NON-NUMERIC MEASUREMENT DEVICES

by

Lev Goldfarb
Sanjay S. Deshpande

TR96-106, March 1996

Preliminary Version

Faculty of Computer Science
University of New Brunswick
Fredericton, N.B. E3B 5A3
Canada

Phone: (506) 453-4566
Fax: (506) 453-3566
E-mail: fcs@unb.ca
www: http://www.cs.unb.ca

Non-numeric measurement devices

Lev Goldfarb and Sanjay S. Deshpande

Faculty of Computer Science

University of New Brunswick,

Fredericton, N.B., Canada E3B 5A3,

Ph.(506)453-4566, FAX.(506)453-3566,

Email: goldfarb, d23d@unb.ca

Abstract

In this report we define a fundamentally new type of measurement device- structured measurement device. The readings of all existing measurement devices are numeric. We propose a measurement device whose readings are non-numeric, or symbolic, i.e. the readings are structured objects, e.g. string, graphs, etc., wiht the geometry defined by the corresponding transformation operations. Thus the set of numbers in the classical devices is replaced by a transformation system.

1 What are symbols: A new approach to image representation

We propose that the fundamental goal of vision which is to construct a (combinative) representation of objects. Now we will discuss how one can construct such a representation starting from the intensity measurements. It is well known that measurements provide a "universal" means to study a phenomena. In case of "image understanding", it is conventionally (implicitly) assumed that the *vector representation* of the intensity measurements - i.e, the representation that is independant of *the combinative structure of the measuring device* - is adequate for understanding the structure of the object. It should now be clear, however, that the combinative structure of the measuring device has to be represented also. Moreover, one should note that it is important to note that numeric measurements (light intensity in particular) are hardly the only means to represent the enviroment, as is the case with all biochemical systems. It is also important to keep in mind that the number itself is a human creation and does not exist in nature. As far as the relationships between various mathematical structures are concerned, and the numeric mathematical structures are very restricted form of the combinative structure the transformation system.

In this section we propose a new form of image representation that *inductively* constructs (from the initial measurements), a combinative object representation. This representation is constructed in two stages: the first stage is called the structured image representation (SIR) and the second stage is called the inductive image representation (IIR).

1.1 The Structured Image Representation (SIR)

In order to construct the structured image representation (SIR) one requires an *adequate abstract specification of the measuring devices*, since the device forms a fundamental integral component of any (human or machine) vision system. We define measurement device in a manner more general than is currently understood.

Definition 1 *A* **measurement unit** *is an abstraction of the elementary/atomic measurement device and will be denoted by* u. *A* **(structured) measurement device** $\mathbf{M}$ *is a triple* $(\mathbf{U}, m, \mathbf{T})$, *where*

$\mathbf{U}$ *is a set of meaurement units,*

$m : \mathbf{U} \to \mathbf{AT}$ *m is a mapping from* $\mathbf{U}$ *into the attribute set* $\mathbf{AT}$ *each element of which is an n-tuple* $m(u) = < a^1, a^2, ..., a^n >$, a^i *characterizes one aspect of the unit (see below)*

and $\mathbf{T} = (S, O, D)$ *is a transformation system (see Section 2) whose structs are built from subsets (possibly of different sizes) of* $\mathbf{U}$.

Each meaurement unit is completely characterised by $m(u)$ - an n-tuple of *attributes of* u. A chosen measurement unit can perform only one type of meaurement, numeric or non-numeric (i.e. symbolic struct measurement), but could also have many other attributes and device M may have units of different types, i.e. performing, for example, different types of measurement at the *same* location. For example, we will assume that attribute a^1 is unit's type (thermal, light, acoustic, etc), a^2 is the unit's location in space l_u, and a^3 is the *range* of the units's measurements which is a set of structs S_u. It is important to stress that the range is also defined as a transformation system $\mathbf{T}_u = (S_u, O_u, D_u)$ that specifies what the measurement are, while the transformation system $\mathbf{T}$ in the abovedefinition specifies the structure of the device M. It is not difficult to see that all present measurement devices are special cases of the above. All present devices have units whose ranges are numeric transformation system, i.e. they all "produce" numbers, which is a very restricted/trivial form of the transformation system (see Section *). It is important to note that some of attributes are *static* (fixed) and some are dynamic (e.g. location). they all produce numbers, which are a very restricted/trivial form of structs.

