

FRACTAL IMAGES FROM $z \leftarrow z^\alpha + c$ IN THE
COMPLEX c -PLANE

by

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ABSTRACT

In this paper, we propose the generalized transformation function $z \mapsto z^\alpha + c$ for generating fractal images. The self-squared function $z \mapsto z^2 + c$, which is discussed extensively in the literature, is a special case of this function. A multitude of interesting, intriguing and rich families of fractals are generated by changing a single parameter, α . Direct relationships are observed between α and the visual characteristics of the fractal images in the c -plane. The exponent α can be represented as $\alpha = \pm(n+\epsilon)$, where n and ϵ are the integer and fractional parts, respectively. It is found that when α is a positive integer number, the resulting image contains lobular structures. The number of major lobes equals $(n-1)$. When α is a negative integer number, the generated fractal image is a planetary structure consisting of overlapping central planets surrounded by satellite structures. The number of satellite structures equals $(n+1)$. A continuous variation of α between two consecutive integers results into a continuous proportional change between the two limiting fractal images. Several conjectures about the visual characteristics of the images and the value of α are stated.

1. INTRODUCTION

The fractals generated from the self squared function $z \leftarrow z^2 + c$ have appeared extensively in the literature [MAND82, PEIT86, PICK88].

In this paper, we consider the generalized transformation function $z \leftarrow z^\alpha + c$ of which the self-squared function is a special case. A multitude of interesting, intriguing and rich families of structures are found to emerge from this function by changing a single parameter, namely, the value of α . The domain of α is chosen to be all real numbers, i.e. both positive and negative, and integer as well as non-integer numbers.

All the images presented in this paper have been generated on an IBM 5080 graphics workstation connected to a vector supercomputer class machine IBM 3090-180VF at the University of New Brunswick. A software system for the interactive generation of fractal objects (IGFO) has been developed in VS FORTRAN using GRAPHIGS [KALR88a]. Extensive experimentations have been carried out to study the various classes of fractals using the IGFO system [KALR88b]. Vectorization of various fractal generating algorithms has been carried out to obtain speedups with the vector facility of the IBM 3090-180VF [KALR87, KALR88c].

The generation process and the classification scheme for fractals is given in Section 2. Then we concentrate on the c -plane fractals in Section 3. Sections 4 and 5 present the fractal images for the positive integer and real values of α . The fractal images for the negative integer and real values of α are given in Sections 6 and 7. Several conjectures about the visual characteristics of the images and their dependence on α are presented. Finally, concluding remarks are given in Section 8.

2. GENERATION PROCESS

The basic principle of generating fractals uses the iterative formula:

$$z_{n+1} = f(z_n),$$

where z_0 = the initial value of z , and

z_i = the value of the complex quantity z at the i -th iteration.

For example, the Mandelbrot's self-squared function for generating fractals is

$$f(z) = z^2 + c,$$

where z and c are both complex quantities.

We consider the following transformation function:

$$f(z) = z^\alpha + c,$$

where z and c are complex quantities and α is a real number. We will show that a rich family of new fractal images are obtained by varying the value of the exponent α from negative to positive real values. Further, a strong correlation is observed between the characteristics of the images and the value of α .

A fractal image consists of a two-dimensional array of pixels. Each pixel is represented by a pair of (x,y) co-ordinates. The complex quantities z and c can be represented as

$$z = z_x + i z_y, \text{ and}$$

$$c = c_x + i c_y,$$

where $i = \sqrt{-1}$, and z_x, c_x are the real parts and z_y, c_y are the imaginary parts of z and c , respectively. The pixel co-ordinates (x,y) may be associated with (c_x, c_y) and/or (z_x, z_y) . Based on this concept, the fractal images can be classified as follows:

- (a) c -plane fractals, wherein (x,y) is associated with (c_x, c_y) .

(b) z-plane fractals, wherein (x,y) is associated with (z_{o_x}, z_{o_y}) .

We further classify each of the above two categories into two subgroups:

(a) c-plane fractals - (i) with $z_o = a$ a complex constant, and

(ii) with $z_o = \phi(x,y)$.

(b) z-plane fractals - (i) with $c = a$ a complex constant, and

(ii) with $c = \psi(x,y)$.

Alternately, one can refer to (a)(ii) and (b)(ii) fractals as cz-plane fractals, since both c and z_o are functions of x and y . It should be noted that the z-plane fractals, strictly, should be referred to as the z_o -plane fractals; however, we drop the suffix for simplicity.

For each pixel, the value of z is computed iteratively. Two criteria are used to terminate this iterative process:

(i) the divergence of z beyond a certain preset limit, L , or

(ii) the allowable maximum number of iterations, M , if the value of z does not diverge.

It should be noted that the pixels which satisfy the condition (i) above represent the stable region while those which satisfy condition (ii) represent the unstable region.

The process of generating a fractal image, based on the concepts discussed above, is as follows:

1. Select the values of α , L and M .
2. Select a pixel, i.e. (x,y) co-ordinates of a pixel.

3. Select c and z_0 based on the type of fractal desired (i.e. c -plane, z -plane, etc.).
4. Set the iteration count, i , to zero.
5. Compute a new value of z using the formula: $z \leftarrow z^\alpha + c$.
6. Increment the iteration count, i , by one.
7. Repeat steps (5) - (6) until the termination condition (based on L or M) is satisfied.
8. Assign a color to the pixel based on the iteration count, i , using a preselected color table.
9. Repeat steps (2) - (8) for all the pixels in the desired image.

