 to 5.7 that the secondary recirculation would have spanwise vorticity, ω_z in the streamwise plane, it also has significant amounts of ω_y . Interestingly, this mechanism is almost identical to how rip currents are formed as ocean waves approach the beach [76]. This is consistent with the large magnitudes of the $\overline{w'w'}$ found near the step face, as shown in Figure 5.6(b), as the flow is directed into and along the step face. Significant variation in the spanwise velocity is also observed near the location of streamwise reattachment in the region of $x/h = 5 - 6$, highlighted by the red ellipses, which is consistent with the high values of $\overline{w'w'}$ close to the wall and near the mean streamwise reattachment point, observed in Figure 5.6(b). After reattachment, the flow continues to sweep side-to-side in the spanwise direction, although the magnitude of the fluctuations eventually decreases as the flow evolves further downstream and re-aligns in the streamwise direction, resulting in the localized peak in $\overline{w'w'}$ near the reattachment point.

The instantaneous spanwise variation in recirculation zone is further

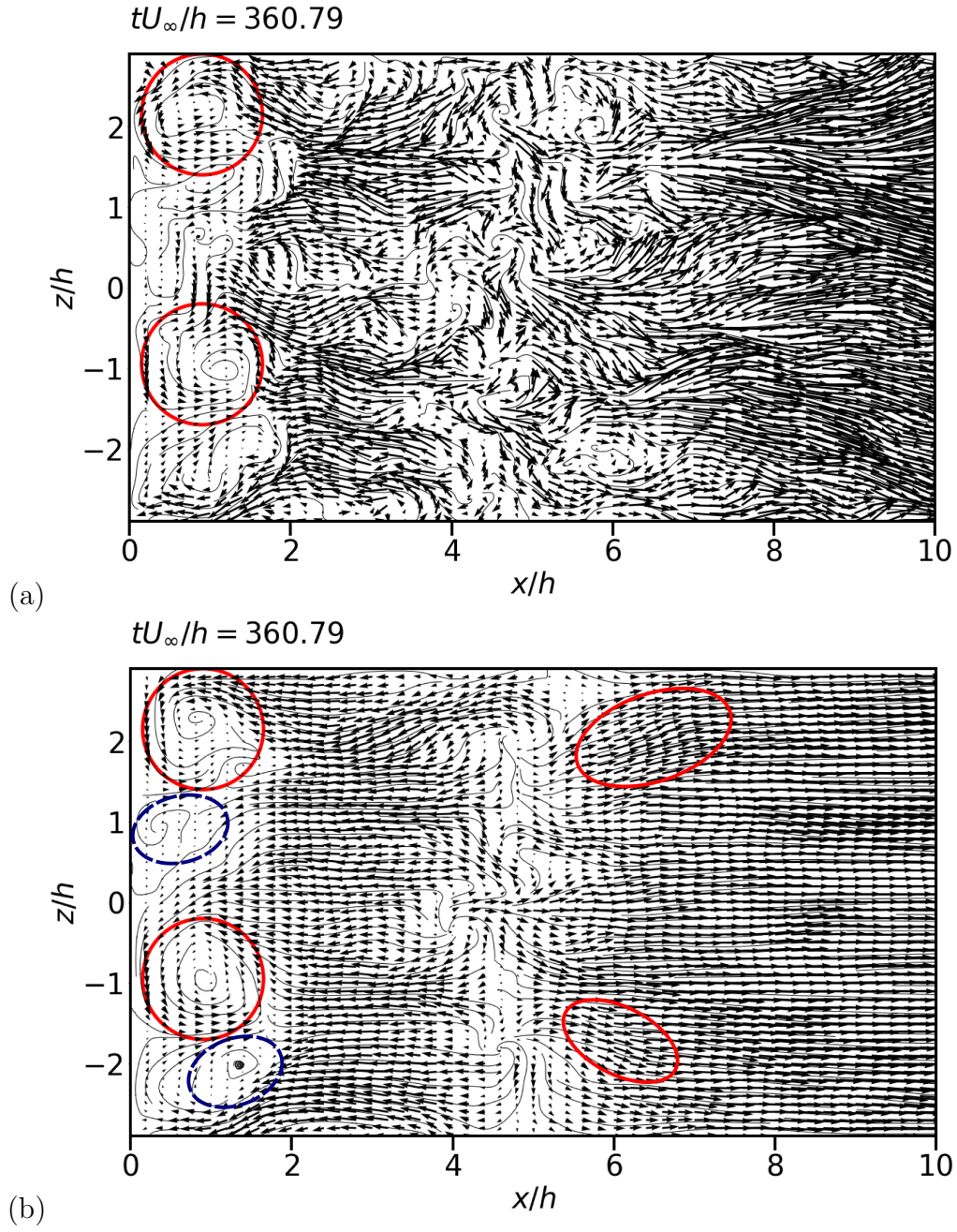


Figure 5.10: Instantaneous Streamwise and Spanwise Velocity Vectors at $y/h = 0.25$: (a) Unfiltered and (b) Low-Pass Filtered.

examined using five (y, z) planes in Figure 5.11, ranging from $z/h = 2$ to $z/h = -2$, for the same instant shown in Figure 5.10. The results at $z/h = 0$ in Figure 5.11(c) shows almost no secondary recirculation, while the outer planes shown in Figure 5.11(a) and (e) show large secondary recirculation zones. The intermediate planes at $z/h = 1$ and $z/h = -1$ show that the secondary recirculation zone is growing larger than the one that is observed on the centre plane. All of this is consistent with the previous results shown in Figure 5.10. The spanwise variation (even in a single instant) and unsteadiness of these opposing recirculation zones are likely the root cause of the low-frequency “flapping” that has long been debated. For example, had these instances been viewed at a single measurement plane of a non-time resolved experimental technique, one may infer that the shear-layer is simply “flapping” up and down in the vertical direction.

The line of $u = 0$ shown in Figures 5.9(b) and 5.10(b) can be used to estimate a spanwise wavelength of the low-pass filtered velocity fields at $y/h = 0.25$. To do this, the waveform is extracted from the low-pass filtered velocity fields at each instant by finding the streamwise location, x/h , where the instantaneous streamwise velocity is zero, $u = 0$, for every spanwise position, z/h . In order to prevent spectral leakage from significantly impacting the results, a standard Hann window function is used [77]. The spanwise averaged location of $u = 0$ is removed from each waveform and the variation of the waveforms are plotted in Figure 5.12 for several instantaneous streamwise velocity fields to get a sense for the spanwise organization in the

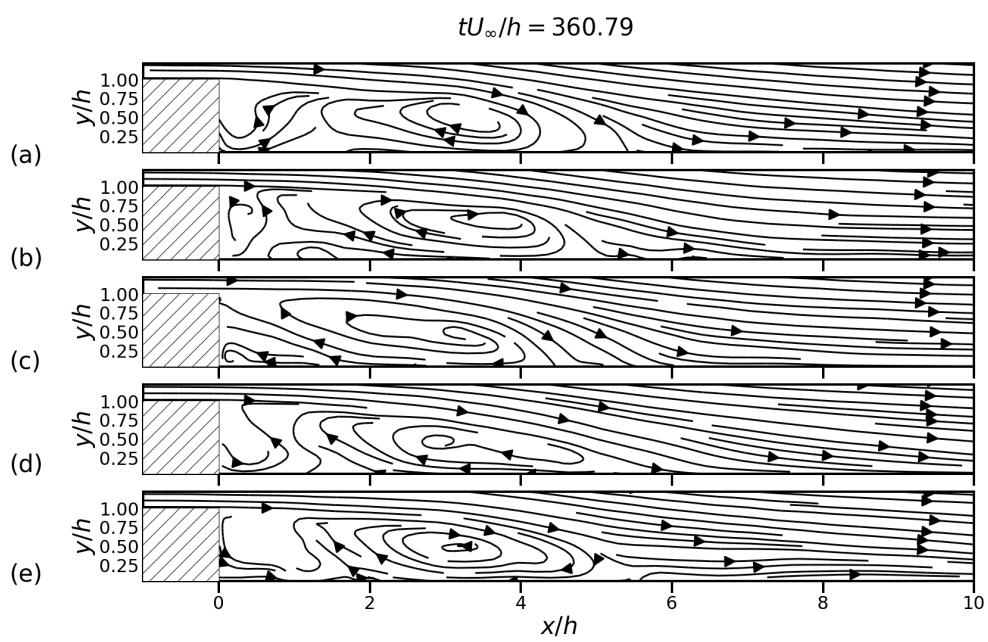


Figure 5.11: Low-Pass Filtered Instantaneous UV Streamlines: (a) $z/h = 2$, (b) $z/h = 1$, (c) $z/h = 0$, (d) $z/h = -1$, (e) $z/h = -2$.

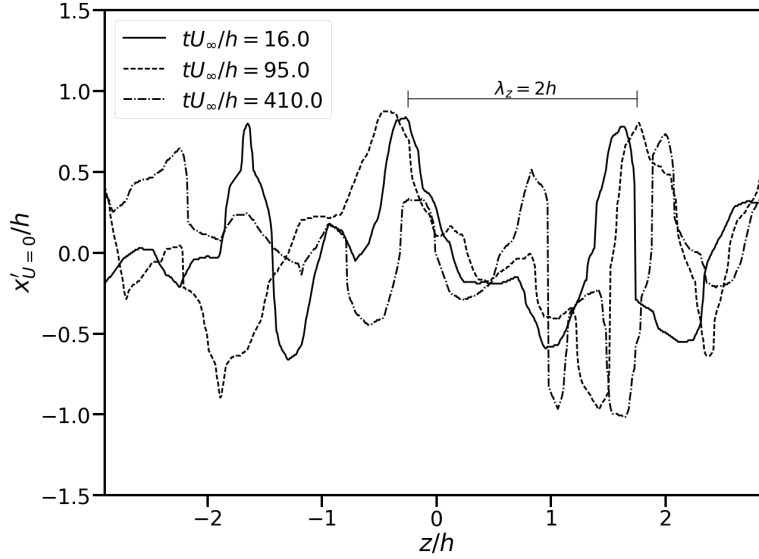


Figure 5.12: Instantaneous Variation in Streamwise Location of $u = 0$, $x'_{u=0}$, For Various Velocity Fields.

separation line. Significant spanwise variation is observed in each waveform, consistent with the spanwise variations in the low-frequency dynamics of the recirculation zones.

The wavenumber spectrum is presented in Figure 5.13 to identify potential periodicity. In order to ensure statistical independence of the instants, only 10 snapshots are extracted from each 1sec block, which results in 1570 independent waveforms. The spectrum rolls-off steeply at the higher wavenumbers as expected, due to the low-pass filtering of the velocity fields. The largest magnitude is found near $\lambda \approx 2h$, which is consistent with the results of Statnikov et al. [20] and Scharnowski et al. [37], who examined unfiltered results at much higher Reynolds and Mach numbers. The results here are

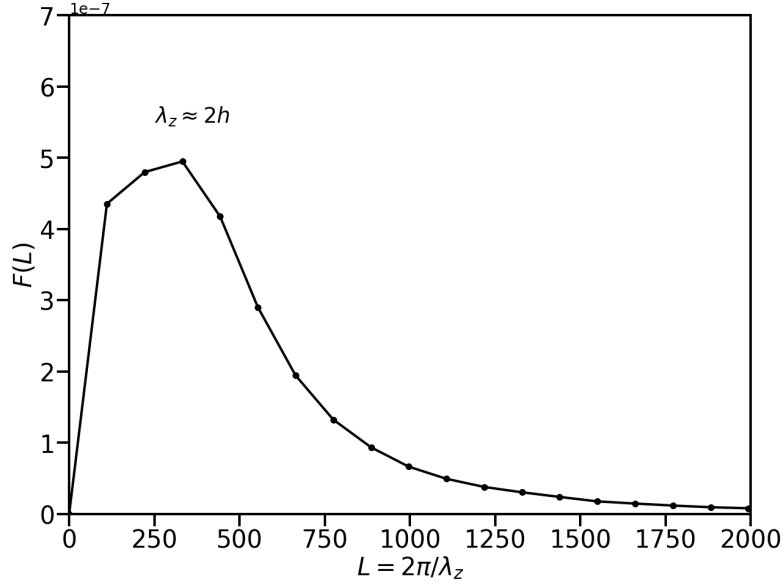


Figure 5.13: Spanwise Wavenumber Spectrum, $F(L)$.

particularly novel as this is the periodicity associated with the line of flow reversal, $u = 0$, itself, and not a related property. It appears as though $\lambda_z \approx 2h$ is the largest stable structure size in these types of flow over BFS studies, despite significant differences in Reynolds and Mach numbers. It is also consistent with the approximate size of the two counter-rotating structures observed in Figures 5.10 and 5.11. As the step geometry is homogeneous in the spanwise direction, this shows that this behavior is not the result of wall or edge effects, which is a conclusion that cannot easily be drawn from experimental studies with limited aspect ratios.