One can easily see that our definition of the measurement devices *fundamentally links* the *concept of the device* to that of the corresponding *mathematical structure.* This link should make it quite clear the differences between various measurement devices. In computational vision, typically, the unit's range $\mathbf{T}_u$ is a subset of reals and $\mathbf{T}$ is a vector space. What was not previously understood at all in the computational vision is that the combinative structure of the objects induces an abstract geometry, defined via the transformation system , which must be fundamentally linked to the structure of the measurement device. Moreover, since, we propose that the objective of vision is to capture the combinative structure of the objects, one can see that the measurement device (including its structure) and vision processes form an integral whole and cannot be disassociated.

Definition 2 *An (instantaneous) measurement m_t by a measurement device $M = (\mathbf{U}, m, \mathbf{T})$ is a mapping*

$$m_t : \mathbf{U} \to \mathbf{AV}, \tag{1}$$

where $m_t(u) \in S_u$ and $\mathbf{AV} = \bigcup_{u \in U} S_u$.

Thus, if all the units measure light intensity, $\mathbf{AV} \subseteq \mathbf{R}$. In view fo the fact that measurements result in the images , we turn next to the process of image formation and its combinative nature.

We propose that images should be thought of as composed of discrete atoms (primitives) and the structure of image, as determined by a chosen combinations of these atoms. Such an atomistic view was proposed and extensively studied by U.Grenander [6]. We postulate (in accordance with Postulate 1) that the combinative structure of the image (or sub-image) can be captured by representing the image (or sub-image) by the corresponding transformation system.

Definition 3 *Given a measurement device* M *and its instantaneous meaurement* m_t, *a* **(structured) image** *is a triple* $I = (m_t(\mathbf{U}), L, T)$, *where* $L = \{l_u \mid u \in U\}$ *is a set of locations and* $\mathbf{T}$ *is a transformation system for* M *(see definition 1); a* **(structured) sub-image** I_1 *is a triple* $(\mathbf{U}_1, L_1, \mathbf{T}_1)$, *where* $\mathbf{U_1} \subseteq \mathbf{U}$, $L_1 \subseteq L$, *and* $\mathbf{T}_1$ *is a sub-transformation* $\mathbf{T_1} \subseteq \mathbf{T}$.

Example in figure 1 illustrates the role of transformation system in image representation. Conventionally, in computational vision an $m \times n$ image region is represented as a vector in a $m \times n$-dimensional vector space. Almost in all the current low-level vision approaches use the vector space is used as the underlying mathematical structure for image representation, i.e. the transformation of the corresponding measurement device is a finite- dimensional vector space. The fundamental drawback of such a vector representation (numeric struct) is related to the fact that the *spatial relations between the pixels and their intensities is completely lost.* This is clearly evident from figure 1. At the same time, the symbolic struct representation together with the operations completely (and explicitly) captures the relationship between the pixels and their intensity values.

A necessity for the choice of the symbolic struct representation can be seen from the following facts. The material properties of the objects are completely determined by its microstructural properties [5]. It is also known that the intensity values depend on the material properties and the surface geometry [4]. Moreover, the image contains objects that have combinative structure. Hence each intensity value represents the microstructural property of the object at that spatial location. Symbolic structs are tailer-made to capture thisstructural information.

Definition 4 *Given a (structured) image* $I = (m_t(\mathbf{U}), L, \mathbf{T})$*, a set of subimages* $\{I_k\}_{k \in K}$ *of* I *is called local (structured) image partitioning of* I *if* $\bigcup_{k \in K} \mathbf{U}_k = \mathbf{U}$ *where* $\mathbf{U}_k \subseteq \mathbf{U}$.

One can think of the above subimages as the receptive fields in the retina (in neurophysiological language), or as image windows (in computational vision terms).

In connection with the above (general) definitions, we will address in this paper *only one aspect* related to the low-level *feature discovery* and representation. As the first step in the process, one has to begin with the construction of the transformation system for a given subimage. It is useful to keep in mind that in this paper we are not addressing the issue of the relationship between the subimage and the image transformation systems as well as between various subimage transformation systems.

2 A new mathematical structure: The Evolving Transformation System

We now turn to the description of the recently proposed evolving transformation system (ETS) model of inductive learning [1] [2] [3], which could be thought of as the formalisation of the *symbolic system* whose role in AI has been so pervasive. ETS is the mathematical structure that is built "on top of" a *basic* mathematical structure – the transformation system.