In this paper we focus on the c -plane fractals with z_0 as a complex constant. The c -plane fractals with $z_0 = \phi(x,y)$ are discussed in [GUJA88a], while the z -plane fractals are discussed in [GUJA88b]. Further, we have experimented with several new divergence tests which produce interesting images; these are discussed in [GUJA88c].

3. c -PLANE FRACTALS

We categorize, in this section, the c -plane fractals with z_0 as a complex constant. Several interesting and intriguing fractal images are generated from the transformation function $z \leftarrow z^\alpha + c$, with positive and negative real values of α . The exponent α can be represented as

$$\alpha = \pm(\eta + \epsilon),$$

where, η - a positive integer, and

ϵ - a fraction, $0 \leq \epsilon < 1$.

We obtain two distinct classes of images depending upon whether α is positive or negative. When $\epsilon = 0$, the images are composed of self-

similar structures, whereas when $\epsilon \neq 0$, an asymmetry is introduced in these images. As the value of ϵ changes from 0 to 1, for a given η , the resulting fractal image smoothly transforms from the image for $\alpha = \pm\eta$ to that for $\alpha = \pm(\eta+1)$, increasing the number of self-similar structures by one. Thus, we classify the c-plane fractal images into the following categories:

- a) Fractal Images for $\alpha = +\eta$.
- b) Fractal Images for $\alpha = +(\eta+\epsilon)$.
- c) Fractal Images for $\alpha = -\eta$.
- d) Fractal Images for $\alpha = -(\eta+\epsilon)$.

We choose the c-plane region for the images as the square region defined by the lower left corner $(-1.25, -1.25)$ and the upper right corner $(1.25, 1.25)$. It is found that with a larger region the relative size of the final structure of interest reduces and with the smaller region the structure gets truncated.

For all the images presented in the ensuing sections, the divergence of z is determined by comparing the magnitude of z with the preset limit value, L , which is chosen as equal to 10. The maximum number of iterations is set to 100.

4. IMAGES FOR $\alpha = +\eta$

In this section we present several fractal images generated for positive integer values of α . The initial value of z is set to $(0,0)$.

Figure 1 shows the fractal structure obtained by setting $\alpha = 4$. It can be observed that the final structure consists of major identical lobes oriented with angular symmetry around the origin. One of these major lobes is situated in the left half of the complex c-plane on the

negative real axis and is symmetrical about the real axis. Further, self-similarity in the various lobules is evident. The portion marked by a square in Figure 1 is zoomed in and shown in Figure 2 wherein the self-similarity property of fractals is clearly seen. Figure 3 zooms in another small square shown in Figure 2, while Figure 4 further zooms inside the region shown in Figure 3. The visual complexity, beauty and self-similarity evident in these structures are comparable to, if not richer than, those of the fractals obtained with the Mandelbrot's self-squared transformation function.

Figures 5 to 8 are generated for $\alpha=15$, $\alpha=6$, $\alpha=5$, and $\alpha=3$, respectively. It can be observed that the number of self-similar major lobes in each of these figures is one less than the value of α ,

We have experimented with several other positive integer values of α . In all these cases we have found that the stable region, represented by the white color, is surrounded by the unstable region. Based on these experiments, we state the following conjecture.

To state the conjecture, we use the following additional notation:

Λ - the total number of major lobes in a fractal image.

λ_s - a self-similar major lobe.

$\Omega(\lambda_s)$ - the angular space spanned by a λ_s around the origin.

CONJECTURE 1. The fractal image generated from the transformation function $z \rightarrow z^\alpha + c$, in the complex c -plane with z_0 as a constant and $\alpha = n$, contains Λ major lobes given by $\Lambda = (n-1)\lambda_s$, with $\Omega(\lambda_s) = 2\pi/(n-1)$. If α is an odd number, then the resulting major lobes are symmetrical around both the real and imaginary axes of the complex c -plane. If α is an even number, then the resulting major lobes are

symmetrical around the real axis, with one of the major lobes situated in the left half of the complex c-plane on the negative real axis.

5. FRACTAL IMAGES FOR $\alpha = +(\eta + \epsilon)$

The case of non-integer positive values of α is considered in this section. The resulting images generated from the even and odd values of η , as ϵ changes from 0 to 1, evolve differently, even though the underlying mathematical formulation is the same. These cases are, therefore, discussed in the following subsections. The initial value of z is set to $(0,0)$, as in the case of $\alpha = \eta$.

5.1 Odd η

Figure 9 is generated for $\alpha = 5.2$. We can see that there are four major self-similar lobes and a small activity between the two major lobes on the left-hand side, thus creating asymmetry. When the value of α is increased to 5.5 we obtain Figure 10. Again we observe four major lobes, with more separation between the two left hand side lobes; an emergence of an embryonic lobe can be identified in this separation. Figure 11 shows the structure obtained for $\alpha = 5.8$. Here, we observe four major lobes and an embryonic lobe between the two left-hand side major lobes. This embryonic lobe develops into a complete self-similar major lobe as α becomes 6 (see Figure 6).

We have generated several images for other odd values of η and observed the changes as ϵ is varied between 0 and 1. Based on these experiments, we state the Conjecture 2, using the following additional notation:

λ_e - an embryonic major lobe.

$\Omega(\lambda_e)$ - the angular space spanned by λ_e .