In order to better understand the three-dimensional dynamics at play in the BFS flow, quiver plots of all three velocity components on two planes

$(z/h = 0 \text{ and } y/h = 0.25)$ are shown in Figure 5.14(a) and (b) for two unrelated instants. Evidence of both the primary and secondary recirculation zones in the $z/h = 0$ plane are evident in Figure 5.14(a), but the smaller secondary zone also has clear rotation in the $y/h = 0.25$ plane as well. Flow can clearly be seen being driven towards the step face near $z/h = -2.5$ on the wall normal plane. This results in a very large region of rotating flow that is accompanied by a counter-rotating region of flow near $z/h = 0$. In Figure 5.14(b) this observation is reversed; the flow near $z/h = 0$ is negative all the way back to the step face and there is little to no secondary recirculation region. Instead the flow impinges on the step face and flows along the step face in both spanwise directions. This mechanism is likely responsible for the large magnitudes of the Reynolds Stress, $\overline{w'w'}/U_\infty^2$, found in Figure 5.6(b) near the step face. This flow being driven into the step face results in a large flow rotation in the $y/h = 0.25$ plane close to this flow impingement near $z/h = 0$. The associated recirculating flow region near $z/h = -2$ to -2.5 is now much smaller. These two counter-rotating regions grow out-of-phase with one another; when the flow is driven straight into the step face there is a large vortex formed near to the impingement which fights against a smaller vortex located approximately 2 step heights away. This pattern repeats along the step-face, but is unsteady in time, as the location of impingement on the step face drifts along the step face. Illustrations highlighting this phenomena are presented in Figure 5.14. The solid color lines correspond to the same colors of the velocity vectors on each

plane.

Figure 5.15 presents an illustration of the proposed low-frequency mechanisms for the turbulent flow over a BFS. Spanwise locations where the flow is almost entirely negative (very large primary recirculation zone, extremely small secondary recirculation zone) and the flow is driven into the step face are highlighted by the large dashed arrows. This results in non-zero magnitudes of instantaneous spanwise velocity, w , which flows along the step face near the point of impingement on the step face (small, horizontal, solid arrows). The u, w components of velocity cause vortex structures with large ω_y to be formed on either side of this impingement location. The large structure is opposed by a smaller counter-rotating structure, which are part of the smaller secondary recirculation zone typically expected in flows over BFS. Regions of positive streamwise velocity encountered between these two structures are highlighted by the smaller dashed arrows. The approximate size of this pair of structures is around $\lambda_z \approx 2h$, as highlighted on the illustration. This pattern repeats in the spanwise direction but these structures wander side-to-side in the spanwise direction due to the unstable nature of the turbulent flow. This spanwise wandering also occurs downstream of reattachment and the flow sweeps side-to-side after impinging on the lower wall. The locations where $u < 0$ all the way to the wall result in shorter reattachment lengths as the flow is being pulled towards the step face. The longer reattachment lengths are caused by the rip-current of positive streamwise velocity formed between the two counter-rotating vortex structures as the

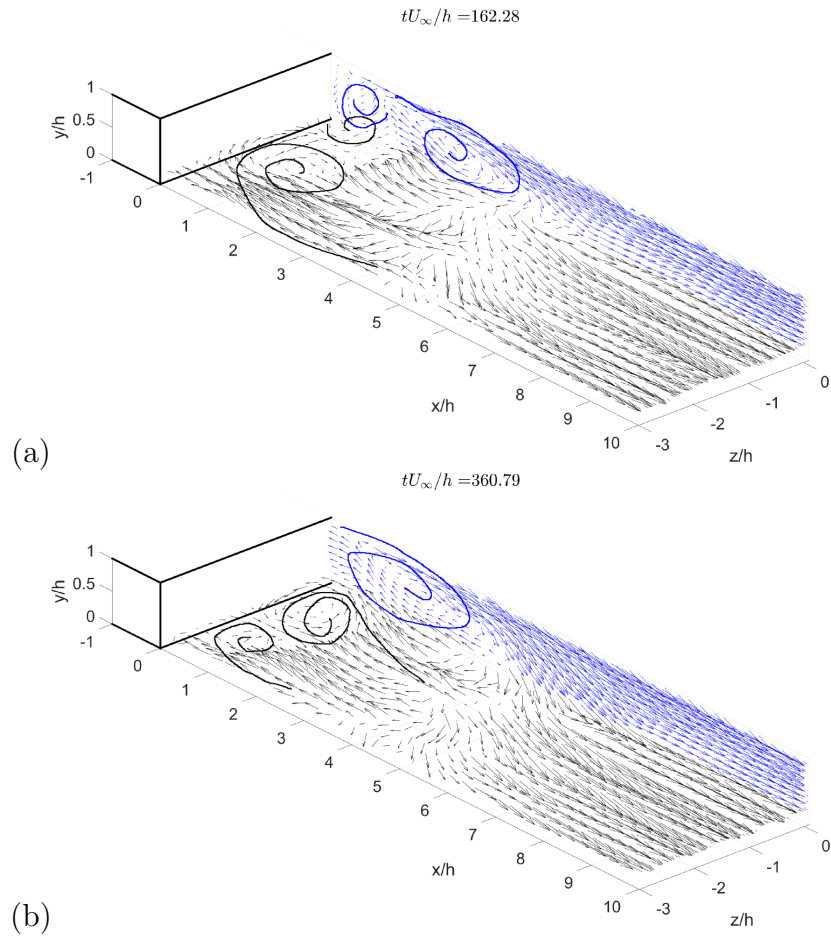


Figure 5.14: Illustration of (a) Primary and Secondary Recirculation Zones on the $z/h = 0$ Plane and (b) Single Recirculation Zone on $z/h = 0$ Plane.

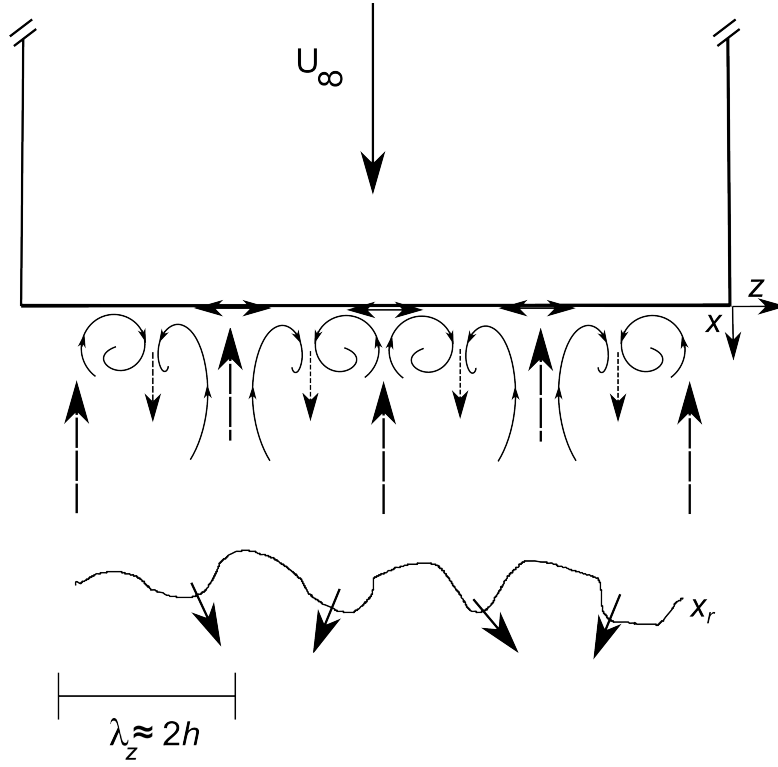


Figure 5.15: Top-View Planar Illustration of Proposed Low-Frequency Recirculation Dynamics in Turbulent Flow Over a BFS Near the Lower-Wall, $y/h > 0$.

flow is shot out in the downstream direction. Naturally these regions are not only restricted in the wall-normal direction, but likely will tilt slightly as well.

5.4 Conclusions

The low-frequency dynamics of turbulent flow over a backwards-facing step at $Re_h = 5100$ has been studied using a large numerical database generated

using a long run-time Large Eddy Simulation using OpenFOAM® 1606. Multiple planes of data were extracted from the simulation database at $500Hz$, and low-pass filtered using 8th-order Butterworth filters with a cut-off non-dimensional frequency of $St < 0.016$. The Reynolds stresses showed a marked reduction in magnitude after filtering, as the high frequency, smaller-scale turbulent structures were removed through the filtering operation. Additionally the shape and location of maximum magnitude of the Reynolds stresses were altered significantly. The Reynolds stresses associated with the low-frequencies were dominant in the two recirculation regions downstream of the step, where the unfiltered stresses tend to be concentrated in the shear-layer originating from the step corner.

Comparison of the instantaneous streamlines, streamwise velocity, u , and skin friction coefficient, C_f showed that the low-pass filtered results generally had a single location of streamwise reattachment, $C_f = 0$, whereas the unfiltered case frequently had multiple locations that could be considered reattachment, which is common for turbulent flows [63]. The low-pass filtered results accounted for almost the entirety of the area of the primary recirculation region and reattachment length.

The location of flow reversal, $u = 0$, for the low-pass filtered velocities on the wall-normal plane, $y/h = 0.25$, showed significant spanwise variation in each instantaneous filtered velocity field. Wavenumber spectrum of the separation line showed a peak in the spectrum concentrated near a wavelength of $\lambda_z \approx 2h$, consistent with other studies at much higher Reynolds

and Mach numbers and was not due to any kind of spanwise wall effects, as the geometry was homogeneous in the spanwise direction. Examination of the filtered three-dimensional velocity vector fields showed evidence of two counter-rotating vortex pairs that are formed by flow impingement and splitting on the step face at $x/h = 0$, which results in elevated levels of the $\overline{w'w'}$ component of the Reynolds stress along the step face. These large structures were balanced by smaller regions of counter-rotating flow at neighboring spanwise locations. These features are tied to the smaller-secondary recirculation zone found in the mean streamlines for flow over BFS geometries. These features wander back and forth in the spanwise direction which results in zero mean flow in the spanwise direction over time.

Chapter 6

Vortex-Shedding in the Backward-Facing Step

This chapter is an article that will soon be submitted to the AIAA Journal, co-authored by Stephen John Wilkins (Principal), Mahdi Hosseinali, and Joseph W. Hall.

6.1 Introduction

Turbulent flow over a BFS is commonly used as a benchmarking geometry, therefore has been investigated extensively using a variety of experimental techniques [7, 6, 31, 51, 32, 33, 34, 18, 35, 36, 1, 20, 37, 38, 58] and numerical simulations [39, 40, 4, 41, 42, 5, 43, 60]. The flow combines a number of interesting flow phenomena including a turbulent boundary layer upstream

of the step, the separated shear-layer and related recirculation region immediately downstream of the step, flow impingement on the lower wall, and the recovery region downstream of flow reattachment. The present investigation uses a Large Eddy Simulation (LES) to investigate both the time-resolved dynamics of the flow, and in particular to understand the instantaneous behaviour of both the field and wall pressure. Additionally, the current simulation is ran for a much longer run-time ($t = 157\text{sec}$) than is typically done for an numerical simulations so that the flow-field statistics can be properly converged. Numerical simulations currently remain the only method able to reliably capture the entire instantaneous pressure field, although experimental techniques to measure the field pressure are in development [2, 78, 79]. Understanding the pressure fluctuations associated with the passage of vortex structures in wall-bounded flows (such as a BFS) is important because the fluctuating pressures create time varying surface forces tied to the generation of acoustic dipole sources, that are known to be dominant in low Mach number flows [80, 81]. For example, these sources are the dominant contributor to the noise inside a vehicle.

Large-scale organized vortex-shedding from the BFS has been found to occur at normalized frequencies of $St \approx 0.06$ to $St \approx 0.09$ [7, 20, 12, 21, 82, 60]. Lee and Sung [7, 21] used wall-pressure measurements and wavelet transforms to study the organized large-scale vortex-shedding at $St \approx 0.065$ and found that these vortices contained the majority of the energy within the wall pressure fluctuations. The formation of these vortex-structures was

hypothesized to be connected to the lower frequency shear-layer oscillations caused by the growth and contraction of the primary recirculation bubble, resulting in a large-scale vortex being ejected from the shear-layer [21]. Hudy et al. [1] used both velocity field and fluctuating wall pressure measurements to study the organized vortex-shedding downstream of a two-dimensional, axisymmetric BFS geometry. Proper Orthogonal Decomposition (POD) and stochastic estimation (SE) were used to isolate the most energetic features of the turbulent flow to demonstrate that regions of vorticity within the flow associated with the formation of discrete vortex structures originating in the shear-layer were linked to the dominant pressure modes on the lower wall. Several investigations have highlighted the development of another large-scale vortex that grows in size at fixed streamwise location, before being convected downstream once it has grown large enough [1, 45]. Higher frequency, smaller-scale structures tied to shear-layer instabilities ($St \approx 0.2$) are also present in the separated shear-layer originating from the sharp corner of the step [35, 38, 65]. Low-frequency, large-scale bulk movements of the shear-layer are found to occur around $St \approx 0.01$, and have been attributed to a “flapping” or “pumping” [7, 12, 19, 20] of the shear-layer. The work presented in Chapter 5 used low-pass filtering of the velocity fields to demonstrate how spanwise variations in the recirculation zone dynamics results in large-scale movement of the shear-layer and instantaneous reattachment point.

The results of Chapter 4 showed that the peak in the spectra associated with vortex-shedding was especially strong outside of the recirculation zone,

while the flow upstream of reattachment tends to be dominated by the lower frequency dynamics of the recirculation bubbles [7, 20, 12, 19, 60]. This chapter aims to elucidate the role of large-scale vortex-shedding in the flow over a BFS independent of the low-frequency flapping, by investigating the band-pass (Fourier) filtered pressure and velocity fields in time. In addition to being one of the foundations of turbulence theory, Fourier based filters are commonly encountered in the study of acoustic pressure fields [27] and time-resolved velocities [26, 28]. The behavior of the structures here is examined using instantaneous pressure fields, fluctuating wall pressures, filtered velocity fields and streamlines. The role of the vortex-shedding, fully de-coupled from the flapping, on the wall-pressure caused by the flow over the step is examined and discussed.

