A Fractional-Bits-per-Symbol Communication System for Low-Power Satellites Using N-Point PSK Constellations

by

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Abstract

Low-Earth orbiting satellites are becoming more common and transmit to ground stations through slowly fading radio channels. Radio links operating in this channel cannot operate efficiently with static communication systems making adaptive communication links critical. A fractional-bits-per-symbol communication system is presented as a possible improvement on current adaptive systems. Fractional-bits-per-symbols are realized by utilizing N-point PSK constellations and sequences. Sequences allow for N-point constellations to transmit whole numbers of bits over a number of grouped symbols called a waveform. Equations governing sequences and waveforms are presented. Mapping schemes are used to map bit-strings to waveforms and are shown to have an effect on the performance. Results of simulations using fractional-bits-per-symbols are presented. Theoretical data throughput is compared for low-Earth orbiting satellites using fractional and non-fractional-bits-per-symbol systems.

Dedication

To my parents.

Acknowledgement

The author would like to acknowledge the following people, groups, and agencies. To my supervisor, Dr. Petersen, who helped and guided me throughout this research. To CubeSat NB, for constant support. To the Canadian Space Agency (CSA), for funding this research and creating the Canadian CubeSat Project (CCP). To the New Brunswick Innovation Foundation (NBIF), for funding this research.

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List of Abbreviations

ACM	Adaptive Coding and Modulation
AC	Adaptive Coding
AF	Adaptive Fractional
AM	Adaptive Modulation
AN	Adaptive Non-fractional
AMSL	Above Mean Sea Level
AMSAT	Radio Amateur Satellite Corporation
AOS	Acquisition Of Signal
ARQ	Automatic Repeat Requests
ASK	Amplitude-Shift Keying
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
bps	bits per second
BPSK	Binary Phase-Shift Keying
CCP	Canadian CubeSat Project
CoM	Computer on Module
CNB	CubeSat New Brunswick
CPBW	Choices Per Base Waveform
CPSP	Choices-Per-Sequence-Position
CSA	Canadian Space Agency

CSI	Channel State Information
CubeSat	Cube Satellite
DPSK	Differential Phase-Shift Keying
ED	Euclidean Distance
FEC	Forward Error Correction
FSK	Frequency-Shift Keying
GNSS	Global Navigation Satellite System
GRIPS	GNSS Receiver for Ionospheric and Position Studies
GS	Ground Station
ISI	Inter-Symbol Interference
ISS	International Space Station
IQ	In-phase and Quadrature
LEO	Low Earth Orbit
LMB	Linear Microwave Board
LOS	Loss Of Signal
LUT	Look Up Table
Mbps	Mega-bits-per-second
Msps	Mega-symbols-per-second
NBIF	New Brunswick Innovation Foundation
NRCSD	NanoRacks CubeSat Deployer
OOK	On-Off Keying
PSK	Phase-Shift Keying
QPSK	Quadrature Phase-Shift Keying
RF	Radio Frequency
RBC	Reflected Binary Code
RX	Receiver
SASI	Spectral Airglow Structure Imager

SBU	S Band Board Unit
SCP	SDR Communications Platform
SDR	Software Defined Radio
SN	Static Non-fractional
SNR	Signal to Noise Ratio
SOI	Symbol Of Interest
SPS	Samples Per Symbol
SRRC	Square-Root Raised Cosine
TDD	Time-Division Duplex
TGC	Trucated Gray Coding
TQAM	Triangular Quadrature Amplitude Modulation
TT&C	Tracking, Telemetry, and Command
TX	Transmitter
UHF	Ultra High Frequency
VHF	Very High Frequency
WOI	Waveform Of Interest

List of Symbols

a_n	A vector of symbols sampled at rate n
a_m	A vector of symbols sampled at rate m
b_i	A vector of input bits
b_o	A vector of output bits
b_{i-D}	A vector of input bits delayed by D samples
b_{sym}	bits-per-symbol
b_{Asym}	bits-per-symbol, actual
b_{Tsym}	bits-per-symbol, theoretical
$b_{[s_1,s_2,\ldots,s_N]}$	Number of bits transmitted in one waveform
β	Excess bandwidth of a square-root raised cosine filter
C	Constellation
$\angle c_\Delta$	Angle between constellation points
c_p	Constellation point p
$\angle c_p$	Angle between the positive real axis and constellation point p
$\mathrm{CPSP}_{[s_1,s_2,\ldots,s_N]}$	Choices-per-sequence-position
e	Error term
E_b	Energy-per-bit
E_{n_m}	Energy of the complex noise sampled at rate m
E_{t_m}	Energy of the transmitted symbols sampled at rate m
ED	Euclidean distance between points

ED_{wave}	Euclidean distance between waveforms
f_m	A vector of received and shaped symbols sampled at rate m
f_n	A vector of received and shaped symbols sampled at rate n
i	Index within a vector
I	Length of a vector
k_n	A vector of quantized symbols sampled at rate n
m	Size of largest constellation in S
M	Mapped waveform used in the ED mapping process
n	Index of position within a sequence
N	Number of symbols within a sequence
n_m	A vector of complex noise sampled at rate m
N_0	Noise power spectral density
N_{sym}	Number of transmitted symbols
$\eta_{[s_1,s_2,\ldots,s_N]}$	Waveform utilization efficiency
p	Index of position within a constellation
P	Number of constellations points contained in C
P_b	Power-per-bit
P_n	Noise power
r_m	A vector of received symbols sampled at rate m
s_n	Number of constellation points at position n
s_m	A vector of symbols sent through a channel sampled at rate m
S	A sequence
SPS	Number of samples per symbol
t	Time
T	A time period
t_m	A vector of transmitted symbols sampled at rate m
U	Unmapped waveform used in the ED mapping process

$W_{[s_1,s_2,\ldots,s_N]}$	Number of waveforms
$W'_{[s_1,s_2,\ldots,s_N]}$	Number of mapped waveforms
$W''_{[s_1,s_2,\ldots,s_N]}$	Number of unmapped waveforms
$W_{A[s_1,s_2,\ldots,s_N]}$	Set of waveforms
$W_{M[s_1,s_2,\ldots,s_N]}$	Subset of mapped waveforms
$W_{U[s_1,s_2,\ldots,s_N]}$	Subset of unmapped waveforms

1. Introduction

1.1 Background

The design challenges of VIOLET, a Cube Satellite (CubeSat) from CubeSat New Brunswick (CNB), inspired much of the work in this thesis. CubeSats, first introduced by California Polytechnic State University, were designed [23] to allow university students easier access to space by being small and lightweight. The size of CubeSats are described in standardized units of 10 cm cubes. VIOLET is a 2U CubeSat having final dimensions of 20 cm by 10 cm by 10 cm. It will weigh less than 3.6 kg making it a member of the nanosatellite (1-10 kg) category. VIOLET is planned to be launched to the International Space Station (ISS) in 2022 and will be deployed soon after into a low-Earth orbit (LEO). The orbit will be initially similar to that of the ISS but because VIOLET will be deployed retrograde to the orbit of the ISS, VIOLET's orbit will drift away. The orbit will have an approximate inclination of 51.6° and will be roughly circular with an initial altitude above mean sea level (AMSL) of between 408 km and 410 km. This gives an approximate time-per-orbit of 92.7 minutes and approximately 15.5 orbits around the Earth per 24-hour period. Many CubeSats, including VIOLET, do not carry any propulsion system on-board making VIOLET's useful life-time based on orbital decay rather than the lifetime of the hardware. As a result, VIOLET's lifetime is not precise and is in the range of months to a few years. During VIOLET's operation, two science missions on-board VIOLET will collect data;

the first mission, GNSS Receiver for Ionospheric and Position Studies (GRIPS), uses a Global Navigation Satellite System (GNSS) receiver to record raw multi-constellation, multi-frequency GNSS observations. These measurements will help advance the understanding of the spatial and temporal variations of the ionosphere caused by space weather. The second mission, Spectral Airglow Structure Imager (SASI), will take images of the airglow produced within the ionosphere from the photon emission of atomic oxygen at 630 nm. SASI's goal is to observe the dynamics of the atmosphere, particularly in the mesosphere to lower thermosphere region. Both missions generate considerable amounts of data for VIOLET to download.

If each mission performed a single data collecting orbit per day and downloaded the data to a ground station (GS), it would require downloading 500 Mbytes per day. As a result, a radio frequency (RF) communication system with a high data rate, for a CubeSat, is required between VIOLET and the GS. Since the orbit of VIOLET is non-geostationary, the amount of time that VIOLET can communicate with the GS per day is given by how many passes occur each day and the duration of the passes. For the GS in Fredericton, the average time per day that VIOLET will be reachable has been simulated to be 30 minutes. This small amount of time that VIOLET can download data each day constrains the communication system.

VIOLET uses a passive approach to attitude control by placing a large permanent magnet in the centre of the CubeSat's structure. This makes VIOLET approximately follow the Earth's magnetic field lines by creating torque between the permanent magnet's magnetic field and Earth's magnetic field [5]. When over the GS in Fredericton, the angle between the spacecraft and a line tangential to the Earth centred on the GS is $68° \pm 5°$. This means that the spacecraft will not be not parallel to the surface of the Earth when passing over the ground station. Additionally, only dampening of VIOLET's roll rate about the Z axis, direction parallel to the 20 cm long face, occurs. This makes directional antennas on VIOLET impractical. The solution that has been chosen is to build four antennas into the four solar panels on VIOLET's larger area faces. These antennas create close to an omnidirectional radiation pattern about the Z-axis so that VIOLET can communicate in most orientations when the larger faces are approximately tangential to the Earth's surface. The choice of the radiation pattern and antenna structure has a dramatic affect on the communication link's maximum performance. A higher performance communication system for VIOLET could be realized with an active attitude system which points in a single highly-directive antenna nadir facing during communication windows. This is one of the many compromises that VIOLET has to overcome.

Two radios are on-board VIOLET; the first radio is a full-duplex, very high frequency (VHF) [19] uplink and ultra high frequency (UHF) downlink radio. This radio is not part of the research in this thesis because it only communicates tracking, telemetry, and command (TT&C) information which can be communicated in a single pass. The other radio is used to communicate science data, is throughput limited, and is the main focus of research. The radio is a half-duplex radio that operates in the S band (2 GHz to 4 GHz) using 2 MHz of bandwidth and has a minimum data rate of 1 Mega-bits-per-second (Mbps) but has the potential for faster rates.

The coordinate system used throughout this thesis is the same as the coordinate system used for the NanoRacks CubeSat Deployer (NRCSD) [32].

1.2 Problem Statement

When designing a radio link for use on a satellite, designers generally use the worst-case link scenario in link budget calculations to ensure that the link operates under all conditions at a specified rate. For satellite links that do not experience significant fading this is acceptable, however for satellites that do experience fading, inefficient communication links can result. Fading reduces the usefulness of a radio link by temporarily reducing the signal-to-noise ratio (SNR) and can be caused by many environmental or human-made phenomena [33]. Some of these phenomena are: changing distance between the transmitter and receiver, rain fading, scintillation, multipath, humidity, temperature variations, and increased noise from nearby transmitters. If a link is determined to be operating in a channel where fading exists, the amount of time the link experiences fading and how deep the fades are should be determined. In the case of LEO satellites, a major fading event occurs during every pass over a GS as the distance between the GS and satellite changes dramatically. For VIOLET, the expected range between the satellite and GS in Fredericton will be between 400 km to 2400 km making the fading change slowly throughout the pass. At the beginning of the pass the radio link will start at the lowest SNR when acquisition-of-signal (AOS) occurs. As the satellite continues its pass, the SNR will increase until VIOLET is at the GS's zenith, or at the largest angle above the horizon when VIOLET does not pass directly over the GS's zenith. After the SNR peaks, the SNR will being dropping until a loss-of-signal (LOS) occurs. VIOLET's communication link is expected to achieve a maximum of approximately 18 dB SNR during a pass allowing for, depending on channel conditions, a phase-shift keying (PSK) modulation scheme to operate at bit-error rates (BER) below 10^{-4} with two, four, or eight-points in the constellation. Two, four, and eight-point PSK constellations are well understood and commonly used in satellite communications but more data could be downloaded during VIOLET's communication passes if a communication system with a consistently varying data rate was available. This communication system would need two capabilities. First, the communications system would need to match the current channel conditions as closely as possible to a data rate that results in a BER below but close to 10^{-4}. Second, the communication system would need the capability of creating more bits-per-symbol values compared to a traditional system.

Within this work, a system is presented that overcomes the fading caused by LEOs and increases the total data throughput of the communication link. This is done by developing a fractional-bits-per-symbol communication system that uses constellations that are non-powers-of-two. Sequences are developed to facilitate these transmissions. The choice of modulation schemes and the choice of constellations within the selected modulation scheme is explored. A description of sequences, the implications of using sequences, and the process for selecting sequences that perform optimally are described. Bit-to-waveform mapping is examined. An adaptive scheme is presented which uses channel state information (CSI) to decide which communication scheme is used. Simulation results of the BERs of fractional-bit-per-symbol systems are compared to systems that use constellations with a number of points that are powers-of-two. Additionally, the performance of a fractional bit rate system using adaptive modulation (AM) is compared to a system using standard constellations with either static or AM links. Finally, recommendations for future work are presented.

1.3 Literature Review

Understanding the impairments that a radio on-board a satellite will experience is of prime importance to ensuring reliable communications. For a communication link in the S band, sky noise, signal depolarization, and tropospheric scintillation do not significantly affect systems below 10 GHz making their contribution to channel impairments low. Refraction and atmospheric multipath are also not considered to be major contributors to channel impairments because communication with the S band radio is only expected to occur at angles 25° above the horizon. Reflection multipath may be a contributor but because the GS uses a highly directive parabolic antenna that will generally have a line-of-sight to the satellite, this will likely not significantly impair the channel. Inter-symbol interference (ISI) is always present in communications systems to some extent but an appropriate matched filtering will limit this effect. Blockages and cloud attenuation will occur but these events are random making them difficult to predict without statistical modelling and out of scope for this thesis. Rain fading, fog attenuation, and signal scintillations caused by the ionosphere will effect this channel but because they are likely to be consistent throughout a pass allowing for their effects to be assumed to be static throughout a pass. For VIOLET, the largest fading event during a pass is caused by the changing range between it and the GS [10, 33, 16].

Many techniques exist to adapt to fading in radio channels. Hayes provides a foundation for adaptive schemes that, with the use of a return channel for CSI, adapt to fading events when multipath occurs [15]. Adaptive schemes for communication systems present challenges when the CSI does not accurately reflect the current channel conditions. In these situations, the system may select a configuration that cannot reliably transmit information or one that results in less efficient communication. To ensure the CSI stays updated, measurements should be taken often and the delay between measurements should be accounted for in the system design [11].

AM is one adaptive method for communication systems. This system decides which constellation and/or modulation scheme is optimal for the current channel conditions [40, 34]. Goldsmith et al. describes another use for AM systems which improves energy efficiency and increases data rates over fading channels [13]. Other variable-energy systems have been presented as a method to maintain communication at a constant SNR by decreasing the transmitted power when optimal channel conditions exits and increasing power during fading events. Early work on this front, using a slowly varying channel was shown to improve the reliability of transmissions and decrease energy requirements [25, 36]. Miodrag et al. proposes a system specific for satellite communications, that uses AM to adapt a satellite's communication system to current channel conditions [27]. However, even for adaptive systems that have access to every modulation scheme and constellation, a perfect match between the bit rate and the channel conditions is difficult to obtain.

Forward error correction (FEC) is a technique used to correct bit errors at the receiver by adding error-correcting information to the bit stream at the transmitter [26]. Adaptive coding (AC) adds an adaptive layer to FEC communication systems which uses CSI to select a coding scheme that matches the channel conditions [39]. AC provides a set of coding schemes that can be selected from to vary the amount of error correcting information present in the bit stream which allows for a balance between the coding rate to data rate [14]. Punctured codes is another type of AC which adapts the amount of error correcting information in the bit stream to the channel conditions while not changing the underlying coding system [35, 24]. AC systems require varying levels of computation resources depending on the coding scheme but all require more than AM methods [9]. This makes AC systems only possible when the computational resources or specialized hardware is available.

Other systems combine AM and AC techniques into an adaptive coding and modulation (ACM) technique which allows the modulation scheme and coding to be varied to match the bit rate to current channel conditions. Goldsmith discusses an ACM system that introduces a simple water-filling technique to be used in adaptive systems [12]. The DVB-S2 standards [7, 8] also employ ACM and is widely used today for transmitting voice and video. Research on ways to improve these standards is ongoing [17].

Some research into communication systems that use non-powers-of-two-point constellations has been completed as a means of achieving fractional-bits-per-symbols. Pierce and Ekanayake et al. provided early independent research that showed that three PSK, can require about 0.75 dB less energy-per-bit than binary-phase-shift keying (BPSK) and quadrature phase-shift keying (QPSK) when the SNR is high [31, 6]. Komo et al. also provided an evaluation of three PSK and five PSK and presented bit rates, bandwidth requirements, and BER curves [21]. Recently, Nakamura et al.

and Abdelaziz et al. have developed coded three PSK which performs similarly or better than coded BPSK [28, 1]. Abdelaziz et al. have also investigated the use of triangular quadrature amplitude modulation (TQAM) using non-power-of-two-point constellations as a means of implementing AM.

In US patent 4941154, Wei describes a system where fractional bit rates are achieved by transmitting many symbols that contain a number of whole bits-per-symbol. An example in the patent describes a systems that transmits four symbols; the first three symbols transmit six bits and the fourth symbol, seven bits. This gives an average bit rate over the four symbols of 6.25 bits-per-symbol [41]. In US patent 5103227, Betts describes a system similar to the one in US patent 4941154 but focused on matching an input bit rate to a baud rate where the baud rate is not a multiple of the bit rate [2]. An example of this system is one where a bit rate of 24000 bits per second is required to be sent over a channel with a baud rate of 3200. The division between the bit rate and baud rate is 24000/3200 and reduces to 15/2. This results in a frame having two words where the total number of bits sent in the two words is 15. The first word contains eight bits and the second word contains seven bits.

Rising numbers of LEO satellites are soon planned for launch. These will require adaptive communications to ensure the resources they provide are optimally used. Many solutions to these LEO satellite communications systems have been produced [3, 9, 42, 37]. Work on LEO constellations using adaptive communications highlighted the challenges of operating LEO satellites in the slowly fading channel with antennas that are not always optimally oriented [22]. As a result, adaptive systems along with antenna optimizations have been developed to increase data throughput [18]. Further research of commercial LEO satellites has examined adaptive communication links in the Ku, K, and Ka bands due to the larger bandwidths available. Satellites that make use of these bands require more sophisticated fading mitigation and adaptive techniques due to a multitude of fading sources [38, 42, 30].

1.4 Thesis Contributions

In this section, three main contributions of this thesis are presented. The first is a method of achieving fractional-bits-per-symbols without the use of coding. Fractional-bits-per-symbols are achieved by using constellations that may have a number of points that are not a power-of-two. Sequences are used to define the number of symbol periods and the number of constellation points that can be selected from during each symbol period. By combining multiple symbols, a set of symbols called a waveform is used to transmit a whole number of bits. Three sequence listings have been computed, generated, and contain general facts about the properties of sequences. All three lists contain sequences from length one to length seven and use constellations that contain two to eight-points. The first listing, is a listing of all sequences. The second listing is a subset of the first listing and contains sequences where the constellation sizes differs by a maximum of two-points. The third listing is a subset of the second listing and contains sequences where the constellation sizes differs by a maximum of one-point.

The second contribution is an exploration into mapping waveforms to bit-strings. Four different mapping schemes, linear, random, truncated Gray coding (TGC), and euclidean distance (ED) mapping, are presented. The advantages and disadvantages of the mapping schemes are compared by determining BER versus E_b/N_0 curves, the complexity in generating the mapping scheme, and different quantization techniques. The third contribution is a look at using this system on a satellite in LEO with a communication system using a fractional-bits-per-symbol system and ideal AM. Additionally, this system is compared to other communication systems using static and traditional adaptive communication systems.

1.5 Thesis Organization

The rest of this thesis is broken down into six chapters. Chapter 2 introduces the major modulation schemes and briefly examines each. The selected modulation scheme and constellations used within the selected scheme are presented. Chapter 3 gives an introduction to sequences and the equations that govern them. Waveform mapping using the four mapping schemes is described. Chapter 4 describes VIOLET's communication system, a satellite pass simulation, and a communication simulation. Chapter 5 presents results from the communication system simulations which compares non-fractional and fractional systems, how sequence variables affects the performance of a sequence, and the effect that mapping schemes have on BER performance. Information from VIOLET's communication system is used in the satellite pass simulator to determine how the SNR of VIOLET changes throughout a pass and the theoretical throughput of different communications systems. Chapter 6 presents conclusions of this thesis and a path for continuing this research.

2. Modulation Schemes and Constellations

2.1 Modulation Schemes

Modulation schemes are the basis for encoding information onto a carrier for transmission from one location to another location. Carriers can exist in various mediums and some common examples are: on a wire, through an optical fibre, within electromagnetic fields, or through compression waves (underwater communications). The medium which information is transmitted over is of prime importance to understanding the propagation methods and limitations in data transmission. In this thesis, electromagnetic radiation at radio frequencies is propagated through an electromagnetic field to transfer information between a spacecraft and a GS located on Earth. In order to encode information onto the carrier, a property of the carrier is changed between symbol periods. For digital communications, the type of communication used in this thesis, these are discrete changes in a property and allows for information to be encoded onto the carrier. Three modulation schemes are examined here: amplitude-shift keying (ASK), frequency-shift keying (FSK), and PSK.

2.1.1 ASK

The ASK modulation scheme uses discrete carrier amplitude levels to transmit information as described in ASK's simplest form of on-off keying (OOK), where a carrier is switched on and off to represent binary ones and zeros. Communication systems implementing ASK can have simple transmitter and receiver designs making them easy to implement. ASK is also a spectrally efficient modulation scheme but it is difficult to justify for power-limited communication systems when compared to other modulation schemes. This is because ASK, generally, requires linear amplification and higher SNRs to accurately distinguish between the transmitted symbols. Noise susceptibility is another area that ASK is more susceptible to compared to other modulation schemes. This is because noise added to the signal can more easily cause changes in the received symbols.

2.1.2 FSK

The FSK modulation scheme uses discrete frequency changes of the carrier to encode information. FSK is good for dealing with noisy channels because it is a constant envelope making added noise affect the communication system less than other schemes. FSK is also power efficient but is not a spectrally efficient. This results in, significantly higher bandwidth requirements for a given data rate than other modulation schemes.

2.1.3 PSK

The PSK modulation scheme uses discrete changes in the phase of a carrier to encode information. Two major types of PSK modulation exist: differential and non-differential. Differential PSK (DPSK) requires examining the phase difference between two consecutive symbols to determine the encoded symbols whereas PSK examines the absolute phase of the carrier to determine the encoded symbols. The main advantage of DPSK is that absolute phase of the carrier does not need to be tracked. Absolute phase tracking can be difficult depending on channel conditions, especially when multipath of the carrier occurs making DPSK significantly easier to perform in the receiver compared to PSK. However, DPSK requires an extra 3 dB of SNR to achieve the same BER. PSK is reasonably spectrally efficient, where BPSK can, at a minimum, achieve a spectral efficiency of 1 bit-per-symbol-per-Hertz. PSK is also a reasonably power efficient modulation scheme and has better noise performance when compared to ASK. PSK has disadvantages though, the algorithms for detecting and recovering the phase information are complex and its spectral efficiency is less than ASK. In this thesis, PSK is the selected modulation scheme.

2.2 Constellations Containing N-Points

2.2.1 Introduction

In this section, PSK constellations containing N-points are described for use in fractional-bits-per-symbol communication systems. Constellations containing a number of points that are a power-of-two will also be used in the fractional and non-fractional systems for comparison. To generate the constellations used in the fractional system, a maximum number of points needs to be determined. During an optimal communication pass of VIOLET, the maximum received SNR has been simulated to be between 16 dB to 20 dB and the maximum allowable BER has been set to be 10^{-4}. While optimal passes achieve this maximum SNR, most passes are not optimal and do not achieve a SNR of 14 dB. This makes the maximum SNR only achieved for small periods of time on optimal passes. At a BER of 10^{-4}, 8-PSK requires approximately 16.5 dB of SNR and 16-PSK requires approximately 22.2 dB of SNR [29]. Comparing the SNR requirements to achieve BERs of less than 10^{-4}, VIOLET is unlikely to be able to utilize constellations that contain more than eight-points.

2.2.2 N-Point PSK

The PSK constellations presented in this thesis only require that the points are contained on a circle with a constant radius and that the points have an equal angle between its neighbours. The angle between points is simply defined as

$$\angle c_{\Delta} = \frac{360^{\circ}}{P}, \tag{2.1}$$

where $\angle c_{\Delta}$ is the angle between two neighbouring constellation points and P is the total number of points within the constellation. Another equation determines the angle between a specific point within the constellation and the positive real axis and is defined as

$$\angle c_{p} = \frac{p\,360^{\circ}}{P}, \tag{2.2}$$

where $\angle c_p$ is absolute angle counted from a counter-clockwise angle between the positive real axis starting at zero degrees and p is the index of the point counted from a counter-clockwise direction.

Equation 2.1 and equation 2.2 were used to create constellations that contain two to eight-points. These constellations are shown below in Figure 2.1 to Figure 2.4. These constellations are not assigned bit values but are numbered from indices 0 to $N-1$. This is because the fractional-bits-per-symbol system does not attach binary values to a specific constellation points and, depending on the mapping process selected, the binary assignments would be altered from what would be presented.

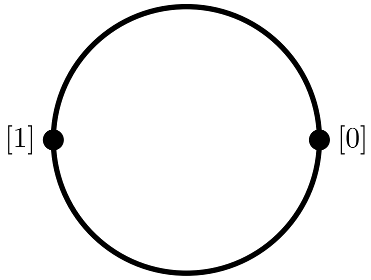

Figure 2.1: 2-PSK Constellation

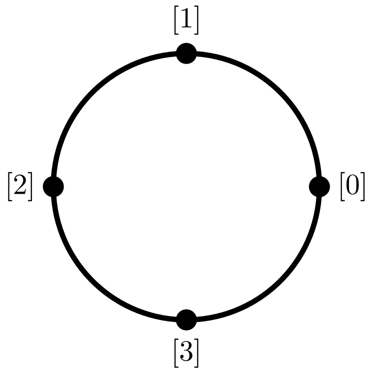

Figure 2.2: 4-PSK Constellation

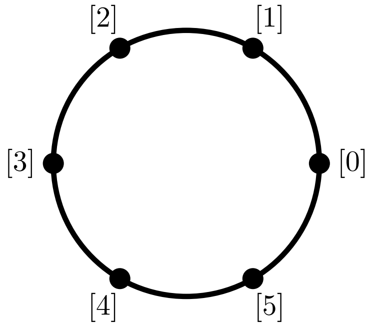

Figure 2.3: 6-PSK Constellation

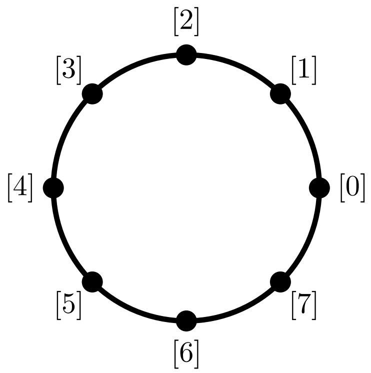

Figure 2.4: 8-PSK Constellation

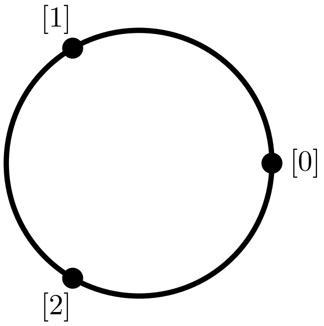

Figure 2.5: 3-PSK Constellation

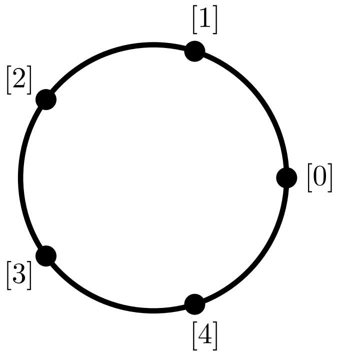

Figure 2.6: 5-PSK Constellation

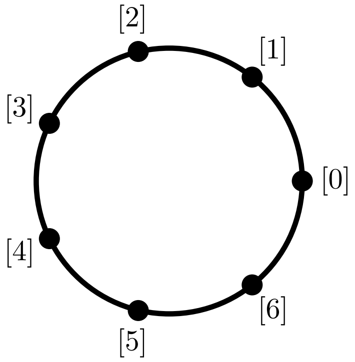

Figure 2.7: 7-PSK Constellation

3. Sequences and Mapping

3.1 Sequences

3.1.1 Sequence Description

Sequences are the basis for this fractional-bits-per-symbol system. They define an integer number of symbols grouped and decoded together. Each position in the sequence describes the number of symbols that can be selected from during a symbol period. Selecting symbols during all symbol periods of a sequence creates a unique selection of symbols called a waveform which is mapped to a unique bit-string. Sequences, however, do not describe what modulation scheme is used for transmission but four rules of sequences are proposed.

1. Sequences are sequential. Each position is selected in the order they appear.
2. Sequences are periodic. Once the end of the sequence is reached, and if the communication link continues, the sequence restarts by re-selecting the constellation described at the first position in the sequence.
3. Sequences can require switching between constellations from one symbol period to the next.
4. The resultant bit-rate of the sequence is determined by how many bits are transmitted per waveform.

In order to create a sequence, two selections must be made. The first selection is the length of the sequence. This must be a whole number. Second, the number of constellation points at each symbol period must be set. These must be whole numbers. The effect of these selections are discussed in Section 3.1.4. The mathematical representation of a sequence is described as

$$S = [s_1, s_2, \ldots, s_N], \tag{3.1}$$

where S is the sequence in use and s_n describes the number of constellation points that are available during each symbol period of S. N defines the number of symbols contained in one waveform period. To determine the number of bits per waveform, the number of unique waveforms must be determined and is

$$W_{[s_1,s_2,\ldots,s_N]} = \prod_{n=1}^{N} s_n, \tag{3.2}$$

where $W_{[s_1,s_2,\ldots,s_N]}$ is the total number of waveforms contained by S. The number of bits transmitted during one waveform period is then given by

$$b_{[s_1,s_2,\ldots,s_N]} = \left\lfloor \log_2(W_{[s_1,s_2,\ldots,s_N]}) \right\rfloor, \tag{3.3}$$

where $b_{[s_1,s_2,\ldots,s_N]}$ is the number of bits that can be transmitted with one waveform of a specific sequence. Flooring $W_{[s_1,s_2,\ldots,s_N]}$ implies that there may be more than a power-of-two number unique waveforms. This is true and has the consequence that some of the unique waveforms will not be selected. This will be further examined in Section 3.2. The number of waveforms that will be selected in the mapping process or while transmitting $b_{[s_1,s_2,\ldots,s_N]}$ bits is given by

$$W'_{[s_1,s_2,\ldots,s_N]} = 2^{b_{[s_1,s_2,\ldots,s_N]}}, \tag{3.4}$$

where $W'_{[s_1,s_2,\ldots,s_N]}$ is the size of the subset of waveforms that will be selected from the full set of waveforms. The number of waveforms that are not selected from $W_{[s_1,s_2,\ldots,s_N]}$ is given by

$$W''_{[s_1,s_2,\ldots,s_N]} = W_{[s_1,s_2,\ldots,s_N]} - W'_{[s_1,s_2,\ldots,s_N]}. \tag{3.5}$$

Lastly, a metric called the waveform utilization efficiency of a sequence, $\eta_{[s_1,s_2,\ldots,s_N]}$, is given by

$$\eta_{[s_1,s_2,\ldots,s_N]} = \frac{2^{b_{[s_1,s_2,\ldots,s_N]}}}{W_{[s_1,s_2,\ldots,s_N]}} \times 100\%. \tag{3.6}$$

$\eta_{[s_1,s_2,\ldots,s_N]}$ is the percentage of the total number of possible waveforms used. Additionally, sequences with low efficiencies are not necessarily poor performing sequences; in some cases they may be the optimal sequence.

3.1.2 Sequence Example

In this section, two sequences are used to show example calculations of how all values discussed in section 3.1.1 are calculated.