CONJECTURE 2. The transformation function $z \rightarrow z^\alpha + c$, for $\alpha = (\eta + \epsilon)$ and η an odd integer, with z_0 as a constant, produces a fractal image in the complex c -plane consisting of Λ self-similar major lobes, given by $\Lambda = (\eta - 1)\lambda_s$, with $\Omega(\lambda_s) = 2\pi/(\eta + \epsilon - 1)$, and one embryonic major lobe (λ_e) with $\Omega(\lambda_e) = 2\pi\epsilon/(\eta + \epsilon - 1)$. Further, the image is symmetrical about the real axis and λ_e is situated in the left half of the complex c -plane, on the negative real axis.

5.2 Even η

Figure 1 shows the image generated with $\alpha = 4$, which consists of 3 major lobes (see Section 4.0). When the value of α is increased to 4.2, i.e. $\eta=4$ and $\epsilon=0.2$, we obtain the image shown in Figure 12. It is seen that the major lobe situated on the negative real axis of the complex c -plane has split; we refer to this major lobe as a composite lobe. Figure 13 shows the image obtained with $\alpha = 4.5$. It is seen that the composite lobe in Figure 12 has a further split. This lobe is about to split into two major lobes in Figure 14 where $\alpha = 4.8$; finally, this splitting is completed when α becomes five (see Figure 7). Thus, it is seen that when $4 < \alpha < 5$, we have two self-similar major lobes and one composite major lobe.

Based on the above and the experimentation for several other even values of η , we state the Conjecture 3, using the following additional notation:

λ_k - a composite major lobe.

$\Omega(\lambda_k)$ - the angular space spanned by λ_k .

CONJECTURE 3. The transformation function $z \leftarrow z^{\alpha+c}$, for $\alpha = (n+\epsilon)$ and n an even integer, with z_0 as a constant, generates a fractal image in the complex c -plane consisting of Λ self-similar major lobes, given by $\Lambda = (n-2)\lambda_s$, with $\Omega(\lambda_s) = 2\pi/(n+\epsilon-1)$ and one composite major lobe (λ_k) with $\Omega(\lambda_k) = 2\pi(1+\epsilon)/(n+\epsilon-1)$. Further, the image is symmetrical about the real axis and λ_k is situated in the left half of the complex c -plane, on the negative real axis.

6. FRACTAL IMAGES FOR $\alpha = -n$

So far, the value of α was constrained to be in the positive domain; now we study the effects of the negative integer α . The initial value of z is set to (1,1). Note that the initial value of z that is used for positive α , i.e. $z_0 = (0,0)$, is not suitable when α is negative since it would cause an overflow.

Figure 15 illustrates the fractal structure obtained by setting $\alpha = -15$. This structure resembles a constellation similar to a planetary arrangement with a central circular planet surrounded by clusters of sixteen satellite structures. These satellite structures are situated symmetrically around the central planet. Note that the stable region, which is represented by the white color, surrounds the constellation (which represents the unstable regions). This is reverse of the case for positive α where the stable region is completely surrounded by the unstable region. Figure 16 is generated for $\alpha=-7$, whereas Figure 17 is obtained for $\alpha=-6$. It is seen that the central

planet decomposes into two overlapping circular planets; these central planets are again surrounded by eight and seven satellite structures. Further, it is observed that the size of the central structure reduces as η reduces, while the size of a satellite structure increases as η reduces. Figures 18 to 20 are generated for $\alpha=-5$, $\alpha=-4$, and $\alpha=-2$, respectively. The satellite structures are again seen to enlarge and diffuse as η decreases; in addition, more central planets, each reduced in size, are seen to emerge.

Based on our experimentation, we state the Conjecture 4, with the following additional notation:

Ξ - the total number of satellite structures around a set of central planets.

ξ_s - a self-similar satellite structure.

$\Omega(\xi_s)$ - the angular space spanned by a ξ_s around the origin.

CONJECTURE 4. A fractal image generated from the transformation function $z \rightarrow a^\alpha + c$, in the complex c -plane with z_0 as a constant and $\alpha = -\eta$, contains Ξ satellite structures given by $\Xi = (\eta+1)\xi_s$, with $\Omega(\xi_s) \cong 2\pi/(\eta+1)$. If α is odd, the resulting satellite structures appear symmetrical around both the real and imaginary axes of the complex c -plane. If α is even, the resulting satellite structures are symmetrical around the real axis, with one of the satellite structures situated in the left-half of the complex c -plane on the negative real axis.

7. FRactal Images for $\alpha = -(\eta + \epsilon)$

Now we illustrate the effects of the non-integer negative values of α on the resulting fractal images. Again, the initial value of z is chosen as $(1,1)$. The cases of the odd and even values of η are discussed in the ensuing subsections.

7.1 Odd η

The fractal image generated for $\alpha=-5$ appears in Figure 18 (see Section 6). In this figure, six satellite structures, situated symmetrically around the central planets, are seen. Moreover, none of these satellite structures are located on the real axis. When the value of α is changed from $\alpha=-5$ to $\alpha=-5.5$, we have observed that the space between the two left-most satellite structures increases, displacing all the satellite structures proportionately. At $\alpha=-5.6$, an embryonic satellite structure is seen to emerge on and around the negative real axis in this space (Figure 21). The size of this embryonic satellite structure increases as the value of α is changed to $\alpha=-5.8$ (Figure 22) and $\alpha=-5.9$ (Figure 23).