6.2 Domain Description, Boundary Conditions, and Numerics

This investigation is focused on the turbulent flow over one-half of a symmetric double expansion BFS, with an expansion ratio of 1.2, as shown in Figure 6.1. The domain was discretized into approximately 1.3×10^6 elements, and the results did not show any significant changes when examined with a finer grid. The side walls were set to periodic boundary conditions to ensure the mean flow remained two-dimensional by having a sufficiently large aspect ratio. The upper boundary of the domain was set to be a sym-

metry plane, which is the same condition of the DNS of Le et al. [5] which studied the same geometry. The outlet plane at $x/h = 20$ is set to zero gradient as the flow is close to fully developed that far downstream, and there was no evidence of any flow returning into the domain. No-slip walls are employed on the wall upstream of the step, the step face, and the lower wall downstream of the step. The cell size normal to the wall is selected such that $y^+ < 1$ in order to avoid the use of any wall models. In order to preserve the fluctuating pressure within the domain, the inlet boundary condition ($x/h = -37.55$) uses the Divergence Free Synthetic Eddy Method proposed by Poletto et al. [29], which has been shown to significantly reduce any non-physical pressure fluctuations associated with traditional Synthetic Eddy Methods (SEM). The use of this inlet condition requires full distributions of the velocity, Reynolds stresses, and turbulent length scales at every location on the inlet face in order to generate the proper eddies. These values were obtained from the online DNS database of a turbulent boundary layer of Schlatter and Örlü [52]. The inlet length of $37.55h$ was selected to enable enough time for the synthetic turbulence to excite the flow, in addition to allowing the boundary layer to develop to match the thickness employed in both the DNS work by Le et al. [83] and the matching experimental work by Jovic and Driver [30].

The total run-time of the simulation was $t = 157sec$ (1.2×10^5 eddy turnover cycles), with a variable time step less than $\Delta t = 2 \times 10^{-5}$. A variable time-step ensured numerical stability by keeping the mean CFL number

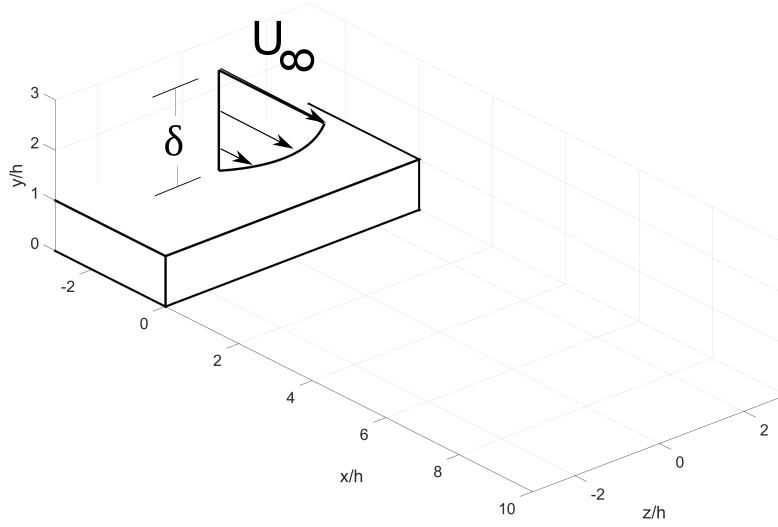


Figure 6.1: Backward Facing Step Geometry

below 0.2. The data was partitioned into 157 individual 1sec blocks prior to computing the spectra or applying any filtering operations. Each one-second block was comprised of 500 sampled instants, which resulted in 7.85×10^4 unique snapshots. A running time averaging was then performed by stepping through all the individual blocks.

The pimpleFoam (merged PISO-SIMPLE) algorithm [53, 54] in OpenFOAM[®] 1606 to perform the LES, and the eddy viscosity is computed by the WALE model [55]. The cube root of the cell volume was used to set the filter length for the LES, and second order discretization was employed for both the advection and diffusion terms. In order to maintain second order accuracy the time scheme used was second order backward.

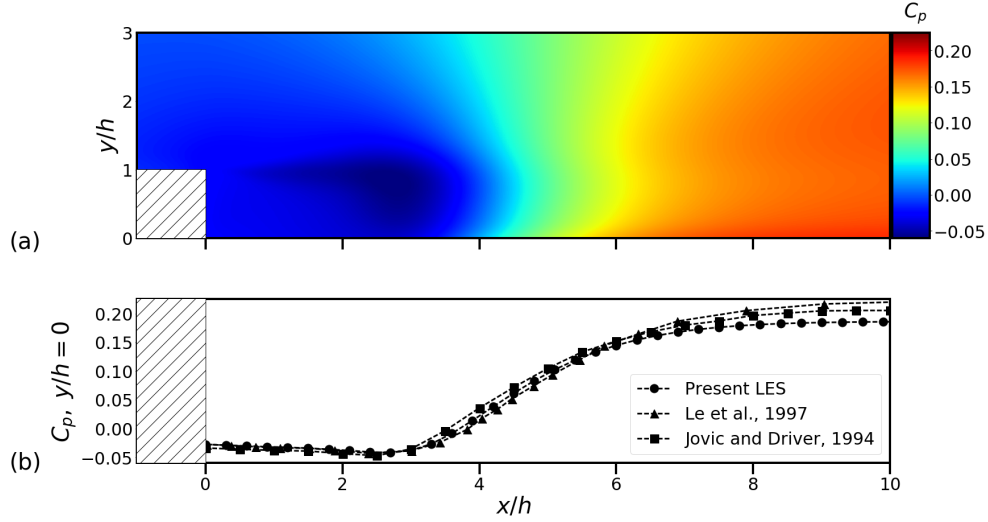


Figure 6.2: Mean Pressure Coefficient, C_p , for (a) Field Pressure and (b) Wall-Pressure, $y/h = 0$, at $z/h = 0$.

6.3 Results and Discussions

The mean pressure coefficient, $C_p = \frac{p-p_0}{\frac{1}{2}\rho U_\infty^2}$, on the $z/h = 0$ centre-plane is presented in Figure 6.2(a) and the pressure coefficient measured on the lower wall at $y/h = 0$ is shown in Figure 6.2. The results for the mean pressure field are in good agreement in both distribution and magnitude with the DNS results presented by Le et al. [5] for the same flow configuration. Excellent agreement in the pressure coefficient is obtained with both the experimental work by Jovic and Driver [30] and the numerical investigation of Le et al. [5].

Figure 6.3 presents wall-pressure spectra downstream of the step, with the largest spectral peak found around $St \approx 0.06 - 0.08$; this peak has traditionally been associated with large-scale vortex-shedding off the step

face [7, 20, 12, 21]. The results indicate that the peak in the spectra increases in frequency at subsequent downstream positions, as previously noted in Chapter 4. However, Chapter 4 only examined the pressure field and wall pressure statistics in a time-averaged sense, and were only able to speculate that the shift in frequency was related to the combined effects of flow accelerations and streamwise elongation of the vortex structures due to the high shear downstream of the step. In order to elucidate the behavior of the flow features occurring around those peak frequencies, the measurements are filtered using an 8th order Butterworth band-pass filter which allowed examination of a normalized frequency range of $0.044 < St < 0.086$ (highlighted by the vertical dashed lines in Figure 6.3). Using a high-order filter ensures a steep roll-off outside of the pass-band frequencies, and reduces the chance for aliasing or a loss of variance. [25, 26]. In order to prevent any phase distortion of the filtered results, the filtering operation is performed both forward and backward. This filter band was chosen to isolate the vortex-shedding by removing the flow features responsible for the other spectral peaks found near $St \approx 0.03$ and $St \approx 0.01$. A discussion of important time scales and frequency is included in Appendix B.

The impact of filtering on a single instant of both the field pressure, and wall pressure, respectively, can be observed in Figure 6.4. The unfiltered instantaneous pressure field normalized by the dynamic head, $q = 0.5\rho U_\infty^2$, is presented in Figure 6.4(a) and the normalized fluctuating pressure field, p'/q , for the same instant is presented in Figure 6.4(c). When the pressure field is

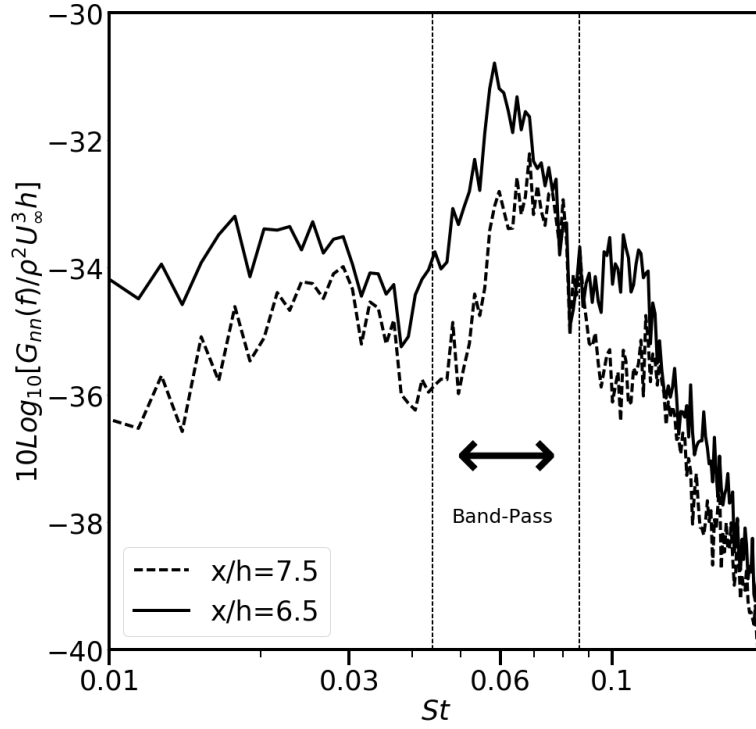


Figure 6.3: Fluctuating Wall-Pressure Spectra Measured at Various Stream-wise Locations. Filter Band-Pass Frequencies are Illustrated by the Vertical Dashed Lines.

passed through a low-pass ($St < 0.01$) filter as shown in Figure 6.4(b), the results closely resemble the mean pressure field shown in Figure 6.2, and are included here to simply demonstrate the reliability of the filtering operations. The band-pass filtered field and wall pressure results in Figure 6.4(d) and (e) capture many of the flow features present in the fluctuating results, particularly the larger pressure disturbances that persist further downstream. The relationship between the fluctuating pressure field above the lower wall and the corresponding wall pressure is evident in Figure 6.4(d) and (e), with the large, organized regions of negative pressure ($x/h \approx 5$) fluctuations within the field resulting in correspondingly large negative wall-pressure fluctuations beneath the structures. Similarly, the large regions of positive pressure fluctuations near $x/h \approx 8$ are also readily captured by the wall-pressure measurements. It is obvious from the results in Figure 6.4(d) and (e) that the fluctuating wall pressures are related to the pressure fluctuations within the flow field above the wall. This is consistent with the time-averaged results from Chapter 4, but also demonstrates that is true at an instant as well.

The fluctuating Root-Mean-Square (RMS) pressure fields for both the unfiltered and band-pass filtered data are shown in Figure 6.5. Filtering the data has altered both the distribution and magnitude of the pressure fluctuations significantly. As expected, the magnitude of the fluctuations has decreased due to the limited signal energy contained in the band-pass region, so the colorbar levels have been adjusted accordingly. The location of the largest magnitudes have been shifted downstream, which is related to the

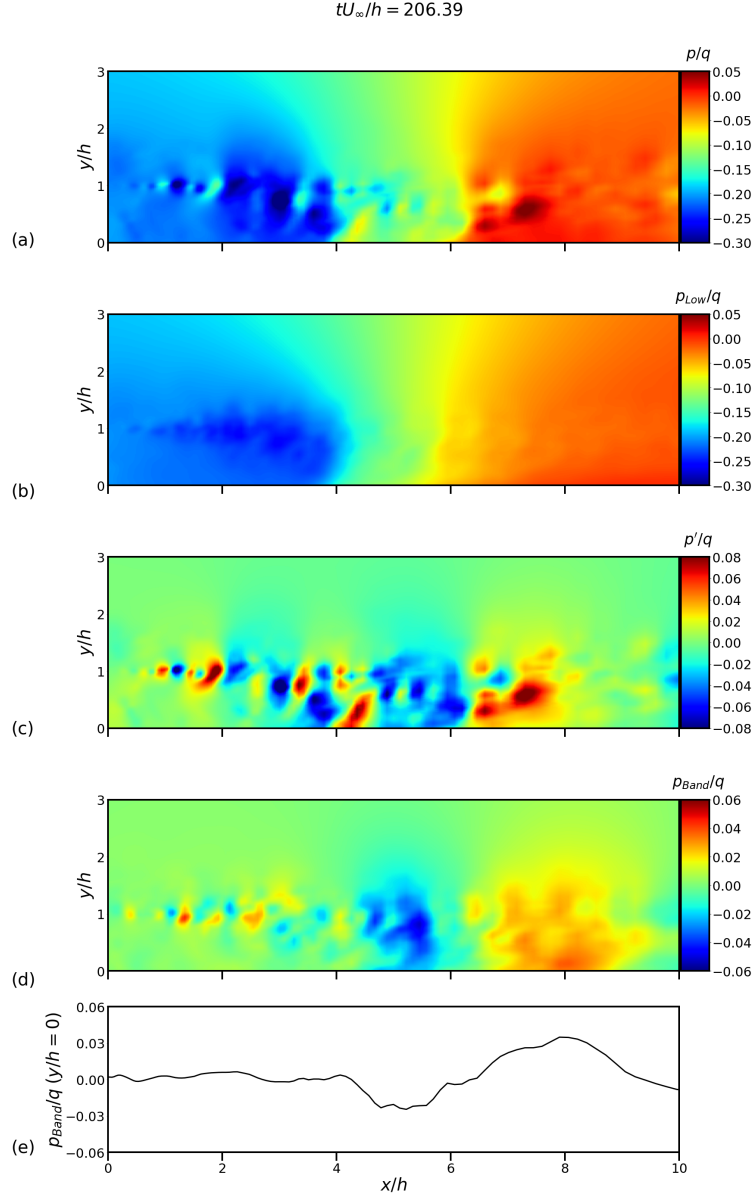


Figure 6.4: Various Properties at an Instant: (a) Unfiltered Field Pressure, (b) Low-Pass Filtered Field Pressure ($St < 0.01$), (c) Fluctuating Field Pressure, (d) Band-Pass Filtered Field Pressure, and (e) Band-Pass Filtered Wall Pressure at $z/h = 0$.