N	m	S	W	b	η
3	4	2,3,4	24	4	67
4	7	6,7,7,7	2058	11	100

Table 3.1: Example Sequence Calculation

Table 3.1 is split into six columns. Column N describes how long a sequence is, column m displays the largest constellation contained in a sequence, column S contains the sequence, column W displays the number of waveforms possible when S is expanded, column b represents how many bits can be transmitted in one waveform, and η is the waveform utilization efficiency. For the first sequence, $S = [2, 3, 4]$, L is three because S contains three symbol periods. Where the first symbol period uses a two-point constellation, the second uses a three-point constellation, the third uses a four-point constellation. m is four because the largest constellation in the sequence is a four-point constellation at symbol period s_3. W is calculated by using equation 3.2, to give

$$W_{[2,3,4]} = \prod_{n=1}^{3} s_n = (2)(3)(4) = 24. \tag{3.7}$$

$W_{[2,3,4]}$ can then be used to determine the number of bits contained in one waveform by using equation 3.3, to give

$$b_{[2,3,4]} = \left\lfloor \log_2(W_{[2,3,4]}) \right\rfloor = \lfloor \log_2(24) \rfloor = 4. \tag{3.8}$$

To determine the waveform utilization efficiency, W' needs to be determined. This is calculated using equation 3.4, to give

$$W'_{[2,3,4]} = 2^{b_{[2,3,4]}} = 2^4 = 16. \tag{3.9}$$

Finally, the waveform utilization efficiency can be calculated by using equation 3.6, to give

$$\eta_{[2,3,4]} = \frac{2^{b_{[2,3,4]}}}{W_{[2,3,4]}} \times 100\% = \frac{16}{24} \times 100\% = 67\%. \tag{3.10}$$

It is worth noting that this waveform utilization efficiency percentage presented here is rounded to the nearest whole number.

For the second sequence in Table 3.1, $S = [6, 7, 7, 7]$, L is four because S contains four symbol periods. Where symbol one uses a constellation with six-points and symbols two, three, and four use constellations with seven-points. m is equal seven to because the largest constellation is a seven-point constellation at symbol periods s_2, s_3, and s_4. The number of possible waveforms using this sequence is given by

$$W_{[6,7,7,7]} = \prod_{n=1}^{4} s_n = (6)(7)(7)(7) = 2058. \tag{3.11}$$

The number of bits that a single waveform can represent is given by

$$b_{[6,7,7,7]} = \left\lfloor \log_2(W_{[6,7,7,7]}) \right\rfloor = \lfloor \log_2(2058) \rfloor = 11. \tag{3.12}$$

The number of waveforms that will be mapped is given by

$$W'_{[6,7,7,7]} = 2^{b_{[6,7,7,7]}} = 2^{11} = 2048. \tag{3.13}$$

Finally, the waveform utilization efficiency is then given by

$$\eta_{[6,7,7,7]} = \frac{2^{b_{[6,7,7,7]}}}{W_{[6,7,7,7]}} \times 100\% = \frac{2048}{2058} \times 100\% = 100\%. \tag{3.14}$$

Rounding to the nearest whole number is also used for this waveform utilization efficiency.

In this section two example sequences were used to show the calculation of the sequence properties and metrics. This section provides examples for how the computer generated sequence values in Appendix A, Appendix B, and Appendix C were calculated. Appendix A is a full listing of sequences from length one to length seven. Appendix B lists a subset of sequences from Appendix A. The sequences contained in this subset are length one to length seven but only differ by a maximum of two-points. Appendix C lists a subset of sequences from Appendix B. The sequences contained in this subset are sequences from length one to length seven but only differ by a maximum of one-point. Rounding of the waveform utilization efficiency to the nearest whole number is also present in the three appendices.

3.1.3 Sequence Expansion

To describe how a sequence is expanded, the sequence [3 4] is expanded in Table 3.2, corresponding to Figure 2.5 and Figure 2.2.

Sequence Expansion for [3 4]												
Waveform #	1	2	3	4	5	6	7	8	9	10	11	12
s_1	0	0	0	0	1	1	1	1	2	2	2	2
s_2	0	1	2	3	0	1	2	3	0	1	2	3

Table 3.2: Sequence [3 4] Expansion Example

3.1.4 Sequence Properties

Sequence Length

The length of a sequence, N, is the number of symbol periods used when transmitting a single waveform. By changing the sequence length, the amount of information transmitted in one waveform is changed. Lengthening the sequence increases the amount of information contained in one waveform and shortening it decreases the amount. A distinction must be made though, increasing or decreasing the sequence length will not necessarily increase or decrease the amount of information transmitted in single symbol period. In fact, if the sequence is increased in length from N to $N+1$ and s_{N+1} was selected to be the average number of points contained in S, assuming that the average of S is a whole number, then the total information transmitted as $t \to \infty$ is identical. To explain this, waveforms can be related to two packet transmissions scenarios. The first scenario is where two packets of information are sent. Each packet requires a time period of T to transmit b bits. The second scenario is where a single packet that requires $2T$ but transmits $2b$ bits. Comparing these two scenarios, neither communication system has transmitted more information but scenario two has halved the number of packets transmitted. This is also true in a fractional-bits-per-symbol communication system. The reader may have arrived at the realization that in many circumstances adding or removing a symbol with the average number of points is not possible when the average number of points is not a whole number. In this scenario, adding a symbol with a number of possible points that has been rounded down to the nearest whole number reduces the information transmitted in each symbol period and adding one that has been rounded up, increases the information.

A specific case is when $N = 1$, which is the traditional method of encoding information onto a single symbol. One consequence of this is that compared to systems that use $N > 1$, soft decisions are not possible. For the other extreme case, where the length of the sequence is arbitrarily long, the possibility of symbol errors increases unless a method of correcting symbol errors is used. When this happens to very long sequences the entire waveform will be decoded incorrectly resulting in many bit errors.

Sequence Points

Just as increasing the number of points in a constellations for a non-fractional system increases the SNR requirement, the same is true for increasing the number of points in a constellation used for a fractional system. As a result, sequences that contain constellations which have a high number of points also need higher SNRs but can achieve higher data rates. Modifying the number of points, like modifying the sequence length, will increase or decrease the amount of information transmitted in one waveform but unlike the sequence length, changing the number of points directly modifies the amount of information per symbol. A balance of bit rate to BER should be achieved to maximize data throughput. This can be achieved by adjusting the number of constellation points available at each symbol period with simulations of the communication system operating in an environment that is statistically similar to the expected real-world environment.

3.1.5 Waveforms

A waveform is the unique pattern of symbols that is created when symbols are selected during each symbol period described by a sequence. While a sequence defines the number of symbol periods and the number of constellation point at each symbol period, the waveform is the specific symbols. For example, in a digitally modulated system using a PSK modulation scheme, a sequence of $S = [5\ 5\ 5]$, a symbol selection of [2 4 3] would describe a unique waveform where during symbol period s_1, the symbol at index two of a five-point constellation was selected, index four of a five-point constellation was selected, and index three of a five-point constellation was selected. A graphical representation can be used to show the symbol selections as a function of time where the number of dimensions required to create the graph is the number of dimensions needed to select a specific symbol plus one dimension for time. A version of this is shown in Figure 3.1 where three five-point constellation labelled s_1, s_2, and s_3 are shown with arced lines connecting the selected symbols.

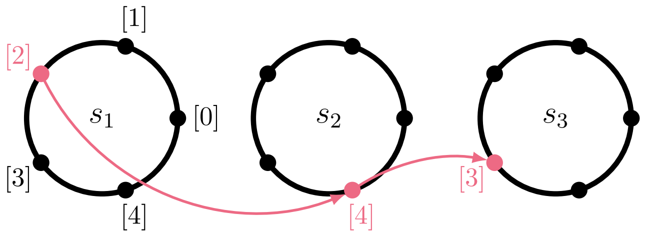

Figure 3.1: One Waveform from the [5 5 5] Sequence

3.2 Mapping

3.2.1 Introduction

An important consideration in a fractional-bits-per-symbol system is how binary strings are mapped to waveforms. This is because, generally, $W_{[s_1,s_2,\ldots,s_N]}$ is not equal to $W'_{[s_1,s_2,\ldots,s_N]}$. As a result, selecting $W'_{[s_1,s_2,\ldots,s_N]}$ waveforms of the total set of waveforms, $W_{A[s_1,s_2,\ldots,s_N]}$, must be performed. The mapped subset of waveforms selected through a mapping process is $W_{M[s_1,s_2,\ldots,s_N]}$ and the unmapped subset is $W_{U[s_1,s_2,\ldots,s_N]}$.

The rest of this section describes four mapping schemes: linear, random, TGC, and ED mapping. These mapping schemes are certainly not the only possible schemes and some may be capable of better performance with low computation complexity.

The process of waveform mapping creates two lookup tables (LUT). One called the transmitter (TX) LUT, located at the transmitter, and the other called the receiver (RX) LUT, located at the receiver. The purpose of the LUTs compared with on-the-fly calculations is to trade off storage space with computational complexity by pre-determining and storing, in memory, which waveform represents a specific bit-string. This is critical for certain mapping scheme and long sequence lengths where determining the waveform to be sent on-the-fly is computationally expensive. Thus the generation and loading of the LUTs into the communication system is done when the system is built. Additionally, two LUTs are generated to remove a search operation in the case where the data is known but the indices of the LUT are not. This allows for one LUT to contain data at the indices and the other LUT to store the reverse. For simulations presented in Chapter 4, the generation of these LUTs is completed as a pre-simulation task. The function of the TX LUT is to convert bit-strings into waveforms that are sent to the transmitter. The TX LUT is setup such that the indices of the TX LUT are equal to the bit-string to be sent and the data at the index is the waveform number. This makes the size of the TX LUT a 1 by $W_{[s_1,s_2,\ldots,s_N]}$. The purpose of the RX LUT is to convert the received waveforms back to bit-strings and is accomplished by making the index of the LUT be the waveform number and data contained at each index be the bit-string. When a communication system contains both a transmitter and a receiver, the communication system requires access to both LUTs. Both LUTs contain one set, $W_{A[s_1,s_2,\ldots,s_N]}$, and two subsets of $W_{A[s_1,s_2,\ldots,s_N]}$, $W_{M[s_1,s_2,\ldots,s_N]}$ and $W_{U[s_1,s_2,\ldots,s_N]}$. $W_{A[s_1,s_2,\ldots,s_N]}$, contains all possible waveforms from the expansion of a sequence. An example expansion is shown in 3.2. The mapped waveforms are contained in the subset $W_{M[s_1,s_2,\ldots,s_N]}$ and are the waveforms that could be transmitted when a bit-string is converted into a waveform. The unmapped waveforms are contained in the subset $W_{U[s_1,s_2,\ldots,s_N]}$ and are invalid waveforms but could be received when valid waveforms are corrupted by noise present in the channel. When a waveform is received from $W_{U[s_1,s_2,\ldots,s_N]}$, bit errors are very likely to occur but not guaranteed. Examples of these LUTs are provided by Table 3.3 and Table 3.4.

TX LUT for Sequence [3 4]										
Index: Data in Decimal+1	1	2	3	4	5	6	7	8	...	12
Waveform #	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	...	W_{12}

Table 3.3: Example of a TX LUT

RX LUT for Sequence [3 4]										
Index: Waveform #	1	2	3	4	5	6	7	8	...	12
Data in Decimal	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	...	D_{12}

Table 3.4: Example of a RX LUT

Two descriptions will be used in this section. The first description is the symbol of interest (SOI) which describes the symbol, selected from the sequence $[s_1, s_2, \ldots, s_N]$, that is of importance at a moment in time. The second description is the waveform of interest (WOI) which describes the waveform that is of importance at a certain moment in time.

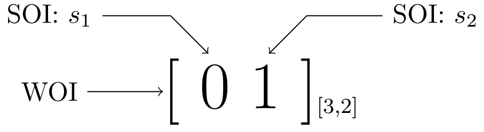

Figure 3.2: SOI and WOI Example using the Sequence [3 2]

3.2.2 Linear Mapping

The linear mapping scheme is the simplest mapping scheme and requires little computational complexity to generate. The generation of the TX LUT and RX LUT can be done in parallel creating two LUTs of length $W_{[s_1,s_2,\ldots,s_N]}$. To perform linear mapping of the TX LUT, the indices are mapped to the waveforms in the set $W_{A[s_1,s_2,\ldots,s_N]}$. This is done by mapping index one to the first waveform, W_{A1}, in the set of $W_{A[s_1,s_2,\ldots,s_N]}$, mapping index two to the second waveform, W_{A2}, and continuing until index $W_{[s_1,s_2,\ldots,s_N]}$ is mapped to the final waveform at position $W_{[s_1,s_2,\ldots,s_N]}$. The process is repeated for the RX LUT except index one, W_{A1} is mapped to the bit-string giving zero, index two is mapped to the bit-string giving one and so on until index $W_{[s_1,s_2,\ldots,s_N]}$ is mapped to the bit-string giving $W_{[s_1,s_2,\ldots,s_N]} - 1$. The valid subset, $W_{M[s_1,s_2,\ldots,s_N]}$, of the RX and TX LUTs occurs in the range of indices 1 to $W'_{[s_1,s_2,\ldots,s_N]}$ and invalid subset $W_{U[s_1,s_2,\ldots,s_N]}$ occurs in the range of indices $W'_{[s_1,s_2,\ldots,s_N]} + 1$ to $W''_{[s_1,s_2,\ldots,s_N]}$ of $W_{A[s_1,s_2,\ldots,s_N]}$. The disadvantage of the linear mapping scheme is that when the waveform utilization efficiency of a sequence is low, it results in causing one or more symbols in some symbol period within the sequence to be used very little or not at all. Table 3.5 shows an example of a linear mapping where rows with white bit-strings represents valid waveforms and rows with shaded bit-strings represents invalid waveforms. Using this example, it is easy to see that valid waveforms only use constellation points zero or one during symbol period s_2. This results in non-equiprobable signalling during symbol period s_2 and has reduced the possible distance between correct waveforms. As a result, less noise can cause a symbol zero or one to be received as symbol two. When this happens, an incorrect waveform is received and an invalid bit-string is decoded. A mapping scheme which maximizes the distance between valid symbols would be a better choice in this case. Generally, the optimal mapping scheme for a sequence with a waveform utilization efficiency close to 50% is not linear a mapping scheme.

Waveform		Bits			
s_2	s_1	b_4	b_3	b_2	b_1
0	0	0	0	0	0
0	1	0	0	0	1
0	2	0	0	1	0
0	3	0	0	1	1
1	0	0	1	0	0
1	1	0	1	0	1
1	2	0	1	1	0
1	3	0	1	1	1
2	0	1	0	0	0
2	1	1	0	0	1
2	2	1	0	1	0
2	3	1	0	1	1

Table 3.5: Linear Mapping Example for Sequence [3 4]

3.2.3 Random Mapping

Similar to linear mapping, random mapping requires little computational complexity to generate and attempts to overcome the issue where sequences with low waveform utilization efficiencies creates non-equal probabilities of symbols during some symbol periods. This implementation of random mapping generates the TX LUT and then converts the TX LUT to the RX LUT. The mapping process begins by randomly assigning the values of 1 to $W_{[s_1,s_2,\ldots,s_N]}$ to any indices in the TX LUT. The TX LUT then contains valid waveforms within indices 1 to $W'_{[s_1,s_2,\ldots,s_N]}$ and non-valid waveforms within indices $W'_{[s_1,s_2,\ldots,s_N]}+1$ to $W_{[s_1,s_2,\ldots,s_N]}$. The RX LUT is reversed from the TX LUT by moving through the RX LUT and searching the TX LUT for which waveform is mapped to each bit-string. Once found, the bit-strings in the TX LUT can be copied to the RX LUT. Waveforms that are part of the invalid subset $W_{U[s_1,s_2,\ldots,s_N]}$ are still required to have valid bit-strings but they require $b_{[s_1,s_2,\ldots,s_N]}+1$ bits to be unique. To make these waveforms only represent $b_{[s_1,s_2,\ldots,s_N]}$ bits, the most significant binary value is truncated so that all received waveforms only produce $b_{[s_1,s_2,\ldots,s_N]}$

bits. While this is the simplest solution it leaves every unmapped waveform with an incorrect bit value and almost always results in bit errors when a waveform from the invalid subset $W_{U[s_1,s_2,\ldots,s_N]}$ is received. A more complicated option would be to remap the unmapped waveforms to represent the same bit-string that a waveform that is only different by one symbol in one symbol position represents. Using Table 3.2 as a reference, if a communication system using the sequence [3 4] was selected giving $W_{[3,4]} = 12$, $W'_{[3,4]} = 8$, and $b_{[3,4]} = 3$. A mapping was performed which mapped the waveform at index nine, the symbol selection of [2 0], to the decimal number of five and the binary number of [0 1 1]. If the waveform at indices 10, having a symbol selection of [2 1], was left unmapped then its mapped binary value would be remapped to be the same as the waveform at index nine. This results in a mapping where the binary values are mapped to waveforms in a pseudo-random fashion. It does not address the problem that when this mapping scheme is used and hard decisions are made, the two waveforms mapped to a single binary value only protects against error at that specific SOI in a specific direction (i.e. the upward direction would be symbol one at SOI one is protected by symbol two at SOI one). If a symbol error occurs at a position that is not the SOI bit errors still occur in the binary stream. Other mappings can help with this problem but they do this at the requirement of more computation complexity during generation. Table 3.6 shows an example of performing random mapping on the sequence [3 4] where white bit-strings row represent valid waveforms and shaded bit-strings rows represent invalid waveforms. Random mapping attempts to overcome the main issue with linear mapping sequences with a low waveform utilization efficiency but this is not always successful. Table 3.6 is a good example that a non-equiprobable situation can still occur with random mapping and mainly occur when the number of possible waveforms for a sequence is low.

Waveform		Bits			
s_2	s_1	b_4	b_3	b_2	b_1
0	0	1	0	1	0
0	1	1	0	0	1
0	2	1	0	0	0
0	3	0	0	1	1
1	0	0	1	0	1
1	1	0	0	0	0
1	2	0	0	1	0
1	3	0	0	0	1
2	0	0	1	1	0
2	1	0	1	1	1
2	2	0	1	0	0
2	3	0	0	0	1

Table 3.6: Random Mapping Example for Sequence [3 4]

3.2.4 Truncated Gray Coding Mapping

In traditional communication systems, where the constellation contains a number of points that are a power-of-two, Gray coding, also called reflected binary coding (RBC), can be used to reduce the number of bit errors when a symbol error is received. This is achieved by assigning binary values which differ by only one bit to the closest neighbouring constellation points as shown in Figure 3.3.

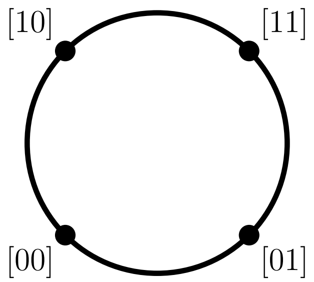

Figure 3.3: Gray Coded Offset QPSK

In the fractional-bits-per-symbol system described in this thesis, Gray coding cannot be fully implemented, in general, due to the non-power-of-two constellations requiring a non-modulo two number of binary string to be assigned to a SOI. As a result, a truncated version, called TGC, has been used which allows for some of the neighbouring waveforms to be Gray coded. The way this is achieved is by fixing all symbols in a waveform except one, the SOI. The full waveforms are then assigned binary strings for all positions in the constellation with the priorities of:

1. Gray code around the entire constellation for the selected SOI;
2. If 1 is not possible, assign a bit value that is a minimum distance away from the bit value at the SOI.

TGC mapping is performed in four steps.

Step one determines two values for the binary string assignments and sets up the initial algorithm state. The first value is a vector of length N which provides the number of choices-per-sequence-position (CPSP). The CPSP values determines how the algorithm will spread out the bit-strings across the symbol positions of S. Two different approaches can be derived. The first approach assigns an approximately equal number of choices to every sequence position to spread the binary strings out evenly between the different symbols positions. Depending on the sequence, there may not be a completely equal distribution of binary values assigned to waveforms, but this approach ensures that the assignment is mostly equal. The second approach creates a prioritized list where symbols with larger numbers of points in their constellation get more coded binary values than symbols with a fewer number of points in their constellation. For the TGC implemented in this thesis, the first option is used and the determination of CPSP is given by

$$\text{CPSP}_{[s_1,s_2,\ldots,s_N]} = \left\lfloor \frac{W'_{[s_1,s_2,\ldots,s_N]}}{N} \right\rfloor . \tag{3.15}$$

Examining equation 3.15, it is clear that there may a number of binary values which still need to be assigned if the value N is not a factor of $W'_{[s_1,s_2,\ldots,s_N]}$. To overcome this, the remainder of this division is added to the CPSP value at SOI s_1. The second value is the choices-per-base-waveform (CPBW) where the base waveform is the part of the waveform that does not change and is shown in Figure 3.4.

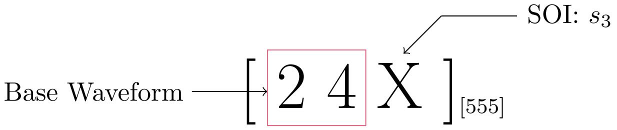

Figure 3.4: Base Waveform Description

The CPBW is also a vector of length N and, in this implementation, has been chosen to be identical to the sequence. This makes the number of choices to be made for each base waveform to be equal to the number of constellation points at the SOI. In the final part of step one, the algorithm is initialized by setting the first CPBW to be index zero for all constellations except the final symbol s_N and by setting the first bit-string to be assigned to be a string of zero bits.

Step two performs the assignment of Gray coded bit-strings to waveforms with a selected base waveform. The binary strings are assigned sequentially starting from index one in the constellation and continuing till point zero is selected. The binary strings are assigned such that they are, or as close to as possible, Hamming distance one from the string assigned to the previous symbol. For the final point, zero in this case, the Hamming distance must be compared to both the bit-string at the previous constellation point and the string at constellation point one. Again this is not always possible. Once all constellation points are assigned a binary string, the CSPS is decremented by the value at the CPBW indexed by the SOI.

Step three checks if the $CPSP_{[s_1,s_2,\ldots,s_N]}$ at n is greater than zero. If it is a new base waveform targeting the same SOI is selected. If it is zero, the next SOI is selected and then a new base waveform is selected. Selection of a new base waveform is done randomly in this implementation. Step two is then repeated till the value at n of $CPSP_{[s_1,s_2,\ldots,s_N]}$ is zero. If all values of $CPSP_{[s_1,s_2,\ldots,s_N]}$ are zero, the mapping process is complete.

Lastly, step four requires that all unmapped waveforms be assigned a bit-string so that upon reception, a bit-string is produced. Of the methods that exist to perform this task some basic ones may be to assign a random value to all unmapped waveforms or assign a fixed value to all unmapped waveforms. In this work, a fixed value is assigned to the unmapped waveforms, typically ones in all of the binary positions. For hard decisions, assigning all ones to each binary number is not ideal but for other decision types these waveforms are never selected.

Table 3.7 shows an example of the TGC mapping scheme where white bit-strings row represent valid waveforms and shaded bit-strings rows represent invalid waveforms. For the TGC scheme, the mapping scheme can perform poorly when using a sequence with a small number of waveforms. This mapping scheme can be a very computationally expensive one. As the number of possible waveforms increases, the computational complexity increases exponentially.

Waveform		Bits			
s_2	s_1	b_4	b_3	b_2	b_1
0	0	0	0	0	0
0	1	0	0	0	1
0	2	0	0	1	1
0	3	0	0	1	0
1	0	1	1	1	1
1	1	0	1	0	0
1	2	0	1	0	0
1	3	1	1	1	1
2	0	0	1	0	1
2	1	0	1	1	0
2	2	0	1	1	1
2	3	1	1	1	1

Table 3.7: TGC Mapping Example for Sequence [3 4]

3.2.5 Euclidean Distance Mapping

The goal of ED mapping is to maximize the distance between valid waveforms so that the chance of noise causing enough corruption to transform a transmitted waveform to an incorrect mapped or unmapped waveform is lowered. To determine the distance between symbols, the ED between two symbols is calculated by

$$ED_{point} = \sqrt{\text{real}(p_2 - p_1)^2 + \text{imag}(p_2 - p_1)^2}, \tag{3.16}$$

where p_1 and p_2 are the selected symbols. To reduce computational complexity, the square-root can be removed and the sum of the difference of squares provides the distance calculation between symbols. Equation 3.16 can then be expanded to include all symbol comparisons of two waveforms

$$ED_{wave} = \sum_{n=1}^{N} [\text{real}(M_n - U_n)^2 + \text{imag}(M_n - U_n)^2], \tag{3.17}$$

where n is the index of the symbol in M and U and N is the length of the sequence. M, called the mapped waveform, is a waveform from the mapped subset selected to determine the distance between itself and the unmapped waveforms U.

The ED mapping scheme occurs in four steps.

Step one initializes the algorithm by making $W_{M[s_1,s_2,\ldots,s_N]}$ be empty and $W_{U[s_1,s_2,\ldots,s_N]}$ to be equal to $W_{A[s_1,s_2,\ldots,s_N]}$. The first waveform of the subset $W_{U[s_1,s_2,\ldots,s_N]}$ is moved to $W_{M[s_1,s_2,\ldots,s_N]}$ to start the mapping process. This is typically waveform $W_{A1[s_1,s_2,\ldots,s_N]}$ but could be any waveform.

Step two computes the distance between the waveforms in M and the waveforms in U. The distances between all symbol is summed and the waveform with the largest distance from the mapped waveforms is selected and moved from $W_{U[s_1,s_2,\ldots,s_N]}$ to $W_{M[s_1,s_2,\ldots,s_N]}$. Step two continues until $W_{M[s_1,s_2,\ldots,s_N]}$ is full.

Step three assigns binary values to the subset $W_{M[s_1,s_2,\ldots,s_N]}$. This involves linearly assigning binary values to the waveforms in the order that the waveform were selected. Lastly, step four requires that all unmapped waveforms be assigned a bit-string so that upon reception of these waveforms a value is produced. Similarly to this process in section 3.2.4, a fixed value is assigned to the unmapped waveforms, typically ones in all of the binary positions. For hard decisions, assigning all ones to each binary number is not ideal but for other decision types these waveforms are never selected. Depending on the efficiency of the sequence, there may be many unmapped waveforms or few. In the case where the number of unmapped waveforms is similar to the number of mapped waveforms, a reasonable amount of distance between waveforms that contain only one symbol error is created. For sequences that have a high waveform utilization efficiency, many waveforms will only have one symbol difference between mapped waveforms making ED mapping unsuitable for these sequences. Other simpler mapping schemes should be used in this case. Additionally, for sequences with large sets of waveforms, determining the subset $W_{M[s_1,s_2,\ldots,s_N]}$ requires a significant computational resources and makes the ED method a computationally expensive mapping scheme. Similarly to the TGC scheme, as the number of possible waveforms increase, the computational complexity increases exponentially. Table 3.8 shows an example of the output of this mapping scheme where white bit-strings row represent valid waveforms and shaded bit-strings rows represent invalid waveforms.

Waveform		Bits			
s_2	s_1	b_4	b_3	b_2	b_1
0	0	0	0	0	0
0	1	0	1	1	0
0	2	1	1	1	1
0	3	0	0	1	1
1	0	0	1	0	0
1	1	1	1	1	1
1	2	0	0	0	1
1	3	0	1	1	1
2	0	1	1	1	1
2	1	0	0	1	0
2	2	0	1	0	1
2	3	1	1	1	1

Table 3.8: ED Mapping Example for Sequence [3 4]

4. Simulations

4.1 Introduction

This chapter introduces VIOLET's S band communication system and the two simulations created to determine the performance of this system. The communication system will be simulated using both fractional and non-fractional communication systems and is based on VIOLET's S band communication system.

4.2 VIOLET's Communication System

4.2.1 VIOLET's Radio

VIOLET's S band radio contains one transmitter, one receiver, and is comprised of five major components: a computer on module (CoM), a software-defined radio (SDR), a baseboard called the SDR communication platform (SCP) that connects the CoM, SDR, and support electronics together, an amplifier and RF chain board called the linear microwave board (LMB), and four antennas. The CoM component of the communication system controls the S band radio, communicates with other systems on-board the satellite, and interfaces with the SDR. The SDR either creates RF signals for transmission from in-phase and quadrature (IQ) data received from the CoM or it creates IQ signals from the received GS transmissions so that the CoM can decode IQ data into binary data. The support electronics of the SCP ensures that the transmitter is disabled during error conditions, power to sensitive components in the S band board unit (SBU) is sequenced correctly, and internal communication buses are routed to components on the SCP. Regardless of whether the radio is transmitting or receiving, all RF signals are conditioned and amplified by the LMB. The LMB contains an RF switch for changing between transmitting and receiving, filters for conditioning the transmit and receive signals, a low noise amplifier for amplifying received signals from the GS, and a three-stage power amplifier for amplifying transmission signals. The four antennas, one located on each solar panel, transmit and receive signals to or from the GS. The antennas create a nearly omnidirectional radiation pattern to allow VIOLET to communicate in most orientations. The radio radiates into a slowly fading additive white Gaussian noise (AWGN) channel using time-division duplexing (TDD).

4.2.2 VIOLET's Radio Channel

During a communication pass, the channel conditions for VIOLET's S band system create a slowly fading AWGN channel. AWGN is a model of corrupting a transmitted signal through the addition of spectrally flat Gaussian noise. This noise is added to the received signal and has some power level compared to the power of the signal. The ratio between the two signal powers at the demodulator's input is the received SNR. The slowly fading portion of this channel arises from the changing range between the satellite and the GS. As a result, the SNR of the received signal changes with the changing path loss of the transmitted signal from VIOLET. The path loss does not change quickly but it changes throughout a pass over the GS. The fading is consistent to the extent that the power of the received signal starts at the lowest SNR at the beginning of a pass and increases as the range between the GS and satellite decreases. Once the spacecraft is as close to the zenith as it will get, the SNR peaks and then decreases as the range between the spacecraft and the GS increases. Other variables could influence this. One example would be transmitting through a clear sky and then transmitting through dense clouds. This type of fading is not necessarily taken into account but could be overcome with an adaptive scheme presented or through automatic repeat requests (ARQ).

4.3 Simulations

4.3.1 Satellite Pass Simulation

A simulation, written in MATLAB®, was used to determine how different passes affect the performance of the communication link. Specifically, this simulation uses predicted future passes of a spacecraft, in this case the ISS, to determine the received SNR at the GS. The ISS was chosen as the spacecraft to base future passes on because VIOLET will be deployed from the ISS and have similar orbital parameters. As described in Chapter 1, VIOLET's orbit will drift away from the ISS but the overall statistical difference between the two orbital parameters is not significant. Additionally, it is unknown what the parameters of VIOLET's deployment will be, making it is impossible to predict the differences between the orbits. Another item that is difficult to overcome is the orbital decay of VIOLET. While the current simulation is a good approximation for the beginning of VIOLET's on-orbit life, the results do not take into account the orbital decay due to atmospheric drag, solar pressure, and other environmental factors. This discrepancy has been accepted and could be improved in future versions of the satellite pass simulator. This simulation has been accepted as a reasonable approximation of VIOLET's potential passes and exists to determine two key communication link characteristics. The first is the approximate angle above the horizon that will result in a high enough SNR to initiate a communication link with a maximum BER of 10^{-4}. In order to accomplish this, a link budget was created to calculate all relevant parameters required to accurately determine the received SNR at the GS's demodulator. The link budget takes into account all components in the RF chain and some worse case radio channel conditions. This link budget was based on King's calculator [20] provided at the AMSAT-UK's website to ensure that a MATLAB® implementation accurately determines the SNR. The second characteristic to be determined is how the SNR changes throughout a pass. This required two elements to be determined. The first element is what part of the radiation pattern of the four antennas is in view of the GS at a given time. This required the radiation pattern on the antennas to be generated and is used in the simulation to determine how attitude affects the received SNR at the GS by using the roll, pitch, and yaw rates of VIOLET. This allows for an SNR margin to be determined that ensures a communication link at a particular SNR would have a low likelihood of causing BERs of above 10^{-4} or a LOS event. The second element is how the range between VIOLET and the GS changes throughout a pass. This is simply how through a pass the satellite starts far away from the GS and gradually decreases it range. Once a peak SNR is reached, the range is at its minimum for the pass and beings to increase the range until a LOS event occurs.

4.3.2 Communication Simulation

Introduction

To determine the performance of a fractional-bits-per-symbol communication system, a simulation has been created in MATLAB®. A Monte-Carlo simulation is used for its ability to simulate statistically random processes to reveal patterns of the process being simulated. For the propose of this simulation, the statistically random process is the channel conditions and the pattern to be revealed is the BER of a specific communication system at a specific E_b/N_0. To do this, many bits are transmitted as symbols shaped by a pulse shaping filter, have noise added, received, matched filtered, quantized, and decoded. The transmitted and received bit streams are compared to determine the number of errors caused by transmission through the channel. The final number of bit errors compared to the number of bits sent gives a close approximation to the expected BER of a real system. To ensure that the BER is accurate, a minimum number of bit errors must be counted. For the simulations performed and presented in Chapter 5, a minimum of 1000 bit errors are counted for BERs at or above 10^{-5}. The simulation can be broken into five major sections, data generation, transmitter, channel, receiver, and metrics. A block diagram of the non-fractional simulation is presented in Figure 4.1 and a block diagram of the fractional simulation is presented in Figure 4.2. Section 4.3.2 gives an overview of the five sections of the non-fractional simulation and Section 4.3.2 presents the differences between the non-fractional and fractional simulations.

Non-Fractional Simulation

1. Data Generation: Data generation contains one block called Random Data Generation. This block generates a random bit-string, b_i in Figure 4.1, of a user-specified length. Depending on the number of bits specified, the block may increase the number of bits to be generated if it is not an integer multiple of the number of bits the communication system transmits in one symbol period. This number is called the smallest transmission bit-string length. Once the length of the bit-string has been checked and potentially corrected, the input data bit-string is generated with a pseudo-random number generator function. The generated bit-string is then output to the transmitter section.