Based on these and other observations for various odd values of η and different ϵ , for negative α , we state the Conjecture 5 using the following additional notation:

ξ_e - an embryonic satellite structure.

$\Omega(\xi_e)$ - the angular space spanned by ξ_e .

CONJECTURE 5. The transformation function $z + z^\alpha + c$, for $\alpha = -(\eta + \epsilon)$ and η an odd integer, with z_0 as a constant, produces a fractal image in

the complex c -plane consisting of Ξ self-similar satellite structures given by $\Xi = (\eta+1)\xi_s$, with $\Omega(\xi_s) \cong 2\pi/(\eta+\epsilon+1)$, and one embryonic satellite structure (ξ_e) with $\Omega(\xi_e) \cong 2\pi\epsilon/(\eta+\epsilon+1)$. Further the image appears symmetrical about the real axis and the embryonic satellite structure is situated in the left-half of the complex c -plane, on the negative real axis.

7.2 Even η

Figure 24 shows the image generated with $\alpha=-6.2$, i.e. $\eta=6$ and $\epsilon=0.2$. In this figure, we can see the six major satellite structures and a composite satellite structure on and around the negative real axis. This composite satellite structure emerges from the splitting of the satellite structure on the negative real axis seen in Figure 17 for $\alpha=-6$. At $\alpha=-6.5$ (Figure 25) this splitting increases and can be observed to be proportional to the value of ϵ . Figure 26 shows the fractal image generated for $\alpha=-6.8$, i.e. $\eta=6$ and $\epsilon=0.8$. The composite satellite structure actually consists of two complete separated satellite structures; this separation keeps on increasing as the value of ϵ approaches one. At $\alpha=-7$, this separation is complete and we see eight fully developed satellite structures (Figure 16, Section 6).

We state the Conjecture 6, based on our experimentation, with the following additional notation:

ξ_k - a composite satellite structure.

$\Omega(\xi_k)$ - the angular space spanned by ξ_k .

CONJECTURE 6. The transformation function $z \rightarrow z^\alpha + c$, for $\alpha = -(\eta+\epsilon)$ and η an even integer, with z_0 as a constant, generates a fractal image

in the complex c -plane consisting of Ξ self-similar satellite structures given by $\Xi = \eta\xi_\delta$, with $\Omega(\xi_\delta) \cong 2\pi/(\eta+\epsilon+1)$, and one composite satellite structure (ξ_k) with $\Omega(\xi_k) \cong 2\pi(1+\epsilon)/(\eta+\epsilon+1)$. Further, the image appears symmetrical about the real axis and ξ_k is situated in the left half of the c -plane, on the negative real axis.

8. CONCLUSION

We have shown in this paper that the transformation function $z + z^\alpha + c$ generates interesting and intriguing fractal images. Direct relationships have been observed between the value of the exponent α and the characteristics of the generated fractal images in the c -plane. When α is a positive integer, the generated image comprises of lobular structures, whereas when α is a negative integer we obtain a planetary structure consisting of overlapping central planets surrounded by satellite structures. The number of lobules or satellite structures are directly proportional to α . When α is varied continuously between two consecutive integer numbers, say β and $(\beta + 1)$, continuous and predictable changes, which represent the intermediate stages in the development from the image for $\alpha = \beta$ to the image for $\alpha = (\beta + 1)$ are observed. Several conjectures about the visual characteristics of the images and the value of α are given.

9. ACKNOWLEDGEMENTS

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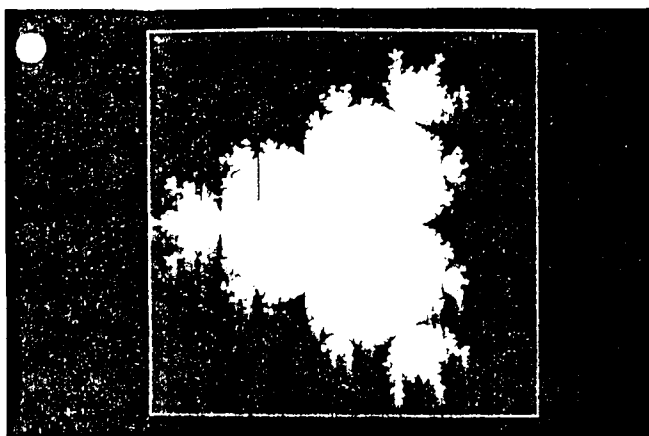