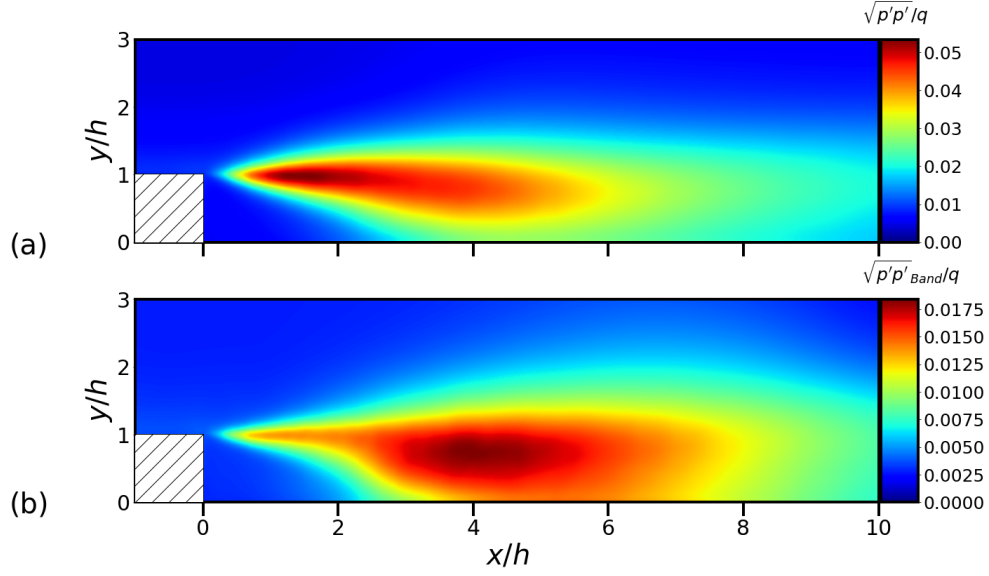


Figure 6.5: Normalized RMS Pressure, $\sqrt{p'p'}/q$, for (a) Unfiltered and (b) Band-Passed Filtered Pressure Fields at $z/h = 0$.

removal of the smaller-scale higher frequency fluctuations in the shear-layer that are present near the step.

Figure 6.6(a) displays a scalar plot of the band-pass filtered wall-normal component of velocity with the mean flow velocity added back on. Additionally, the plot is overlaid with the two-dimensional streamlines of $U_{Band} + \bar{U}$ and $V_{Band} + \bar{V}$. The streamlines indicate that there are two regions of flow rotating in the clockwise direction near $x/h \approx 2$ and $x/h \approx 5$, and these regions are bound on the left and right by negative and positive wall-normal velocities. As the mean wall-normal velocity is very low, the same regions of positive and negative fluctuating wall-normal velocity are clearly visible in the band-pass filtered wall-normal velocity fluctuations, as well, particularly

at $x/h \approx 2$. Flows with high mean shear (such as a BFS) normally have the mean removed during analysis as it can obscure the more localized shear associated with smaller-scale features; however, the subsequent loss of an inertial frame of reference can make interpretation of the corresponding results difficult. These results indicate that alternating regions of fluctuating wall-normal velocity are indicative of the rotational flow associated with organized vortex-shedding, as the mean flow does not have as large of an impact on the wall-normal velocity component. In Figure 6.6(c) the band-pass filtered pressure field clearly shows regions of low pressure associated with the core of those rotating structures at $x/h \approx 2$ and $x/h \approx 5$. It is normally very challenging to identify vortex structures through the use of pressure in turbulent three-dimensional cases [84], but the band-pass filtered results clearly yields much better insight into these features. The large negative pressure fluctuations near $x/h \approx 5$ are well captured in the wall-pressure measurements presented in Figure 6.6(d); however, the fluctuations observed near $x/h \approx 2$ away from the wall in the shear-layer are not well captured by the wall pressure. The fact that the wall-pressure fluctuations in this frequency range are connected to large-scale regions of rotating flow substantiate the estimation techniques of Hudy et al. [1] and all pressure based estimation as it proves that the wall pressure is a reliable means of estimating large-scale coherent structures in this flow.

The temporal evolution of a vortex-shedding cycle is examined using a sequence of images in Figure 6.7 of the instantaneous band-pass filtered

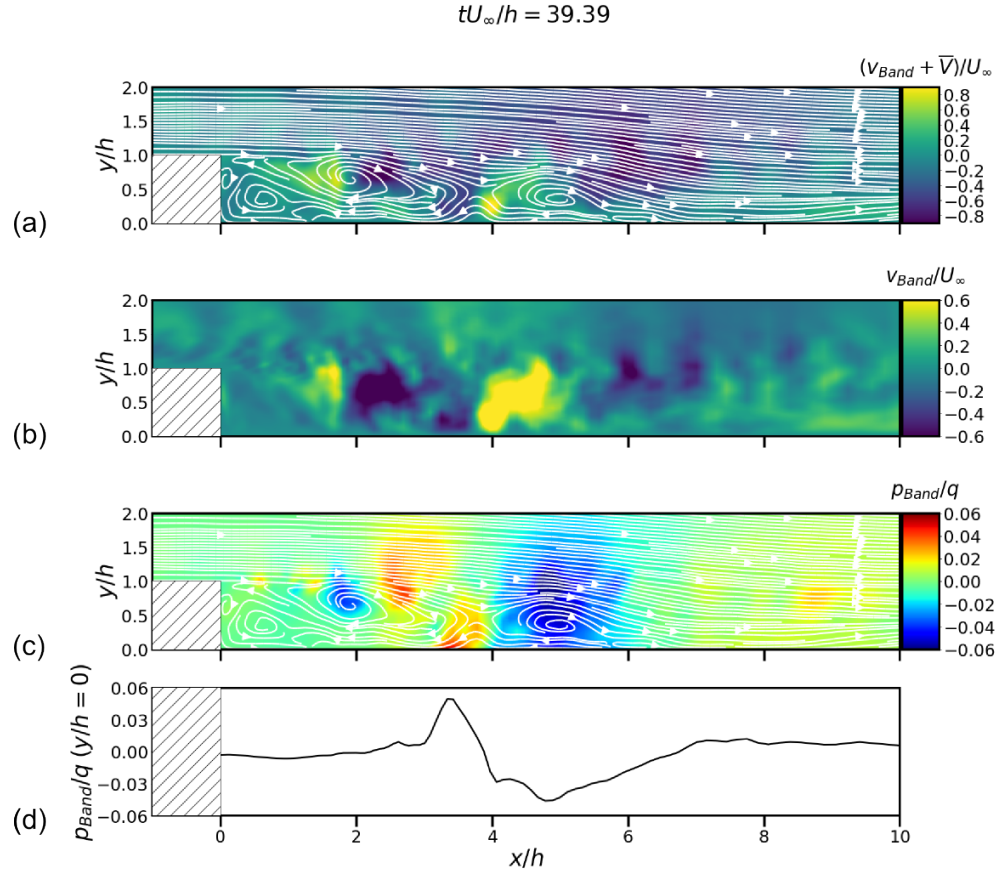


Figure 6.6: Instantaneous Normalized: (a) Band-Pass Filtered Wall-Normal Velocity with the Mean Added, (b) Band-Pass Filtered Velocity, (c) Band-Pass Filtered Pressure, and (d) Band-Pass Filtered Wall Pressure. Streamlines correspond to $U_{Band} + \bar{U}$ and $V_{Band} + \bar{V}$ at $z/h = 0$.

pressure fields. In order to make the connection between these pressure fluctuations and the velocity field, the streamlines of $U_{Band} + \bar{U}$ and $V_{Band} + \bar{V}$ are overlaid in white. Strong negative pressure fluctuations and clear clockwise rotation of the streamlines (both strong indicators of a vortex) are observed to be forming in the shear-layer originating off the step around $x/h \approx 2$. This structure convects downstream through the remaining instants, getting closer to the lower wall as it moves further downstream. Eventually in Figure 6.7(i) and (j) a second vortex is beginning to roll-up and repeat the process. Although the growth of the vortex makes it more challenging to see as the structures convect downstream, the streamwise displacement between subsequent images appears to be increasing, and this is particularly evident in the positive fluctuations around $x/h \approx 4 - 6$ between Figure 6.7(d) and (e). As the timing between images is constant, this indicates that the structures are undergoing acceleration in convective velocity as the mean velocity is increasing as the flow moves downstream.

Evidence of this streamwise elongation is shown in the band-pass filtered field and wall pressure in Figure 6.8 for two representative instants. The band-pass fluctuations in the field are significantly larger further downstream as indicated by the lines, and the streamwise footprint (*i.e* the length) of these features is significantly increased as well. The clear streamwise elongation of the pressure fluctuations within the field and the corresponding increase in the length of the wall-pressure footprint, combined with the increasing mean flow velocity with increasing distance from the recirculation region, causes

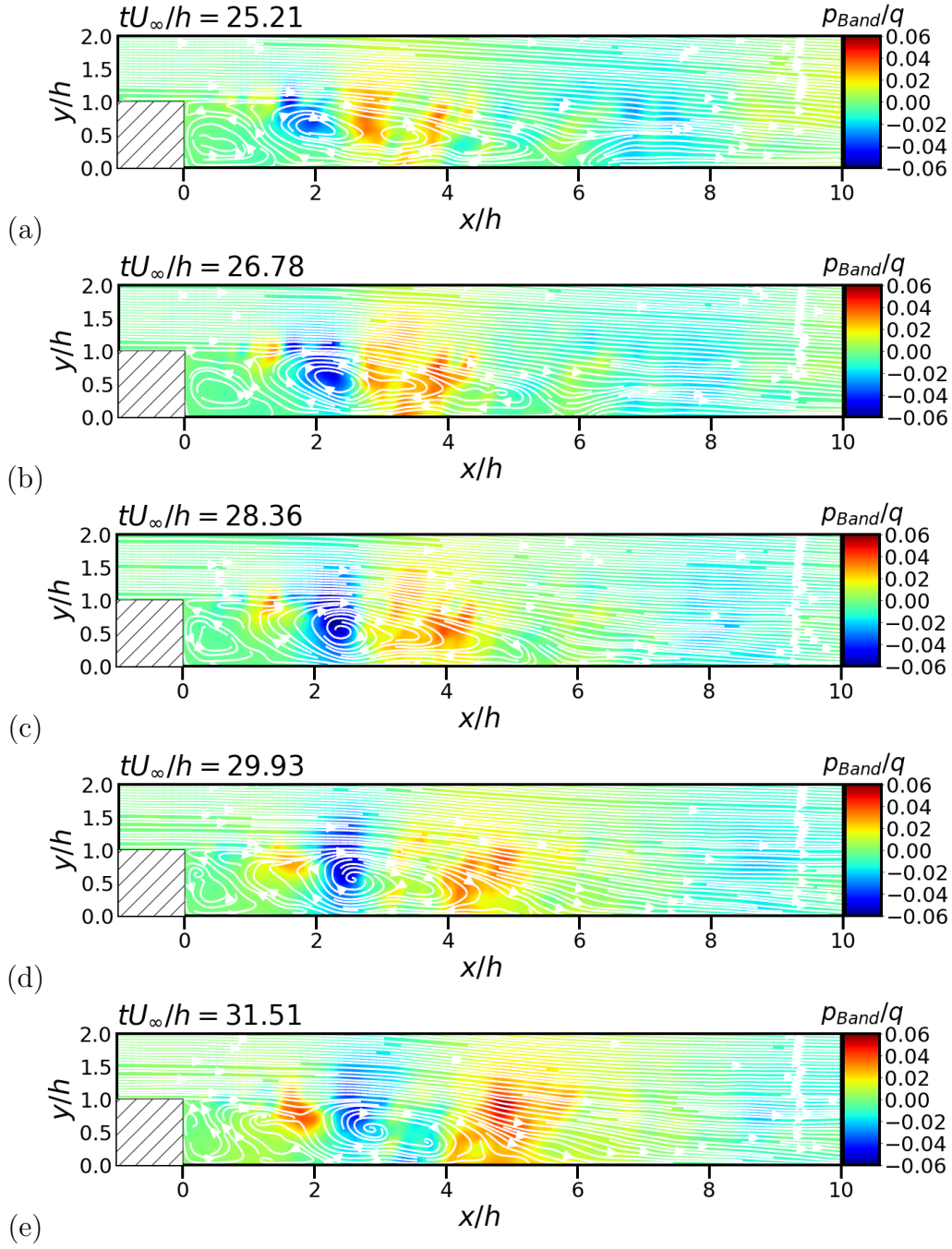


Figure 6.7: Sequence of Instantaneous Band-Pass Filtered Pressure Fields Illustrating the Development of Vortex Structures. Streamlines correspond to the Band-Pass Filtered Streamwise and Wall-Normal with the Mean Velocities Added at $z/h = 0$.