2. Transmitter: The transmitter section contains three blocks, Symbol Selection, Interpolation, and Pulse Shaping. Symbol Selection determines the modulation scheme and constellation that the user has selected and converts the input bit-string, b_i, into a vector of complex-valued symbols. To do this, bit-strings of the smallest transmission bit length are isolated from b_i and converted to

complex-valued symbols by use of the TX LUT. The complex-valued symbols, a_n, are output to the interpolation block at the sampling rate of n, where n is equal to one sample per symbol (SPS). To convert a_n to a new SPS specified by the user, the interpolation block spreads out the symbol pulses by placing zero samples between the symbol pulses. a_m is output and passed to the Pulse Shaping block where the symbols pass through one half of a square-root raised cosine (SRRC) filter. The SRRC filter spreads out the energy in the symbols to multiple samples to reduce ISI. The filter requires, at minimum, 2 SPS to account for the bandwidth expansion caused by the pulse shaping filter. The filter length and excess bandwidth are user defined options and are used to generate the filter taps as a pre-simulation task. At this point, the final complex-valued output vector, t_m, has been created is sent to the channel section and a power calculation block. The power calculation is used to generate the required E_b/N_0 of the received signal and is calculated by

$$P_b = \frac{\sum_{i=1}^{I} |t_m^2|}{b_{sym} N_{sym} \mathrm{SPS}}, \tag{4.1}$$

where I is the size of the symbol vector, N_{sym} is the number of transmitted symbols, and b_{sym} is the number of bits transmitted per symbol.

3. Channel: The channel section contains the blocks: Channel, E_b/N_0 Calc, and AWGN Generation. The output from the transmitter section, t_m, is received by the Channel block where it is convolved with the channel impulse response to create the vector s_m. Noise is now added to s_m to achieve the required E_b/N_0. To do this, the AWGN Generation block generates a complex noise vector, n_m, of size s_m. The noise is generated with an initial, pre-defined, variance and the noise power calculated by

$$P_n = \frac{\sum_{i=1}^{I} |n_m^2|}{N_{sym} \mathrm{SPS}}, \tag{4.2}$$

where I is the size of the noise vector, i is the current index, SPS is the number of samples per symbol, and N_{sym} is the number of symbols. The E_b/N_0 is now determined by feeding both the noise and the signal power to the E_b/N_0 Calc block. The E_b/N_0 calculation is then given by

$$\frac{E_b}{N_0} = 10\log_{10}\left(\frac{P_b}{P_n}\right). \tag{4.3}$$

If the E_b/N_0 is not within the tolerance for the simulation cycle, the variance of the noise is adjusted by the SNR Calc Block and the noise is regenerated until the required E_b/N_0 is achieved. Once the target E_b/N_0 is achieved, n_m is added to s_m forming r_m which passes the output to the receiver section.

4. Receiver: The receiver section contains the Pulse Shaping, Decimation, Quantizer, and Decode blocks. The pulse shaping filter receives r_m from the output of the channel section where it is passed through the second half of the SRRC filter. This creates a matched filter and forms f_m. f_m contains the symbols that have been corrupted by the channel impulse response and the channel noise. f_m is then decimated by the Decimation block to rate n by keeping 1 sample for every n samples. This transforms f_m into f_n with rate n. Quantization of f_n can now be performed by passing f_n through the Quantizer block which examines every received symbol, decides what the closest symbol was and outputs a corrected symbol as if it was sent ideally. Once all of the symbols have been quantized, the symbols, k_n, are passed to the decoder, where a direct mapping between the constellation points and the data contained in each symbol is performed. All binary values are placed into the output data bit-string, b_o for comparison.

5. Simulations Metrics: The metrics section contains the Delay, XOR gate, and BER Metric blocks. To delay b_i, it is passed through the Delay block to align b_o and the delayed input data bit-string, b_{i-D}. b_{i-D} and b_o are then passed to

the XOR gate which outputs a binary string with one at the location of every bit error between b_{i-D} and b_o or zeros for every correct bit as $b_{i-D} \oplus b_o$. The BER metric block counts the ones in the bit stream, divides this by the total number of bits compared and reports a final BER for a simulation run at a specified E_b/N_0.

Fractional Simulation

Three changes exist between the communication simulations presented in Section 4.1 and in Section 4.2.

1. In the data generation section, the Random Data Generation block may increase the number of bits to be generated if it is not an integer multiple of the number of bits in one waveform period rather than one symbol period.

2. In the transmitter section, the Symbol Selection block has been replaced with a Waveform Selection block. This block, instead of choosing individual symbols to be transmitted, picks an entire waveform of symbols by accessing data in the TX LUT at the index given by the current bit-string.

3. Similarly to item two, the receiver section, replaces the Symbol Decode block with a Waveform Decode block. This block examines a number of symbols given by the sequence and decodes the bit-string sent during one waveform.

4. When E_b/N_0 is calculated for fractional systems, the number of bits-per-symbol must still be defined. This can be using two methods. The first method uses the actual number of bits per waveform period and is described by

$$b_{Asym} = \frac{b_{[s_1,s_2,\ldots,s_N]}}{N}. \tag{4.4}$$

The second method uses the theoretical number of bits that could be sent in one waveform period. This is determined by

$$b_{Tsym} = \frac{\log_2(W_{[s_1,s_2,\ldots,s_N]})}{N}. \tag{4.5}$$

The difference between both methods is discussed in Chapter 5.

Other Features

Other features of the communication simulation are briefly discussed.

1. E_b/N_0 Options: Two other E_b/N_0 options are present in the communication simulation. The first is a single E_b/N_0 simulation. This simulation only checks the BER of a single simulation at a single E_b/N_0. It is useful for debugging and specific simulations situations where only one E_b/N_0 matters. The second E_b/N_0 option uses the E_b/N_0 curve output from the satellite pass simulation to determine throughput of static and adaptive communication systems using either non-fractional or fractional systems. This allows for satellite passes to be simulated ahead of time and throughput to be determined. From these results adaptive options can be adjusted to optimize the throughput of the satellite.

2. Constellation options: Constellations within the simulation are not fixed and can be changed easily. This allows the user to quickly try new constellations which may provide better performance or represent different modulation schemes.

3. Quantization options: Two major quantization/decisions options exist within this simulation. The first option is to quantize symbols using a hard quantizer. This is the standard type of quantizer used in the communication simulation. It examines each symbol and decides the closest matching symbol. This decision type is called hard decisions since the value selected at this stage cannot be changed regardless of other factors which may cause suspicion to be placed on

the symbol decision. The advantage of this method is that it is computationally inexpensive compared to other methods. The second method available is soft decisions. It is only available to the fractional-bits-per-symbol system and, generally, performs better than hard decisions. The cost of this method is additional computation complexity. Soft decisions, in this simulator, works by receiving a number of symbols determined by the sequence and calculating the ED between the received symbol and the possible symbols. The result is that an error term is calculated for each symbol and the waveform that has the smallest error term is selected. The waveform is then checked to ensure it is a member of the $W_{M[s_1,s_2,\ldots,s_N]}$ subset. If it is not, the waveform with the second smallest error term is selected. This process continues until a valid waveform is found. This waveform is then decoded and the bit-string is output. The advantage of this decision method is that some symbol errors can be detected and corrected.

4. Mapping Options: As described in section 3.2, a few mapping options are present in the communication simulation. They vary in computation complexity and performance but, generally, there is one mapping option that is ideal for every sequence.

4.3.3 Non-Fractional Simulation Block Diagram

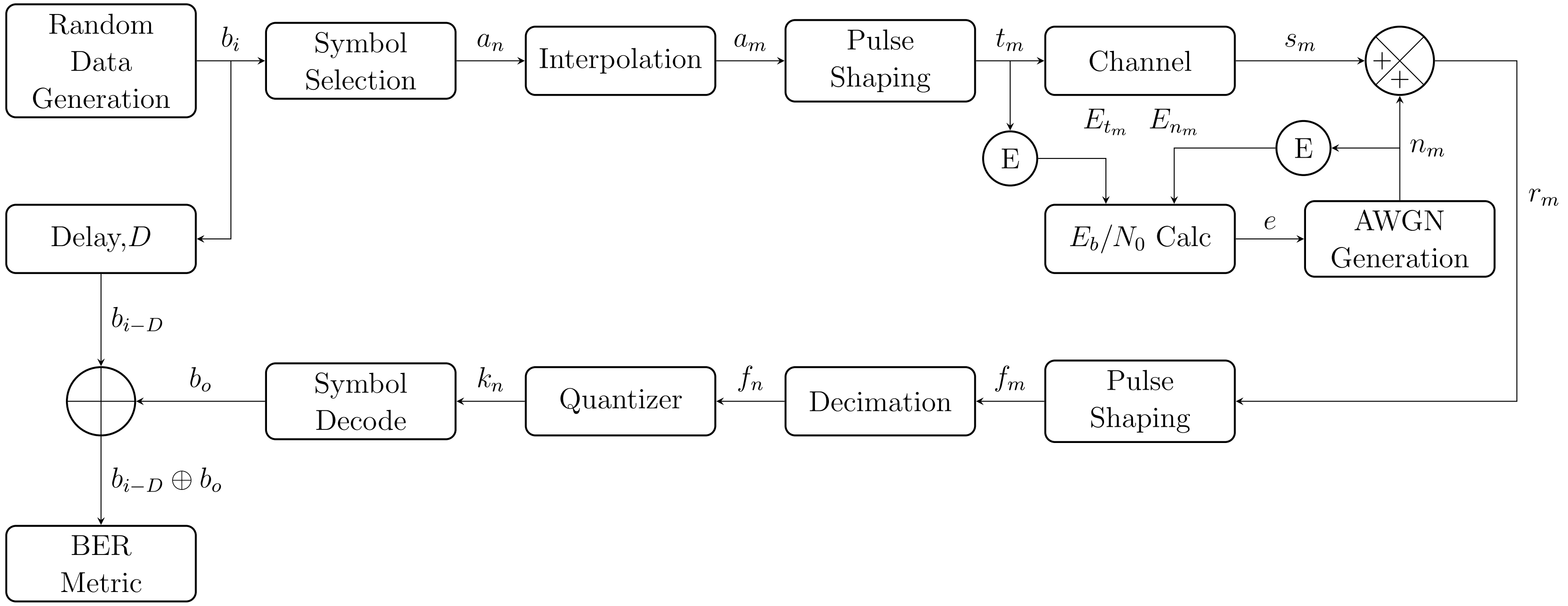

Figure 4.1: Non-Fractional Communication Simulation Block Diagram

4.3.4 Fractional Simulation Block Diagram

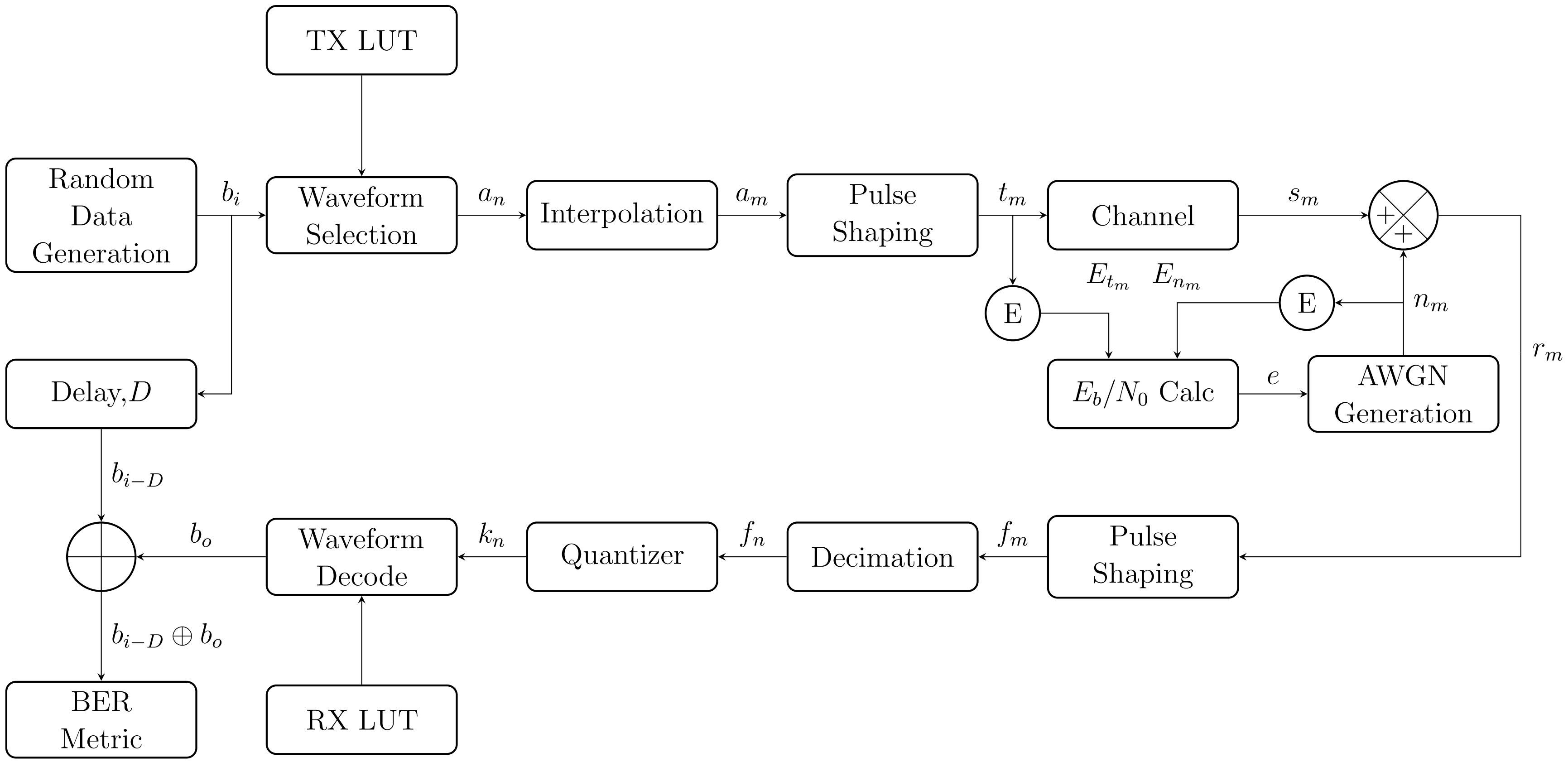

Figure 4.2: Fractional Communication Simulation Block Diagram

5. Results

5.1 Introduction

This chapter presents the results of simulating non-fractional and fraction-bits-per-symbol communication systems. Simulations were run using the communication simulator described in 4.3.2. The target of the simulations was to create BER versus E_b/N_0 curves to determine the performance of fractional-bits-per-symbol communication systems above BERs of 10^{-5}. These curves were compared to the non-fractional systems BER curves. Unless otherwise described, the settings used for all simulations were: two SPS, an E_b/N_0 sweep of 0 dB to 14 dB with 2 dB steps, the minimum of number bits that were sent for a single E_b/N_0 step was 2^{25} for BERs equal to or above 10^{-5}, at least 1000 bit errors were counted before selecting a new E_b/N_0, and a SRRC filter with 256 taps and an excess bandwidth parameter, β, of 0.5 was used for pulse shaping. Fractional-bits-per-symbol simulations that do not specify the mapping scheme or decision type, used random mapping and hard decisions as defaults. Finally, when plotting the results, all lines drawn between data points are linear.

5.2 Theoretical versus Actual Bits-per-Symbol

The method used for determining the number of bits-per-symbol in E_b/N_0 calculations changes the resultant BER curve in fractional-bits-per-symbol systems. Both methods were presented in Section 4.3.2 but to show the difference between generated BER curves simulations were run with both methods. Sequences labelled with the -A tag and plotted with unbroken BER curves used the actual bits-per-symbol calculation method, equation 4.4, and sequences labelled with the -T tag and plotted with dashed BER curves used the theoretical number of bits-per-symbol calculation method, equation 4.5. Figure 5.1 shows the difference in BER curves between the theoretical and actual bits-per-symbol calculation methods. Examining the sequences that use three-point constellations and calculate E_b/N_0 using the theoretical number of bits-per-symbol, shows that as the sequence grows in length, the E_b/N_0 requirements increase for longer length sequences. Additionally, the gap between the BER curves with

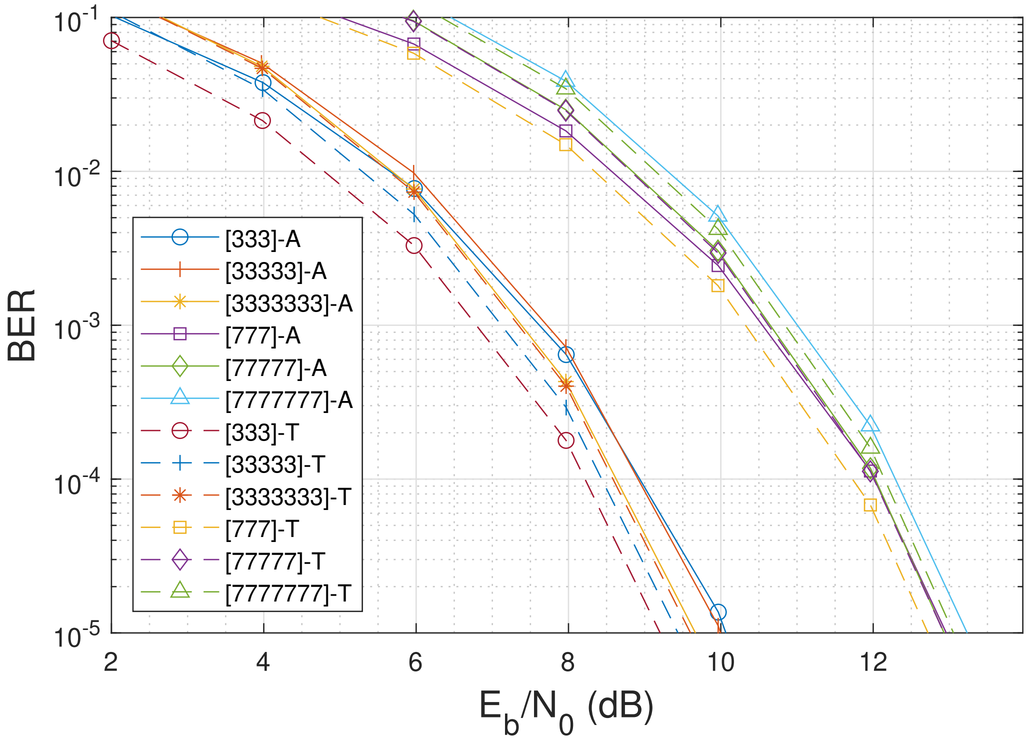

Figure 5.1: Theoretical versus Actual Bits-per-Symbol BERs

different length sequences only slightly decreases as the E_b/N_0 increases. Conversely, for the same sequences but using the actual number of bits-per-symbol for calculating E_b/N_0, the BER of longer length sequences reduces much faster than shorter sequences. This results in some longer length sequences to out-perform shorter length sequences as shown with sequences [3 3 3] and [3 3 3 3 3 3 3] between 8 dB and 10 dB.

Even though it is possible to receive $W_{[s_1,s_2,\ldots,s_N]}$ different waveforms, the receiver receives any symbol equally regardless of how the mapping process has assigned waveforms. The actual number of bits-per-symbol are used in E_b/N_0 calculations. To explain why this was chosen, a comparison between a fractional-bits-per-symbol system and a system using coding can be made. In systems using coding, E_b/N_0 is calculated using the number of data bits sent rather than the number of data bits and parity bits. In this sense, the energy of the parity bits is used to decrease the energy requirements of correctly receiving the data bits. This is coding gain when compared to uncoded systems and is achieved by increasing the distance between code words. Comparatively in a fractional-bits-per-symbol system the distance between some neighbouring waveforms is increased by only selecting some of the possible waveforms through a mapping process.

5.3 Sequence Properties

5.3.1 Sequence Length

The first sequence property examined is how the length of a sequence affects the performance of a communication system. To achieve this, sequences of length three, five, and seven using constellations with three, five, and seven-points for each symbol period were simulated. Figure 5.2 shows that increasing the sequence length does not necessary increase the probability of BER depending on the number of points and the energy-per-bit received. For example, the [3 3 3] sequence performs better than the

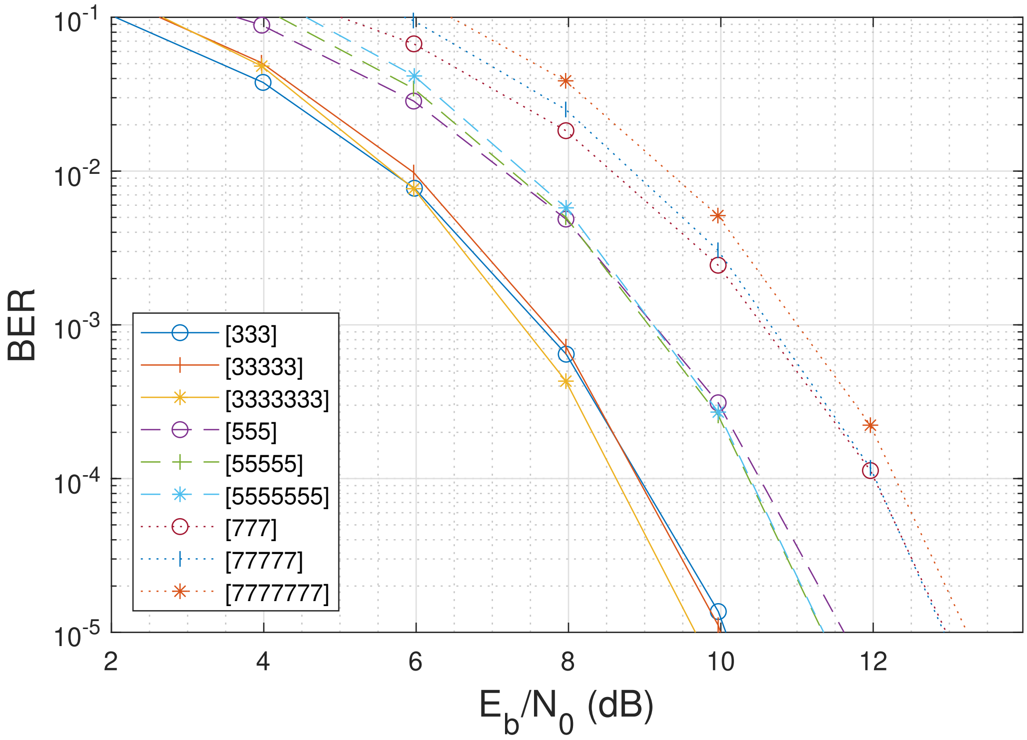

Figure 5.2: Sequence BERs with Varying Lengths

[3 3 3 3 3 3 3] sequence between 0 dB and approximately 6 dB. Above approximately 6 dB, the [3 3 3 3 3 3 3] sequence performs better than the [3 3 3] sequence. This result is also similar in the sequences with five-points in their constellations and with the sequences [7 7 7] and [7 7 7 7 7] but at different energies-per-bit.

Next, a single constellation containing three-points for all symbol periods but having sequence lengths of two to seven were simulated. Figure 5.3 again shows that shorter sequences compared to longer sequences do not always give lower BERs for higher E_b/N_0. However the [3 3] sequence is always the best performing sequence.

5.3.2 Number of Points

The second sequence property examined is how the number of constellation points for all symbol periods affects system performance. To show this, seven sequences of length three with an increasing number of constellation points were simulated. Figure 5.4 shows that as the number of points in the constellation increases, so to does

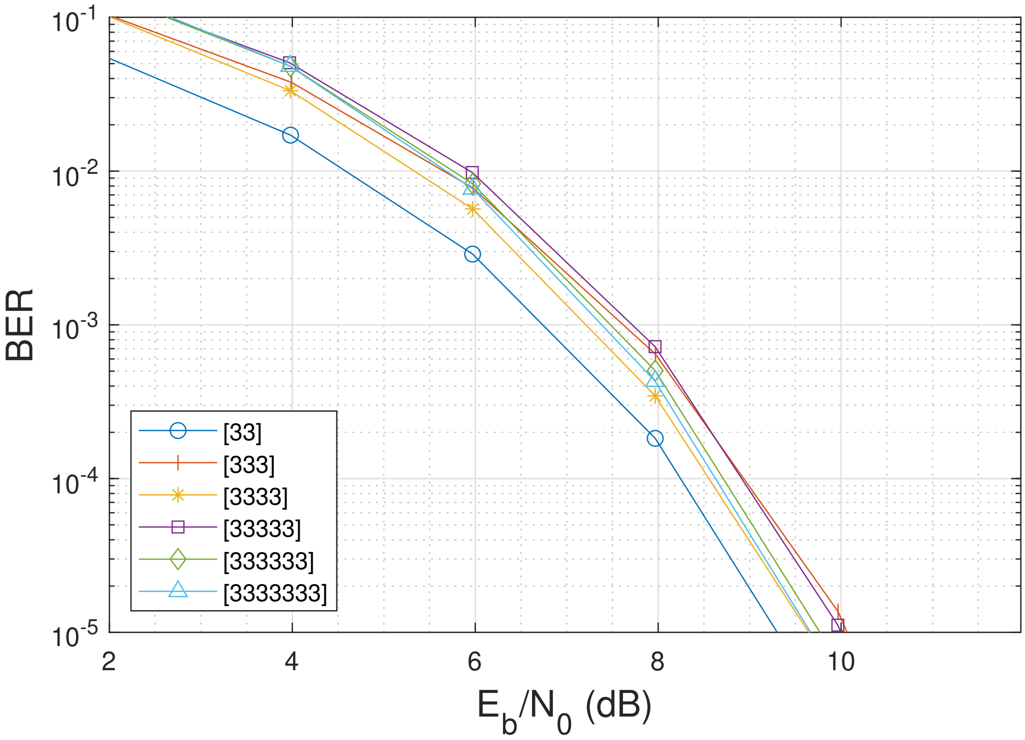

Figure 5.3: BERs of Sequences with Lengths 2 to 7 and 3-Point Constellations

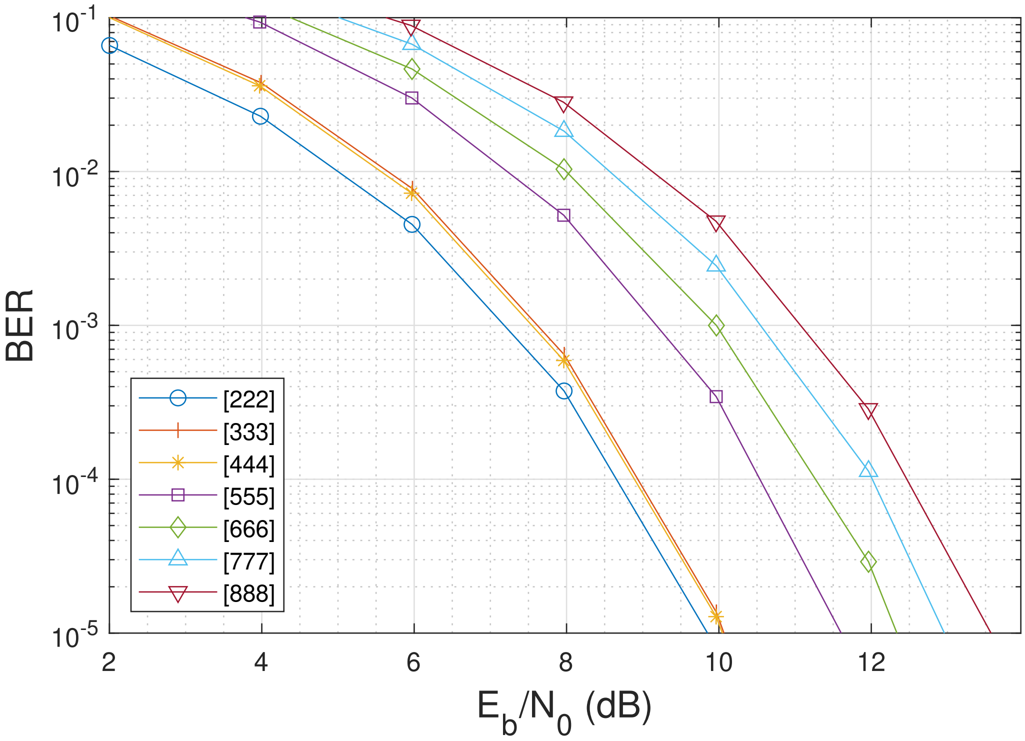

Figure 5.4: Sequence BERs for Varying Constellation Points

the energy-per-bit requirement. This is expected since tighter packed constellations require higher energies to resolve points accurately.

5.3.3 Sequences Symbols that Differ-by-at-Most-One

Figure 5.5 shows ten, length four sequences. The starting sequence uses a constellation with four-points. The sequence is then sequentially modified, such that each new sequence created contains one, two, three, then four symbols that use a five-point constellation. These sequences are simulated using hard decisions and then re-simulated using soft decisions.

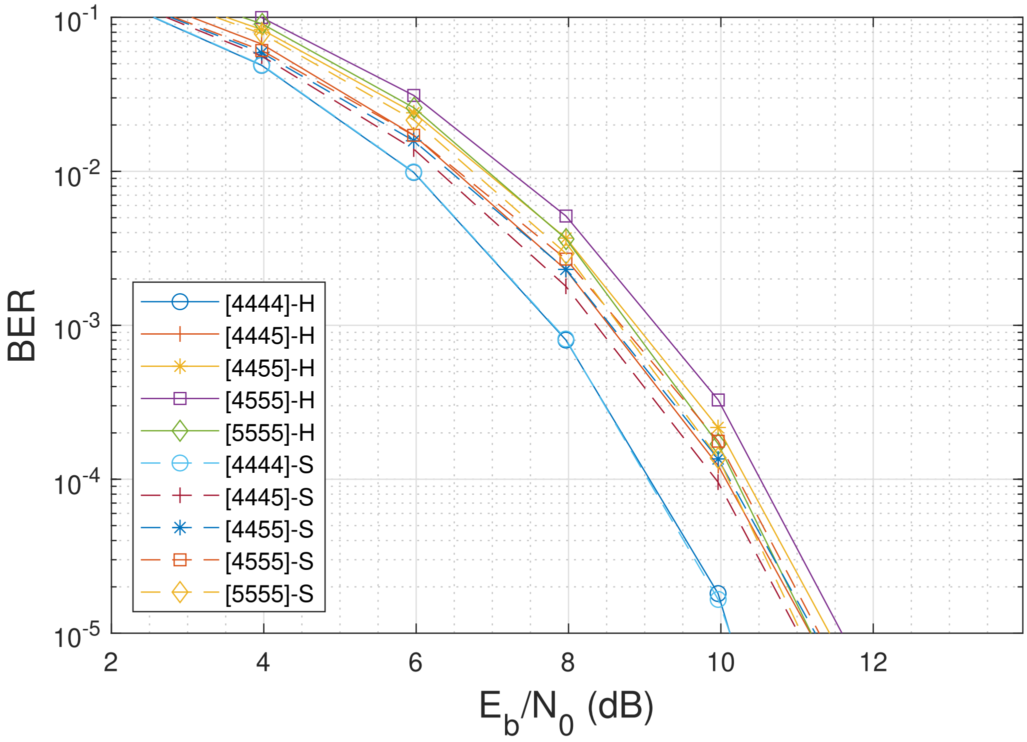

Figure 5.5: Sequences from [4 4 4 4] to [5 5 5 5] BERs

Figure 5.5 shows five sequences using both hard and soft decisions. The [4 4 4 4] sequence has the lowest energy-per-bit requirement. Neither decision type had a significant impact on performance making the addition computational requirements of soft decisions not recommended for the [4 4 4 4] sequence. Sequence [4 4 4 5], contains a single symbol using the five-point constellation and increased E_b/N_0 requirements compared to the [4 4 4 4] sequence of approximately 1 dB at a BER of $10^{-4}$when hard decisions were used. Sequences [4 4 5 5], [4 5 5 5], and [5 5 5 5] also require higher energies compared to the [4 4 4 5] sequence of 0.21 dB, 0.35 dB, and 0.52 dB respectively. Conversely to the [4 4 4 4] sequence, sequences [4 4 4 5], [4 4 5 5], [4 5 5 5], and [5 5 5 5] performed slightly better when soft decisions were used with a reduction of energy-per-bit of 0.092 dB, 0.25 dB, 0.33 dB, and 0.081 dB respectively. However, as presented in Table 5.1, the total number of bits transmitted per waveform for sequences [4 4 4 5], [4 4 5 5], and [4 5 5 5] is the same as the [4 4 4 4] sequence. As a result, only the [5 5 5 5] sequence transmits more bits per waveform and requires approximately 1.2 dB more energy-per-bit compared to the [4 4 4 4] sequence.

Sequence	TX Bits	η
[4 4 4 4]	8	100
[4 4 4 5]	8	80
[4 4 5 5]	8	64
[4 5 5 5]	8	51
[5 5 5 5]	9	82

Table 5.1: Bits-per-Waveform Based on Figure 5.5

Examining Figure 5.5 and Table 5.1, only two sequences, [4 4 4 4] and [5 5 5 5], are viable selections from this subset of sequences.