A transformation system (TS) is a triple

$$T = (\mathbf{S}, \mathbf{O}, \mathbf{D}) \qquad (2)$$

where

refer to the original paper for constructive definition *

S is a set of *structs* (e.g. strings, trees, etc.) which are representations of analogously structured (discrete ?) objects; *(use the phrase structs are bulit from units)*

$\mathbf{O} = \{O_i\}_{i=1}^{m}$ is a finite set of multivalued functions, $O_i : S \to S$, satisfying the following two axioms:

(i) $\forall O \in \mathbf{O} \quad \forall s \in \mathbf{S} \quad \exists O^{-1} \in \mathbf{O}$ such that $s \in \bigcap_{\bar{s} \in O(s)} O^{-1}(\bar{s})$

(ii) for every pair of structs there exists a sequence of operations that transforms one into the other,

the set $\mathbf{O}$ specifies *permissible* operations for transforming one object into another (e.g. deletion-insertion, substitution operations), and can be thought of as a postulated set of object features and

$\mathbf{D} = \{\Delta_\omega\}_{\omega \in \Omega}$ is *competing family of distance functions* defined on $\mathbf{S}$ whose parameter set Ω is the $(m-1)$ -dimensional unit simplex in $\Re^m$

$$\Omega = \left\{ \omega = (w^1, w^2, \ldots, w^m) \mid w^i \geq 0, \ \sum_{i=1}^{m} w^i = 1 \right\}. \tag{3}$$

Each of the distance functions Δ_ω is defined as follows: Weight w^i is assigned to the operation O_i and

$$\Delta_\omega(s_1, s_2) = \min_{o_j \in \mathbf{O}} \sum_{n=1}^{k} w^i_{(j)} \tag{4}$$

where the minimum is taken over all possible sequences $o_j = \left(O_i^{(j)} \ldots . O_k^{(j)}\right)$ of operations that transform struct s_1 into struct s_2.

To compute the distances $\Delta_w(s_1, s_2)$, the system must use its set of operations in a *cooperative and competitive* manner. Thus, all the properties of the system resulting from this definition can be viewed as *emergent* properties.

Learning in a TS can be reduced to the following optimization problem:

$$\max_{\omega \in \Omega} f(\omega), \quad f(\omega) = \frac{f_1(\omega)}{c + f_2(\omega)} \tag{5}$$

where $f_1(\omega)$ is the Δ_ω-distance *between* $\mathbf{C}^+$ and $\mathbf{C}^-$, $f_2(\omega)$ is the average Δ_ω-distance *within* $\mathbf{C}^+$, and c is a small positive constant to prevent the overflow condition (when the values of $f_2(\omega)$ approach 0). Since $f(\omega)$ gives the measure of the separation of $\mathbf{C}^+$ with respect to $\mathbf{C}^-$, it is called the *quality of (learning) class perception.*

For a given concept $\mathbf{C}$ and given set $\mathbf{O}$ of operations, every optimal weighting scheme $\omega^* \in \Omega$ generates the 'best' metric configuration of the training set C^+. In other words, the distance function Δ_{ω^*} gives the 'best' separation of the positive class with respect to the negative class.

Evolving transformation system (ETS) is a mathematical structure which is a finite or infinite sequence of TS's with a common set of structured objects

$$T_i = (\mathbf{S}, \mathbf{O_i}, \mathbf{D_i}, \mathbf{R}) \tag{6}$$

in which each set of operations $\mathbf{O}_i$, except $\mathbf{O}_0$, is obtained from $\mathbf{O}_{i-1}$ by adding to it one or several operations that are constructed from the operations in $\mathbf{O}_{i-1}$ with the help of a small set $\mathbf{R}$ of *composition rules.* Each rule $R \in \mathbf{R}$ allows one to construct new operations from existing operations.

From the above definition of ETS it follows that at stage t we have

$$\mathbf{O}_0 \subseteq \mathbf{O}_1 \subsetneq \mathbf{O}_2 \subsetneq \ldots \subsetneq \mathbf{O}_t \Omega_0 \subsetneq \Omega_1 \subseteq \Omega_2 \subsetneq \ldots \subsetneq \Omega_t \tag{7}$$

Roughly speaking, the inductive learning process for the ETS proceeds by constructing a sequence of $\mathbf{O}_i$'s in such a way that, for each consecutive T_i, the minimum value of f_2 decreases (while making sure that the value of f_1 is not zero). In essence, the interdistances in $\mathbf{C}^+$ gradually shrink to 0 while the distance between $\mathbf{C}^+$ and $\mathbf{C}^-$ remains non-zero. For a complete example of an inductive learning process in ETS see [3].

In view of the above inductive learning process the inductive class structure can be defined by the following triple

$$\Sigma = (\bar{\mathbf{C}}^+, \mathbf{O_f}, \bar{\mathbf{\Omega}}_\mathbf{f}) \tag{8}$$

where $\bar{\mathbf{C}}^+$ is a subset of $\mathbf{C}^+$, $\mathbf{O_f}$ is the final set of operations at the end of the learning process, and $\bar{\Omega}_f$

subseteqΩ is the set of optimal weight vectors for the final transformation system.