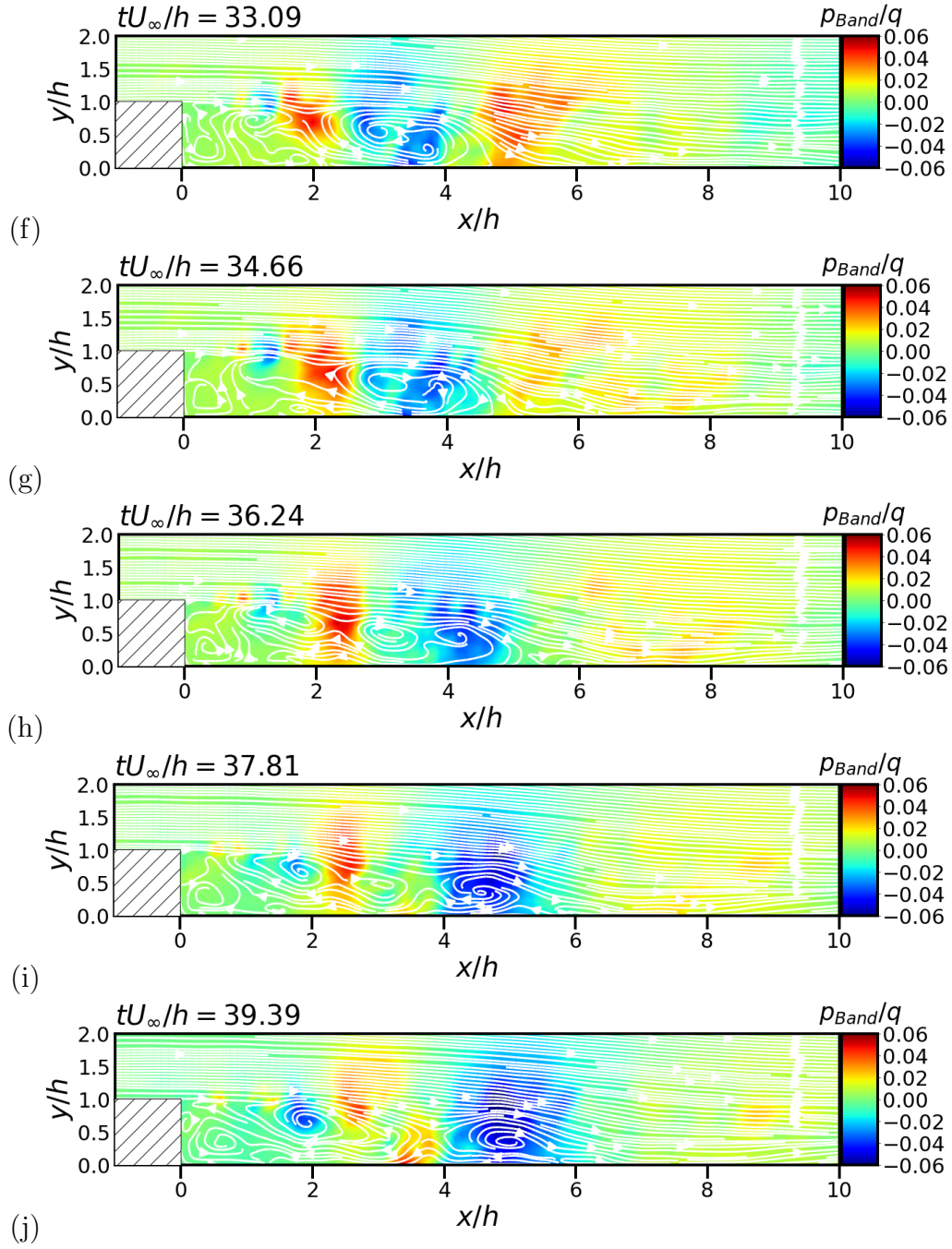


Figure 6.7: (cont.) Sequence of Instantaneous Band-Pass Filtered Pressure Fields Illustrating the Development of Vortex Structures. Streamlines correspond to the Band-Pass Filtered Streamwise and Wall-Normal with the Mean Velocities Added at $z/h = 0$.

the differences in peak frequency of the power spectra discussed in Chapter 4.

Extremely large pressure events are occasionally observed in close proximity to the lower wall, as shown in Figure 6.9(a)(i) near $x/h \approx 3.5$ and near $x/h \approx 4.5$ in Figure 6.9(b)(i). By examining the band-pass filtered vorticity, using the same method as the streamlines, it is apparent that the passage of the larger-scale structures associated with vortex-shedding can induce regions of counter-rotating flow near the lower wall that is tied to these sudden localized pressure fluctuations. Counter-clockwise vorticity is observed near the lower wall just past $x/h \approx 3.5$ in Figure 6.9(a)(ii) and just downstream of $x/h \approx 4.5$, the same locations of the sharp pressure increases. These structures close to the wall are likely formed to balance the vorticity of the vortex structures passing over the lower wall. The sudden formation of these counter-rotating structures so close to the lower wall can result in significant wall pressure fluctuations, as shown in Figure 6.9(a)(iii) and (b)(iii). As they are very close to the wall, these fluctuations become a dominant intermittent feature in the pressure signals, and thus a potentially strong intermittent source of noise [80]. Examination of the adverse pressure gradient (with the mean pressure included) indicated that the induced separation was not likely caused by a unsteady adverse pressure gradient (like that is known to occur in wall jets [85]).

Instantaneous pressure fields from two wall-normal planes at $y/h = 0.75$ and $y/h = 0$ are presented in Figure 6.10. These two planes were selected to highlight the impact that the fluctuating pressures above the lower wall at

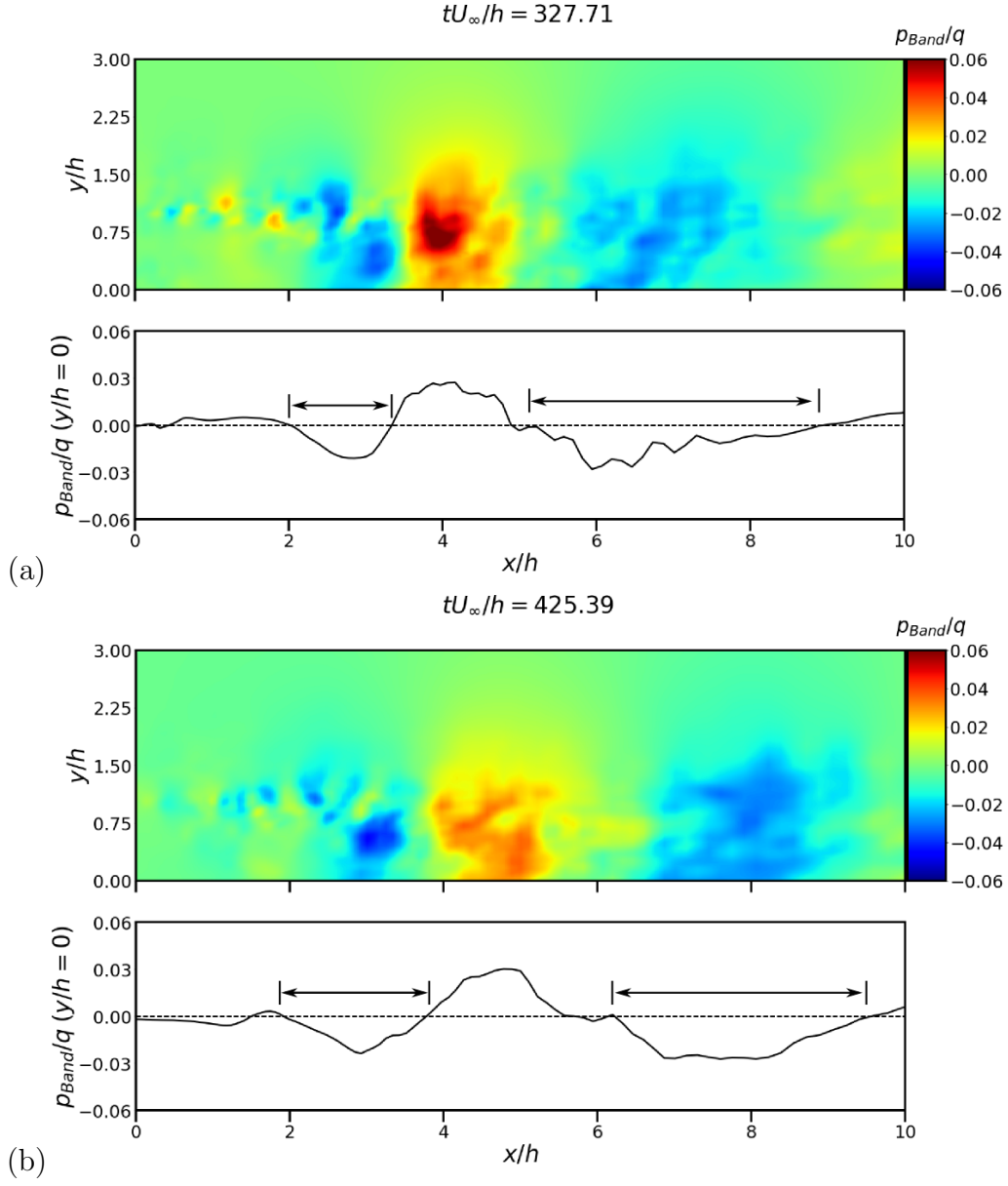


Figure 6.8: Instantaneous Band-Pass Filtered Field Pressure and Wall Pressure at $z/h = 0$.

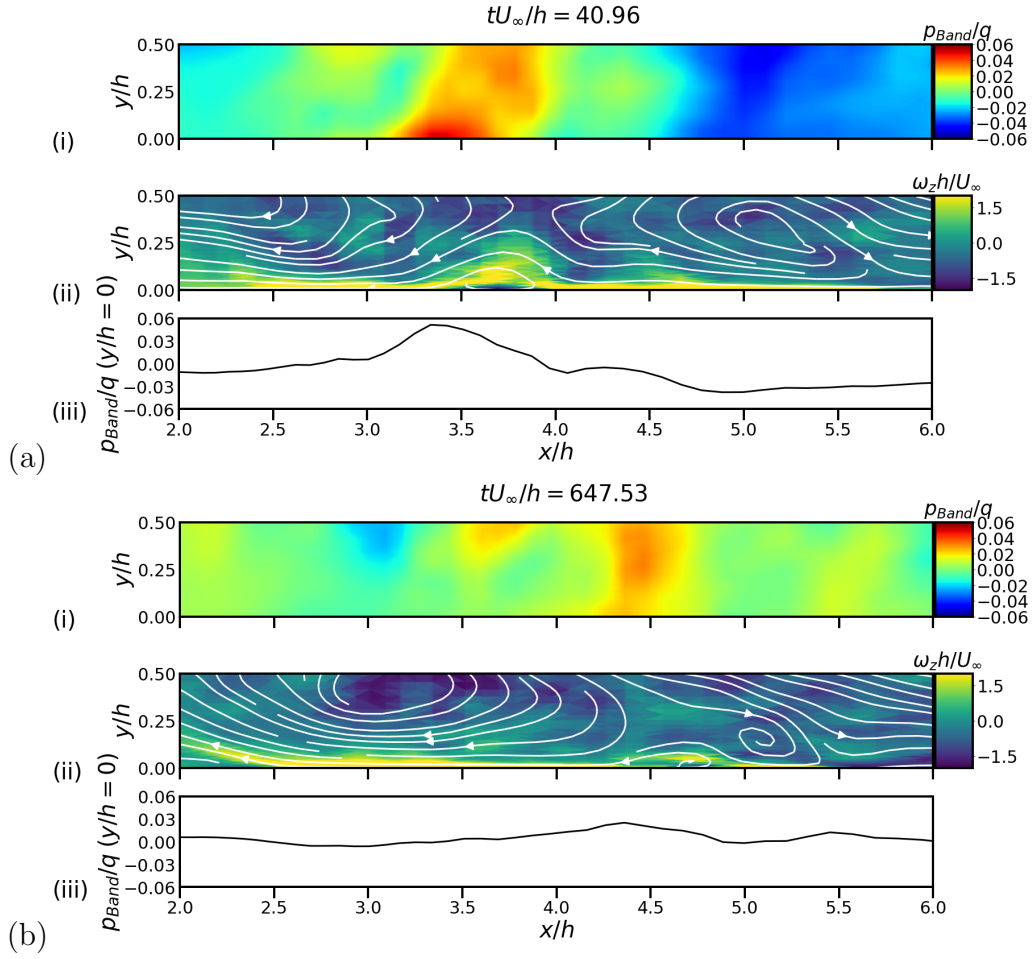


Figure 6.9: Instantaneous (i) Band-Pass Filtered Pressure Fields, (ii) Vorticity and Streamlines, and (iii) Band-Pass Filtered Wall Pressure for Two Separate Instants, (a) and (b), Taken at $z/h = 0$.