Fig. 1: $\alpha=4$

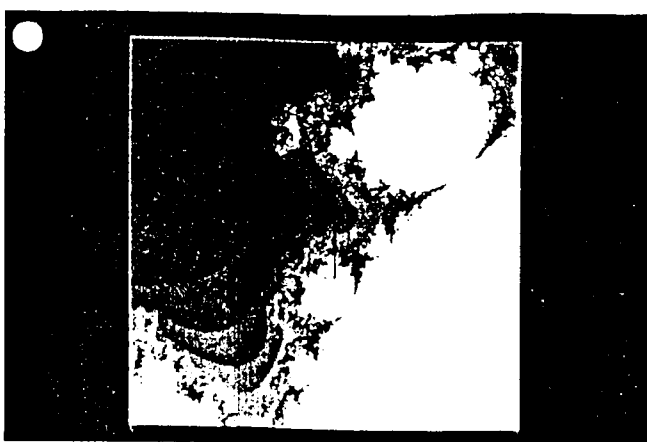


Fig. 2: $\alpha=4$, zoomed



Fig. 3: $\alpha=4$, zoomed

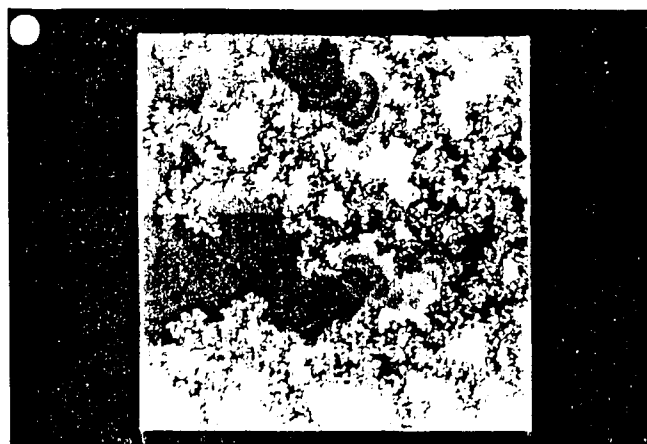


Fig. 4: $\alpha=4$, zoomed

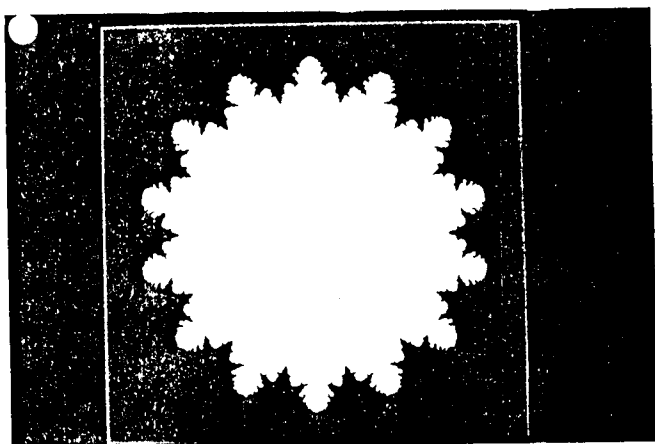


Fig. 5: $\alpha=15$

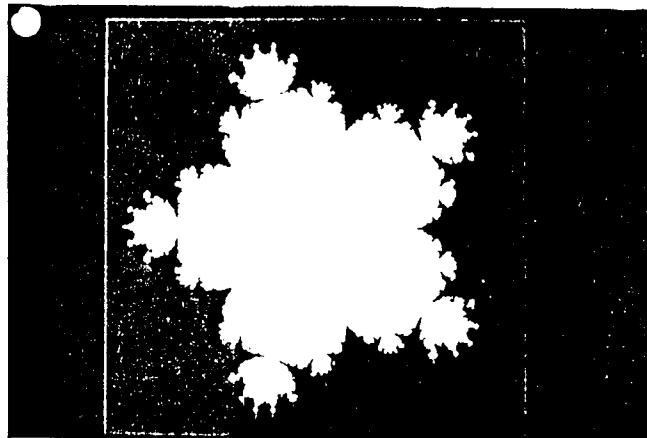


Fig. 6: $\alpha=6$

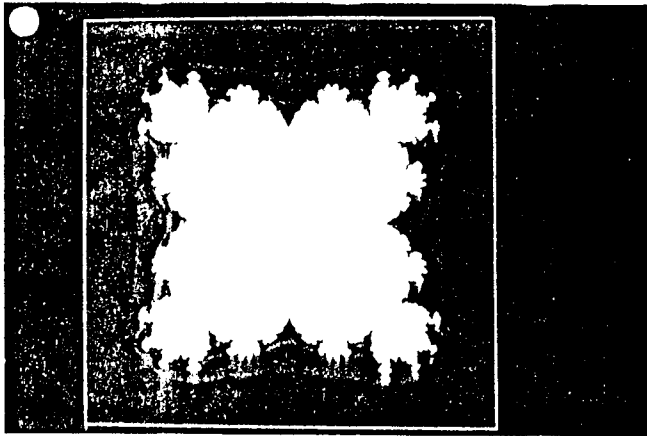


Fig. 7: $\alpha=5$



Fig. 8: $\alpha=3$