5.3.4 Waveform Utilization Efficiency

To determine the effect that the waveform utilization efficiency, η, has on sequence performance, five sequences were selected with the same length, a similar number of points for all symbol periods, but efficiencies varying from 57% to 100%. Figure 5.6 displays the results of simulations run for sequences with varying efficiencies. Three groupings of sequences that perform similarly can be seen in the results. Group one contains sequences [4 3 3 3] and [4 4 4 4], group two containing sequences [4 3 5 5] and [4 4 5 5], and group three containing sequences [5 5 6 6] and [5 6 6 6]. These

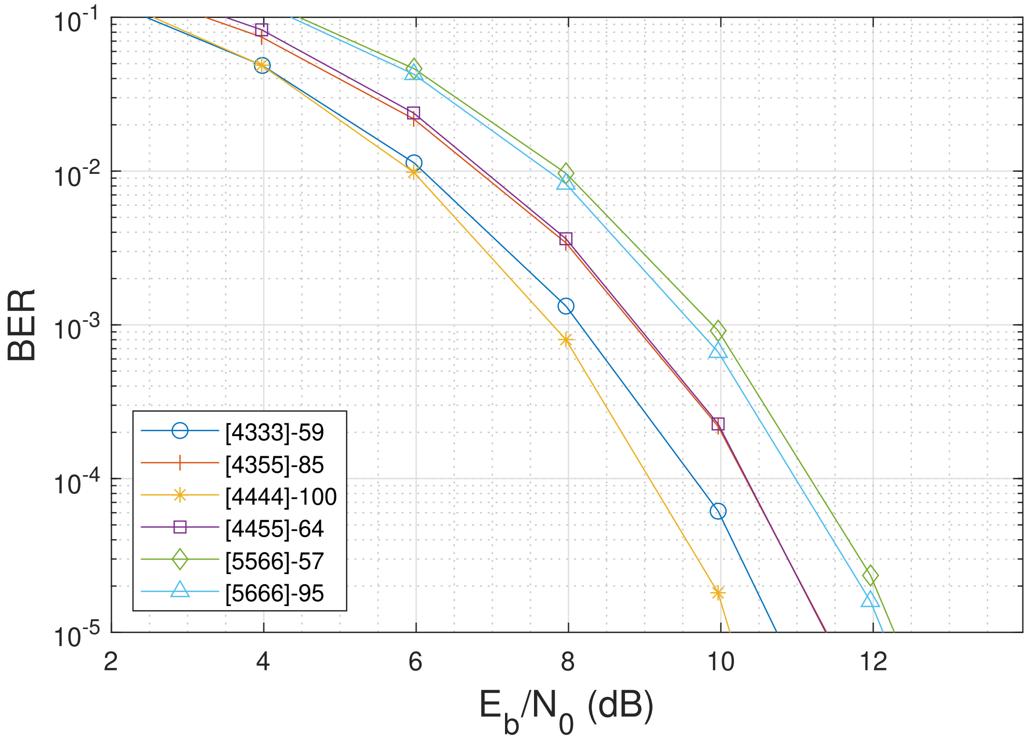

Figure 5.6: Sequence BERs with Varying η

groups exist even though the waveform utilization efficiencies of the sequences are significantly different. This shows that the performance of sequences is more affected by number of bits sent per waveform and largest constellation size.

5.4 Hard versus Soft Decisions

To determine the gain that can be obtained from using soft decisions and the sequence properties that allow soft decisions to perform best, four sequences were simulated using hard and soft decisions. In section 5.5, this will be continued to determine the affect that mapping schemes have on soft decisions. The sequences used and their waveform utilization efficiencies are: [3 3 3]-59%, [3 3 4]-89%, [5 5 5 6 6]-91%, and [5 5 6 6 6]-76%. Figure 5.7 shows the effect of soft decisions compared to hard decisions on a fractional-bits-per-symbol system. Performance gain can be achieved by using soft decisions and achieves the most performance gain at lower BER. Inefficient

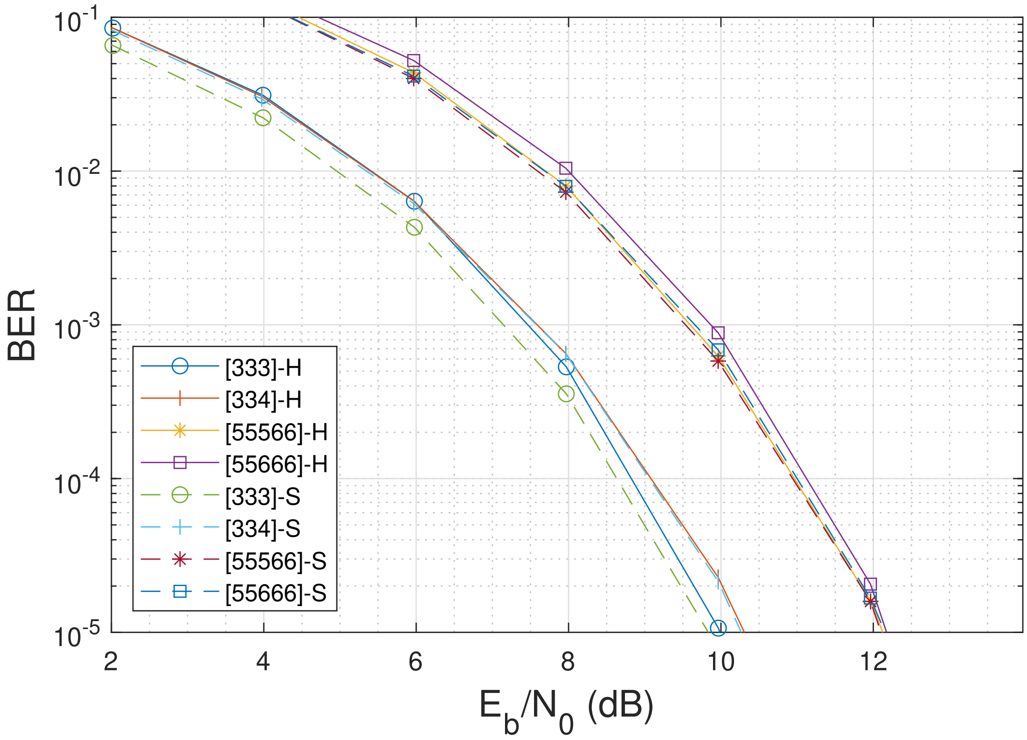

Figure 5.7: Hard and Soft Decision BERs

sequences gain the most performance, making these potentially worth the extra computational complexity. For highly efficient sequence very little performance is gained. The best comparison is shown with the sequence [3 3 3], having an efficiency of 59%, and [5 5 5 6 6], having an efficiency of 91%. For the [3 3 3] sequence at 6 dB using hard decisions, the achieved BER is 6.5×10^{-3}, while the same sequence using soft decisions achieves a BER of 4.3×10^{-3}. At 8 dB using hard decisions, the BER is 5.4×10^{-4} and when using soft decisions the BER is 3.4×10^{-4}. Comparing these values to the [5 5 5 6 6] where virtually no performance difference was achieved at an E_b/N_0.

5.5 Mapping Comparisons

5.5.1 Length 2 Sequences

Sequences [3 3] and [7 8] were simulated using either linear or ED mapping schemes. The sequences were then re-simulated using soft decision. The two sequences, [3 3] and [7 8], have waveform utilization efficiencies of 89% and 57% respectively.

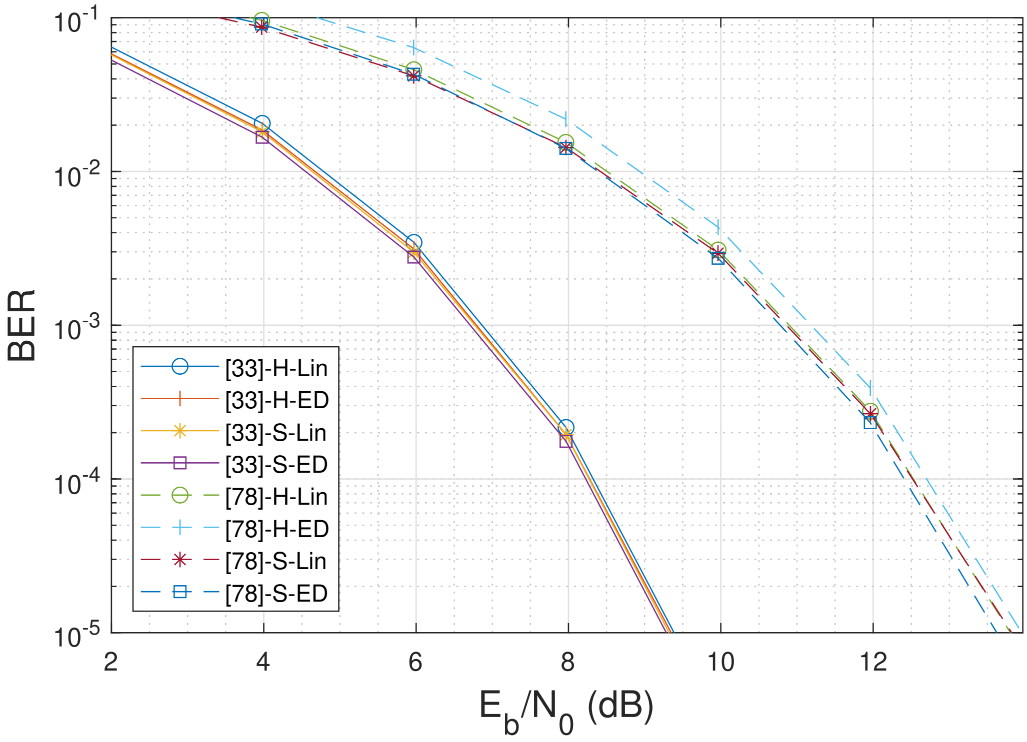

Figure 5.8: Sequences [3 3] and [7 8] Mapping BERs

Figure 5.8 shows the simulation results of two length two sequences using different mapping schemes. For the higher efficiency sequence, [3 3], the effect that the mapping scheme has on performance is not significant at high E_b/N_0. At low E_b/N_0 the ED mapping scheme using soft decision performed best. For the lower efficiency sequence, [7 8], the results were similar but the BER curve using hard decisions and ED mapping showed significantly worse performance compared to the other options.

5.5.2 Length 3 Sequences

Sequences [3 3 3] and [3 3 4] were simulated with all four mapping schemes using both hard and soft decision methods. Sequences [3 3 3] and [3 3 4] have waveform utilization efficiencies of 59% and 89% respectively. Figure 5.9 shows the simulation results of the [3 3 3] and [3 3 4] sequences with all four mapping schemes and both decisions types. In the next four plots, narrower E_b/N_0 ranges and individual sequences will be isolated to highlight the performance differences.

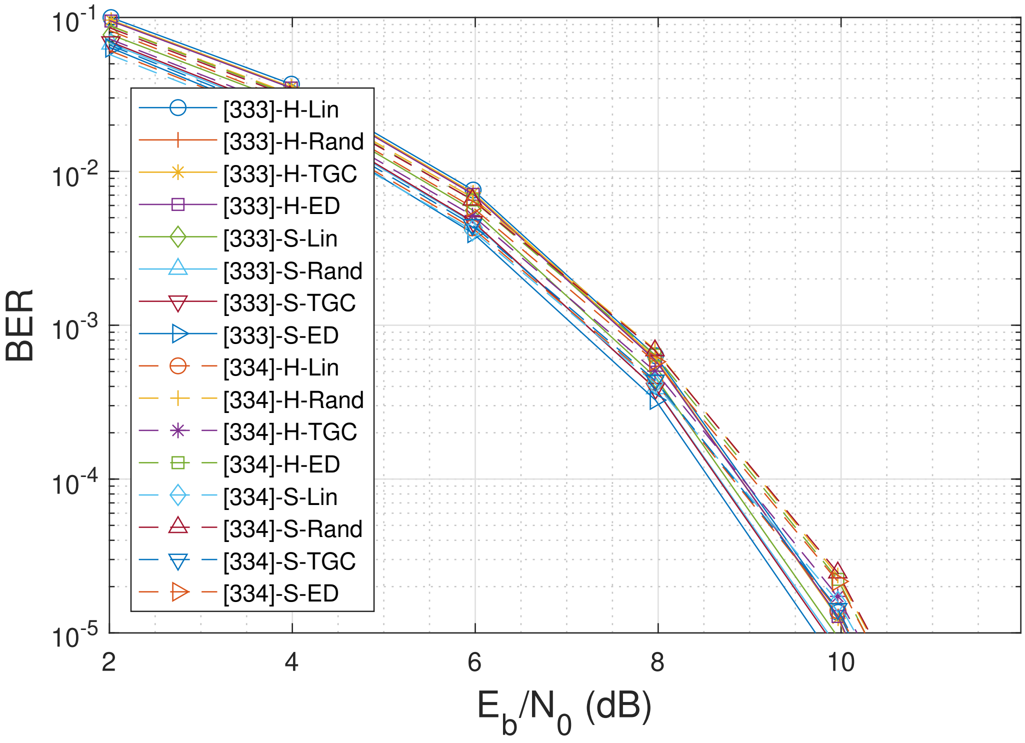

Figure 5.9: Sequences [3 3 3] and [3 3 4] Mapping BERs

Figure 5.10 and Figure 5.11 are expanded views of Figure 5.9 at 6 dB and 8 dB to better show the separation between different mapping schemes and decision methods with the [3 3 3] sequence. These figures show a clear performance improvement when using soft decisions. When using soft or hard decisions, the ED mapping scheme performs best.

Figure 5.12 and Figure 5.13 are expanded views of Figure 5.9 at 6 dB and 8 dB to

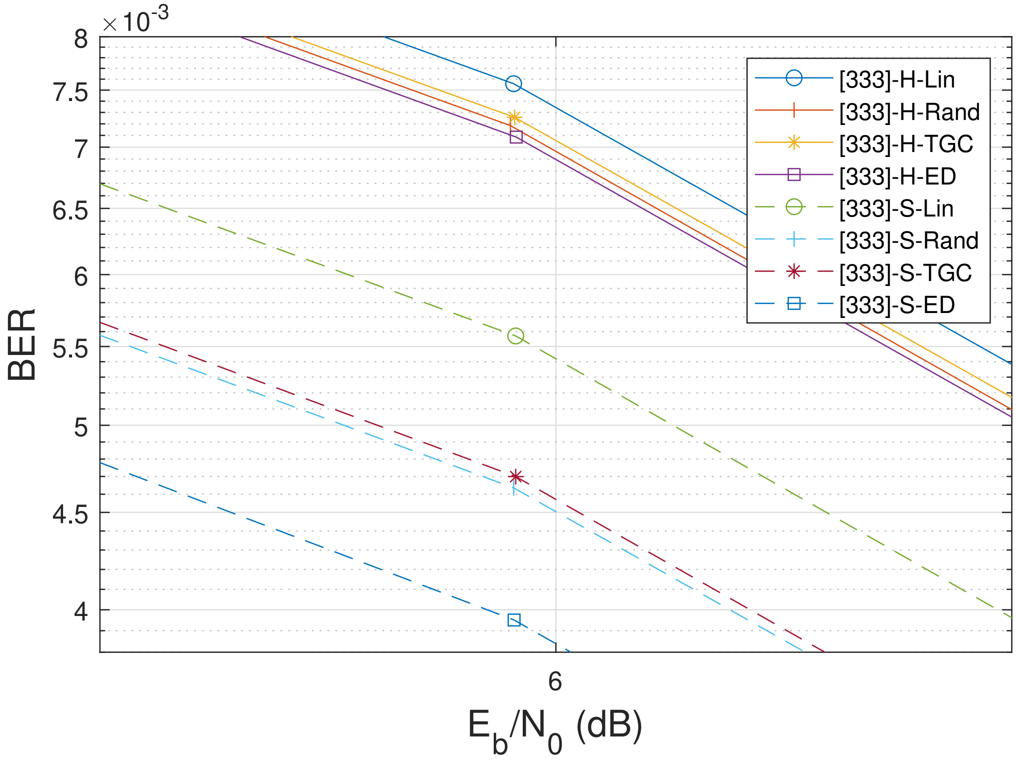

Figure 5.10: Sequence [3 3 3] Mapping BERs at 6 dB

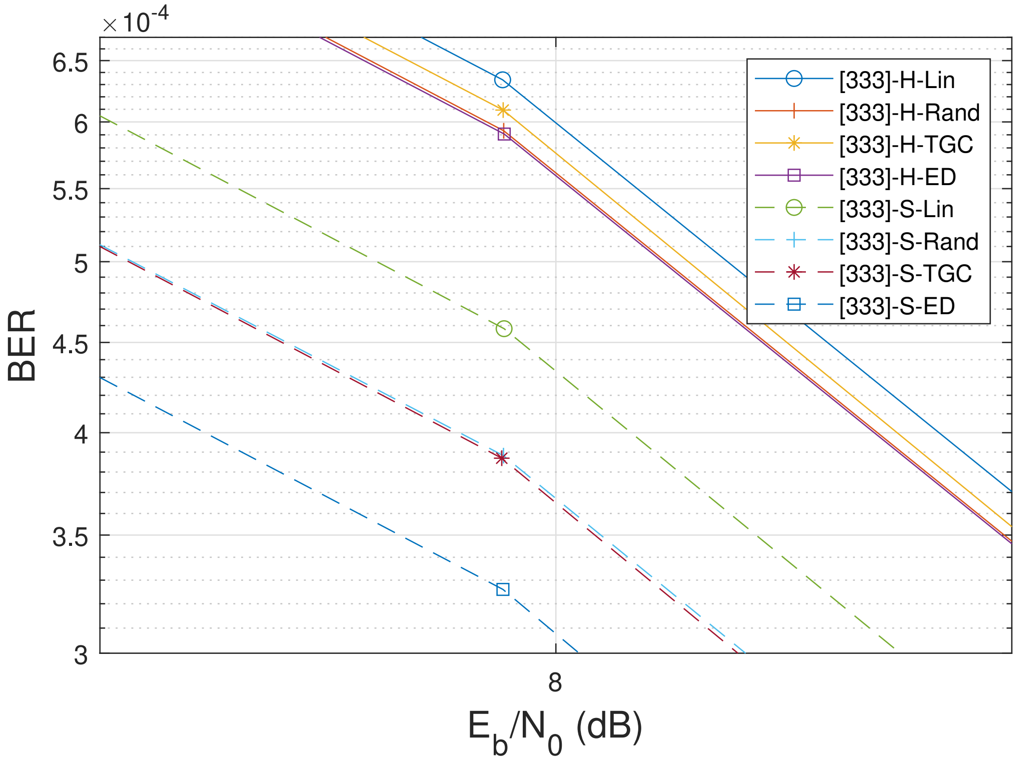

Figure 5.11: Sequence [3 3 3] Mapping BERs at 8 dB

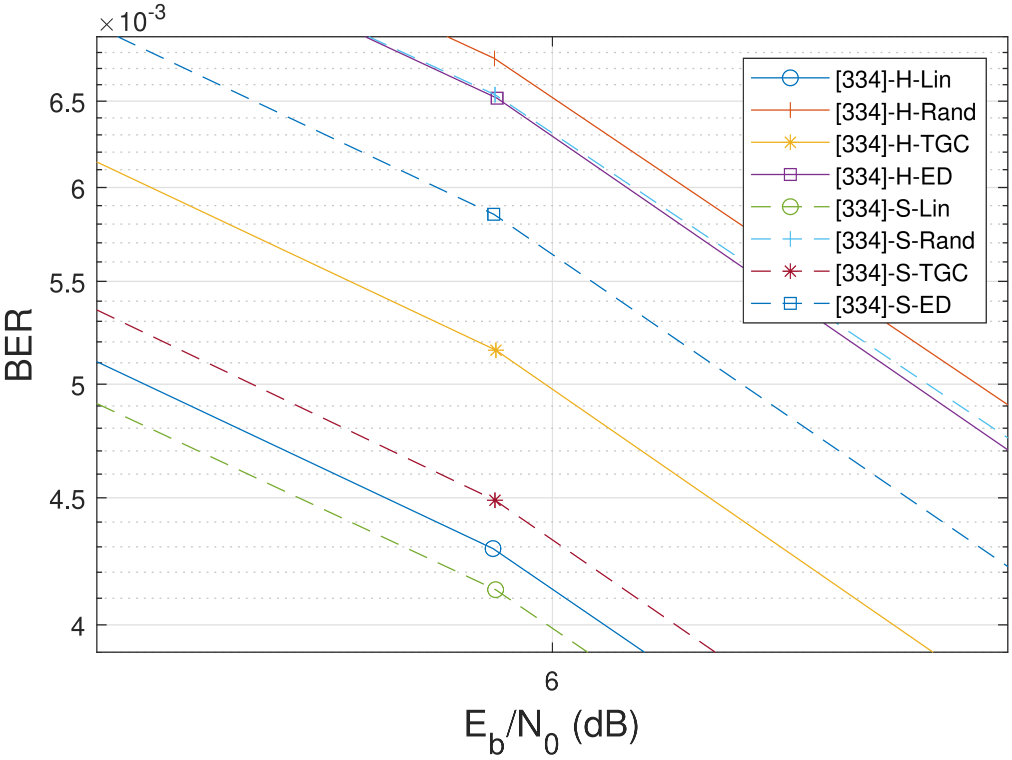

Figure 5.12: Sequence [3 3 4] Mapping BERs at 6 dB

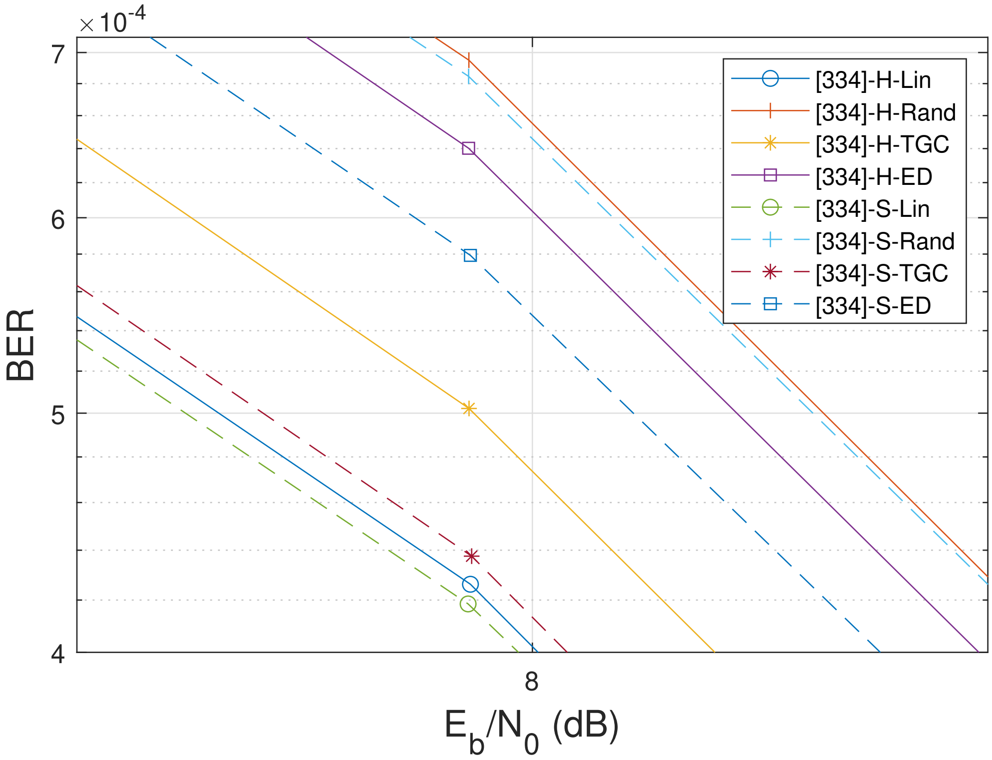

Figure 5.13: Sequence [3 3 4] Mapping BERs at 8 dB

better show the separation between different mapping schemes and decision methods with the [3 3 4] sequence. For this sequence, the selected mapping scheme makes a larger impact on performance than the decision type. The best performing mapping schemes are linear and TGC compared to random and ED mappings. Soft decisions do provide a small improvement compared to hard decisions for the same mapping scheme but do not impact the performance as much as with the [3 3 3] sequence. The system using linear mapping and hard decisions is one of the best performing systems with the smallest computational requirements. This makes it one of the best overall choices since it has better performance than many systems using soft decisions.

Comparing these results to Figures 5.10 and 5.11, it is clear that waveform utilization efficiency plays a role in which mapping scheme and decision type is optimal.

5.5.3 Length 4 Sequences

Sequences [4 3 5 5] and [4 4 5 5] were simulated with all four mapping schemes and hard and soft decision types. Sequences [4 3 5 5] and [4 4 5 5] have waveform utilization efficiencies of 85% and 64% respectively. Expanded views at 8 dB and 10 dB are also presented.

Figure 5.14 shows simulations of the [4 3 5 5] and [4 4 5 5] sequences with all four mapping schemes using hard and soft decisions.

Figure 5.15 and Figure 5.16 are expanded views of Figure 5.14 at 8dB and 10 dB to better show the separation between different mapping schemes and decision methods with the [4 3 5 5] sequence. Sequence [4 3 5 5] shows a similar result to the [3 3 4] sequence shown in Figure 5.9, where linear and TGC mapping schemes outperform random and ED. Figure 5.17 and Figure 5.18 are expanded views of Figure 5.14 at 8 dB and 10 dB to better show the separation between different mapping schemes and decision methods with the [4 5 5 5] sequence. For this sequence, a combination of the mapping scheme and decision type gives rise to the the performance variation.