$y/h = 0.75$ have on the wall pressures recorded at $y/h = 0$. It is clear that there is large-scale spanwise organization associated with the vortex-shedding both on and away from the wall; the regions of negative pressure fluctuations are instantaneously on the order of 2–3 step heights in the spanwise direction, consistent with the results of Statnikov et al. [20] and the results presented in Chapter 5. This strong spanwise coherence means that these structures should be effective radiators of sound [80]. The wall pressure fluctuations are clearly linked with those fluctuations occurring above them, appearing as just a more orderly representation of the $y/h = 0.75$ fluctuations. The negative fluctuations near $x/h \approx 2$ in Figure 6.10 are not being observed near the lower wall as they remain far from the lower wall, as shown in the $x - y$ plane results in Figure 6.7(j). As the structures grow and convect downstream they move closer to the lower wall, which results in a greater footprint on the lower wall. Immediately downstream of the step, the vortex structures are physically much smaller and are far away from the wall, and their impact on the wall pressure is minimal. This is consistent with the wall pressure distributions presented and discussed in Figures 6.7 and 6.8.

6.4 Conclusions

In order to highlight the dynamics of large-scale (on the order of the step height) vortex-shedding, the time-resolved instantaneous pressure and velocity fields from a LES of turbulent flow over a BFS were examined using

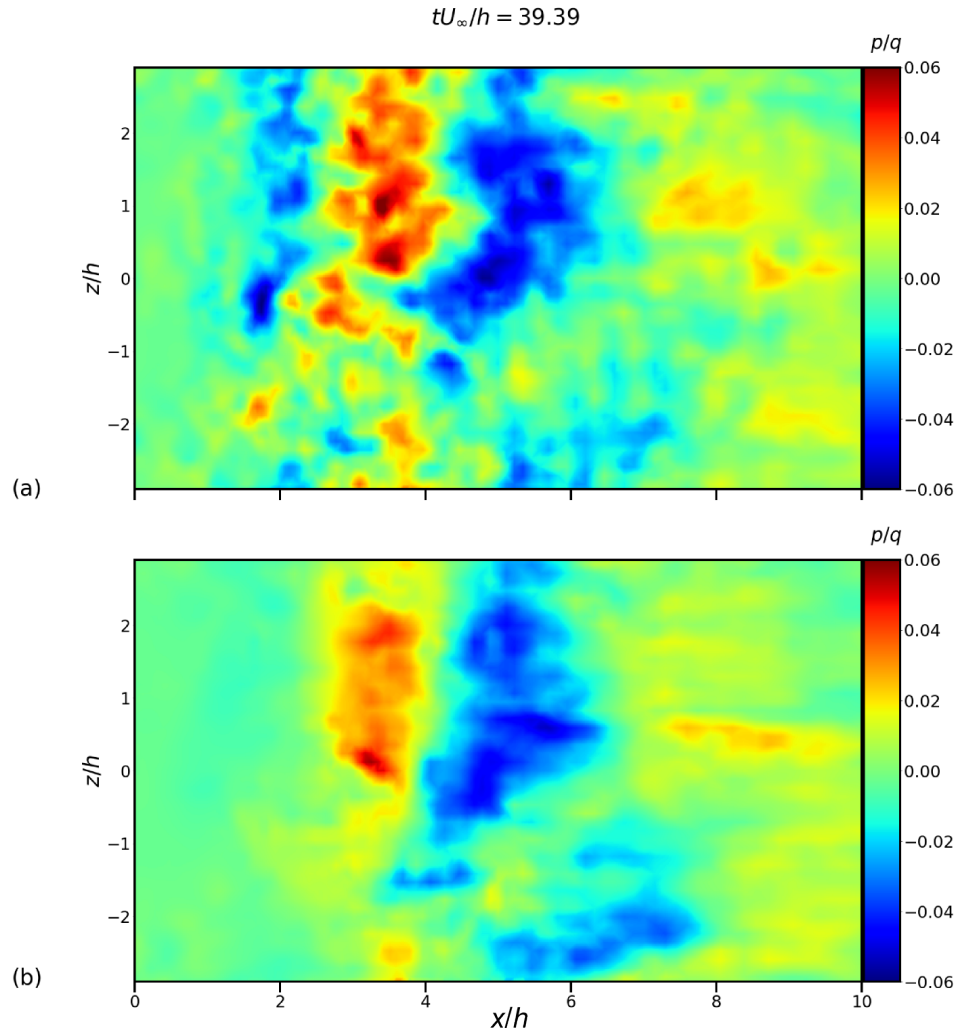


Figure 6.10: Instantaneous Band-Pass Filtered Pressure Fields at (a) $y/h = 0.75$ and (b) $y/h = 0$.

band-pass filtering in the range of $0.044 < St < 0.086$. This allowed the clearest investigation of the isolated dynamics of the vortex-shedding. Filtering in this band allowed the pressure fluctuations associated with vortex-shedding to be captured and their role highlighted. Investigation of the temporal behavior of the shedding showed vortex formation, growth and convection. It was shown that the pressure fluctuations remained organized further downstream.

The coherent regions of pressure fluctuations were shown to instantaneously undergo significant streamwise elongation as they convected downstream due to the high mean shear downstream of the step and this was also observed in the wall pressure results, as well. Acceleration of the vortex structures near the mean reattachment point was observed. The combined streamwise elongation and acceleration of these structures provides a good explanation of the shift in peak spectral frequency previously discussed in Chapter 4.

Significant spanwise organization of the large scale vortex structures on the order of 2-3 step heights was observed, further indicating that these structures would be a strong source of noise in this flow. When these structures pass close enough to the lower wall, they can also excite counter-rotating regions of flow very close to the wall itself. The sudden formation of these structures in such close proximity to the wall can strong pressure fluctuations in their vicinity and would certainly be strong contributors to aeroacoustic noise.

Chapter 7

Conclusions and Recommendations

7.1 Summary of Results

Turbulent flow over a BFS at $Re_h = 5100$ has been examined using a long run-time LES ($t = 157sec$) in order to better understand the underlying organized physical motions. The statistical results using the full data were well validated against previous experimental and numerical studies, in terms of both pressure and velocity statistics, in both time and frequency domains. Although experimental examinations often struggle to resolve low frequency features do the frequency response limitations of most available sensors, and numerical studies typically do not have enough temporal data to accurately compute the statistics of these low frequency features, the long run-time em-

ployed here allowed accurate and detailed investigations of the low frequency oscillations and the mechanisms responsible for them that have long been debated within the BFS literature. Due to the extensive amount of high resolution spatial and temporal data available within the results from the simulation thorough investigation of many higher order statistical moments and two-point, two-time correlations were able to be computed with a high degree of accuracy. The most important conclusions from each results chapter will be summarized here.

7.1.1 Wall Pressure, Field Pressure, and Velocity Field Statistics

The measured field and wall pressure spectra within the database highlighted the existence of numerous organized flow features downstream of the step. The wall pressure spectra showed peaks that were in good agreement with previous investigations and captured the low frequency “flapping” or “pumping” ($St \approx 0.01$), large scale organized vortex-shedding $St \approx 0.06 - 0.09$, as well as potential combinations of the two mechanism, $St \approx 0.03$. It was demonstrated that the probe locations on the lower wall downstream of the step were generally well suited to capturing the nature of the pressure fluctuations within the field above the lower wall, at all but the highest frequencies. This was attributed to the fact that the smaller-scale, high frequency fluctuations tied to the instability of shear-layer structures were

being obscured by the larger-scale fluctuations that were occurring in closer proximity to lower wall. Although it has been generally accepted that wall-pressure measurements are able to capture the pressure fluctuations away from the wall, confirmation of that had been challenging due to the difficulty of determining the instantaneous pressure field within the domain. Due to the preservation of the pressure fluctuations within this simulation, the relationship between field and wall-pressure fluctuations has been demonstrated for the larger scale structures that produce large pressure footprints on the wall.

The peak frequency in the spectra was found to depend heavily on the streamwise location of the measurement probe. Upstream of reattachment at $x/h = 4.5$ the peak frequency was $St \approx 0.01$ and was dominated by the low-frequency dynamics of the reattachment and recirculation zone. Just downstream of reattachment, $x/h = 6.5$, the peak frequency was found to occur around $St \approx 0.06$, but this peak frequency had increased to $St \approx 0.09$. Many studies have shown significant variation in their peak spectral frequency, which is likely tied to the different streamwise locations of their respective measurements. The variation in spectral measurements based on streamwise location is a critical point to consider when designing or evaluating LSE investigations, as those results are likely to benefit from effective sensor placement.

In order to examine the frequency shift of the spectral peaks downstream of reattachment in greater detail, multi-point statistics were em-

ployed, including cross-spectral magnitude and phase (convective velocity), cross-correlation, and coherence. The peak associated with the shedding off the step occurring around $St \approx 0.06 - 0.09$ demonstrated significant coherence, indicating significant organization within the mechanism. The convective velocity of the structures was found to increase as the flow moved downstream. Previous researchers attributed this increase in velocity to the fact that the flow velocity is increasing as it moves downstream. By examining both the change in spectral peak and convective velocity in this examination, it was observed that the increase in convective velocity of approximately 33% was accompanied by a much smaller increase in peak frequency of only 7 – 11%. If the increase in convective velocity was tied only to flow acceleration, the peak frequency would increase at an equivalent rate. The fact that the peak frequency did not increase by an equivalent percentage indicates that it is likely undergoing an elongation of the structure by the strong shear downstream of the step. The distortion of the vortex structures is also a key feature in terms of assessing the SE results from a given step, as it would be important to capture this deformation of the structures in order to provide accurate estimations. Single-point, single-time estimation techniques will likely have a hard time resolving this phenomena as they take only a single velocity snapshot into account when building the correlations, and the variation in the physical dimension of the structures will likely be averaged out over the ensemble of snapshots.

7.1.2 Low-Pass Filtered Velocity Fields, $St < 0.016$

The low frequency “flapping” downstream of the step has long been debated within the literature. In order to clarify the flow mechanisms responsible for the spectral peak near $St \approx 0.01$, the velocity fields at various streamwise planes from $z/h = -2$ to $z/h = 2$, and a plane just above the lower wall at $y/h = 0.25$, are low-pass filtered using a 8th-order digital Butterworth filter with a non-dimensional cut-off frequency of $St < 0.016$. Examination of the Reynolds stress distributions downstream of the step showed that the filtering operation had significantly reduced the magnitude of the fluctuations, and altered their distribution. The low-pass filtered results showed that the majority of the fluctuations were concentrated within the primary and secondary recirculation zones instead of the shear-layer, as was the case for the unfiltered results.

Instantaneous snapshots of the streamlines, streamwise velocity and skin friction coefficient, C_f , for the low-pass filtered results were able to identify a single location for instantaneous reattachment and were found to capture almost the entirety of the primary recirculation bubble, indicating that the lowest frequencies encountered within the step flow are tied to the unsteadiness in the recirculation zones and are largely responsible for setting the instantaneous location of flow reattachment on the lower wall. The unfiltered results did not generally allow for instantaneous identification of a single reattachment point and had to rely on time averaged values for determining the mean location of reattachment. The various methods employed for identification of

reattachment used in other examinations are likely to play a significant role in the conclusions drawn from those results. The low-frequency behavior is a prime candidate for flow-control applications in order to adjust the reattachment point downstream of the step. As such, it would be critical for any SE investigations to be able to replicate this behavior. If the reattachment point is the key feature one is seeking to control, then the sensor location and quantity can be adjusted to give the best estimates of the low-frequency recirculation zone dynamics.

As the location of $u = 0$ could readily be found in the low-pass filtered results, the spanwise variation in the streamwise reattachment could be extracted from the results just above the lower wall at $y/h = 0.25$ for the first time. A spanwise line of reattachment was extracted from a large number of independent instants and used to determine a spanwise wavenumber spectrum. This novel approach to examining the spanwise dynamics demonstrated that the spanwise flow exhibited a peak in the spectrum near $\lambda_z \approx 2h$, which was consistent with previous researchers that employed other methods to estimate the wavelength. This wavelength appears to be Reynolds and Mach number independent, at least for studied turbulent cases in the range of $5100 < Re < 1.8 \times 10^5$. The fact that the flow showed instantaneous spanwise variation, indicates that SE applications would benefit from having a grid of sensors spread out in both the streamwise and spanwise directions to adequately capture the instantaneous three-dimensionality of the flow.

Investigation of sequences of the time resolved data from multiple planes

showed that the $\lambda_z \approx 2h$ wavelength was tied to the formation of two counter-rotating regions of velocity that were formed when the flow in the primary recirculation zone was driven all the way back into the step face. Upon impinging on the step face the flow splits, and is diverted side-to-side along the step face. This produces not only ω_z vorticity, but also ω_y vorticity normal to the wall. These two structures were found to grow and shrink in opposition to one another, and their spanwise location was found to sweep back and forth, and covered regions of approximately $2h$, which was consistent with the results obtained from other measures. The side-to-side sweeping of the flow was also found to occur downstream of the instantaneous location of flow reattachment, resulting in elevated levels of the $\overline{w'w'}$ component of the Reynolds stresses in that region. The low-frequency dynamics downstream of the step are much more complicated than a simple “flapping” of a shear-layer! The ability of SE to capture these key dynamics would be an important tool in assessing the performance of the estimation.