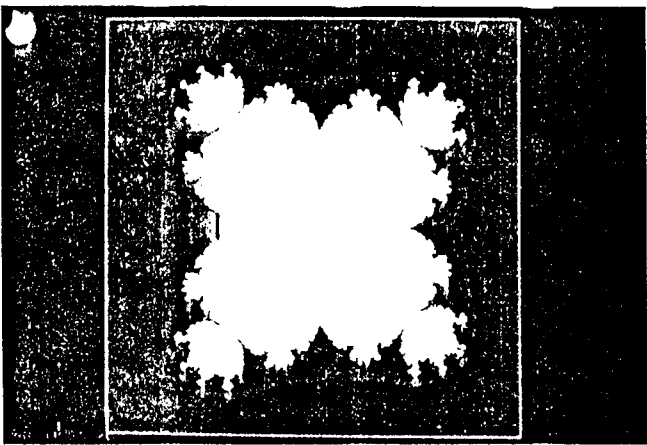


Fig. 9: $\alpha=5.2$

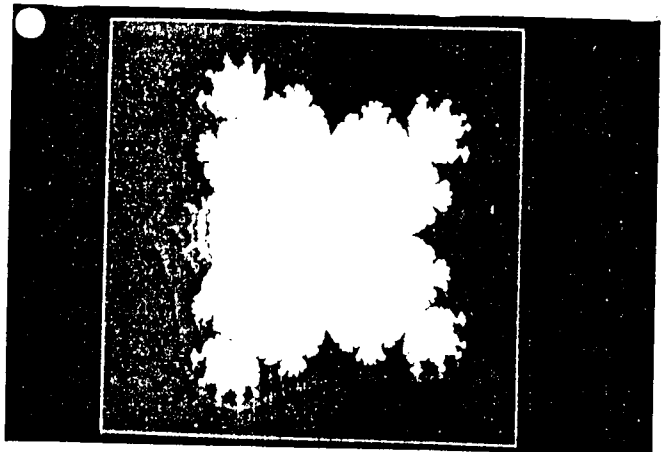


Fig. 10: $\alpha=5.5$

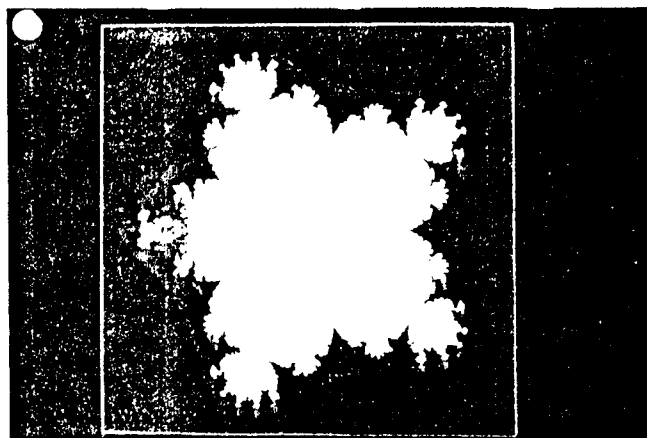


Fig. 11: $\alpha=5.8$

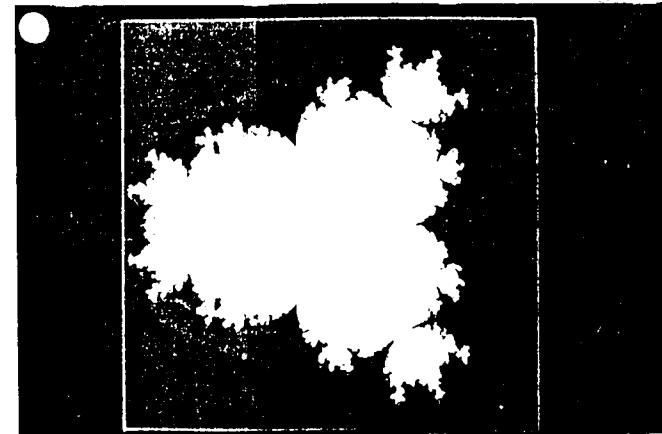


Fig. 12: $\alpha=4.2$



Fig. 13: $\alpha=4.5$

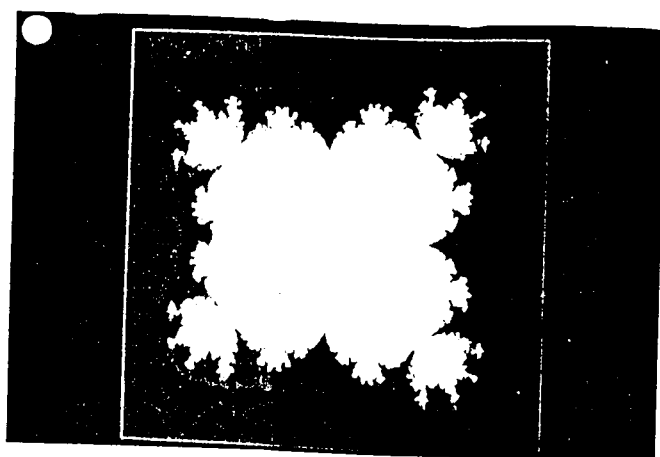


Fig. 14: $\alpha=4.8$

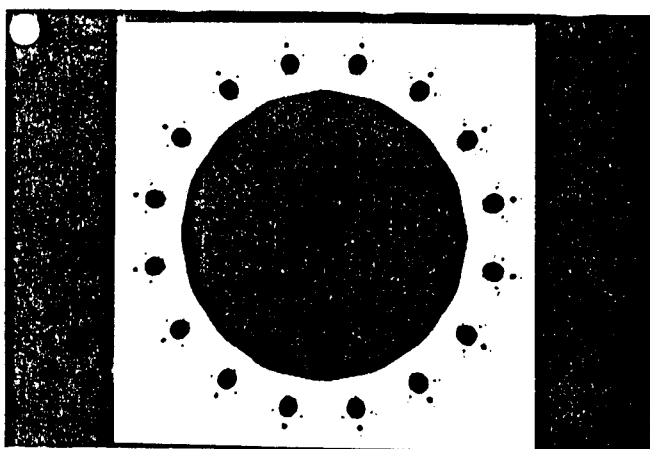