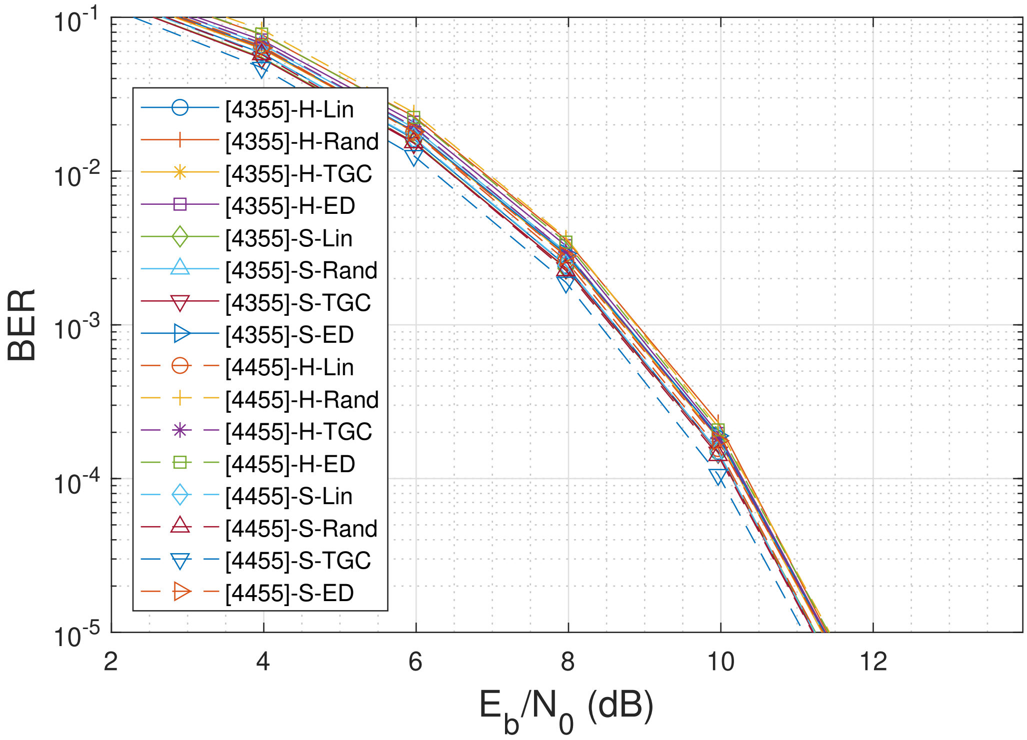

Figure 5.14: Sequences [4 3 5 5] and [4 4 5 5] Mapping BERs

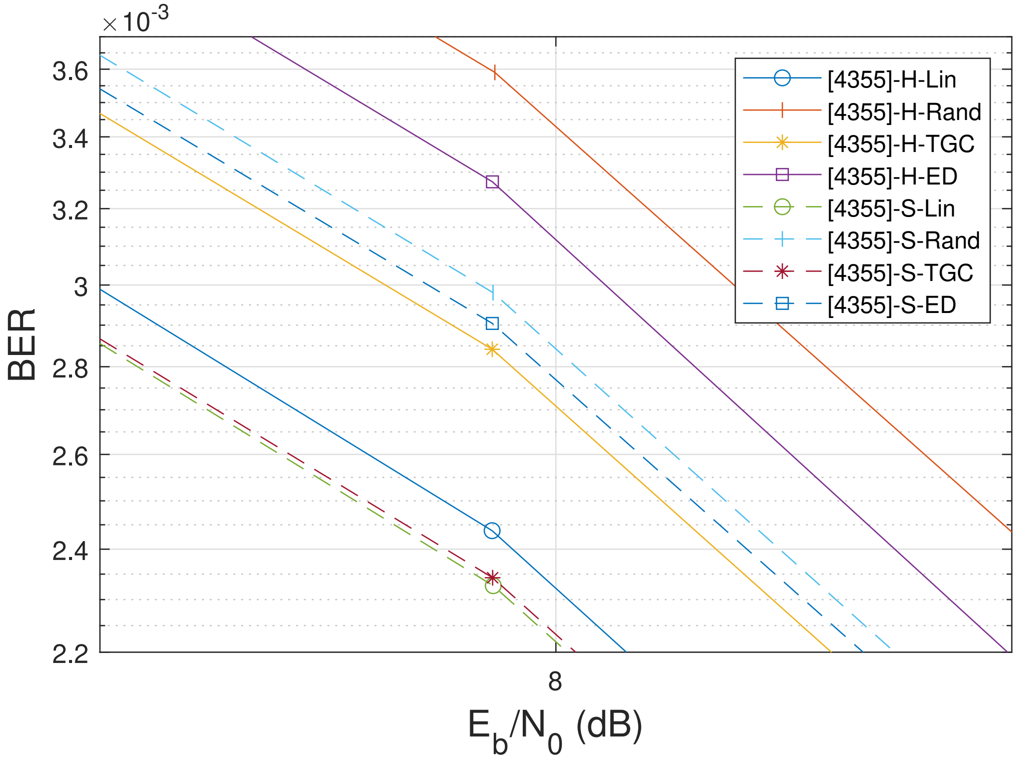

Figure 5.15: Sequence [4 3 5 5] Mapping BERs at 8 dB

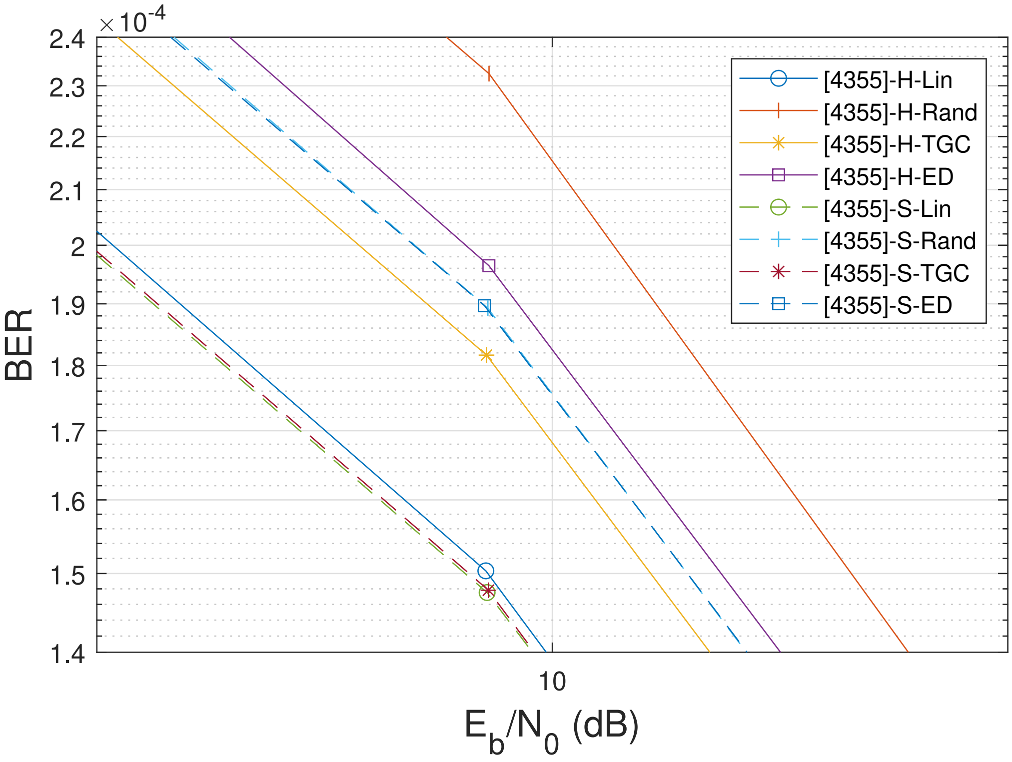

Figure 5.16: Sequence [4 3 5 5] Mapping BERs at 10 dB

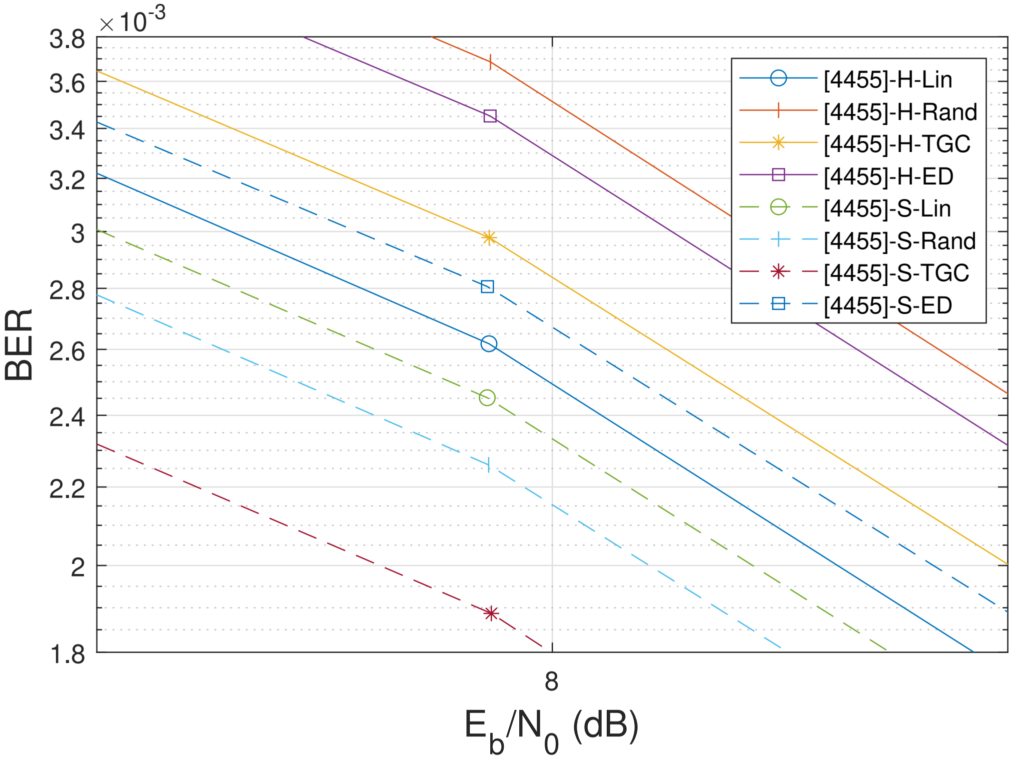

Figure 5.17: Sequence [4 4 5 5] Mapping BERs at 8 dB

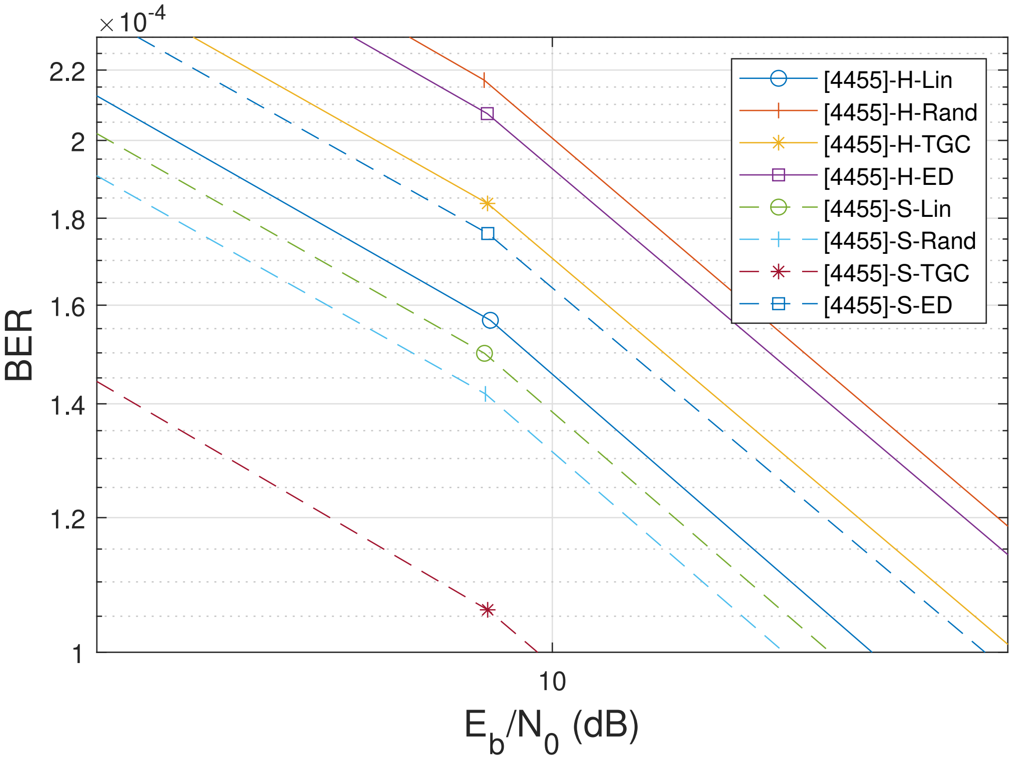

Figure 5.18: Sequence [4 4 5 5] Mapping BERs at 10 dB

The TGC sequence with soft decisions performs best but if only hard decisions are available, linear mapping is the best performing scheme.

In this section, the selection of a mapping scheme has been shown to have an effect on the performance of a fractional-bits-per-symbol system. Sequences with waveform utilization efficiencies of less than 65%, [7 8], [3 3 3], and [4 4 5 5], were shown to perform, generally, better with soft decisions and either ED or TGC mappings. As the efficiency increases, the performance difference between hard and soft decisions makes less of an impact. The [4 4 5 5] sequence begins showing this in Figures 5.17 and 5.18, where the results show that some of the mapping schemes using hard decisions perform similarly or better compared to others using soft decisions. This trend continues in the highly efficient sequences of [3 3], [3 3 4], and [4 3 5 5], where the choice of decisions type can have very little impact on the performance compared with other mapping schemes. Additionally, these highly efficient mapping schemes perform best with linear mapping schemes using either hard or soft decisions and then, generally, followed closely by TGC mapping with soft decisions.

5.6 Fractional versus Non-Fractional Systems

In this section both fractional and non-fractional systems are compared. The non-fractional systems simulated were BPSK, QPSK, and 8-PSK and are shown in Figures 5.19 through Figure 5.22. Figure 5.19 shows the results of simulating non-fractional and length 2 sequences with hard decisions and random mapping. From the results present in sections 5.3.4 and 5.5, it is known that sequences with low waveform utilization efficiencies, [5 5]-64% and [7 7]-65%, generally perform best when paired with the ED mapping scheme. For sequences that have high efficiencies, [6 6]-89% and [8 8]-100%, linear or TGC mapping generally perform best. This makes the BER curves for sequences [5 5] and [7 7] likely the least optimal and the results of

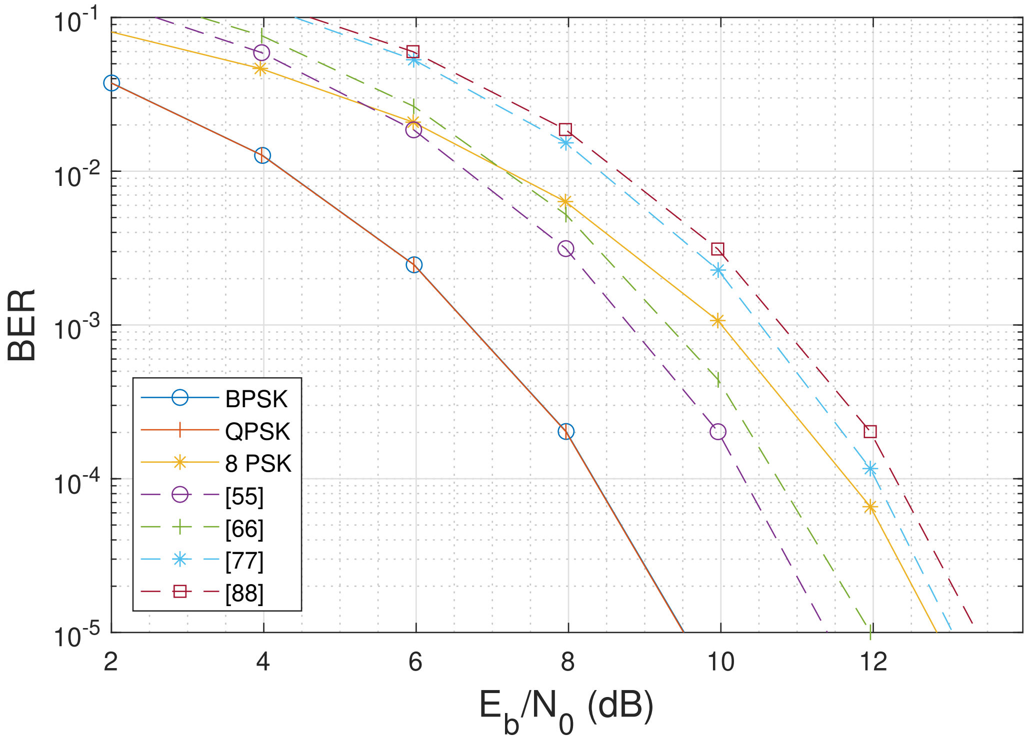

Figure 5.19: Length 2 Fractional Sequences and Non-Fractional Systems BERs

sequences [6 6] and [8 8] the most optimal. Comparing these four sequences to BPSK, QPSK, and 8-PSK shows that, at BER rates lower than 10−3, the performance of sequences [5 5] and [6 6] is in between BPSK/QPSK and 8-PSK and the sequences [7 7] and [8 8] generally performs worse than 8-PSK. As a result, it is almost always better to use 8-PSK rather than the [7 7] or [8 8] sequences.

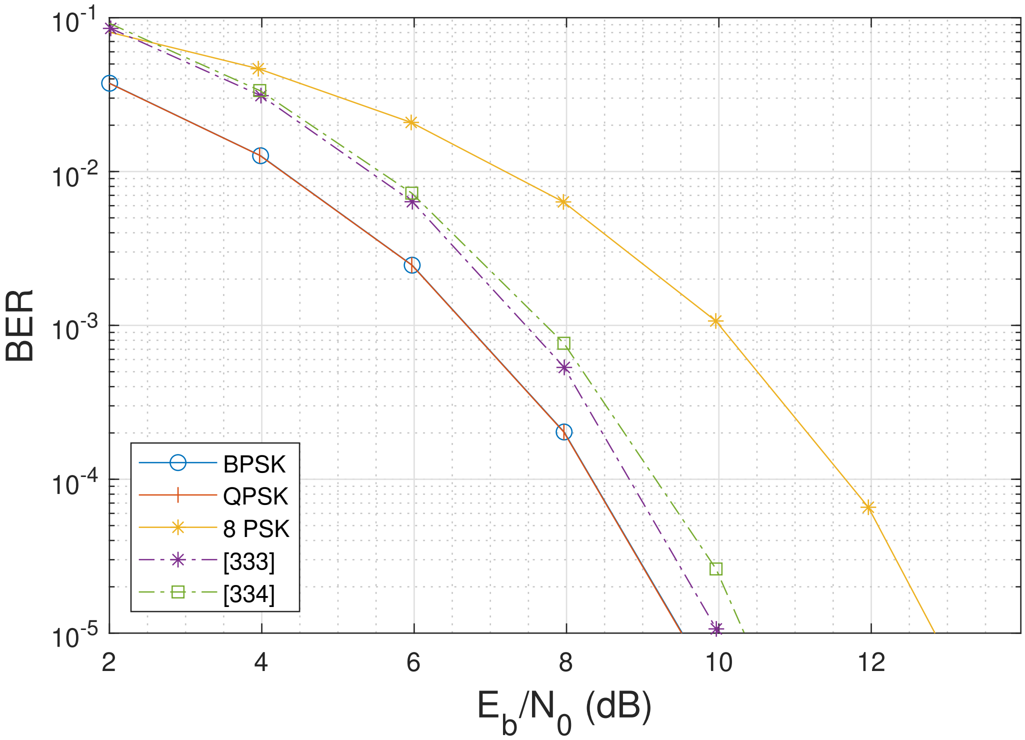

Figure 5.20: Length 3 Fractional Sequences and Non-Fractional Systems BERs

Figure 5.20 shows a comparison of BPSK, QPSK, and 8-PSK schemes and two sequences, [3 3 3] and [3 3 4], using hard decisions and random mapping. Sequence [3 3 3] has an efficiency of 59% and transmits 4 bits per waveform. Sequence [3 3 4] has an efficiency of 89% and transmits 5 bits per waveform. For the [3 3 4] sequence, above approximately 8 dB, the energy-per-bit requirement decreases compared to the [3 3 3] sequence. Neither the [3 3 3] or the [3 3 4] have lower E_b/N_0 than the QPSK scheme.

Figure 5.21 shows a comparison of three PSK schemes and four sequences, [5 5 5 5], [6 6 6 6], [7 7 7 7], and [8 8 8 8]. As the number of points increase for each sequence,

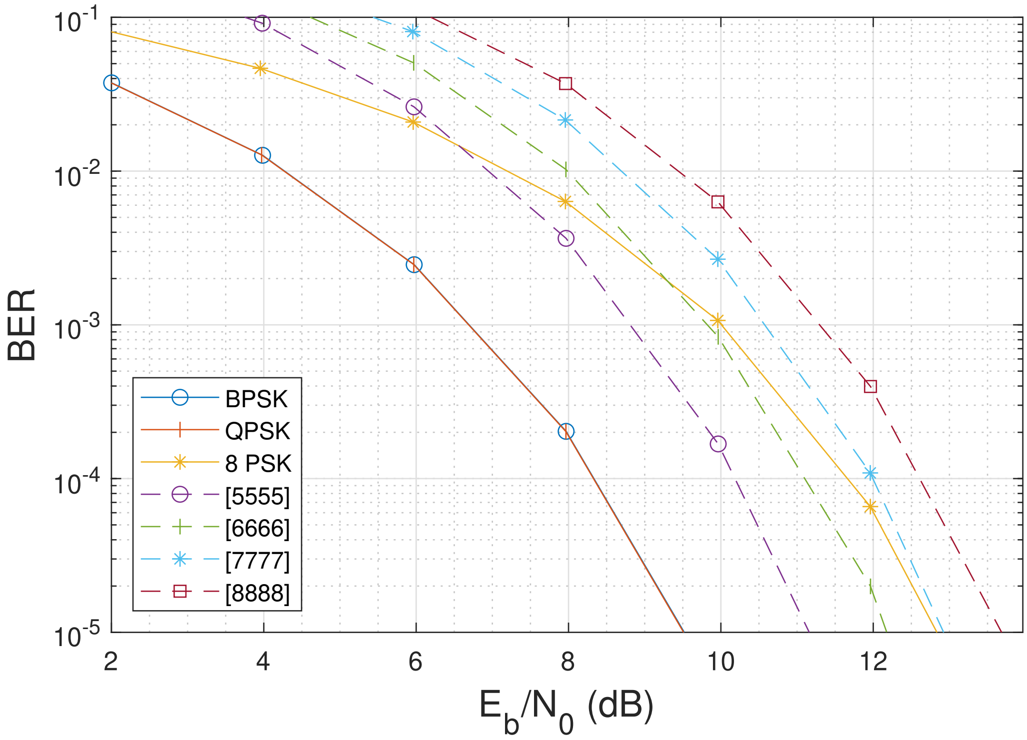

Figure 5.21: Length 4 Fractional Sequences and Non-Fractional Systems BERs

higher energies are required to achieve a specific BER. Sequences [5 5 5 5] and [6 6 6 6] are shown to have performance in between QPSK and 8-PSK making them candidates for increasing data throughput. Comparing these two to sequences [5 5] and [6 6], shown in Figure 5.19, at a BER of 10^{-4}, a reduction of 0.20 dB exists for the [5 5 5 5] sequence compared to the [5 5] sequence. For the [6 6 6 6] sequence, an increase of 0.32 dB exists when compared to the [6 6] sequence. Sequences [5 5], [6 6], [5 5 5 5], and [6 6 6 6] transmit 2, 2.5, 2.25, and 2.5 bits-per-symbol respectfully. A communication system could then use sequence [5 5 5 5] and [6 6], to gradually increase the number of bits per symbol as SNR increases. Sequences [7 7 7 7] and [8 8 8 8] had worse performance than 8-PSK.

Figure 5.22 shows that a slight gain can be achieved through the use of optimal decision types and an optimal mapping scheme compared to Figure 5.21. However, these sequences have a waveform utilization efficiency of greater than 75% making soft decisions have diminishing returns on reducing the BER.

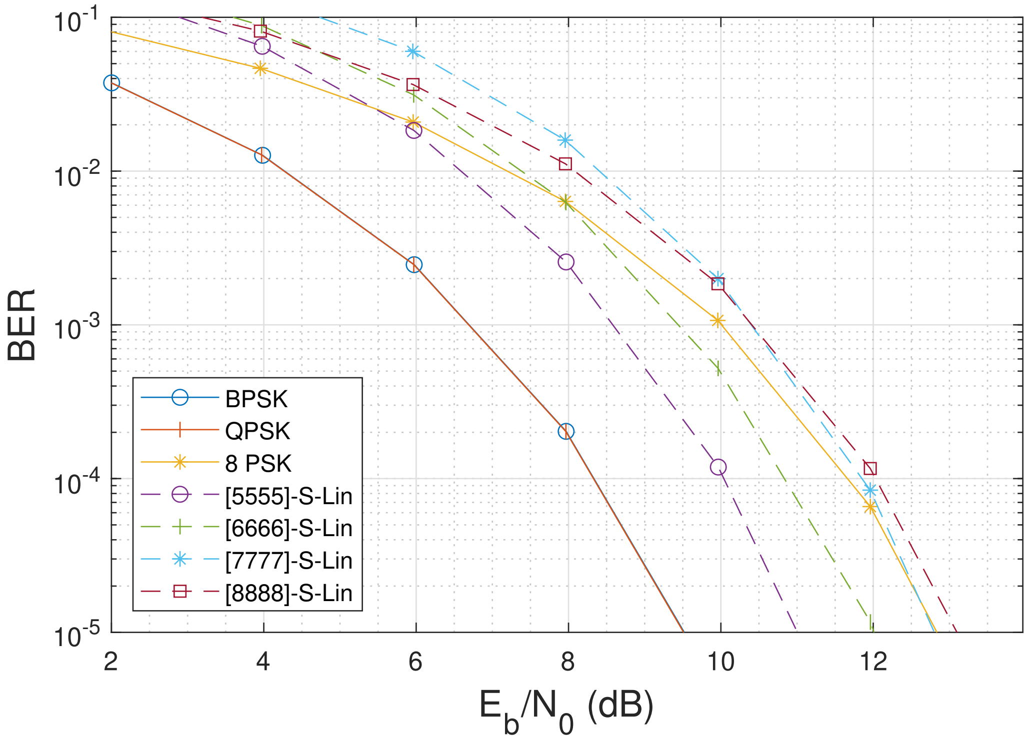

Figure 5.22: Optimally Mapped Length 4 Fractional Sequences and Non-Fractional Systems using Soft Decisions BERs

5.7 Throughput

In this section, three pass types are used to determine the throughput of three different types of communication systems: static non-fractional (SN), adaptive non-fractional (AN), and adaptive fractional (AF). The satellite pass simulator, described in Section 4.3.1, was used to generate the received SNR curve at the input of GS demodulator for the three pass types [4]. The settings of the simulator were: a transmit frequency of 2.4054 GHz, bandwidth of 2 MHz, a symbol rate of 1.33 Mega-symbols-per-second (Msps), and the RF chain details of VIOLET [4]. The first pass, shown in Figure 5.23, is an optimal pass which occurs when the satellite passes very close or through the zenith of the GS. It has the longest pass time of 657 s, reaches the highest SNR of 18.1 dB at the minimum range of 424 km. The second pass is an average pass and was generated by averaging all simulated passes of VIOLET. The average pass duration is 645 s, the highest SNR is 14.5 dB at the minimum range of 645 km. The third pass is a poor pass that occurs when communication is possible but VIOLET does not make an angle with the horizon that is 10° above 25° [4]. The poor pass has a duration of 610 s, the highest SNR is 13.6 dB at the minimum range of 711 km. Passes which do not have the capability to start the communication link are not described.

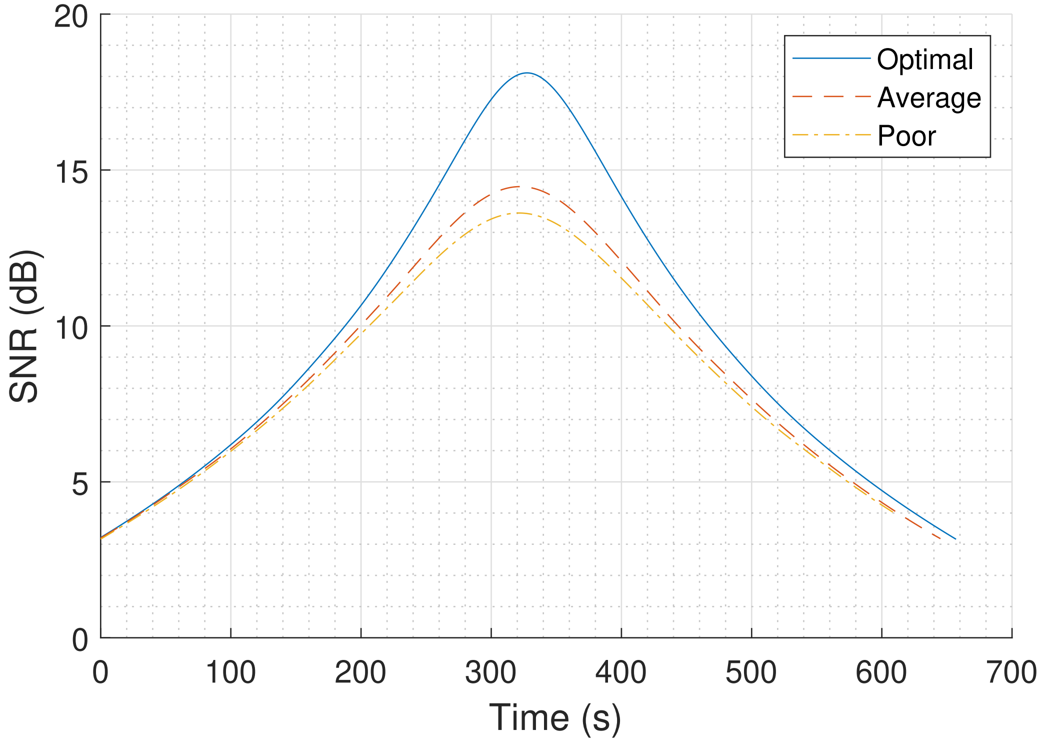

Figure 5.23: VIOLET's Expected Poor, Average, and Optimal Passes from the Satellite Pass Simulator

To determine which modulation schemes can be used on-board VIOLET, the results of the communication simulations presented in Section 5.2 to Section 5.6 need to be converted into SNR at the input of the GS demodulator. These values are used along with the pass results in Figure 5.23 to determine which modulation schemes can be used, when they can be switched to, and for how long they will be active for. Table 5.2 provides the required SNR for the selected modulation schemes to achieve a BER of 10^{-4}. The selected modulation schemes are BPSK, QPSK, 8-PSK

Modulation Scheme	b_{Tsym}	b_{Asym}	$\frac{E_b}{N_0}$ (dB)	Required SNR (dB)	Decision Type	Mapping
BPSK	1	1	8.32	8.32	Hard	-
QPSK	2	2	8.32	11.3	Hard	-
8-PSK	3	3	11.7	16.5	Hard	-
[3 3]	1.59	1.50	8.23	9.99	Soft	ED
[3 3]	1.59	1.50	8.32	10.1	Hard	Random
[6 6 6]	2.59	2.33	10.0	13.7	Hard	Random

Table 5.2: Decision SNRs for a BER of 10^{-4}

for non-fractional systems and [3 3] using soft decisions with ED mapping and [6 6 6] using hard decisions and random mapping for fractional systems. The [3 3] sequence using hard decisions and random mapping has also been included in Table 5.2 because as a replacement for the [3 3] sequence with soft decisions and ED mapping, it would only increase the required SNR for the [3 3] sequence by 0.1 dB and would remove the requirement for soft decisions to be made in the receiver.

For the SN system type, two systems were analyzed. System one uses BPSK and system two uses QPSK. For the AN system type, the system uses BPSK, QPSK, and 8-PSK modulation schemes. For the AF system type, the system uses BPSK, the [3 3] sequence with soft decisions and ED mapping, QPSK, the [6 6 6] sequence with hard decisions and random mapping, and 8-PSK modulations schemes. When determining the throughput of a system ideal conditions are assumed including, a maximum BER of 10^{-4}, no other significant fading sources exist, CSI is perfect and delay-free, the communication system transmits continuously, and the time to change modulation schemes is not noticeable. Table 5.3 shows the throughput results of various communication systems. Comparing the AN system to the SN system using QPSK, the throughput was increased by 42.8% for the poor pass, 36.2% for the average pass, and 45.3% for the optimal pass. Comparing the AF system to the AN system, the throughput was increased by 6.47.% for the poor pass, 5.47% for the average pass, and 7.67% for the optimal pass.

System Type	Modulation Scheme	Poor Time (s)	Data (Mb)	Average Time (s)	Data (Mb)	Optimal Time (s)	Data (Mb)
SN	BPSK	310	412	322	428	347	462
	QPSK	167	444	187	497	231	601
AN	BPSK	143	190	135	180	116	154
	QPSK	167	444	187	497	152	404
	8-PSK	-	-	-	-	79	315
	Total	310	634	322	677	347	873
AF	BPSK	82	109	79	105	68	90.4
	[3 3]	61	122	56	112	48	95.8
	QPSK	167	444	187	497	74	197
	[6 6 6]	-	-	-	-	78	242
	8-PSK	-	-	-	-	79	315
	Total	310	675	322	714	347	940

Table 5.3: Throughput of Various Communication Systems for VIOLET

6. Conclusions and Future Work

6.1 Conclusions

In conclusion, BER curves were generated and presented for the fractional bits-per-symbol communication scheme. Sequence properties were shown to have a significant effect on sequence performance. Sequence length was shown to, in some cases, increase the energy-per-bit requirements with longer sequences and in the other cases at higher energies, decrease the requirement. The number of points in the sequence has a large effect on sequence performance. Both fractional and non-fractional systems require higher energies to resolve constellation points that are tightly packed. Waveform utilization efficiency, was not shown to affect performance greatly however, it can be used as a tool for determining the optimal mapping scheme or decision technique for a specific sequence. Sequence mapping was shown to have a varying effect on the performance of sequences and is driven by the waveform utilization efficiency. Very efficient sequences, generally, performed the best when paired with a linear mapping scheme compared to inefficient sequences that, generally, performed best with a TGC or ED mapping scheme. This relationship of decreasing efficiency increases the effect that a mapping scheme can have on a sequence is due to the additional distance that can be gained when more specialized mapping scheme can make better use of the non-mapped waveforms to increase distance between mapped waveforms. Hard and soft decision techniques were shown to have a varying effect on sequence performance depending on the waveform utilization efficiency. Sequences with higher efficiencies do not gain as much as sequences with lower efficiencies when soft decisions were implemented. Sequences with constellations that change the number of points between each symbol can be useful to increase or decrease the waveform utilization efficiency to create systems that use a specific decision technique or mapping scheme but can come at a cost depending on the selected constellation size. Data throughput of static and adaptive communications systems were determined in ideal scenarios. Adaptive communications systems improved the data throughput compared to static communications systems. Adding fractional-bits-per-symbol sequences to an adaptive system was shown to increase data throughput when compared to non-fractional systems. This gain was found by better adjusting the data rate to the channel conditions.

6.2 Future Work

For future work, three areas could be focused on. The first would be to improvement the current TGC and ED mapping schemes. A demonstration of the mapping schemes characteristics has been shown but the algorithms for selecting the optimal set of mapped waveforms for ED mapping and Gray-coded bit-strings for TGC could be improved. The second would be to research applying error correcting codes in addition to or as a process during waveform selection. It is believed that this could result in improvements to the performance of a fractional-bits-per-symbol communication system. The third would be to implement a fractional-bits-per-symbol system on a physical communication system and corroborate the findings presented with real-world testing results. This system would allow for requirements to be determined for a carrier-loop tracking and phase tracking algorithms that may need to be specifically designed for use with sequences.

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Appendices

Appendix A: Sequence Listing

Note: This is a computer generated table. Within this table, efficiency, η, is rounded to the nearest whole number. As a result, the efficiency shown may not be accurate but, if the true efficiency is needed, a division of the number of mapped waveforms, 2^b, over the total number of waveforms, W, can be performed for a specific sequence. See 3.1.2 for a usage example.