Thus, since the class membership of a struct s will be determined on the basis of its Δ_{ω^*} distance from $\bar{\mathbf{C}}^+$, the class structure embodies the symbiosis of both the classical discrete and continous formalisms.

Postulate 2: Inductive learning processes are evolving processes that capture the object (class) structure mentioned in Postulate 1. *Hence, the corresonding mathematical structure used in the model of inductive learning should have the capabilities to capture this structure.* (It turns out that the corresponding mathematical structure is fundamentally different from the classical mathematical structure.)

References

[1] L. Goldfarb (1990). On the foundations of intelligent processes - I. An evolving model for pattern learning. *Pattern Recognition,* 23, 596–616.

[2] L. Goldfarb (1992). What is distance and why we need the metric model for pattern learning. *Pattern Recognition,* 25, 431–438.

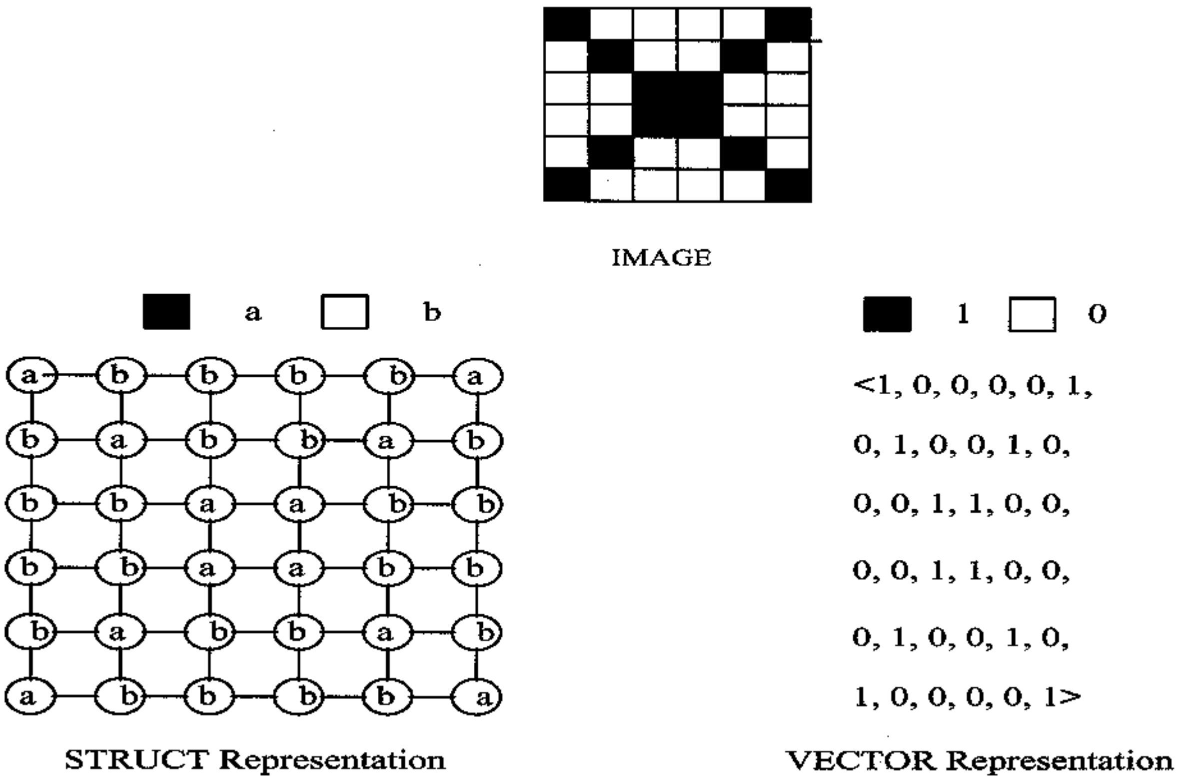

Figure 1: Image Representation

[3] L. Goldfarb and S. Nigam (1994). The unified learning paradigm: A fopundation for AI. In *Artificial Intelligence and Neural Networks: Steps toward Principled Integration,* eds. V. Honavar and L. Uhr, Academic Press, Boston.

[4] B. K. P. Horn and R. W. Sjoberg (1979). Calculating the reflectance map, *Applied Optics,* Vol. 18, No. 11, 1770–1779.

[5] W. D. Callister, Jr (1991). *Material Science and Engineering.* John Wiley and Sons, Inc., New York.

[6] U. Grenander (1981). *Regular Structures. Lectures in pattern theory Vol. 3* Springer-Verlag, New York.