Fig. 15: $\alpha=-15$

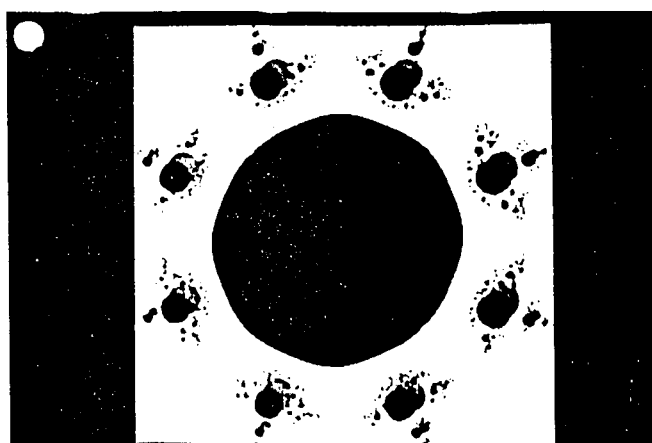


Fig. 16: $\alpha=-7$

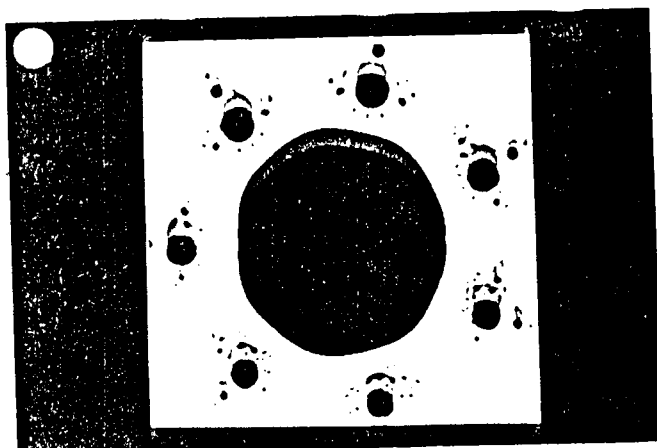


Fig. 17: $\alpha=-6$

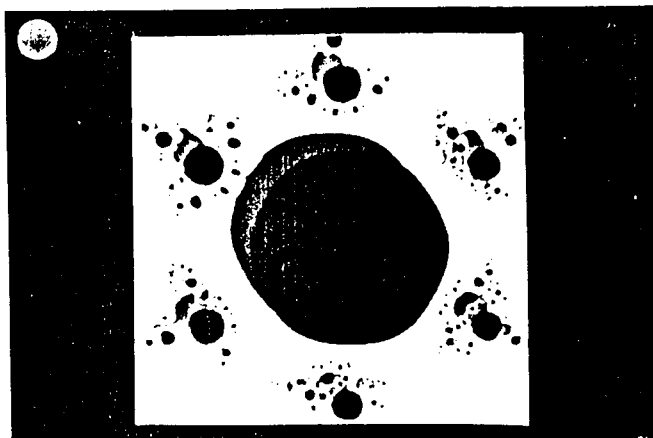


Fig. 18: $\alpha=-5$

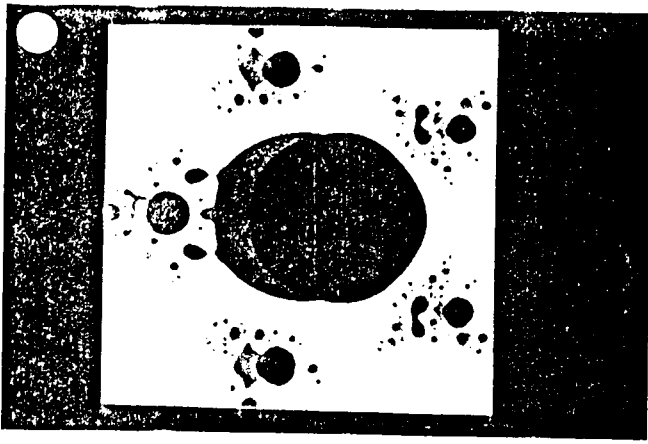


Fig. 19: $\alpha=-4$

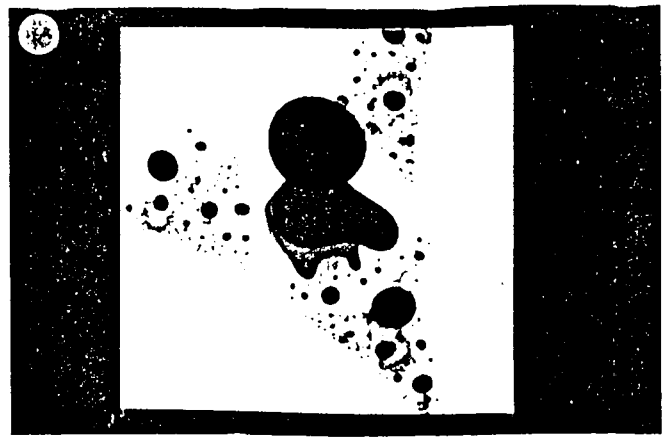


Fig. 20: $\alpha=-2$

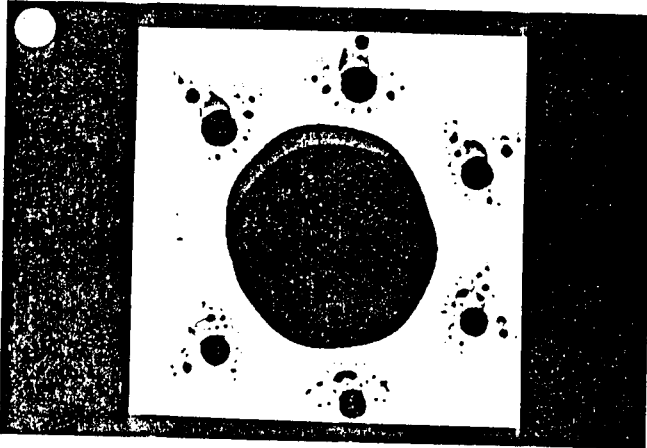