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
1	2	2	2	1	100	1	3	3	3	1	67	1	4	4	4	2	100
1	5	5	5	2	80	1	6	6	6	2	67	1	7	7	7	2	57
1	8	8	8	3	100	2	2	2,2	4	2	100	2	3	2,3	6	2	67
2	4	2,4	8	3	100	2	5	2,5	10	3	80	2	6	2,6	12	3	67
2	7	2,7	14	3	57	2	8	2,8	16	4	100	2	3	3,3	9	3	89
2	4	3,4	12	3	67	2	5	3,5	15	3	53	2	6	3,6	18	4	89
2	7	3,7	21	4	76	2	8	3,8	24	4	67	2	4	4,4	16	4	100
2	5	4,5	20	4	80	2	6	4,6	24	4	67	2	7	4,7	28	4	57
2	8	4,8	32	5	100	2	5	5,5	25	4	64	2	6	5,6	30	4	53
2	7	5,7	35	5	91	2	8	5,8	40	5	80	2	6	6,6	36	5	89
2	7	6,7	42	5	76	2	8	6,8	48	5	67	2	7	7,7	49	5	65
2	8	7,8	56	5	57	2	8	8,8	64	6	100	3	2	2,2,2	8	3	100
3	3	2,2,3	12	3	67	3	4	2,2,4	16	4	100	3	5	2,2,5	20	4	80
3	6	2,2,6	24	4	67	3	7	2,2,7	28	4	57	3	8	2,2,8	32	5	100
3	3	2,3,3	18	4	89	3	4	2,3,4	24	4	67	3	5	2,3,5	30	4	53
3	6	2,3,6	36	5	89	3	7	2,3,7	42	5	76	3	8	2,3,8	48	5	67
3	4	2,4,4	32	5	100	3	5	2,4,5	40	5	80	3	6	2,4,6	48	5	67
3	7	2,4,7	56	5	57	3	8	2,4,8	64	6	100	3	5	2,5,5	50	5	64

N m	S	W	b	η	N m	S	W	b	η	N m	S	W	b	η
3 6	2,5,6	60	5	53	3 7	2,5,7	70	6	91	3 8	2,5,8	80	6	80
3 6	2,6,6	72	6	89	3 7	2,6,7	84	6	76	3 8	2,6,8	96	6	67
3 7	2,7,7	98	6	65	3 8	2,7,8	112	6	57	3 8	2,8,8	128	7	100
3 3	3,3,3	27	4	59	3 4	3,3,4	36	5	89	3 5	3,3,5	45	5	71
3 6	3,3,6	54	5	59	3 7	3,3,7	63	5	51	3 8	3,3,8	72	6	89
3 4	3,4,4	48	5	67	3 5	3,4,5	60	5	53	3 6	3,4,6	72	6	89
3 7	3,4,7	84	6	76	3 8	3,4,8	96	6	67	3 5	3,5,5	75	6	85
3 6	3,5,6	90	6	71	3 7	3,5,7	105	6	61	3 8	3,5,8	120	6	53
3 6	3,6,6	108	6	59	3 7	3,6,7	126	6	51	3 8	3,6,8	144	7	89
3 7	3,7,7	147	7	87	3 8	3,7,8	168	7	76	3 8	3,8,8	192	7	67
3 4	4,4,4	64	6	100	3 5	4,4,5	80	6	80	3 6	4,4,6	96	6	67
3 7	4,4,7	112	6	57	3 8	4,4,8	128	7	100	3 5	4,5,5	100	6	64
3 6	4,5,6	120	6	53	3 7	4,5,7	140	7	91	3 8	4,5,8	160	7	80
3 6	4,6,6	144	7	89	3 7	4,6,7	168	7	76	3 8	4,6,8	192	7	67
3 7	4,7,7	196	7	65	3 8	4,7,8	224	7	57	3 8	4,8,8	256	8	100
3 5	5,5,5	125	6	51	3 6	5,5,6	150	7	85	3 7	5,5,7	175	7	73
3 8	5,5,8	200	7	64	3 6	5,6,6	180	7	71	3 7	5,6,7	210	7	61
3 8	5,6,8	240	7	53	3 7	5,7,7	245	7	52	3 8	5,7,8	280	8	91
3 8	5,8,8	320	8	80	3 6	6,6,6	216	7	59	3 7	6,6,7	252	7	51
3 8	6,6,8	288	8	89	3 7	6,7,7	294	8	87	3 8	6,7,8	336	8	76
3 8	6,8,8	384	8	67	3 7	7,7,7	343	8	75	3 8	7,7,8	392	8	65
3 8	7,8,8	448	8	57	3 8	8,8,8	512	9	100	4 2	2,2,2,2	16	4	100
4 3	2,2,2,3	24	4	67	4 4	2,2,2,4	32	5	100	4 5	2,2,2,5	40	5	80
4 6	2,2,2,6	48	5	67	4 7	2,2,2,7	56	5	57	4 8	2,2,2,8	64	6	100
4 3	2,2,3,3	36	5	89	4 4	2,2,3,4	48	5	67	4 5	2,2,3,5	60	5	53
4 6	2,2,3,6	72	6	89	4 7	2,2,3,7	84	6	76	4 8	2,2,3,8	96	6	67
4 4	2,2,4,4	64	6	100	4 5	2,2,4,5	80	6	80	4 6	2,2,4,6	96	6	67
4 7	2,2,4,7	112	6	57	4 8	2,2,4,8	128	7	100	4 5	2,2,5,5	100	6	64
4 6	2,2,5,6	120	6	53	4 7	2,2,5,7	140	7	91	4 8	2,2,5,8	160	7	80
4 6	2,2,6,6	144	7	89	4 7	2,2,6,7	168	7	76	4 8	2,2,6,8	192	7	67
4 7	2,2,7,7	196	7	65	4 8	2,2,7,8	224	7	57	4 8	2,2,8,8	256	8	100
4 3	2,3,3,3	54	5	59	4 4	2,3,3,4	72	6	89	4 5	2,3,3,5	90	6	71

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
4	6	2,3,3,6	108	6	59	4	7	2,3,3,7	126	6	51	4	8	2,3,3,8	144	7	89
4	4	2,3,4,4	96	6	67	4	5	2,3,4,5	120	6	53	4	6	2,3,4,6	144	7	89
4	7	2,3,4,7	168	7	76	4	8	2,3,4,8	192	7	67	4	5	2,3,5,5	150	7	85
4	6	2,3,5,6	180	7	71	4	7	2,3,5,7	210	7	61	4	8	2,3,5,8	240	7	53
4	6	2,3,6,6	216	7	59	4	7	2,3,6,7	252	7	51	4	8	2,3,6,8	288	8	89
4	7	2,3,7,7	294	8	87	4	8	2,3,7,8	336	8	76	4	8	2,3,8,8	384	8	67
4	4	2,4,4,4	128	7	100	4	5	2,4,4,5	160	7	80	4	6	2,4,4,6	192	7	67
4	7	2,4,4,7	224	7	57	4	8	2,4,4,8	256	8	100	4	5	2,4,5,5	200	7	64
4	6	2,4,5,6	240	7	53	4	7	2,4,5,7	280	8	91	4	8	2,4,5,8	320	8	80
4	6	2,4,6,6	288	8	89	4	7	2,4,6,7	336	8	76	4	8	2,4,6,8	384	8	67
4	7	2,4,7,7	392	8	65	4	8	2,4,7,8	448	8	57	4	8	2,4,8,8	512	9	100
4	5	2,5,5,5	250	7	51	4	6	2,5,5,6	300	8	85	4	7	2,5,5,7	350	8	73
4	8	2,5,5,8	400	8	64	4	6	2,5,6,6	360	8	71	4	7	2,5,6,7	420	8	61
4	8	2,5,6,8	480	8	53	4	7	2,5,7,7	490	8	52	4	8	2,5,7,8	560	9	91
4	8	2,5,8,8	640	9	80	4	6	2,6,6,6	432	8	59	4	7	2,6,6,7	504	8	51
4	8	2,6,6,8	576	9	89	4	7	2,6,7,7	588	9	87	4	8	2,6,7,8	672	9	76
4	8	2,6,8,8	768	9	67	4	7	2,7,7,7	686	9	75	4	8	2,7,7,8	784	9	65
4	8	2,7,8,8	896	9	57	4	8	2,8,8,8	1024	10	100	4	3	3,3,3,3	81	6	79
4	4	3,3,3,4	108	6	59	4	5	3,3,3,5	135	7	95	4	6	3,3,3,6	162	7	79
4	7	3,3,3,7	189	7	68	4	8	3,3,3,8	216	7	59	4	4	3,3,4,4	144	7	89
4	5	3,3,4,5	180	7	71	4	6	3,3,4,6	216	7	59	4	7	3,3,4,7	252	7	51
4	8	3,3,4,8	288	8	89	4	5	3,3,5,5	225	7	57	4	6	3,3,5,6	270	8	95
4	7	3,3,5,7	315	8	81	4	8	3,3,5,8	360	8	71	4	6	3,3,6,6	324	8	79
4	7	3,3,6,7	378	8	68	4	8	3,3,6,8	432	8	59	4	7	3,3,7,7	441	8	58
4	8	3,3,7,8	504	8	51	4	8	3,3,8,8	576	9	89	4	4	3,4,4,4	192	7	67
4	5	3,4,4,5	240	7	53	4	6	3,4,4,6	288	8	89	4	7	3,4,4,7	336	8	76
4	8	3,4,4,8	384	8	67	4	5	3,4,5,5	300	8	85	4	6	3,4,5,6	360	8	71
4	7	3,4,5,7	420	8	61	4	8	3,4,5,8	480	8	53	4	6	3,4,6,6	432	8	59
4	7	3,4,6,7	504	8	51	4	8	3,4,6,8	576	9	89	4	7	3,4,7,7	588	9	87
4	8	3,4,7,8	672	9	76	4	8	3,4,8,8	768	9	67	4	5	3,5,5,5	375	8	68
4	6	3,5,5,6	450	8	57	4	7	3,5,5,7	525	9	98	4	8	3,5,5,8	600	9	85
4	6	3,5,6,6	540	9	95	4	7	3,5,6,7	630	9	81	4	8	3,5,6,8	720	9	71

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
4	7	3,5,7,7	735	9	70	4	8	3,5,7,8	840	9	61	4	8	3,5,8,8	960	9	53
4	6	3,6,6,6	648	9	79	4	7	3,6,6,7	756	9	68	4	8	3,6,6,8	864	9	59
4	7	3,6,7,7	882	9	58	4	8	3,6,7,8	1008	9	51	4	8	3,6,8,8	1152	10	89
4	7	3,7,7,7	1029	10	100	4	8	3,7,7,8	1176	10	87	4	8	3,7,8,8	1344	10	76
4	8	3,8,8,8	1536	10	67	4	4	4,4,4,4	256	8	100	4	5	4,4,4,5	320	8	80
4	6	4,4,4,6	384	8	67	4	7	4,4,4,7	448	8	57	4	8	4,4,4,8	512	9	100
4	5	4,4,5,5	400	8	64	4	6	4,4,5,6	480	8	53	4	7	4,4,5,7	560	9	91
4	8	4,4,5,8	640	9	80	4	6	4,4,6,6	576	9	89	4	7	4,4,6,7	672	9	76
4	8	4,4,6,8	768	9	67	4	7	4,4,7,7	784	9	65	4	8	4,4,7,8	896	9	57
4	8	4,4,8,8	1024	10	100	4	5	4,5,5,5	500	8	51	4	6	4,5,5,6	600	9	85
4	7	4,5,5,7	700	9	73	4	8	4,5,5,8	800	9	64	4	6	4,5,6,6	720	9	71
4	7	4,5,6,7	840	9	61	4	8	4,5,6,8	960	9	53	4	7	4,5,7,7	980	9	52
4	8	4,5,7,8	1120	10	91	4	8	4,5,8,8	1280	10	80	4	6	4,6,6,6	864	9	59
4	7	4,6,6,7	1008	9	51	4	8	4,6,6,8	1152	10	89	4	7	4,6,7,7	1176	10	87
4	8	4,6,7,8	1344	10	76	4	8	4,6,8,8	1536	10	67	4	7	4,7,7,7	1372	10	75
4	8	4,7,7,8	1568	10	65	4	8	4,7,8,8	1792	10	57	4	8	4,8,8,8	2048	11	100
4	5	5,5,5,5	625	9	82	4	6	5,5,5,6	750	9	68	4	7	5,5,5,7	875	9	59
4	8	5,5,5,8	1000	9	51	4	6	5,5,6,6	900	9	57	4	7	5,5,6,7	1050	10	98
4	8	5,5,6,8	1200	10	85	4	7	5,5,7,7	1225	10	84	4	8	5,5,7,8	1400	10	73
4	8	5,5,8,8	1600	10	64	4	6	5,6,6,6	1080	10	95	4	7	5,6,6,7	1260	10	81
4	8	5,6,6,8	1440	10	71	4	7	5,6,7,7	1470	10	70	4	8	5,6,7,8	1680	10	61
4	8	5,6,8,8	1920	10	53	4	7	5,7,7,7	1715	10	60	4	8	5,7,7,8	1960	10	52
4	8	5,7,8,8	2240	11	91	4	8	5,8,8,8	2560	11	80	4	6	6,6,6,6	1296	10	79
4	7	6,6,6,7	1512	10	68	4	8	6,6,6,8	1728	10	59	4	7	6,6,7,7	1764	10	58
4	8	6,6,7,8	2016	10	51	4	8	6,6,8,8	2304	11	89	4	7	6,7,7,7	2058	11	100
4	8	6,7,7,8	2352	11	87	4	8	6,7,8,8	2688	11	76	4	8	6,8,8,8	3072	11	67
4	7	7,7,7,7	2401	11	85	4	8	7,7,7,8	2744	11	75	4	8	7,7,8,8	3136	11	65
4	8	7,8,8,8	3584	11	57	4	8	8,8,8,8	4096	12	100	5	2	2,2,2,2,2	32	5	100
5	3	2,2,2,2,3	48	5	67	5	4	2,2,2,2,4	64	6	100	5	5	2,2,2,2,5	80	6	80
5	6	2,2,2,2,6	96	6	67	5	7	2,2,2,2,7	112	6	57	5	8	2,2,2,2,8	128	7	100
5	3	2,2,2,3,3	72	6	89	5	4	2,2,2,3,4	96	6	67	5	5	2,2,2,3,5	120	6	53
5	6	2,2,2,3,6	144	7	89	5	7	2,2,2,3,7	168	7	76	5	8	2,2,2,3,8	192	7	67

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
5	4	2,2,2,4,4	128	7	100	5	5	2,2,2,4,5	160	7	80	5	6	2,2,2,4,6	192	7	67
5	7	2,2,2,4,7	224	7	57	5	8	2,2,2,4,8	256	8	100	5	5	2,2,2,5,5	200	7	64
5	6	2,2,2,5,6	240	7	53	5	7	2,2,2,5,7	280	8	91	5	8	2,2,2,5,8	320	8	80
5	6	2,2,2,6,6	288	8	89	5	7	2,2,2,6,7	336	8	76	5	8	2,2,2,6,8	384	8	67
5	7	2,2,2,7,7	392	8	65	5	8	2,2,2,7,8	448	8	57	5	8	2,2,2,8,8	512	9	100
5	3	2,2,3,3,3	108	6	59	5	4	2,2,3,3,4	144	7	89	5	5	2,2,3,3,5	180	7	71
5	6	2,2,3,3,6	216	7	59	5	7	2,2,3,3,7	252	7	51	5	8	2,2,3,3,8	288	8	89
5	4	2,2,3,4,4	192	7	67	5	5	2,2,3,4,5	240	7	53	5	6	2,2,3,4,6	288	8	89
5	7	2,2,3,4,7	336	8	76	5	8	2,2,3,4,8	384	8	67	5	5	2,2,3,5,5	300	8	85
5	6	2,2,3,5,6	360	8	71	5	7	2,2,3,5,7	420	8	61	5	8	2,2,3,5,8	480	8	53
5	6	2,2,3,6,6	432	8	59	5	7	2,2,3,6,7	504	8	51	5	8	2,2,3,6,8	576	9	89
5	7	2,2,3,7,7	588	9	87	5	8	2,2,3,7,8	672	9	76	5	8	2,2,3,8,8	768	9	67
5	4	2,2,4,4,4	256	8	100	5	5	2,2,4,4,5	320	8	80	5	6	2,2,4,4,6	384	8	67
5	7	2,2,4,4,7	448	8	57	5	8	2,2,4,4,8	512	9	100	5	5	2,2,4,5,5	400	8	64
5	6	2,2,4,5,6	480	8	53	5	7	2,2,4,5,7	560	9	91	5	8	2,2,4,5,8	640	9	80
5	6	2,2,4,6,6	576	9	89	5	7	2,2,4,6,7	672	9	76	5	8	2,2,4,6,8	768	9	67
5	7	2,2,4,7,7	784	9	65	5	8	2,2,4,7,8	896	9	57	5	8	2,2,4,8,8	1024	10	100
5	5	2,2,5,5,5	500	8	51	5	6	2,2,5,5,6	600	9	85	5	7	2,2,5,5,7	700	9	73
5	8	2,2,5,5,8	800	9	64	5	6	2,2,5,6,6	720	9	71	5	7	2,2,5,6,7	840	9	61
5	8	2,2,5,6,8	960	9	53	5	7	2,2,5,7,7	980	9	52	5	8	2,2,5,7,8	1120	10	91
5	8	2,2,5,8,8	1280	10	80	5	6	2,2,6,6,6	864	9	59	5	7	2,2,6,6,7	1008	9	51
5	8	2,2,6,6,8	1152	10	89	5	7	2,2,6,7,7	1176	10	87	5	8	2,2,6,7,8	1344	10	76
5	8	2,2,6,8,8	1536	10	67	5	7	2,2,7,7,7	1372	10	75	5	8	2,2,7,7,8	1568	10	65
5	8	2,2,7,8,8	1792	10	57	5	8	2,2,8,8,8	2048	11	100	5	3	2,3,3,3,3	162	7	79
5	4	2,3,3,3,4	216	7	59	5	5	2,3,3,3,5	270	8	95	5	6	2,3,3,3,6	324	8	79
5	7	2,3,3,3,7	378	8	68	5	8	2,3,3,3,8	432	8	59	5	4	2,3,3,4,4	288	8	89
5	5	2,3,3,4,5	360	8	71	5	6	2,3,3,4,6	432	8	59	5	7	2,3,3,4,7	504	8	51
5	8	2,3,3,4,8	576	9	89	5	5	2,3,3,5,5	450	8	57	5	6	2,3,3,5,6	540	9	95
5	7	2,3,3,5,7	630	9	81	5	8	2,3,3,5,8	720	9	71	5	6	2,3,3,6,6	648	9	79
5	7	2,3,3,6,7	756	9	68	5	8	2,3,3,6,8	864	9	59	5	7	2,3,3,7,7	882	9	58
5	8	2,3,3,7,8	1008	9	51	5	8	2,3,3,8,8	1152	10	89	5	4	2,3,4,4,4	384	8	67
5	5	2,3,4,4,5	480	8	53	5	6	2,3,4,4,6	576	9	89	5	7	2,3,4,4,7	672	9	76

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
5	8	2,3,4,4,8	768	9	67	5	5	2,3,4,5,5	600	9	85	5	6	2,3,4,5,6	720	9	71
5	7	2,3,4,5,7	840	9	61	5	8	2,3,4,5,8	960	9	53	5	6	2,3,4,6,6	864	9	59
5	7	2,3,4,6,7	1008	9	51	5	8	2,3,4,6,8	1152	10	89	5	7	2,3,4,7,7	1176	10	87
5	8	2,3,4,7,8	1344	10	76	5	8	2,3,4,8,8	1536	10	67	5	5	2,3,5,5,5	750	9	68
5	6	2,3,5,5,6	900	9	57	5	7	2,3,5,5,7	1050	10	98	5	8	2,3,5,5,8	1200	10	85
5	6	2,3,5,6,6	1080	10	95	5	7	2,3,5,6,7	1260	10	81	5	8	2,3,5,6,8	1440	10	71
5	7	2,3,5,7,7	1470	10	70	5	8	2,3,5,7,8	1680	10	61	5	8	2,3,5,8,8	1920	10	53
5	6	2,3,6,6,6	1296	10	79	5	7	2,3,6,6,7	1512	10	68	5	8	2,3,6,6,8	1728	10	59
5	7	2,3,6,7,7	1764	10	58	5	8	2,3,6,7,8	2016	10	51	5	8	2,3,6,8,8	2304	11	89
5	7	2,3,7,7,7	2058	11	100	5	8	2,3,7,7,8	2352	11	87	5	8	2,3,7,8,8	2688	11	76
5	8	2,3,8,8,8	3072	11	67	5	4	2,4,4,4,4	512	9	100	5	5	2,4,4,4,5	640	9	80
5	6	2,4,4,4,6	768	9	67	5	7	2,4,4,4,7	896	9	57	5	8	2,4,4,4,8	1024	10	100
5	5	2,4,4,5,5	800	9	64	5	6	2,4,4,5,6	960	9	53	5	7	2,4,4,5,7	1120	10	91
5	8	2,4,4,5,8	1280	10	80	5	6	2,4,4,6,6	1152	10	89	5	7	2,4,4,6,7	1344	10	76
5	8	2,4,4,6,8	1536	10	67	5	7	2,4,4,7,7	1568	10	65	5	8	2,4,4,7,8	1792	10	57
5	8	2,4,4,8,8	2048	11	100	5	5	2,4,5,5,5	1000	9	51	5	6	2,4,5,5,6	1200	10	85
5	7	2,4,5,5,7	1400	10	73	5	8	2,4,5,5,8	1600	10	64	5	6	2,4,5,6,6	1440	10	71
5	7	2,4,5,6,7	1680	10	61	5	8	2,4,5,6,8	1920	10	53	5	7	2,4,5,7,7	1960	10	52
5	8	2,4,5,7,8	2240	11	91	5	8	2,4,5,8,8	2560	11	80	5	6	2,4,6,6,6	1728	10	59
5	7	2,4,6,6,7	2016	10	51	5	8	2,4,6,6,8	2304	11	89	5	7	2,4,6,7,7	2352	11	87
5	8	2,4,6,7,8	2688	11	76	5	8	2,4,6,8,8	3072	11	67	5	7	2,4,7,7,7	2744	11	75
5	8	2,4,7,7,8	3136	11	65	5	8	2,4,7,8,8	3584	11	57	5	8	2,4,8,8,8	4096	12	100
5	5	2,5,5,5,5	1250	10	82	5	6	2,5,5,5,6	1500	10	68	5	7	2,5,5,5,7	1750	10	59
5	8	2,5,5,5,8	2000	10	51	5	6	2,5,5,6,6	1800	10	57	5	7	2,5,5,6,7	2100	11	98
5	8	2,5,5,6,8	2400	11	85	5	7	2,5,5,7,7	2450	11	84	5	8	2,5,5,7,8	2800	11	73
5	8	2,5,5,8,8	3200	11	64	5	6	2,5,6,6,6	2160	11	95	5	7	2,5,6,6,7	2520	11	81
5	8	2,5,6,6,8	2880	11	71	5	7	2,5,6,7,7	2940	11	70	5	8	2,5,6,7,8	3360	11	61
5	8	2,5,6,8,8	3840	11	53	5	7	2,5,7,7,7	3430	11	60	5	8	2,5,7,7,8	3920	11	52
5	8	2,5,7,8,8	4480	12	91	5	8	2,5,8,8,8	5120	12	80	5	6	2,6,6,6,6	2592	11	79
5	7	2,6,6,6,7	3024	11	68	5	8	2,6,6,6,8	3456	11	59	5	7	2,6,6,7,7	3528	11	58
5	8	2,6,6,7,8	4032	11	51	5	8	2,6,6,8,8	4608	12	89	5	7	2,6,7,7,7	4116	12	100
5	8	2,6,7,7,8	4704	12	87	5	8	2,6,7,8,8	5376	12	76	5	8	2,6,8,8,8	6144	12	67

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
5	7	2,7,7,7,7	4802	12	85	5	8	2,7,7,7,8	5488	12	75	5	8	2,7,7,8,8	6272	12	65
5	8	2,7,8,8,8	7168	12	57	5	8	2,8,8,8,8	8192	13	100	5	3	3,3,3,3,3	243	7	53
5	4	3,3,3,3,4	324	8	79	5	5	3,3,3,3,5	405	8	63	5	6	3,3,3,3,6	486	8	53
5	7	3,3,3,3,7	567	9	90	5	8	3,3,3,3,8	648	9	79	5	4	3,3,3,4,4	432	8	59
5	5	3,3,3,4,5	540	9	95	5	6	3,3,3,4,6	648	9	79	5	7	3,3,3,4,7	756	9	68
5	8	3,3,3,4,8	864	9	59	5	5	3,3,3,5,5	675	9	76	5	6	3,3,3,5,6	810	9	63
5	7	3,3,3,5,7	945	9	54	5	8	3,3,3,5,8	1080	10	95	5	6	3,3,3,6,6	972	9	53
5	7	3,3,3,6,7	1134	10	90	5	8	3,3,3,6,8	1296	10	79	5	7	3,3,3,7,7	1323	10	77
5	8	3,3,3,7,8	1512	10	68	5	8	3,3,3,8,8	1728	10	59	5	4	3,3,4,4,4	576	9	89
5	5	3,3,4,4,5	720	9	71	5	6	3,3,4,4,6	864	9	59	5	7	3,3,4,4,7	1008	9	51
5	8	3,3,4,4,8	1152	10	89	5	5	3,3,4,5,5	900	9	57	5	6	3,3,4,5,6	1080	10	95
5	7	3,3,4,5,7	1260	10	81	5	8	3,3,4,5,8	1440	10	71	5	6	3,3,4,6,6	1296	10	79
5	7	3,3,4,6,7	1512	10	68	5	8	3,3,4,6,8	1728	10	59	5	7	3,3,4,7,7	1764	10	58
5	8	3,3,4,7,8	2016	10	51	5	8	3,3,4,8,8	2304	11	89	5	5	3,3,5,5,5	1125	10	91
5	6	3,3,5,5,6	1350	10	76	5	7	3,3,5,5,7	1575	10	65	5	8	3,3,5,5,8	1800	10	57
5	6	3,3,5,6,6	1620	10	63	5	7	3,3,5,6,7	1890	10	54	5	8	3,3,5,6,8	2160	11	95
5	7	3,3,5,7,7	2205	11	93	5	8	3,3,5,7,8	2520	11	81	5	8	3,3,5,8,8	2880	11	71
5	6	3,3,6,6,6	1944	10	53	5	7	3,3,6,6,7	2268	11	90	5	8	3,3,6,6,8	2592	11	79
5	7	3,3,6,7,7	2646	11	77	5	8	3,3,6,7,8	3024	11	68	5	8	3,3,6,8,8	3456	11	59
5	7	3,3,7,7,7	3087	11	66	5	8	3,3,7,7,8	3528	11	58	5	8	3,3,7,8,8	4032	11	51
5	8	3,3,8,8,8	4608	12	89	5	4	3,4,4,4,4	768	9	67	5	5	3,4,4,4,5	960	9	53
5	6	3,4,4,4,6	1152	10	89	5	7	3,4,4,4,7	1344	10	76	5	8	3,4,4,4,8	1536	10	67
5	5	3,4,4,5,5	1200	10	85	5	6	3,4,4,5,6	1440	10	71	5	7	3,4,4,5,7	1680	10	61
5	8	3,4,4,5,8	1920	10	53	5	6	3,4,4,6,6	1728	10	59	5	7	3,4,4,6,7	2016	10	51
5	8	3,4,4,6,8	2304	11	89	5	7	3,4,4,7,7	2352	11	87	5	8	3,4,4,7,8	2688	11	76
5	8	3,4,4,8,8	3072	11	67	5	5	3,4,5,5,5	1500	10	68	5	6	3,4,5,5,6	1800	10	57
5	7	3,4,5,5,7	2100	11	98	5	8	3,4,5,5,8	2400	11	85	5	6	3,4,5,6,6	2160	11	95
5	7	3,4,5,6,7	2520	11	81	5	8	3,4,5,6,8	2880	11	71	5	7	3,4,5,7,7	2940	11	70
5	8	3,4,5,7,8	3360	11	61	5	8	3,4,5,8,8	3840	11	53	5	6	3,4,6,6,6	2592	11	79
5	7	3,4,6,6,7	3024	11	68	5	8	3,4,6,6,8	3456	11	59	5	7	3,4,6,7,7	3528	11	58
5	8	3,4,6,7,8	4032	11	51	5	8	3,4,6,8,8	4608	12	89	5	7	3,4,7,7,7	4116	12	100
5	8	3,4,7,7,8	4704	12	87	5	8	3,4,7,8,8	5376	12	76	5	8	3,4,8,8,8	6144	12	67

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
5	5	3,5,5,5,5	1875	10	55	5	6	3,5,5,5,6	2250	11	91	5	7	3,5,5,5,7	2625	11	78
5	8	3,5,5,5,8	3000	11	68	5	6	3,5,5,6,6	2700	11	76	5	7	3,5,5,6,7	3150	11	65
5	8	3,5,5,6,8	3600	11	57	5	7	3,5,5,7,7	3675	11	56	5	8	3,5,5,7,8	4200	12	98
5	8	3,5,5,8,8	4800	12	85	5	6	3,5,6,6,6	3240	11	63	5	7	3,5,6,6,7	3780	11	54
5	8	3,5,6,6,8	4320	12	95	5	7	3,5,6,7,7	4410	12	93	5	8	3,5,6,7,8	5040	12	81
5	8	3,5,6,8,8	5760	12	71	5	7	3,5,7,7,7	5145	12	80	5	8	3,5,7,7,8	5880	12	70
5	8	3,5,7,8,8	6720	12	61	5	8	3,5,8,8,8	7680	12	53	5	6	3,6,6,6,6	3888	11	53
5	7	3,6,6,6,7	4536	12	90	5	8	3,6,6,6,8	5184	12	79	5	7	3,6,6,7,7	5292	12	77
5	8	3,6,6,7,8	6048	12	68	5	8	3,6,6,8,8	6912	12	59	5	7	3,6,7,7,7	6174	12	66
5	8	3,6,7,7,8	7056	12	58	5	8	3,6,7,8,8	8064	12	51	5	8	3,6,8,8,8	9216	13	89
5	7	3,7,7,7,7	7203	12	57	5	8	3,7,7,7,8	8232	13	100	5	8	3,7,7,8,8	9408	13	87
5	8	3,7,8,8,8	10752	13	76	5	8	3,8,8,8,8	12288	13	67	5	4	4,4,4,4,4	1024	10	100
5	5	4,4,4,4,5	1280	10	80	5	6	4,4,4,4,6	1536	10	67	5	7	4,4,4,4,7	1792	10	57
5	8	4,4,4,4,8	2048	11	100	5	5	4,4,4,5,5	1600	10	64	5	6	4,4,4,5,6	1920	10	53
5	7	4,4,4,5,7	2240	11	91	5	8	4,4,4,5,8	2560	11	80	5	6	4,4,4,6,6	2304	11	89
5	7	4,4,4,6,7	2688	11	76	5	8	4,4,4,6,8	3072	11	67	5	7	4,4,4,7,7	3136	11	65
5	8	4,4,4,7,8	3584	11	57	5	8	4,4,4,8,8	4096	12	100	5	5	4,4,5,5,5	2000	10	51
5	6	4,4,5,5,6	2400	11	85	5	7	4,4,5,5,7	2800	11	73	5	8	4,4,5,5,8	3200	11	64
5	6	4,4,5,6,6	2880	11	71	5	7	4,4,5,6,7	3360	11	61	5	8	4,4,5,6,8	3840	11	53
5	7	4,4,5,7,7	3920	11	52	5	8	4,4,5,7,8	4480	12	91	5	8	4,4,5,8,8	5120	12	80
5	6	4,4,6,6,6	3456	11	59	5	7	4,4,6,6,7	4032	11	51	5	8	4,4,6,6,8	4608	12	89
5	7	4,4,6,7,7	4704	12	87	5	8	4,4,6,7,8	5376	12	76	5	8	4,4,6,8,8	6144	12	67
5	7	4,4,7,7,7	5488	12	75	5	8	4,4,7,7,8	6272	12	65	5	8	4,4,7,8,8	7168	12	57
5	8	4,4,8,8,8	8192	13	100	5	5	4,5,5,5,5	2500	11	82	5	6	4,5,5,5,6	3000	11	68
5	7	4,5,5,5,7	3500	11	59	5	8	4,5,5,5,8	4000	11	51	5	6	4,5,5,6,6	3600	11	57
5	7	4,5,5,6,7	4200	12	98	5	8	4,5,5,6,8	4800	12	85	5	7	4,5,5,7,7	4900	12	84
5	8	4,5,5,7,8	5600	12	73	5	8	4,5,5,8,8	6400	12	64	5	6	4,5,6,6,6	4320	12	95
5	7	4,5,6,6,7	5040	12	81	5	8	4,5,6,6,8	5760	12	71	5	7	4,5,6,7,7	5880	12	70
5	8	4,5,6,7,8	6720	12	61	5	8	4,5,6,8,8	7680	12	53	5	7	4,5,7,7,7	6860	12	60
5	8	4,5,7,7,8	7840	12	52	5	8	4,5,7,8,8	8960	13	91	5	8	4,5,8,8,8	10240	13	80
5	6	4,6,6,6,6	5184	12	79	5	7	4,6,6,6,7	6048	12	68	5	8	4,6,6,6,8	6912	12	59
5	7	4,6,6,7,7	7056	12	58	5	8	4,6,6,7,8	8064	12	51	5	8	4,6,6,8,8	9216	13	89

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
5	7	4,6,7,7,7	8232	13	100	5	8	4,6,7,7,8	9408	13	87	5	8	4,6,7,8,8	10752	13	76
5	8	4,6,8,8,8	12288	13	67	5	7	4,7,7,7,7	9604	13	85	5	8	4,7,7,7,8	10976	13	75
5	8	4,7,7,8,8	12544	13	65	5	8	4,7,8,8,8	14336	13	57	5	8	4,8,8,8,8	16384	14	100
5	5	5,5,5,5,5	3125	11	66	5	6	5,5,5,5,6	3750	11	55	5	7	5,5,5,5,7	4375	12	94
5	8	5,5,5,5,8	5000	12	82	5	6	5,5,5,6,6	4500	12	91	5	7	5,5,5,6,7	5250	12	78
5	8	5,5,5,6,8	6000	12	68	5	7	5,5,5,7,7	6125	12	67	5	8	5,5,5,7,8	7000	12	59
5	8	5,5,5,8,8	8000	12	51	5	6	5,5,6,6,6	5400	12	76	5	7	5,5,6,6,7	6300	12	65
5	8	5,5,6,6,8	7200	12	57	5	7	5,5,6,7,7	7350	12	56	5	8	5,5,6,7,8	8400	13	98
5	8	5,5,6,8,8	9600	13	85	5	7	5,5,7,7,7	8575	13	96	5	8	5,5,7,7,8	9800	13	84
5	8	5,5,7,8,8	11200	13	73	5	8	5,5,8,8,8	12800	13	64	5	6	5,6,6,6,6	6480	12	63
5	7	5,6,6,6,7	7560	12	54	5	8	5,6,6,6,8	8640	13	95	5	7	5,6,6,7,7	8820	13	93
5	8	5,6,6,7,8	10080	13	81	5	8	5,6,6,8,8	11520	13	71	5	7	5,6,7,7,7	10290	13	80
5	8	5,6,7,7,8	11760	13	70	5	8	5,6,7,8,8	13440	13	61	5	8	5,6,8,8,8	15360	13	53
5	7	5,7,7,7,7	12005	13	68	5	8	5,7,7,7,8	13720	13	60	5	8	5,7,7,8,8	15680	13	52
5	8	5,7,8,8,8	17920	14	91	5	8	5,8,8,8,8	20480	14	80	5	6	6,6,6,6,6	7776	12	53
5	7	6,6,6,6,7	9072	13	90	5	8	6,6,6,6,8	10368	13	79	5	7	6,6,6,7,7	10584	13	77
5	8	6,6,6,7,8	12096	13	68	5	8	6,6,6,8,8	13824	13	59	5	7	6,6,7,7,7	12348	13	66
5	8	6,6,7,7,8	14112	13	58	5	8	6,6,7,8,8	16128	13	51	5	8	6,6,8,8,8	18432	14	89
5	7	6,7,7,7,7	14406	13	57	5	8	6,7,7,7,8	16464	14	100	5	8	6,7,7,8,8	18816	14	87
5	8	6,7,8,8,8	21504	14	76	5	8	6,8,8,8,8	24576	14	67	5	7	7,7,7,7,7	16807	14	97
5	8	7,7,7,7,8	19208	14	85	5	8	7,7,7,8,8	21952	14	75	5	8	7,7,8,8,8	25088	14	65
5	8	7,8,8,8,8	28672	14	57	5	8	8,8,8,8,8	32768	15	100	6	2	2,2,2,2,2,2	64	6	100
6	3	2,2,2,2,2,3	96	6	67	6	4	2,2,2,2,2,4	128	7	100	6	5	2,2,2,2,2,5	160	7	80
6	6	2,2,2,2,2,6	192	7	67	6	7	2,2,2,2,2,7	224	7	57	6	8	2,2,2,2,2,8	256	8	100
6	3	2,2,2,2,3,3	144	7	89	6	4	2,2,2,2,3,4	192	7	67	6	5	2,2,2,2,3,5	240	7	53
6	6	2,2,2,2,3,6	288	8	89	6	7	2,2,2,2,3,7	336	8	76	6	8	2,2,2,2,3,8	384	8	67
6	4	2,2,2,2,4,4	256	8	100	6	5	2,2,2,2,4,5	320	8	80	6	6	2,2,2,2,4,6	384	8	67
6	7	2,2,2,2,4,7	448	8	57	6	8	2,2,2,2,4,8	512	9	100	6	5	2,2,2,2,5,5	400	8	64
6	6	2,2,2,2,5,6	480	8	53	6	7	2,2,2,2,5,7	560	9	91	6	8	2,2,2,2,5,8	640	9	80
6	6	2,2,2,2,6,6	576	9	89	6	7	2,2,2,2,6,7	672	9	76	6	8	2,2,2,2,6,8	768	9	67
6	7	2,2,2,2,7,7	784	9	65	6	8	2,2,2,2,7,8	896	9	57	6	8	2,2,2,2,8,8	1024	10	100
6	3	2,2,2,3,3,3	216	7	59	6	4	2,2,2,3,3,4	288	8	89	6	5	2,2,2,3,3,5	360	8	71