7.1.3 Band-Pass Filtered Flow Fields, $0.044 < St < 0.086$

To elucidate the role of large-scale organized vortex-shedding that is known to persist in the BFS flow, the pressure and velocity fields are band-pass filtered using an 8th-order digital Butterworth filter with a pass band of $0.045 < St < 0.085$ to better understand the dynamics and contributions

of vortex-shedding, independent of the low-frequency behavior. Significant spectral energy was found in this frequency range, and the filtered RMS pressure fluctuations were shown to be significant. The band-pass pressure fluctuations were also found to persist further downstream than the unfiltered results.

Large-scale (on the order of the step height) structures were observed in both the band-pass filtered velocity and pressure fields and regions of strong negative pressure fluctuations were often found coincident with regions of high flow rotation, evidence of the passage of low-pressure vortex cores. These organized regions of pressure fluctuations were found to increase in size dramatically as they convect downstream, as well as evidence of increasing convective velocity. The resulting wall-pressure footprints beneath these features showed clear elongation in the streamwise direction. The combined effect of elongation and acceleration is consistent with the spectral results discussed in Chapter 4. The fact that the instantaneous band-pass filtered pressure fluctuations within the domain were extremely well captured by the wall-pressure measurements clearly demonstrates the effectiveness of wall-pressure measurements representing the key pressure features above the wall.

The band-pass filtered wall pressure fluctuations were found to represent most of the large-scale pressure fluctuations extremely well, particularly when these features passed in close proximity to the lower wall. However, smaller features found near the step and associated with the initial formation of these

vortex structures were not captured in the filtered wall pressure, as they are simply too far away from the wall to have an appreciable footprint. The passage of the large-scale features in close proximity to the lower wall were occasionally found to induce instantaneous and localized flow separation off the lower wall. This highly unsteady flow feature in such close proximity to the lower wall results in abrupt changes in the field and wall-pressure in those areas, and would likely contribute significantly to aeroacoustic noise generation due to the formation of strong acoustic dipole sources.

7.2 Future Work

The original motivation of this thesis was to examine the impact of experimental sensor configurations on SE investigations by allowing easy and simple adjustments to probe locations, type and quantity. In order to achieve this, a well understood benchmark flow was necessary. Unfortunately it was soon discovered that some of the important mechanisms underlying flow over a BFS were still the subject of great debate within the literature. As such, it was necessary to take a step back from the original goal and spend some further effort towards determining the key features of turbulent flow over a BFS that would be the logical targets for investigation using either LSE or POD. This numerical database and the results presented in this work have provided ample evidence and analysis of two significant organized flow mechanisms: (1) low-frequency recirculation zone dynamics around $St \approx 0.01$ and

(2) large-scale vortex-shedding around $St \approx 0.06$. With the ever expanding abilities to capture and analyze turbulent flow data and with this improved understanding of the low-frequency recirculation zone dynamics as well as the vortex-shedding phenomena, the ability of the complementary techniques to adequately capture these flow features will hopefully continue to develop. Proper estimations can significantly improve the viability of accurately controlling flow with active means.

The next logical step for this work is to begin to test the effectiveness of SE and POD to capture the low-frequency recirculation zone behaviors, as well as the vortex-shedding phenomena. The ability of SE to capture the low-frequency behaviors can be examined by filtering the velocity fields using Fourier-based filters prior to the calculation of the correlations, and similarly by using the band-pass filtered velocities to isolate the vortex-shedding. It is imperative that the SE be able to reliably capture these flow features, as they are the most likely targets for flow-control.

In order to determine the effectiveness of the estimations to capture the flow field dynamics, the filtered velocity fields can be down-sampled to replicate the acquisition rates of typical PIV experiments, then used to build the correlations necessary for the SE. The effectiveness of the SE to adequately capture the fields used to build the correlations can be examined, a so-called *training set*. The impact of sensor type, location and quantities on the estimation accuracy can be studied, and guidelines established. If the estimates can be shown to be effective at replicating the velocity fields from

which the correlations were built on, the ability for SE to replicate new data can be observed by using the data taken from instants not employed in the original training set.

Additionally, the interpretations and relations between POD modes and actual physical features can be examined by looking at the time variation of the POD coefficients. Spectra of coefficients could be used to link them to the spectral features measured in this examination to provide some physical basis for the mode shapes produced by the POD algorithm. Reconstructions employing modes that were shown to occur at critical frequencies could then be used to examine those flow features and see if they are consistent with the known mechanisms that occur at those same frequencies.

It would also be interesting to examine the spanwise dynamics of the flow using a wider computational domain. Although these results satisfied the general recommendation of twice the size of the spanwise structures, it would be interesting to see if a wider spanwise domain would show any larger features within the pressure field, which tends to be more low-dimensional than velocity and might be more organized.

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Appendix A

Overview of Stochastic Estimation

A.1 Stochastic Estimation Techniques

The techniques of conditional averaging and stochastic estimation were first applied to the field of turbulence by Adrian [86] as a means to study the underlying structures of turbulent flows. The fundamental goal of employing these techniques is to develop estimation coefficients that can use the results from point sensors to predict the resulting fluctuating flow field over the entire region of interest. The fluctuating velocity components $u(x, y, l)$ and $v(x, y, l)$, corresponding to a two-dimensional vector field (x, y) and the corresponding vector field snapshot number, l , are defined as:

$$u'(x, y, l) = u(x, y, l) - \langle U(x, y) \rangle \quad (\text{A.1})$$

and

$$v'(x, y, l) = v(x, y, l) - \langle V(x, y) \rangle \quad (\text{A.2})$$

where u and u are the instantaneous velocity vectors, and *ensemble* averaging over all statistically independent snapshots is indicated by $\langle \rangle$. In this study, the time-resolved two-dimensional (x, y) *fluctuating* velocity field, $u_{est}(x, y)$ and $v_{est}(x, y)$, will be estimated using N individual surface pressure sensors measuring the fluctuating surface pressures, p , defined as:

$$p'(n, t) = p_n(t) - \overline{p_n} \quad (\text{A.3})$$

where $p(n, t)$ is the instantaneous pressure reading at position n and time t , and the over-line denotes *time* averaging. Figure A.1 illustrates a typical generalized experimental setup where the fluctuating velocity components, $u'(x, y)$ and $v'(x, y)$, have been taken from a velocity field snapshot and the fluctuating pressure readings have been recorded at the same instant as the snapshot.

Using a Taylor Series expansion and retaining only the linear terms (hence *Linear* Stochastic Estimation), the fluctuating velocity can be estimated at each individual (x, y) location by:

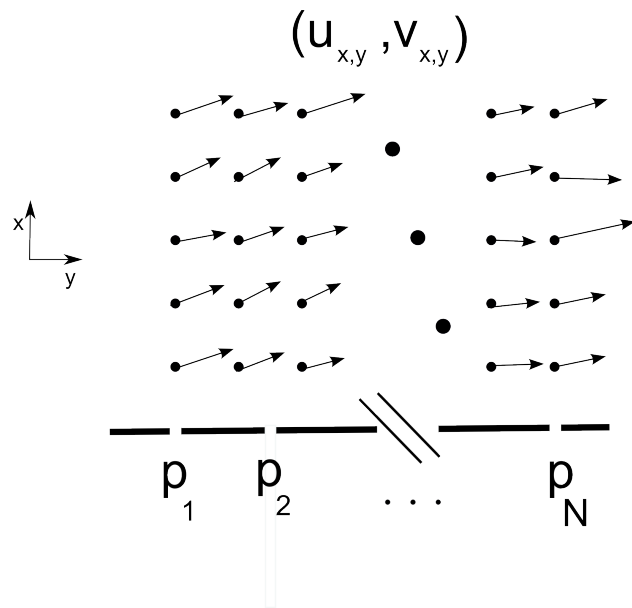


Figure A.1: General Stochastic Estimation Configuration for Velocity Field and Surface Pressure Measurements

$$u_{est}(x, y, t) = \sum_{n=1}^N Ax_n(x, y)p'_n(t) \quad (\text{A.4})$$

$$v_{est}(x, y, t) = \sum_{n=1}^N Ay_n(x, y)p'_n(t) \quad (\text{A.5})$$

where the unknown coefficients for each sensor are labelled $Ax_n(x, y)$ and $Ay_n(x, y)$ and are calculated at each individual (x, y) location. There has been interest in studying the effects of higher order terms upon the estimations, but simply increasing the order of the expansion does not guarantee any improvements in the estimates [87, 88]. Other stochastic estimation approaches include the effect of pressure signals that occur within a certain period of time at from when the snapshot was taken, namely *multi-time delay* linear stochastic estimation [89, 90] and *spectral* linear stochastic estimation [50, 46]. The greatest advantages of these techniques is the fact that the pressure-velocity correlations might be significantly improved at a certain time delay, τ . The incorporation of additional time-resolved information from the point sensors preserves information regarding the *phase angle* between the readings [90, 50]. The mathematics behind the techniques are significantly more involved, and their detailed discussion is out of the scope of the current work, which is limited to the linear estimates.

The goal is to find coefficients that minimize the mean squared error between the estimate the actual fluctuating velocity field recorded in each independent vector field. Defining the squared error between estimate and

actual fluctuating vector field for a single snapshot l recorded at time t^l for the simple case of $N = 2$ pressure sensors as:

$$E_{LSE,x}(x, y, l) = (u'_{est}(x, y, t^l) - u'(x, y, l))^2 \quad (\text{A.6})$$

$$E_{LSE,x}(x, y, l) = (Ax_1(x, y)p'_1(t^l) + Ax_2(x, y)p'_2(t^l) - u'(x, y, l))^2 \quad (\text{A.7})$$

$$E_{LSE,y}(x, y, l) = (v'_{est}(x, y, t^l) - v'(x, y, l))^2 \quad (\text{A.8})$$

$$E_{LSE,y}(x, y, l) = (Ay_1(x, y)p'_1(t^l) + Ay_2(x, y)p'_2(t^l) - v'(x, y, l))^2 \quad (\text{A.9})$$

then taking the partial derivative of the ensemble-mean squared error with respect to each unknown coefficient and setting it equal to zero yields the following system of equations:

$$0 = Ax_1(x, y) \langle p'_1 p'_1 \rangle + Ax_2(x, y) \langle p'_1 p'_2 \rangle - \langle u'(x, y) p'_1 \rangle \quad (\text{A.10})$$

$$0 = Ax_1(x, y) \langle p'_1 p'_2 \rangle + Ax_2(x, y) \langle p'_2 p'_2 \rangle - \langle u'(x, y) p'_2 \rangle \quad (\text{A.11})$$

$$0 = Ay_1(x, y) \langle p'_1 p'_1 \rangle + Ay_2(x, y) \langle p'_1 p'_2 \rangle - \langle v'(x, y) p'_1 \rangle \quad (\text{A.12})$$

$$0 = Ay_1(x, y) \langle p'_1 p'_2 \rangle + Ay_2(x, y) \langle p'_2 p'_2 \rangle - \langle v'(x, y) p'_2 \rangle \quad (\text{A.13})$$

Putting the equations into matrix form yields the following matrices:

$$\begin{bmatrix} \langle p'_1 p'_1 \rangle & \langle p'_1 p'_2 \rangle \\ \langle p'_2 p'_1 \rangle & \langle p'_2 p'_2 \rangle \end{bmatrix} \begin{bmatrix} Ax_1(x, y) \\ Ax_2(x, y) \end{bmatrix} = \begin{bmatrix} \langle u'(x, y) p'_1 \rangle \\ \langle u'(x, y) p'_2 \rangle \end{bmatrix}$$

and

$$\begin{bmatrix} \langle p'_1 p'_1 \rangle & \langle p'_1 p'_2 \rangle \\ \langle p'_2 p'_1 \rangle & \langle p'_2 p'_2 \rangle \end{bmatrix} \begin{bmatrix} Ay_1(x, y) \\ Ay_2(x, y) \end{bmatrix} = \begin{bmatrix} \langle v'(x, y) p'_1 \rangle \\ \langle v'(x, y) p'_2 \rangle \end{bmatrix}$$

The solution for the unknown coefficients depends on both the averaged fluctuating pressure-pressure correlations, $\langle pp \rangle$, and the averaged pressure-velocity correlations $\langle pu \rangle$ and $\langle pv \rangle$ at each (x, y) location. That means these systems of equations must be solved M times, where M is the total number of vector positions in the snapshot. Expanding the analysis to include N point sensors, the matrices can be determined as follows:

$$\begin{bmatrix} \langle p'_1 p'_1 \rangle & \cdots & \langle p'_1 p'_N \rangle \\ \vdots & \ddots & \vdots \\ \langle p'_N p'_1 \rangle & \cdots & \langle p'_N p'_N \rangle \end{bmatrix} \begin{bmatrix} Ax_1(x, y) \\ \vdots \\ Ax_N(x, y) \end{bmatrix} = \begin{bmatrix} \langle u'(x, y) p'_1 \rangle \\ \vdots \\ \langle u'(x, y) p'_N \rangle \end{bmatrix}$$

and

$$\begin{bmatrix} \langle p'_1 p'_1 \rangle & \cdots & \langle p'_1 p'_N \rangle \\ \vdots & \ddots & \vdots \\ \langle p'_N p'_1 \rangle & \cdots & \langle p'_N p'_N \rangle \end{bmatrix} \begin{bmatrix} Ay_1(x, y) \\ \vdots \\ Ay_N(x, y) \end{bmatrix} = \begin{bmatrix} \langle v'(x, y) p'_1 \rangle \\ \vdots \\ \langle v'(x, y) p'_N \rangle \end{bmatrix}$$

One of the first applications of linear stochastic estimation in fluid mechanics was the study of homogeneous shear flows by Adrian and Moin [91], which illustrated the presence of hairpin vortices and first demonstrated the ability of linear stochastic estimation to resolve and elucidate the presence of turbulent structures. A study of the mixing layer of a jet was performed by Cole et al. [3] and they determined that the number of point measures and their respective location was “critical” [3]. Unfortunately, more recent studies have neglected this fact and pay little to no attention to the selection and positioning of the sensors. The quantification of estimation error is an important consideration and was also investigated by Cole et al. [3] and Brereton [88], with the latter investigating the impact of including higher order terms within the Taylor series. Brereton [88] found that merely increasing the number of terms of the Taylor series did not guarantee an increase in estimation performance; it depended on the statistical distribution of the velocity fluctuations. Strong positive or negative skewness can actually cause

the higher order estimates to perform worse than the linear case [88]. Shortly after the first applications of linear stochastic estimation, methods to improve its performance were being investigated. One of the earliest additions was that of *Proper Orthogonal Decomposition*, a complementary technique that also made use of the fluctuating velocity components.