Fig. 21: $\alpha=-5.6$

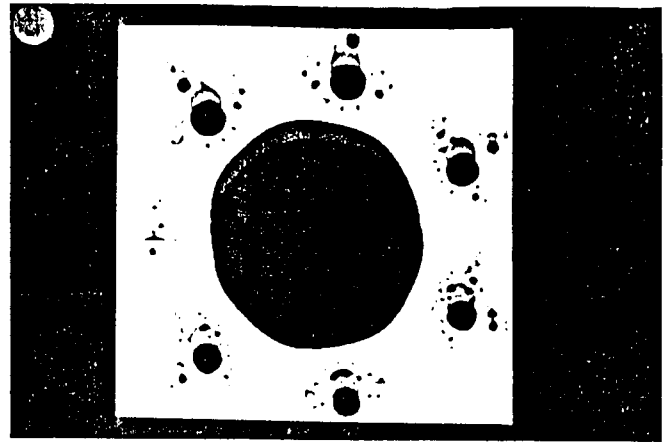


Fig. 22: $\alpha=-5.8$

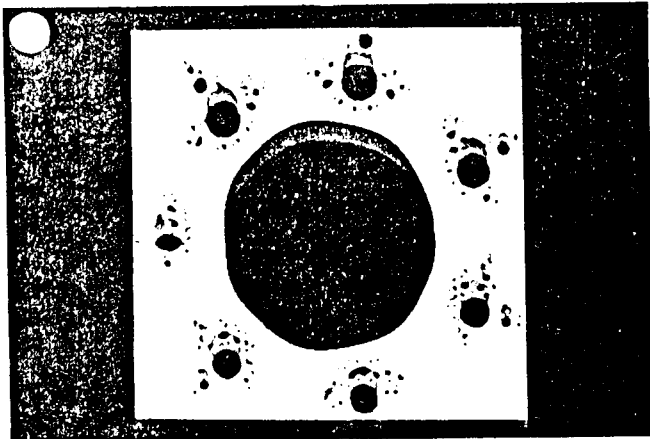


Fig. 23: $\alpha=-5.9$

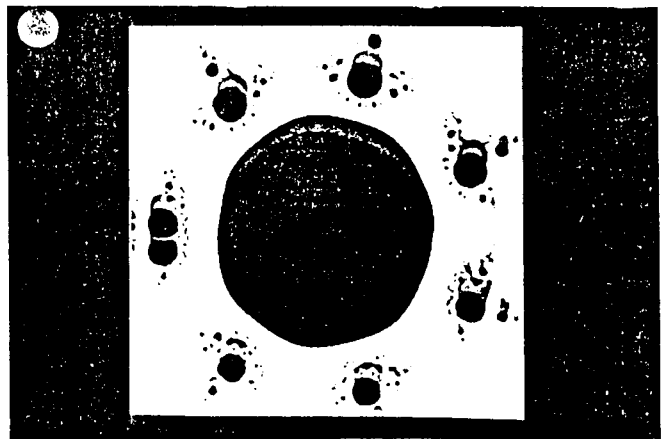


Fig. 24: $\alpha=-6.2$

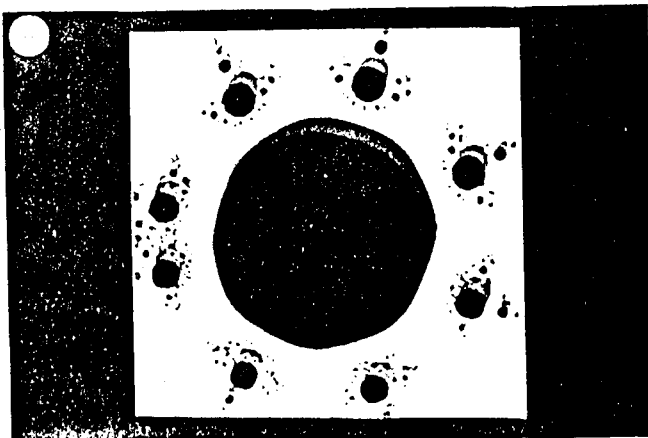


Fig. 25: $\alpha=-6.5$

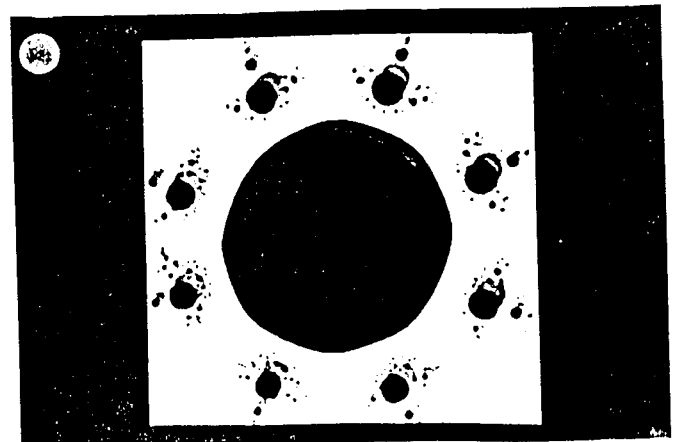


Fig. 26: $\alpha=-6.8$

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