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
6	6	2,2,2,3,3,6	432	8	59	6	7	2,2,2,3,3,7	504	8	51	6	8	2,2,2,3,3,8	576	9	89
6	4	2,2,2,3,4,4	384	8	67	6	5	2,2,2,3,4,5	480	8	53	6	6	2,2,2,3,4,6	576	9	89
6	7	2,2,2,3,4,7	672	9	76	6	8	2,2,2,3,4,8	768	9	67	6	5	2,2,2,3,5,5	600	9	85
6	6	2,2,2,3,5,6	720	9	71	6	7	2,2,2,3,5,7	840	9	61	6	8	2,2,2,3,5,8	960	9	53
6	6	2,2,2,3,6,6	864	9	59	6	7	2,2,2,3,6,7	1008	9	51	6	8	2,2,2,3,6,8	1152	10	89
6	7	2,2,2,3,7,7	1176	10	87	6	8	2,2,2,3,7,8	1344	10	76	6	8	2,2,2,3,8,8	1536	10	67
6	4	2,2,2,4,4,4	512	9	100	6	5	2,2,2,4,4,5	640	9	80	6	6	2,2,2,4,4,6	768	9	67
6	7	2,2,2,4,4,7	896	9	57	6	8	2,2,2,4,4,8	1024	10	100	6	5	2,2,2,4,5,5	800	9	64
6	6	2,2,2,4,5,6	960	9	53	6	7	2,2,2,4,5,7	1120	10	91	6	8	2,2,2,4,5,8	1280	10	80
6	6	2,2,2,4,6,6	1152	10	89	6	7	2,2,2,4,6,7	1344	10	76	6	8	2,2,2,4,6,8	1536	10	67
6	7	2,2,2,4,7,7	1568	10	65	6	8	2,2,2,4,7,8	1792	10	57	6	8	2,2,2,4,8,8	2048	11	100
6	5	2,2,2,5,5,5	1000	9	51	6	6	2,2,2,5,5,6	1200	10	85	6	7	2,2,2,5,5,7	1400	10	73
6	8	2,2,2,5,5,8	1600	10	64	6	6	2,2,2,5,6,6	1440	10	71	6	7	2,2,2,5,6,7	1680	10	61
6	8	2,2,2,5,6,8	1920	10	53	6	7	2,2,2,5,7,7	1960	10	52	6	8	2,2,2,5,7,8	2240	11	91
6	8	2,2,2,5,8,8	2560	11	80	6	6	2,2,2,6,6,6	1728	10	59	6	7	2,2,2,6,6,7	2016	10	51
6	8	2,2,2,6,6,8	2304	11	89	6	7	2,2,2,6,7,7	2352	11	87	6	8	2,2,2,6,7,8	2688	11	76
6	8	2,2,2,6,8,8	3072	11	67	6	7	2,2,2,7,7,7	2744	11	75	6	8	2,2,2,7,7,8	3136	11	65
6	8	2,2,2,7,8,8	3584	11	57	6	8	2,2,2,8,8,8	4096	12	100	6	3	2,2,3,3,3,3	324	8	79
6	4	2,2,3,3,3,4	432	8	59	6	5	2,2,3,3,3,5	540	9	95	6	6	2,2,3,3,3,6	648	9	79
6	7	2,2,3,3,3,7	756	9	68	6	8	2,2,3,3,3,8	864	9	59	6	4	2,2,3,3,4,4	576	9	89
6	5	2,2,3,3,4,5	720	9	71	6	6	2,2,3,3,4,6	864	9	59	6	7	2,2,3,3,4,7	1008	9	51
6	8	2,2,3,3,4,8	1152	10	89	6	5	2,2,3,3,5,5	900	9	57	6	6	2,2,3,3,5,6	1080	10	95
6	7	2,2,3,3,5,7	1260	10	81	6	8	2,2,3,3,5,8	1440	10	71	6	6	2,2,3,3,6,6	1296	10	79
6	7	2,2,3,3,6,7	1512	10	68	6	8	2,2,3,3,6,8	1728	10	59	6	7	2,2,3,3,7,7	1764	10	58
6	8	2,2,3,3,7,8	2016	10	51	6	8	2,2,3,3,8,8	2304	11	89	6	4	2,2,3,4,4,4	768	9	67
6	5	2,2,3,4,4,5	960	9	53	6	6	2,2,3,4,4,6	1152	10	89	6	7	2,2,3,4,4,7	1344	10	76
6	8	2,2,3,4,4,8	1536	10	67	6	5	2,2,3,4,5,5	1200	10	85	6	6	2,2,3,4,5,6	1440	10	71
6	7	2,2,3,4,5,7	1680	10	61	6	8	2,2,3,4,5,8	1920	10	53	6	6	2,2,3,4,6,6	1728	10	59
6	7	2,2,3,4,6,7	2016	10	51	6	8	2,2,3,4,6,8	2304	11	89	6	7	2,2,3,4,7,7	2352	11	87
6	8	2,2,3,4,7,8	2688	11	76	6	8	2,2,3,4,8,8	3072	11	67	6	5	2,2,3,5,5,5	1500	10	68
6	6	2,2,3,5,5,6	1800	10	57	6	7	2,2,3,5,5,7	2100	11	98	6	8	2,2,3,5,5,8	2400	11	85
6	6	2,2,3,5,6,6	2160	11	95	6	7	2,2,3,5,6,7	2520	11	81	6	8	2,2,3,5,6,8	2880	11	71

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
6	7	2,2,3,5,7,7	2940	11	70	6	8	2,2,3,5,7,8	3360	11	61	6	8	2,2,3,5,8,8	3840	11	53
6	6	2,2,3,6,6,6	2592	11	79	6	7	2,2,3,6,6,7	3024	11	68	6	8	2,2,3,6,6,8	3456	11	59
6	7	2,2,3,6,7,7	3528	11	58	6	8	2,2,3,6,7,8	4032	11	51	6	8	2,2,3,6,8,8	4608	12	89
6	7	2,2,3,7,7,7	4116	12	100	6	8	2,2,3,7,7,8	4704	12	87	6	8	2,2,3,7,8,8	5376	12	76
6	8	2,2,3,8,8,8	6144	12	67	6	4	2,2,4,4,4,4	1024	10	100	6	5	2,2,4,4,4,5	1280	10	80
6	6	2,2,4,4,4,6	1536	10	67	6	7	2,2,4,4,4,7	1792	10	57	6	8	2,2,4,4,4,8	2048	11	100
6	5	2,2,4,4,5,5	1600	10	64	6	6	2,2,4,4,5,6	1920	10	53	6	7	2,2,4,4,5,7	2240	11	91
6	8	2,2,4,4,5,8	2560	11	80	6	6	2,2,4,4,6,6	2304	11	89	6	7	2,2,4,4,6,7	2688	11	76
6	8	2,2,4,4,6,8	3072	11	67	6	7	2,2,4,4,7,7	3136	11	65	6	8	2,2,4,4,7,8	3584	11	57
6	8	2,2,4,4,8,8	4096	12	100	6	5	2,2,4,5,5,5	2000	10	51	6	6	2,2,4,5,5,6	2400	11	85
6	7	2,2,4,5,5,7	2800	11	73	6	8	2,2,4,5,5,8	3200	11	64	6	6	2,2,4,5,6,6	2880	11	71
6	7	2,2,4,5,6,7	3360	11	61	6	8	2,2,4,5,6,8	3840	11	53	6	7	2,2,4,5,7,7	3920	11	52
6	8	2,2,4,5,7,8	4480	12	91	6	8	2,2,4,5,8,8	5120	12	80	6	6	2,2,4,6,6,6	3456	11	59
6	7	2,2,4,6,6,7	4032	11	51	6	8	2,2,4,6,6,8	4608	12	89	6	7	2,2,4,6,7,7	4704	12	87
6	8	2,2,4,6,7,8	5376	12	76	6	8	2,2,4,6,8,8	6144	12	67	6	7	2,2,4,7,7,7	5488	12	75
6	8	2,2,4,7,7,8	6272	12	65	6	8	2,2,4,7,8,8	7168	12	57	6	8	2,2,4,8,8,8	8192	13	100
6	5	2,2,5,5,5,5	2500	11	82	6	6	2,2,5,5,5,6	3000	11	68	6	7	2,2,5,5,5,7	3500	11	59
6	8	2,2,5,5,5,8	4000	11	51	6	6	2,2,5,5,6,6	3600	11	57	6	7	2,2,5,5,6,7	4200	12	98
6	8	2,2,5,5,6,8	4800	12	85	6	7	2,2,5,5,7,7	4900	12	84	6	8	2,2,5,5,7,8	5600	12	73
6	8	2,2,5,5,8,8	6400	12	64	6	6	2,2,5,6,6,6	4320	12	95	6	7	2,2,5,6,6,7	5040	12	81
6	8	2,2,5,6,6,8	5760	12	71	6	7	2,2,5,6,7,7	5880	12	70	6	8	2,2,5,6,7,8	6720	12	61
6	8	2,2,5,6,8,8	7680	12	53	6	7	2,2,5,7,7,7	6860	12	60	6	8	2,2,5,7,7,8	7840	12	52
6	8	2,2,5,7,8,8	8960	13	91	6	8	2,2,5,8,8,8	10240	13	80	6	6	2,2,6,6,6,6	5184	12	79
6	7	2,2,6,6,6,7	6048	12	68	6	8	2,2,6,6,6,8	6912	12	59	6	7	2,2,6,6,7,7	7056	12	58
6	8	2,2,6,6,7,8	8064	12	51	6	8	2,2,6,6,8,8	9216	13	89	6	7	2,2,6,7,7,7	8232	13	100
6	8	2,2,6,7,7,8	9408	13	87	6	8	2,2,6,7,8,8	10752	13	76	6	8	2,2,6,8,8,8	12288	13	67
6	7	2,2,7,7,7,7	9604	13	85	6	8	2,2,7,7,7,8	10976	13	75	6	8	2,2,7,7,8,8	12544	13	65
6	8	2,2,7,8,8,8	14336	13	57	6	8	2,2,8,8,8,8	16384	14	100	6	3	2,3,3,3,3,3	486	8	53
6	4	2,3,3,3,3,4	648	9	79	6	5	2,3,3,3,3,5	810	9	63	6	6	2,3,3,3,3,6	972	9	53
6	7	2,3,3,3,3,7	1134	10	90	6	8	2,3,3,3,3,8	1296	10	79	6	4	2,3,3,3,4,4	864	9	59
6	5	2,3,3,3,4,5	1080	10	95	6	6	2,3,3,3,4,6	1296	10	79	6	7	2,3,3,3,4,7	1512	10	68
6	8	2,3,3,3,4,8	1728	10	59	6	5	2,3,3,3,5,5	1350	10	76	6	6	2,3,3,3,5,6	1620	10	63

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
6	7	2,3,3,3,5,7	1890	10	54	6	8	2,3,3,3,5,8	2160	11	95	6	6	2,3,3,3,6,6	1944	10	53
6	7	2,3,3,3,6,7	2268	11	90	6	8	2,3,3,3,6,8	2592	11	79	6	7	2,3,3,3,7,7	2646	11	77
6	8	2,3,3,3,7,8	3024	11	68	6	8	2,3,3,3,8,8	3456	11	59	6	4	2,3,3,4,4,4	1152	10	89
6	5	2,3,3,4,4,5	1440	10	71	6	6	2,3,3,4,4,6	1728	10	59	6	7	2,3,3,4,4,7	2016	10	51
6	8	2,3,3,4,4,8	2304	11	89	6	5	2,3,3,4,5,5	1800	10	57	6	6	2,3,3,4,5,6	2160	11	95
6	7	2,3,3,4,5,7	2520	11	81	6	8	2,3,3,4,5,8	2880	11	71	6	6	2,3,3,4,6,6	2592	11	79
6	7	2,3,3,4,6,7	3024	11	68	6	8	2,3,3,4,6,8	3456	11	59	6	7	2,3,3,4,7,7	3528	11	58
6	8	2,3,3,4,7,8	4032	11	51	6	8	2,3,3,4,8,8	4608	12	89	6	5	2,3,3,5,5,5	2250	11	91
6	6	2,3,3,5,5,6	2700	11	76	6	7	2,3,3,5,5,7	3150	11	65	6	8	2,3,3,5,5,8	3600	11	57
6	6	2,3,3,5,6,6	3240	11	63	6	7	2,3,3,5,6,7	3780	11	54	6	8	2,3,3,5,6,8	4320	12	95
6	7	2,3,3,5,7,7	4410	12	93	6	8	2,3,3,5,7,8	5040	12	81	6	8	2,3,3,5,8,8	5760	12	71
6	6	2,3,3,6,6,6	3888	11	53	6	7	2,3,3,6,6,7	4536	12	90	6	8	2,3,3,6,6,8	5184	12	79
6	7	2,3,3,6,7,7	5292	12	77	6	8	2,3,3,6,7,8	6048	12	68	6	8	2,3,3,6,8,8	6912	12	59
6	7	2,3,3,7,7,7	6174	12	66	6	8	2,3,3,7,7,8	7056	12	58	6	8	2,3,3,7,8,8	8064	12	51
6	8	2,3,3,8,8,8	9216	13	89	6	4	2,3,4,4,4,4	1536	10	67	6	5	2,3,4,4,4,5	1920	10	53
6	6	2,3,4,4,4,6	2304	11	89	6	7	2,3,4,4,4,7	2688	11	76	6	8	2,3,4,4,4,8	3072	11	67
6	5	2,3,4,4,5,5	2400	11	85	6	6	2,3,4,4,5,6	2880	11	71	6	7	2,3,4,4,5,7	3360	11	61
6	8	2,3,4,4,5,8	3840	11	53	6	6	2,3,4,4,6,6	3456	11	59	6	7	2,3,4,4,6,7	4032	11	51
6	8	2,3,4,4,6,8	4608	12	89	6	7	2,3,4,4,7,7	4704	12	87	6	8	2,3,4,4,7,8	5376	12	76
6	8	2,3,4,4,8,8	6144	12	67	6	5	2,3,4,5,5,5	3000	11	68	6	6	2,3,4,5,5,6	3600	11	57
6	7	2,3,4,5,5,7	4200	12	98	6	8	2,3,4,5,5,8	4800	12	85	6	6	2,3,4,5,6,6	4320	12	95
6	7	2,3,4,5,6,7	5040	12	81	6	8	2,3,4,5,6,8	5760	12	71	6	7	2,3,4,5,7,7	5880	12	70
6	8	2,3,4,5,7,8	6720	12	61	6	8	2,3,4,5,8,8	7680	12	53	6	6	2,3,4,6,6,6	5184	12	79
6	7	2,3,4,6,6,7	6048	12	68	6	8	2,3,4,6,6,8	6912	12	59	6	7	2,3,4,6,7,7	7056	12	58
6	8	2,3,4,6,7,8	8064	12	51	6	8	2,3,4,6,8,8	9216	13	89	6	7	2,3,4,7,7,7	8232	13	100
6	8	2,3,4,7,7,8	9408	13	87	6	8	2,3,4,7,8,8	10752	13	76	6	8	2,3,4,8,8,8	12288	13	67
6	5	2,3,5,5,5,5	3750	11	55	6	6	2,3,5,5,5,6	4500	12	91	6	7	2,3,5,5,5,7	5250	12	78
6	8	2,3,5,5,5,8	6000	12	68	6	6	2,3,5,5,6,6	5400	12	76	6	7	2,3,5,5,6,7	6300	12	65
6	8	2,3,5,5,6,8	7200	12	57	6	7	2,3,5,5,7,7	7350	12	56	6	8	2,3,5,5,7,8	8400	13	98
6	8	2,3,5,5,8,8	9600	13	85	6	6	2,3,5,6,6,6	6480	12	63	6	7	2,3,5,6,6,7	7560	12	54
6	8	2,3,5,6,6,8	8640	13	95	6	7	2,3,5,6,7,7	8820	13	93	6	8	2,3,5,6,7,8	10080	13	81
6	8	2,3,5,6,8,8	11520	13	71	6	7	2,3,5,7,7,7	10290	13	80	6	8	2,3,5,7,7,8	11760	13	70

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
6	8	2,3,5,7,8,8	13440	13	61	6	8	2,3,5,8,8,8	15360	13	53	6	6	2,3,6,6,6,6	7776	12	53
6	7	2,3,6,6,6,7	9072	13	90	6	8	2,3,6,6,6,8	10368	13	79	6	7	2,3,6,6,7,7	10584	13	77
6	8	2,3,6,6,7,8	12096	13	68	6	8	2,3,6,6,8,8	13824	13	59	6	7	2,3,6,7,7,7	12348	13	66
6	8	2,3,6,7,7,8	14112	13	58	6	8	2,3,6,7,8,8	16128	13	51	6	8	2,3,6,8,8,8	18432	14	89
6	7	2,3,7,7,7,7	14406	13	57	6	8	2,3,7,7,7,8	16464	14	100	6	8	2,3,7,7,8,8	18816	14	87
6	8	2,3,7,8,8,8	21504	14	76	6	8	2,3,8,8,8,8	24576	14	67	6	4	2,4,4,4,4,4	2048	11	100
6	5	2,4,4,4,4,5	2560	11	80	6	6	2,4,4,4,4,6	3072	11	67	6	7	2,4,4,4,4,7	3584	11	57
6	8	2,4,4,4,4,8	4096	12	100	6	5	2,4,4,4,5,5	3200	11	64	6	6	2,4,4,4,5,6	3840	11	53
6	7	2,4,4,4,5,7	4480	12	91	6	8	2,4,4,4,5,8	5120	12	80	6	6	2,4,4,4,6,6	4608	12	89
6	7	2,4,4,4,6,7	5376	12	76	6	8	2,4,4,4,6,8	6144	12	67	6	7	2,4,4,4,7,7	6272	12	65
6	8	2,4,4,4,7,8	7168	12	57	6	8	2,4,4,4,8,8	8192	13	100	6	5	2,4,4,5,5,5	4000	11	51
6	6	2,4,4,5,5,6	4800	12	85	6	7	2,4,4,5,5,7	5600	12	73	6	8	2,4,4,5,5,8	6400	12	64
6	6	2,4,4,5,6,6	5760	12	71	6	7	2,4,4,5,6,7	6720	12	61	6	8	2,4,4,5,6,8	7680	12	53
6	7	2,4,4,5,7,7	7840	12	52	6	8	2,4,4,5,7,8	8960	13	91	6	8	2,4,4,5,8,8	10240	13	80
6	6	2,4,4,6,6,6	6912	12	59	6	7	2,4,4,6,6,7	8064	12	51	6	8	2,4,4,6,6,8	9216	13	89
6	7	2,4,4,6,7,7	9408	13	87	6	8	2,4,4,6,7,8	10752	13	76	6	8	2,4,4,6,8,8	12288	13	67
6	7	2,4,4,7,7,7	10976	13	75	6	8	2,4,4,7,7,8	12544	13	65	6	8	2,4,4,7,8,8	14336	13	57
6	8	2,4,4,8,8,8	16384	14	100	6	5	2,4,5,5,5,5	5000	12	82	6	6	2,4,5,5,5,6	6000	12	68
6	7	2,4,5,5,5,7	7000	12	59	6	8	2,4,5,5,5,8	8000	12	51	6	6	2,4,5,5,6,6	7200	12	57
6	7	2,4,5,5,6,7	8400	13	98	6	8	2,4,5,5,6,8	9600	13	85	6	7	2,4,5,5,7,7	9800	13	84
6	8	2,4,5,5,7,8	11200	13	73	6	8	2,4,5,5,8,8	12800	13	64	6	6	2,4,5,6,6,6	8640	13	95
6	7	2,4,5,6,6,7	10080	13	81	6	8	2,4,5,6,6,8	11520	13	71	6	7	2,4,5,6,7,7	11760	13	70
6	8	2,4,5,6,7,8	13440	13	61	6	8	2,4,5,6,8,8	15360	13	53	6	7	2,4,5,7,7,7	13720	13	60
6	8	2,4,5,7,7,8	15680	13	52	6	8	2,4,5,7,8,8	17920	14	91	6	8	2,4,5,8,8,8	20480	14	80
6	6	2,4,6,6,6,6	10368	13	79	6	7	2,4,6,6,6,7	12096	13	68	6	8	2,4,6,6,6,8	13824	13	59
6	7	2,4,6,6,7,7	14112	13	58	6	8	2,4,6,6,7,8	16128	13	51	6	8	2,4,6,6,8,8	18432	14	89
6	7	2,4,6,7,7,7	16464	14	100	6	8	2,4,6,7,7,8	18816	14	87	6	8	2,4,6,7,8,8	21504	14	76
6	8	2,4,6,8,8,8	24576	14	67	6	7	2,4,7,7,7,7	19208	14	85	6	8	2,4,7,7,7,8	21952	14	75
6	8	2,4,7,7,8,8	25088	14	65	6	8	2,4,7,8,8,8	28672	14	57	6	8	2,4,8,8,8,8	32768	15	100
6	5	2,5,5,5,5,5	6250	12	66	6	6	2,5,5,5,5,6	7500	12	55	6	7	2,5,5,5,5,7	8750	13	94
6	8	2,5,5,5,5,8	10000	13	82	6	6	2,5,5,5,6,6	9000	13	91	6	7	2,5,5,5,6,7	10500	13	78
6	8	2,5,5,5,6,8	12000	13	68	6	7	2,5,5,5,7,7	12250	13	67	6	8	2,5,5,5,7,8	14000	13	59

N m	S	W	b	η	N m	S	W	b	η	N m	S	W	b	η
6 8	2,5,5,5,8,8	16000	13	51	6 6	2,5,5,6,6,6	10800	13	76	6 7	2,5,5,6,6,7	12600	13	65
6 8	2,5,5,6,6,8	14400	13	57	6 7	2,5,5,6,7,7	14700	13	56	6 8	2,5,5,6,7,8	16800	14	98
6 8	2,5,5,6,8,8	19200	14	85	6 7	2,5,5,7,7,7	17150	14	96	6 8	2,5,5,7,7,8	19600	14	84
6 8	2,5,5,7,8,8	22400	14	73	6 8	2,5,5,8,8,8	25600	14	64	6 6	2,5,6,6,6,6	12960	13	63
6 7	2,5,6,6,6,7	15120	13	54	6 8	2,5,6,6,6,8	17280	14	95	6 7	2,5,6,6,7,7	17640	14	93
6 8	2,5,6,6,7,8	20160	14	81	6 8	2,5,6,6,8,8	23040	14	71	6 7	2,5,6,7,7,7	20580	14	80
6 8	2,5,6,7,7,8	23520	14	70	6 8	2,5,6,7,8,8	26880	14	61	6 8	2,5,6,8,8,8	30720	14	53
6 7	2,5,7,7,7,7	24010	14	68	6 8	2,5,7,7,7,8	27440	14	60	6 8	2,5,7,7,8,8	31360	14	52
6 8	2,5,7,8,8,8	35840	15	91	6 8	2,5,8,8,8,8	40960	15	80	6 6	2,6,6,6,6,6	15552	13	53
6 7	2,6,6,6,6,7	18144	14	90	6 8	2,6,6,6,6,8	20736	14	79	6 7	2,6,6,6,7,7	21168	14	77
6 8	2,6,6,6,7,8	24192	14	68	6 8	2,6,6,6,8,8	27648	14	59	6 7	2,6,6,7,7,7	24696	14	66
6 8	2,6,6,7,7,8	28224	14	58	6 8	2,6,6,7,8,8	32256	14	51	6 8	2,6,6,8,8,8	36864	15	89
6 7	2,6,7,7,7,7	28812	14	57	6 8	2,6,7,7,7,8	32928	15	100	6 8	2,6,7,7,8,8	37632	15	87
6 8	2,6,7,8,8,8	43008	15	76	6 8	2,6,8,8,8,8	49152	15	67	6 7	2,7,7,7,7,7	33614	15	97
6 8	2,7,7,7,7,8	38416	15	85	6 8	2,7,7,7,8,8	43904	15	75	6 8	2,7,7,8,8,8	50176	15	65
6 8	2,7,8,8,8,8	57344	15	57	6 8	2,8,8,8,8,8	65536	16	100	6 3	3,3,3,3,3,3	729	9	70
6 4	3,3,3,3,3,4	972	9	53	6 5	3,3,3,3,3,5	1215	10	84	6 6	3,3,3,3,3,6	1458	10	70
6 7	3,3,3,3,3,7	1701	10	60	6 8	3,3,3,3,3,8	1944	10	53	6 4	3,3,3,3,4,4	1296	10	79
6 5	3,3,3,3,4,5	1620	10	63	6 6	3,3,3,3,4,6	1944	10	53	6 7	3,3,3,3,4,7	2268	11	90
6 8	3,3,3,3,4,8	2592	11	79	6 5	3,3,3,3,5,5	2025	10	51	6 6	3,3,3,3,5,6	2430	11	84
6 7	3,3,3,3,5,7	2835	11	72	6 8	3,3,3,3,5,8	3240	11	63	6 6	3,3,3,3,6,6	2916	11	70
6 7	3,3,3,3,6,7	3402	11	60	6 8	3,3,3,3,6,8	3888	11	53	6 7	3,3,3,3,7,7	3969	11	52
6 8	3,3,3,3,7,8	4536	12	90	6 8	3,3,3,3,8,8	5184	12	79	6 4	3,3,3,4,4,4	1728	10	59
6 5	3,3,3,4,4,5	2160	11	95	6 6	3,3,3,4,4,6	2592	11	79	6 7	3,3,3,4,4,7	3024	11	68
6 8	3,3,3,4,4,8	3456	11	59	6 5	3,3,3,4,5,5	2700	11	76	6 6	3,3,3,4,5,6	3240	11	63
6 7	3,3,3,4,5,7	3780	11	54	6 8	3,3,3,4,5,8	4320	12	95	6 6	3,3,3,4,6,6	3888	11	53
6 7	3,3,3,4,6,7	4536	12	90	6 8	3,3,3,4,6,8	5184	12	79	6 7	3,3,3,4,7,7	5292	12	77
6 8	3,3,3,4,7,8	6048	12	68	6 8	3,3,3,4,8,8	6912	12	59	6 5	3,3,3,5,5,5	3375	11	61
6 6	3,3,3,5,5,6	4050	11	51	6 7	3,3,3,5,5,7	4725	12	87	6 8	3,3,3,5,5,8	5400	12	76
6 6	3,3,3,5,6,6	4860	12	84	6 7	3,3,3,5,6,7	5670	12	72	6 8	3,3,3,5,6,8	6480	12	63
6 7	3,3,3,5,7,7	6615	12	62	6 8	3,3,3,5,7,8	7560	12	54	6 8	3,3,3,5,8,8	8640	13	95
6 6	3,3,3,6,6,6	5832	12	70	6 7	3,3,3,6,6,7	6804	12	60	6 8	3,3,3,6,6,8	7776	12	53

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
6	7	3,3,3,6,7,7	7938	12	52	6	8	3,3,3,6,7,8	9072	13	90	6	8	3,3,3,6,8,8	10368	13	79
6	7	3,3,3,7,7,7	9261	13	88	6	8	3,3,3,7,7,8	10584	13	77	6	8	3,3,3,7,8,8	12096	13	68
6	8	3,3,3,8,8,8	13824	13	59	6	4	3,3,4,4,4,4	2304	11	89	6	5	3,3,4,4,4,5	2880	11	71
6	6	3,3,4,4,4,6	3456	11	59	6	7	3,3,4,4,4,7	4032	11	51	6	8	3,3,4,4,4,8	4608	12	89
6	5	3,3,4,4,5,5	3600	11	57	6	6	3,3,4,4,5,6	4320	12	95	6	7	3,3,4,4,5,7	5040	12	81
6	8	3,3,4,4,5,8	5760	12	71	6	6	3,3,4,4,6,6	5184	12	79	6	7	3,3,4,4,6,7	6048	12	68
6	8	3,3,4,4,6,8	6912	12	59	6	7	3,3,4,4,7,7	7056	12	58	6	8	3,3,4,4,7,8	8064	12	51
6	8	3,3,4,4,8,8	9216	13	89	6	5	3,3,4,5,5,5	4500	12	91	6	6	3,3,4,5,5,6	5400	12	76
6	7	3,3,4,5,5,7	6300	12	65	6	8	3,3,4,5,5,8	7200	12	57	6	6	3,3,4,5,6,6	6480	12	63
6	7	3,3,4,5,6,7	7560	12	54	6	8	3,3,4,5,6,8	8640	13	95	6	7	3,3,4,5,7,7	8820	13	93
6	8	3,3,4,5,7,8	10080	13	81	6	8	3,3,4,5,8,8	11520	13	71	6	6	3,3,4,6,6,6	7776	12	53
6	7	3,3,4,6,6,7	9072	13	90	6	8	3,3,4,6,6,8	10368	13	79	6	7	3,3,4,6,7,7	10584	13	77
6	8	3,3,4,6,7,8	12096	13	68	6	8	3,3,4,6,8,8	13824	13	59	6	7	3,3,4,7,7,7	12348	13	66
6	8	3,3,4,7,7,8	14112	13	58	6	8	3,3,4,7,8,8	16128	13	51	6	8	3,3,4,8,8,8	18432	14	89
6	5	3,3,5,5,5,5	5625	12	73	6	6	3,3,5,5,5,6	6750	12	61	6	7	3,3,5,5,5,7	7875	12	52
6	8	3,3,5,5,5,8	9000	13	91	6	6	3,3,5,5,6,6	8100	12	51	6	7	3,3,5,5,6,7	9450	13	87
6	8	3,3,5,5,6,8	10800	13	76	6	7	3,3,5,5,7,7	11025	13	74	6	8	3,3,5,5,7,8	12600	13	65
6	8	3,3,5,5,8,8	14400	13	57	6	6	3,3,5,6,6,6	9720	13	84	6	7	3,3,5,6,6,7	11340	13	72
6	8	3,3,5,6,6,8	12960	13	63	6	7	3,3,5,6,7,7	13230	13	62	6	8	3,3,5,6,7,8	15120	13	54
6	8	3,3,5,6,8,8	17280	14	95	6	7	3,3,5,7,7,7	15435	13	53	6	8	3,3,5,7,7,8	17640	14	93
6	8	3,3,5,7,8,8	20160	14	81	6	8	3,3,5,8,8,8	23040	14	71	6	6	3,3,6,6,6,6	11664	13	70
6	7	3,3,6,6,6,7	13608	13	60	6	8	3,3,6,6,6,8	15552	13	53	6	7	3,3,6,6,7,7	15876	13	52
6	8	3,3,6,6,7,8	18144	14	90	6	8	3,3,6,6,8,8	20736	14	79	6	7	3,3,6,7,7,7	18522	14	88
6	8	3,3,6,7,7,8	21168	14	77	6	8	3,3,6,7,8,8	24192	14	68	6	8	3,3,6,8,8,8	27648	14	59
6	7	3,3,7,7,7,7	21609	14	76	6	8	3,3,7,7,7,8	24696	14	66	6	8	3,3,7,7,8,8	28224	14	58
6	8	3,3,7,8,8,8	32256	14	51	6	8	3,3,8,8,8,8	36864	15	89	6	4	3,4,4,4,4,4	3072	11	67
6	5	3,4,4,4,4,5	3840	11	53	6	6	3,4,4,4,4,6	4608	12	89	6	7	3,4,4,4,4,7	5376	12	76
6	8	3,4,4,4,4,8	6144	12	67	6	5	3,4,4,4,5,5	4800	12	85	6	6	3,4,4,4,5,6	5760	12	71
6	7	3,4,4,4,5,7	6720	12	61	6	8	3,4,4,4,5,8	7680	12	53	6	6	3,4,4,4,6,6	6912	12	59
6	7	3,4,4,4,6,7	8064	12	51	6	8	3,4,4,4,6,8	9216	13	89	6	7	3,4,4,4,7,7	9408	13	87
6	8	3,4,4,4,7,8	10752	13	76	6	8	3,4,4,4,8,8	12288	13	67	6	5	3,4,4,5,5,5	6000	12	68
6	6	3,4,4,5,5,6	7200	12	57	6	7	3,4,4,5,5,7	8400	13	98	6	8	3,4,4,5,5,8	9600	13	85