A.2 Complementary Technique: Proper Orthogonal Decomposition (POD)

In order to improve the estimates, Bonnet et al. [48] suggested combining the stochastic estimations with a method for extracting the dominant features of a turbulent flow field known as POD, originally proposed by Lumley [66]. Although all features and scales of a turbulent flow contribute to the flow field as a whole, in terms flow control, it would be ideal to be able to identify the major physical features of the flow and then to pass only these dominant features on to the estimates. This could potentially improve the pressure-velocity correlations by removing non-essential information from the velocity fields, which could lead to an improvement in the estimates. Primarily used in the study of time-resolved data commonly in the form of hot-wire or pressure measurements at relatively few positions, it is known today as *classical POD*. A method introduced by Sirovich [68] proved that the eigenvalue problem can be reduced from an $M \times M$ problem (again where M is the number of vector positions), to that of an $L \times L$ problem where L is the number of

snapshots. Normally, the number of snapshots is much smaller than the number of measurement locations from the velocity fields, so that $L \ll M$, reducing the required number of computations greatly. Entitled the *snapshot* method or *snaphot* POD, it is commonly used today on high spatial resolution data. The basis of all POD techniques involves decomposing the vector fields into a series of L *modes* that can be considered to be representative features of the flow. The resulting modes are arranged in descending order such that the modes with the highest portion of the turbulent kinetic energy occur first. These modes can then be used to reconstruct the fluctuating velocity field; if all modes are employed, the original vector field will be reconstructed. Typically, a smaller number of modes is selected to effectively filter the vector field based on turbulent kinetic energy, thereby retaining the features of the flow that contain the greatest contributions to the turbulent kinetic energy and removing the features of the flow that possess little turbulent kinetic energy. The procedure for snapshot POD is outlined below:

The autocovariance matrix, R , is composed of the all the fluctuating

velocity components organized vectorially as follows:

$$R = \begin{bmatrix} u_1'^1 & u_1'^2 & \cdots & u_1'^L \\ \vdots & \vdots & \vdots & \vdots \\ u_M'^1 & u_M'^2 & \cdots & u_M'^L \\ v_1'^1 & v_1'^2 & \cdots & v_1'^L \\ \vdots & \vdots & \vdots & \vdots \\ v_M'^1 & v_M'^2 & \cdots & v_M'^L \end{bmatrix}$$

where the subscripts correspond to the vector location and the superscripts indicate the vector field number. The procedure for performing snapshot POD is outlined as follows:

$$C = R^T R \tag{A.14}$$

An eigenvalue problem of the following form must be solved:

$$C A^i = \lambda^i A^i \tag{A.15}$$

where $i = 1, \dots, L$ and the resulting eigenvalues λ ordered such that:

$$\lambda^1 > \lambda^2 > \lambda^3 > \cdots > \lambda^L = 0$$

In this way, the modes that possess the largest portion of the are encountered first. The resulting eigenvectors, A^i , form the basis for the POD

modes, ϕ^i , shown below:

$$\phi^i = \frac{\sum_{l=1}^L A^i u^l}{\|\sum_{l=1}^L A^i u^l\|} \quad (\text{A.16})$$

where the discrete norm $\|\cdot\|$, is defined as:

$$\|b\| = \sqrt{b_1^2 + b_2^2 + \dots + b_M^2} \quad (\text{A.17})$$

The fluctuating velocity components can then be reconstructed using any number of POD modes via:

$$u^l = \sum_{i=1}^L a_i^l \phi^i \quad (\text{A.18})$$

The unknown POD coefficients, a_i^l , can be found by projecting the POD modes upon the fluctuating vector fields:

$$\Psi = [\phi^1 \ \phi^2 \ \dots \ \phi^l] \quad (\text{A.19})$$

$$a_i^l = \Psi^T u^l \quad (\text{A.20})$$

By selecting a limited number of POD modes, ($l \ll L$), a low-order model of the flow has been developed that ensures that the most energetic turbulent features are captured within as few modes as possible. The impact of these features can now be more easily studied, as they have been extracted, and the small-scale features of the flow that possess minimal amounts of turbulent kinetic energy have been effectively filtered out. Within all POD

analyses, the experimenter must decide upon a reasonable balance of selecting a sufficient number of modes to accurately reconstruct the physics of the flow while limiting the number of modes to include only what are deemed to be the most important features of the flow.

Stochastic estimation and POD are known as complementary techniques, (also known as *modified* stochastic estimation), as they both make use of the fluctuating velocity components. Their use in conjunction was originally discussed by Bonnet et. al [48] in an axisymmetric jet shear-layer of $Re = 110,000$ and a two-dimensional mixing layer. The axisymmetric jet was revisited by Ewing and Citriniti [49], ($Re = 80,000$), using hot-wires. Ewing and Citriniti [49] make the comment that the most important feature of estimation techniques was the ability to capture the large scale dynamics, rather than to reduce the error. They noted some difficulty in accurately capturing the velocity field using only $N = 2$ sensors to estimate, achieving only about 50% of the fluctuating velocity [49]. Perhaps the results could improve if the location or number of point measures could be optimized? Taylor and Glauser [92] studied flow over an adjustable backward facing ramp for Reynolds numbers from $Re = 2 \times 10^5 - 5 \times 10^5$, using PIV and $N = 22$ surface pressure measurements, where the classical POD was performed on the pressure measurements. The authors mentioned that the relatively low frequency response of the system ($< 10Hz$) limited the ability of the system to resolve the higher modes [92]. As the goal of that study was to control the low frequency structures contained within the first few modes, that was sufficient,

but improvements could be seen with a different sensor configuration.

The previous studies focused on employing the classical approach to time resolved data, but with the advent and increased availability of experimental equipment that can provide vector field data on an extremely dense grid of fine resolution (such as PIV), the computational demands of classical POD have been prohibitive for dense measurement grids. Most studies have focused on applying only the classical POD definition to the time resolved measurements, but of greater interest here is the effect of snapshot POD to extract dominant modes from the PIV velocity fields! Relatively few papers have examined the combination of employing POD by the method of snapshots and stochastic estimation. In the work of Durgesh and Naughton [89], snapshot POD was employed to decompose the PIV vector fields into modes before employing them to develop the estimation coefficients for flow over a wedge at $Re = 9.8 \times 10^5$ (albeit for a multi-time delay estimation approach). The fluctuating pressure on the model was measured at $N = 4$ positions. The authors commented that the estimations should work well in conjunction with POD when a large proportion of the turbulent kinetic energy is contained in a few modes [89]. Tu et al. [93] employed snapshot POD to study the wake behind an elliptical splitter plate at $Re = 50,000$, based on the the length of the model, using both standard PIV and time-resolved PIV. Interestingly, the time resolved point sensor data ($N = 1$) was taken at specific locations within the time resolved PIV results, rather than a point sensor like a microphone or hot-wire. The velocity field was then estimated

stochastically using the measurements from the selected point velocity. This illustrates a common issue within the literature regarding stochastic estimation; widely varying selection of the number of sensors employed with little to no discussion of the potential impact on the accuracy of the estimates. Studies employ anywhere from $N = 1$ [93] to $N = 32$ [1] sensors. That is a huge variation, and rarely is any discussion or justification provided.

The work by Cole et al. [3] and later studies by Tu et al. [93] and Lasagna et al. [90] on shear flow over a cavity ($Re = 2.77 \times 10^4$) looked at the performance of the estimations in terms of an estimator error defined as:

$$\epsilon = \frac{\langle (u'_{est} - u')^2 \rangle}{\langle u'^2 \rangle} \quad (\text{A.21})$$

Lasagna et al. [90] examined the impact of the number of sensors ($N = 1 - 6$) and the impact of the utilizing different time delays in the estimations of flow over a cavity. The work indicated that there was a slight improvement in the accuracy of the estimations as the number of sensors was increased, meaning the error was decreased [90]. The slight increase in estimation performance was attributed to the fact that the mechanisms of a cavity flow are likely strongly organized, and that a single sensor may be sufficient to capture enough of the dynamics [90]. The paper by Clark et al. [51] looked at using different error minimization algorithms other than least squares in a backward-facing step flow and found that using a separate training set of PIV images to build the pressure-velocity correlations was important [51].

However, there does not appear to be much (if any) interest within recent experiments with regard to experimental design that seeks to understand the physical aspects of the experiment on the estimation accuracy. Can the experiments themselves be better designed to improve estimation accuracy? One of the greatest limitations to the technique of stochastic estimation is the lack of rigorous experimental guidelines on parameters such as number of sensors and the placement of sensors and their corresponding impact on the resulting estimates. Discussion of how the placements and number of sensors impact the estimates are exceedingly rare, and it means that someone setting out to design an experiment has little knowledge or guidance on the initial design of the experiment in order to yield the best possible estimations. One example of the lack of guidelines in regards to sensor placement is evident when comparing the works on flow over a cavity by Murray and Ukeiley [94, 47] and that of the work on backward-facing steps by Hudy [1]. In the cavity work, wall pressure sensors are installed on both the vertical and horizontal faces of the cavity for a total number of sensors of $N = 14$ [94, 47], whereas the backward facing step experimental setup has pressures on the horizontal plane only ($N = 32$) [1]. Whether these sensors locations and number of sensors were selected specifically or not was not reported. If one were set to embark on the experimental design of a similar geometry, one would be left pondering whether to instrument both faces or not. The impact on the estimates is unclear in the current literature and would likely depend heavily on the experimental geometry in question. Nevertheless, per-

haps some simple guidelines may be established by an appropriately designed investigation. The goal of this research project is to design a suitable case to study the accuracy of stochastic estimation techniques and to study how experimental design parameters (such as number of sensors, sensor location, and correlation thresholds, etc.) have an effect on the errors and accuracy associated with the estimates.

Appendix B

Important Frequencies

Table B.1 presents the relevant frequencies of interest for this BFS geometry and simulation parameters. The vortex-shedding frequency is found to occur around a normalized frequency of $St \approx 0.06$ for turbulent flow over a BFS [7, 20, 12, 21, 82, 60]. The data sampling rate used in this investigation was selected to be equal to approximately 10 times the vortex-shedding frequency to ensure that the frequencies associated with this feature could be accurately resolved. The strictest frequency cut-off present in this examination corresponds to the Nyquist frequency, which is equal to one half of the $f_s = 500\text{Hz}$ sampling rate. In this examination, no frequencies greater than 250Hz can reliably be resolved using the current data sampling rates. The large eddy turnover scale is an estimate of the largest streamwise turbulent length scale of the flow which is approximately equal to the step height of $h = 0.0098\text{m}$. The convective velocity of $U_c = 0.5U_\infty$ is used to account for

Table B.1: Various Length Scales and Frequencies

Description	Equation	Time-Scale (sec)	Frequency (Hz)
Vortex-Shedding	$f = \frac{StU_\infty}{L}$	0.02	50
Nyquist Criterion	$f_s/2$	4×10^{-4}	250
Large Eddy Turnover	$t_L = \frac{L}{U_c} = \frac{h}{U_c}$	2.5×10^{-3}	400
Subgrid Scale	$t_{SGS} = \frac{\mathbb{V}^{1/3}}{U_c}$	2.6×10^{-3}	3860
Kolmogorov Scale	$t_\eta \approx \frac{t_L}{Re_L^{1/2}}$	5×10^{-5}	2×10^4

the streamwise convection of the large-scale structures, and the time-scale is a measure of the time required for a large eddy to complete a rotation. The subgrid scale is based on the cube root of the volume, $\mathbb{V}$, used in setting the LES filter length and sets the limit between the scales that are resolved and those that are handled by the SGS model. The Kolmogorov scale, η_L , corresponds to the smallest eddies in the turbulent flow where dissipation occurs. Resolution of the Kolmogorov scales are only possible using DNS, and the computational costs are extreme due to the severe restrictions placed on the size of the grid elements and time-step requirements to capture the effect of these scales.

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