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
6	6	3,4,4,5,6,6	8640	13	95	6	7	3,4,4,5,6,7	10080	13	81	6	8	3,4,4,5,6,8	11520	13	71
6	7	3,4,4,5,7,7	11760	13	70	6	8	3,4,4,5,7,8	13440	13	61	6	8	3,4,4,5,8,8	15360	13	53
6	6	3,4,4,6,6,6	10368	13	79	6	7	3,4,4,6,6,7	12096	13	68	6	8	3,4,4,6,6,8	13824	13	59
6	7	3,4,4,6,7,7	14112	13	58	6	8	3,4,4,6,7,8	16128	13	51	6	8	3,4,4,6,8,8	18432	14	89
6	7	3,4,4,7,7,7	16464	14	100	6	8	3,4,4,7,7,8	18816	14	87	6	8	3,4,4,7,8,8	21504	14	76
6	8	3,4,4,8,8,8	24576	14	67	6	5	3,4,5,5,5,5	7500	12	55	6	6	3,4,5,5,5,6	9000	13	91
6	7	3,4,5,5,5,7	10500	13	78	6	8	3,4,5,5,5,8	12000	13	68	6	6	3,4,5,5,6,6	10800	13	76
6	7	3,4,5,5,6,7	12600	13	65	6	8	3,4,5,5,6,8	14400	13	57	6	7	3,4,5,5,7,7	14700	13	56
6	8	3,4,5,5,7,8	16800	14	98	6	8	3,4,5,5,8,8	19200	14	85	6	6	3,4,5,6,6,6	12960	13	63
6	7	3,4,5,6,6,7	15120	13	54	6	8	3,4,5,6,6,8	17280	14	95	6	7	3,4,5,6,7,7	17640	14	93
6	8	3,4,5,6,7,8	20160	14	81	6	8	3,4,5,6,8,8	23040	14	71	6	7	3,4,5,7,7,7	20580	14	80
6	8	3,4,5,7,7,8	23520	14	70	6	8	3,4,5,7,8,8	26880	14	61	6	8	3,4,5,8,8,8	30720	14	53
6	6	3,4,6,6,6,6	15552	13	53	6	7	3,4,6,6,6,7	18144	14	90	6	8	3,4,6,6,6,8	20736	14	79
6	7	3,4,6,6,7,7	21168	14	77	6	8	3,4,6,6,7,8	24192	14	68	6	8	3,4,6,6,8,8	27648	14	59
6	7	3,4,6,7,7,7	24696	14	66	6	8	3,4,6,7,7,8	28224	14	58	6	8	3,4,6,7,8,8	32256	14	51
6	8	3,4,6,8,8,8	36864	15	89	6	7	3,4,7,7,7,7	28812	14	57	6	8	3,4,7,7,7,8	32928	15	100
6	8	3,4,7,7,8,8	37632	15	87	6	8	3,4,7,8,8,8	43008	15	76	6	8	3,4,8,8,8,8	49152	15	67
6	5	3,5,5,5,5,5	9375	13	87	6	6	3,5,5,5,5,6	11250	13	73	6	7	3,5,5,5,5,7	13125	13	62
6	8	3,5,5,5,5,8	15000	13	55	6	6	3,5,5,5,6,6	13500	13	61	6	7	3,5,5,5,6,7	15750	13	52
6	8	3,5,5,5,6,8	18000	14	91	6	7	3,5,5,5,7,7	18375	14	89	6	8	3,5,5,5,7,8	21000	14	78
6	8	3,5,5,5,8,8	24000	14	68	6	6	3,5,5,6,6,6	16200	13	51	6	7	3,5,5,6,6,7	18900	14	87
6	8	3,5,5,6,6,8	21600	14	76	6	7	3,5,5,6,7,7	22050	14	74	6	8	3,5,5,6,7,8	25200	14	65
6	8	3,5,5,6,8,8	28800	14	57	6	7	3,5,5,7,7,7	25725	14	64	6	8	3,5,5,7,7,8	29400	14	56
6	8	3,5,5,7,8,8	33600	15	98	6	8	3,5,5,8,8,8	38400	15	85	6	6	3,5,6,6,6,6	19440	14	84
6	7	3,5,6,6,6,7	22680	14	72	6	8	3,5,6,6,6,8	25920	14	63	6	7	3,5,6,6,7,7	26460	14	62
6	8	3,5,6,6,7,8	30240	14	54	6	8	3,5,6,6,8,8	34560	15	95	6	7	3,5,6,7,7,7	30870	14	53
6	8	3,5,6,7,7,8	35280	15	93	6	8	3,5,6,7,8,8	40320	15	81	6	8	3,5,6,8,8,8	46080	15	71
6	7	3,5,7,7,7,7	36015	15	91	6	8	3,5,7,7,7,8	41160	15	80	6	8	3,5,7,7,8,8	47040	15	70
6	8	3,5,7,8,8,8	53760	15	61	6	8	3,5,8,8,8,8	61440	15	53	6	6	3,6,6,6,6,6	23328	14	70
6	7	3,6,6,6,6,7	27216	14	60	6	8	3,6,6,6,6,8	31104	14	53	6	7	3,6,6,6,7,7	31752	14	52
6	8	3,6,6,6,7,8	36288	15	90	6	8	3,6,6,6,8,8	41472	15	79	6	7	3,6,6,7,7,7	37044	15	88
6	8	3,6,6,7,7,8	42336	15	77	6	8	3,6,6,7,8,8	48384	15	68	6	8	3,6,6,8,8,8	55296	15	59

N m	S	W	b	η	N m	S	W	b	η	N m	S	W	b	η
6 7	3,6,7,7,7,7	43218	15	76	6 8	3,6,7,7,7,8	49392	15	66	6 8	3,6,7,7,8,8	56448	15	58
6 8	3,6,7,8,8,8	64512	15	51	6 8	3,6,8,8,8,8	73728	16	89	6 7	3,7,7,7,7,7	50421	15	65
6 8	3,7,7,7,7,8	57624	15	57	6 8	3,7,7,7,8,8	65856	16	100	6 8	3,7,7,8,8,8	75264	16	87
6 8	3,7,8,8,8,8	86016	16	76	6 8	3,8,8,8,8,8	98304	16	67	6 4	4,4,4,4,4,4	4096	12	100
6 5	4,4,4,4,4,5	5120	12	80	6 6	4,4,4,4,4,6	6144	12	67	6 7	4,4,4,4,4,7	7168	12	57
6 8	4,4,4,4,4,8	8192	13	100	6 5	4,4,4,4,5,5	6400	12	64	6 6	4,4,4,4,5,6	7680	12	53
6 7	4,4,4,4,5,7	8960	13	91	6 8	4,4,4,4,5,8	10240	13	80	6 6	4,4,4,4,6,6	9216	13	89
6 7	4,4,4,4,6,7	10752	13	76	6 8	4,4,4,4,6,8	12288	13	67	6 7	4,4,4,4,7,7	12544	13	65
6 8	4,4,4,4,7,8	14336	13	57	6 8	4,4,4,4,8,8	16384	14	100	6 5	4,4,4,5,5,5	8000	12	51
6 6	4,4,4,5,5,6	9600	13	85	6 7	4,4,4,5,5,7	11200	13	73	6 8	4,4,4,5,5,8	12800	13	64
6 6	4,4,4,5,6,6	11520	13	71	6 7	4,4,4,5,6,7	13440	13	61	6 8	4,4,4,5,6,8	15360	13	53
6 7	4,4,4,5,7,7	15680	13	52	6 8	4,4,4,5,7,8	17920	14	91	6 8	4,4,4,5,8,8	20480	14	80
6 6	4,4,4,6,6,6	13824	13	59	6 7	4,4,4,6,6,7	16128	13	51	6 8	4,4,4,6,6,8	18432	14	89
6 7	4,4,4,6,7,7	18816	14	87	6 8	4,4,4,6,7,8	21504	14	76	6 8	4,4,4,6,8,8	24576	14	67
6 7	4,4,4,7,7,7	21952	14	75	6 8	4,4,4,7,7,8	25088	14	65	6 8	4,4,4,7,8,8	28672	14	57
6 8	4,4,4,8,8,8	32768	15	100	6 5	4,4,5,5,5,5	10000	13	82	6 6	4,4,5,5,5,6	12000	13	68
6 7	4,4,5,5,5,7	14000	13	59	6 8	4,4,5,5,5,8	16000	13	51	6 6	4,4,5,5,6,6	14400	13	57
6 7	4,4,5,5,6,7	16800	14	98	6 8	4,4,5,5,6,8	19200	14	85	6 7	4,4,5,5,7,7	19600	14	84
6 8	4,4,5,5,7,8	22400	14	73	6 8	4,4,5,5,8,8	25600	14	64	6 6	4,4,5,6,6,6	17280	14	95
6 7	4,4,5,6,6,7	20160	14	81	6 8	4,4,5,6,6,8	23040	14	71	6 7	4,4,5,6,7,7	23520	14	70
6 8	4,4,5,6,7,8	26880	14	61	6 8	4,4,5,6,8,8	30720	14	53	6 7	4,4,5,7,7,7	27440	14	60
6 8	4,4,5,7,7,8	31360	14	52	6 8	4,4,5,7,8,8	35840	15	91	6 8	4,4,5,8,8,8	40960	15	80
6 6	4,4,6,6,6,6	20736	14	79	6 7	4,4,6,6,6,7	24192	14	68	6 8	4,4,6,6,6,8	27648	14	59
6 7	4,4,6,6,7,7	28224	14	58	6 8	4,4,6,6,7,8	32256	14	51	6 8	4,4,6,6,8,8	36864	15	89
6 7	4,4,6,7,7,7	32928	15	100	6 8	4,4,6,7,7,8	37632	15	87	6 8	4,4,6,7,8,8	43008	15	76
6 8	4,4,6,8,8,8	49152	15	67	6 7	4,4,7,7,7,7	38416	15	85	6 8	4,4,7,7,7,8	43904	15	75
6 8	4,4,7,7,8,8	50176	15	65	6 8	4,4,7,8,8,8	57344	15	57	6 8	4,4,8,8,8,8	65536	16	100
6 5	4,5,5,5,5,5	12500	13	66	6 6	4,5,5,5,5,6	15000	13	55	6 7	4,5,5,5,5,7	17500	14	94
6 8	4,5,5,5,5,8	20000	14	82	6 6	4,5,5,5,6,6	18000	14	91	6 7	4,5,5,5,6,7	21000	14	78
6 8	4,5,5,5,6,8	24000	14	68	6 7	4,5,5,5,7,7	24500	14	67	6 8	4,5,5,5,7,8	28000	14	59
6 8	4,5,5,5,8,8	32000	14	51	6 6	4,5,5,6,6,6	21600	14	76	6 7	4,5,5,6,6,7	25200	14	65
6 8	4,5,5,6,6,8	28800	14	57	6 7	4,5,5,6,7,7	29400	14	56	6 8	4,5,5,6,7,8	33600	15	98

N m	S	W	b	η	N m	S	W	b	η	N m	S	W	b	η
6 8	4,5,5,6,8,8	38400	15	85	6 7	4,5,5,7,7,7	34300	15	96	6 8	4,5,5,7,7,8	39200	15	84
6 8	4,5,5,7,8,8	44800	15	73	6 8	4,5,5,8,8,8	51200	15	64	6 6	4,5,6,6,6,6	25920	14	63
6 7	4,5,6,6,6,7	30240	14	54	6 8	4,5,6,6,6,8	34560	15	95	6 7	4,5,6,6,7,7	35280	15	93
6 8	4,5,6,6,7,8	40320	15	81	6 8	4,5,6,6,8,8	46080	15	71	6 7	4,5,6,7,7,7	41160	15	80
6 8	4,5,6,7,7,8	47040	15	70	6 8	4,5,6,7,8,8	53760	15	61	6 8	4,5,6,8,8,8	61440	15	53
6 7	4,5,7,7,7,7	48020	15	68	6 8	4,5,7,7,7,8	54880	15	60	6 8	4,5,7,7,8,8	62720	15	52
6 8	4,5,7,8,8,8	71680	16	91	6 8	4,5,8,8,8,8	81920	16	80	6 6	4,6,6,6,6,6	31104	14	53
6 7	4,6,6,6,6,7	36288	15	90	6 8	4,6,6,6,6,8	41472	15	79	6 7	4,6,6,6,7,7	42336	15	77
6 8	4,6,6,6,7,8	48384	15	68	6 8	4,6,6,6,8,8	55296	15	59	6 7	4,6,6,7,7,7	49392	15	66
6 8	4,6,6,7,7,8	56448	15	58	6 8	4,6,6,7,8,8	64512	15	51	6 8	4,6,6,8,8,8	73728	16	89
6 7	4,6,7,7,7,7	57624	15	57	6 8	4,6,7,7,7,8	65856	16	100	6 8	4,6,7,7,8,8	75264	16	87
6 8	4,6,7,8,8,8	86016	16	76	6 8	4,6,8,8,8,8	98304	16	67	6 7	4,7,7,7,7,7	67228	16	97
6 8	4,7,7,7,7,8	76832	16	85	6 8	4,7,7,7,8,8	87808	16	75	6 8	4,7,7,8,8,8	100352	16	65
6 8	4,7,8,8,8,8	114688	16	57	6 8	4,8,8,8,8,8	131072	17	100	6 5	5,5,5,5,5,5	15625	13	52
6 6	5,5,5,5,5,6	18750	14	87	6 7	5,5,5,5,5,7	21875	14	75	6 8	5,5,5,5,5,8	25000	14	66
6 6	5,5,5,5,6,6	22500	14	73	6 7	5,5,5,5,6,7	26250	14	62	6 8	5,5,5,5,6,8	30000	14	55
6 7	5,5,5,5,7,7	30625	14	53	6 8	5,5,5,5,7,8	35000	15	94	6 8	5,5,5,5,8,8	40000	15	82
6 6	5,5,5,6,6,6	27000	14	61	6 7	5,5,5,6,6,7	31500	14	52	6 8	5,5,5,6,6,8	36000	15	91
6 7	5,5,5,6,7,7	36750	15	89	6 8	5,5,5,6,7,8	42000	15	78	6 8	5,5,5,6,8,8	48000	15	68
6 7	5,5,5,7,7,7	42875	15	76	6 8	5,5,5,7,7,8	49000	15	67	6 8	5,5,5,7,8,8	56000	15	59
6 8	5,5,5,8,8,8	64000	15	51	6 6	5,5,6,6,6,6	32400	14	51	6 7	5,5,6,6,6,7	37800	15	87
6 8	5,5,6,6,6,8	43200	15	76	6 7	5,5,6,6,7,7	44100	15	74	6 8	5,5,6,6,7,8	50400	15	65
6 8	5,5,6,6,8,8	57600	15	57	6 7	5,5,6,7,7,7	51450	15	64	6 8	5,5,6,7,7,8	58800	15	56
6 8	5,5,6,7,8,8	67200	16	98	6 8	5,5,6,8,8,8	76800	16	85	6 7	5,5,7,7,7,7	60025	15	55
6 8	5,5,7,7,7,8	68600	16	96	6 8	5,5,7,7,8,8	78400	16	84	6 8	5,5,7,8,8,8	89600	16	73
6 8	5,5,8,8,8,8	102400	16	64	6 6	5,6,6,6,6,6	38880	15	84	6 7	5,6,6,6,6,7	45360	15	72
6 8	5,6,6,6,6,8	51840	15	63	6 7	5,6,6,6,7,7	52920	15	62	6 8	5,6,6,6,7,8	60480	15	54
6 8	5,6,6,6,8,8	69120	16	95	6 7	5,6,6,7,7,7	61740	15	53	6 8	5,6,6,7,7,8	70560	16	93
6 8	5,6,6,7,8,8	80640	16	81	6 8	5,6,6,8,8,8	92160	16	71	6 7	5,6,7,7,7,7	72030	16	91
6 8	5,6,7,7,7,8	82320	16	80	6 8	5,6,7,7,8,8	94080	16	70	6 8	5,6,7,8,8,8	107520	16	61
6 8	5,6,8,8,8,8	122880	16	53	6 7	5,7,7,7,7,7	84035	16	78	6 8	5,7,7,7,7,8	96040	16	68
6 8	5,7,7,7,8,8	109760	16	60	6 8	5,7,7,8,8,8	125440	16	52	6 8	5,7,8,8,8,8	143360	17	91

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
6	8	5,8,8,8,8,8	163840	17	80	6	6	6,6,6,6,6,6	46656	15	70	6	7	6,6,6,6,6,7	54432	15	60
6	8	6,6,6,6,6,8	62208	15	53	6	7	6,6,6,6,7,7	63504	15	52	6	8	6,6,6,6,7,8	72576	16	90
6	8	6,6,6,6,8,8	82944	16	79	6	7	6,6,6,7,7,7	74088	16	88	6	8	6,6,6,7,7,8	84672	16	77
6	8	6,6,6,7,8,8	96768	16	68	6	8	6,6,6,8,8,8	110592	16	59	6	7	6,6,7,7,7,7	86436	16	76
6	8	6,6,7,7,7,8	98784	16	66	6	8	6,6,7,7,8,8	112896	16	58	6	8	6,6,7,8,8,8	129024	16	51
6	8	6,6,8,8,8,8	147456	17	89	6	7	6,7,7,7,7,7	100842	16	65	6	8	6,7,7,7,7,8	115248	16	57
6	8	6,7,7,7,8,8	131712	17	100	6	8	6,7,7,8,8,8	150528	17	87	6	8	6,7,8,8,8,8	172032	17	76
6	8	6,8,8,8,8,8	196608	17	67	6	7	7,7,7,7,7,7	117649	16	56	6	8	7,7,7,7,7,8	134456	17	97
6	8	7,7,7,7,8,8	153664	17	85	6	8	7,7,7,8,8,8	175616	17	75	6	8	7,7,8,8,8,8	200704	17	65
6	8	7,8,8,8,8,8	229376	17	57	6	8	8,8,8,8,8,8	262144	18	100	7	2	2,2,2,2,2,2,2	128	7	100
7	3	2,2,2,2,2,2,3	192	7	67	7	4	2,2,2,2,2,2,4	256	8	100	7	5	2,2,2,2,2,2,5	320	8	80
7	6	2,2,2,2,2,2,6	384	8	67	7	7	2,2,2,2,2,2,7	448	8	57	7	8	2,2,2,2,2,2,8	512	9	100
7	3	2,2,2,2,2,3,3	288	8	89	7	4	2,2,2,2,2,3,4	384	8	67	7	5	2,2,2,2,2,3,5	480	8	53
7	6	2,2,2,2,2,3,6	576	9	89	7	7	2,2,2,2,2,3,7	672	9	76	7	8	2,2,2,2,2,3,8	768	9	67
7	4	2,2,2,2,2,4,4	512	9	100	7	5	2,2,2,2,2,4,5	640	9	80	7	6	2,2,2,2,2,4,6	768	9	67
7	7	2,2,2,2,2,4,7	896	9	57	7	8	2,2,2,2,2,4,8	1024	10	100	7	5	2,2,2,2,2,5,5	800	9	64
7	6	2,2,2,2,2,5,6	960	9	53	7	7	2,2,2,2,2,5,7	1120	10	91	7	8	2,2,2,2,2,5,8	1280	10	80
7	6	2,2,2,2,2,6,6	1152	10	89	7	7	2,2,2,2,2,6,7	1344	10	76	7	8	2,2,2,2,2,6,8	1536	10	67
7	7	2,2,2,2,2,7,7	1568	10	65	7	8	2,2,2,2,2,7,8	1792	10	57	7	8	2,2,2,2,2,8,8	2048	11	100
7	3	2,2,2,2,3,3,3	432	8	59	7	4	2,2,2,2,3,3,4	576	9	89	7	5	2,2,2,2,3,3,5	720	9	71
7	6	2,2,2,2,3,3,6	864	9	59	7	7	2,2,2,2,3,3,7	1008	9	51	7	8	2,2,2,2,3,3,8	1152	10	89
7	4	2,2,2,2,3,4,4	768	9	67	7	5	2,2,2,2,3,4,5	960	9	53	7	6	2,2,2,2,3,4,6	1152	10	89
7	7	2,2,2,2,3,4,7	1344	10	76	7	8	2,2,2,2,3,4,8	1536	10	67	7	5	2,2,2,2,3,5,5	1200	10	85
7	6	2,2,2,2,3,5,6	1440	10	71	7	7	2,2,2,2,3,5,7	1680	10	61	7	8	2,2,2,2,3,5,8	1920	10	53
7	6	2,2,2,2,3,6,6	1728	10	59	7	7	2,2,2,2,3,6,7	2016	10	51	7	8	2,2,2,2,3,6,8	2304	11	89
7	7	2,2,2,2,3,7,7	2352	11	87	7	8	2,2,2,2,3,7,8	2688	11	76	7	8	2,2,2,2,3,8,8	3072	11	67
7	4	2,2,2,2,4,4,4	1024	10	100	7	5	2,2,2,2,4,4,5	1280	10	80	7	6	2,2,2,2,4,4,6	1536	10	67
7	7	2,2,2,2,4,4,7	1792	10	57	7	8	2,2,2,2,4,4,8	2048	11	100	7	5	2,2,2,2,4,5,5	1600	10	64
7	6	2,2,2,2,4,5,6	1920	10	53	7	7	2,2,2,2,4,5,7	2240	11	91	7	8	2,2,2,2,4,5,8	2560	11	80
7	6	2,2,2,2,4,6,6	2304	11	89	7	7	2,2,2,2,4,6,7	2688	11	76	7	8	2,2,2,2,4,6,8	3072	11	67
7	7	2,2,2,2,4,7,7	3136	11	65	7	8	2,2,2,2,4,7,8	3584	11	57	7	8	2,2,2,2,4,8,8	4096	12	100
7	5	2,2,2,2,5,5,5	2000	10	51	7	6	2,2,2,2,5,5,6	2400	11	85	7	7	2,2,2,2,5,5,7	2800	11	73

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	2,2,2,2,5,5,8	3200	11	64	7	6	2,2,2,2,5,6,6	2880	11	71	7	7	2,2,2,2,5,6,7	3360	11	61
7	8	2,2,2,2,5,6,8	3840	11	53	7	7	2,2,2,2,5,7,7	3920	11	52	7	8	2,2,2,2,5,7,8	4480	12	91
7	8	2,2,2,2,5,8,8	5120	12	80	7	6	2,2,2,2,6,6,6	3456	11	59	7	7	2,2,2,2,6,6,7	4032	11	51
7	8	2,2,2,2,6,6,8	4608	12	89	7	7	2,2,2,2,6,7,7	4704	12	87	7	8	2,2,2,2,6,7,8	5376	12	76
7	8	2,2,2,2,6,8,8	6144	12	67	7	7	2,2,2,2,7,7,7	5488	12	75	7	8	2,2,2,2,7,7,8	6272	12	65
7	8	2,2,2,2,7,8,8	7168	12	57	7	8	2,2,2,2,8,8,8	8192	13	100	7	3	2,2,2,3,3,3,3	648	9	79
7	4	2,2,2,3,3,3,4	864	9	59	7	5	2,2,2,3,3,3,5	1080	10	95	7	6	2,2,2,3,3,3,6	1296	10	79
7	7	2,2,2,3,3,3,7	1512	10	68	7	8	2,2,2,3,3,3,8	1728	10	59	7	4	2,2,2,3,3,4,4	1152	10	89
7	5	2,2,2,3,3,4,5	1440	10	71	7	6	2,2,2,3,3,4,6	1728	10	59	7	7	2,2,2,3,3,4,7	2016	10	51
7	8	2,2,2,3,3,4,8	2304	11	89	7	5	2,2,2,3,3,5,5	1800	10	57	7	6	2,2,2,3,3,5,6	2160	11	95
7	7	2,2,2,3,3,5,7	2520	11	81	7	8	2,2,2,3,3,5,8	2880	11	71	7	6	2,2,2,3,3,6,6	2592	11	79
7	7	2,2,2,3,3,6,7	3024	11	68	7	8	2,2,2,3,3,6,8	3456	11	59	7	7	2,2,2,3,3,7,7	3528	11	58
7	8	2,2,2,3,3,7,8	4032	11	51	7	8	2,2,2,3,3,8,8	4608	12	89	7	4	2,2,2,3,4,4,4	1536	10	67
7	5	2,2,2,3,4,4,5	1920	10	53	7	6	2,2,2,3,4,4,6	2304	11	89	7	7	2,2,2,3,4,4,7	2688	11	76
7	8	2,2,2,3,4,4,8	3072	11	67	7	5	2,2,2,3,4,5,5	2400	11	85	7	6	2,2,2,3,4,5,6	2880	11	71
7	7	2,2,2,3,4,5,7	3360	11	61	7	8	2,2,2,3,4,5,8	3840	11	53	7	6	2,2,2,3,4,6,6	3456	11	59
7	7	2,2,2,3,4,6,7	4032	11	51	7	8	2,2,2,3,4,6,8	4608	12	89	7	7	2,2,2,3,4,7,7	4704	12	87
7	8	2,2,2,3,4,7,8	5376	12	76	7	8	2,2,2,3,4,8,8	6144	12	67	7	5	2,2,2,3,5,5,5	3000	11	68
7	6	2,2,2,3,5,5,6	3600	11	57	7	7	2,2,2,3,5,5,7	4200	12	98	7	8	2,2,2,3,5,5,8	4800	12	85
7	6	2,2,2,3,5,6,6	4320	12	95	7	7	2,2,2,3,5,6,7	5040	12	81	7	8	2,2,2,3,5,6,8	5760	12	71
7	7	2,2,2,3,5,7,7	5880	12	70	7	8	2,2,2,3,5,7,8	6720	12	61	7	8	2,2,2,3,5,8,8	7680	12	53
7	6	2,2,2,3,6,6,6	5184	12	79	7	7	2,2,2,3,6,6,7	6048	12	68	7	8	2,2,2,3,6,6,8	6912	12	59
7	7	2,2,2,3,6,7,7	7056	12	58	7	8	2,2,2,3,6,7,8	8064	12	51	7	8	2,2,2,3,6,8,8	9216	13	89
7	7	2,2,2,3,7,7,7	8232	13	100	7	8	2,2,2,3,7,7,8	9408	13	87	7	8	2,2,2,3,7,8,8	10752	13	76
7	8	2,2,2,3,8,8,8	12288	13	67	7	4	2,2,2,4,4,4,4	2048	11	100	7	5	2,2,2,4,4,4,5	2560	11	80
7	6	2,2,2,4,4,4,6	3072	11	67	7	7	2,2,2,4,4,4,7	3584	11	57	7	8	2,2,2,4,4,4,8	4096	12	100
7	5	2,2,2,4,4,5,5	3200	11	64	7	6	2,2,2,4,4,5,6	3840	11	53	7	7	2,2,2,4,4,5,7	4480	12	91
7	8	2,2,2,4,4,5,8	5120	12	80	7	6	2,2,2,4,4,6,6	4608	12	89	7	7	2,2,2,4,4,6,7	5376	12	76
7	8	2,2,2,4,4,6,8	6144	12	67	7	7	2,2,2,4,4,7,7	6272	12	65	7	8	2,2,2,4,4,7,8	7168	12	57
7	8	2,2,2,4,4,8,8	8192	13	100	7	5	2,2,2,4,5,5,5	4000	11	51	7	6	2,2,2,4,5,5,6	4800	12	85
7	7	2,2,2,4,5,5,7	5600	12	73	7	8	2,2,2,4,5,5,8	6400	12	64	7	6	2,2,2,4,5,6,6	5760	12	71
7	7	2,2,2,4,5,6,7	6720	12	61	7	8	2,2,2,4,5,6,8	7680	12	53	7	7	2,2,2,4,5,7,7	7840	12	52

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	2,2,2,4,5,7,8	8960	13	91	7	8	2,2,2,4,5,8,8	10240	13	80	7	6	2,2,2,4,6,6,6	6912	12	59
7	7	2,2,2,4,6,6,7	8064	12	51	7	8	2,2,2,4,6,6,8	9216	13	89	7	7	2,2,2,4,6,7,7	9408	13	87
7	8	2,2,2,4,6,7,8	10752	13	76	7	8	2,2,2,4,6,8,8	12288	13	67	7	7	2,2,2,4,7,7,7	10976	13	75
7	8	2,2,2,4,7,7,8	12544	13	65	7	8	2,2,2,4,7,8,8	14336	13	57	7	8	2,2,2,4,8,8,8	16384	14	100
7	5	2,2,2,5,5,5,5	5000	12	82	7	6	2,2,2,5,5,5,6	6000	12	68	7	7	2,2,2,5,5,5,7	7000	12	59
7	8	2,2,2,5,5,5,8	8000	12	51	7	6	2,2,2,5,5,6,6	7200	12	57	7	7	2,2,2,5,5,6,7	8400	13	98
7	8	2,2,2,5,5,6,8	9600	13	85	7	7	2,2,2,5,5,7,7	9800	13	84	7	8	2,2,2,5,5,7,8	11200	13	73
7	8	2,2,2,5,5,8,8	12800	13	64	7	6	2,2,2,5,6,6,6	8640	13	95	7	7	2,2,2,5,6,6,7	10080	13	81
7	8	2,2,2,5,6,6,8	11520	13	71	7	7	2,2,2,5,6,7,7	11760	13	70	7	8	2,2,2,5,6,7,8	13440	13	61
7	8	2,2,2,5,6,8,8	15360	13	53	7	7	2,2,2,5,7,7,7	13720	13	60	7	8	2,2,2,5,7,7,8	15680	13	52
7	8	2,2,2,5,7,8,8	17920	14	91	7	8	2,2,2,5,8,8,8	20480	14	80	7	6	2,2,2,6,6,6,6	10368	13	79
7	7	2,2,2,6,6,6,7	12096	13	68	7	8	2,2,2,6,6,6,8	13824	13	59	7	7	2,2,2,6,6,7,7	14112	13	58
7	8	2,2,2,6,6,7,8	16128	13	51	7	8	2,2,2,6,6,8,8	18432	14	89	7	7	2,2,2,6,7,7,7	16464	14	100
7	8	2,2,2,6,7,7,8	18816	14	87	7	8	2,2,2,6,7,8,8	21504	14	76	7	8	2,2,2,6,8,8,8	24576	14	67
7	7	2,2,2,7,7,7,7	19208	14	85	7	8	2,2,2,7,7,7,8	21952	14	75	7	8	2,2,2,7,7,8,8	25088	14	65
7	8	2,2,2,7,8,8,8	28672	14	57	7	8	2,2,2,8,8,8,8	32768	15	100	7	3	2,2,3,3,3,3,3	972	9	53
7	4	2,2,3,3,3,3,4	1296	10	79	7	5	2,2,3,3,3,3,5	1620	10	63	7	6	2,2,3,3,3,3,6	1944	10	53
7	7	2,2,3,3,3,3,7	2268	11	90	7	8	2,2,3,3,3,3,8	2592	11	79	7	4	2,2,3,3,3,4,4	1728	10	59
7	5	2,2,3,3,3,4,5	2160	11	95	7	6	2,2,3,3,3,4,6	2592	11	79	7	7	2,2,3,3,3,4,7	3024	11	68
7	8	2,2,3,3,3,4,8	3456	11	59	7	5	2,2,3,3,3,5,5	2700	11	76	7	6	2,2,3,3,3,5,6	3240	11	63
7	7	2,2,3,3,3,5,7	3780	11	54	7	8	2,2,3,3,3,5,8	4320	12	95	7	6	2,2,3,3,3,6,6	3888	11	53
7	7	2,2,3,3,3,6,7	4536	12	90	7	8	2,2,3,3,3,6,8	5184	12	79	7	7	2,2,3,3,3,7,7	5292	12	77
7	8	2,2,3,3,3,7,8	6048	12	68	7	8	2,2,3,3,3,8,8	6912	12	59	7	4	2,2,3,3,4,4,4	2304	11	89
7	5	2,2,3,3,4,4,5	2880	11	71	7	6	2,2,3,3,4,4,6	3456	11	59	7	7	2,2,3,3,4,4,7	4032	11	51
7	8	2,2,3,3,4,4,8	4608	12	89	7	5	2,2,3,3,4,5,5	3600	11	57	7	6	2,2,3,3,4,5,6	4320	12	95
7	7	2,2,3,3,4,5,7	5040	12	81	7	8	2,2,3,3,4,5,8	5760	12	71	7	6	2,2,3,3,4,6,6	5184	12	79
7	7	2,2,3,3,4,6,7	6048	12	68	7	8	2,2,3,3,4,6,8	6912	12	59	7	7	2,2,3,3,4,7,7	7056	12	58
7	8	2,2,3,3,4,7,8	8064	12	51	7	8	2,2,3,3,4,8,8	9216	13	89	7	5	2,2,3,3,5,5,5	4500	12	91
7	6	2,2,3,3,5,5,6	5400	12	76	7	7	2,2,3,3,5,5,7	6300	12	65	7	8	2,2,3,3,5,5,8	7200	12	57
7	6	2,2,3,3,5,6,6	6480	12	63	7	7	2,2,3,3,5,6,7	7560	12	54	7	8	2,2,3,3,5,6,8	8640	13	95
7	7	2,2,3,3,5,7,7	8820	13	93	7	8	2,2,3,3,5,7,8	10080	13	81	7	8	2,2,3,3,5,8,8	11520	13	71
7	6	2,2,3,3,6,6,6	7776	12	53	7	7	2,2,3,3,6,6,7	9072	13	90	7	8	2,2,3,3,6,6,8	10368	13	79

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	7	2,2,3,3,6,7,7	10584	13	77	7	8	2,2,3,3,6,7,8	12096	13	68	7	8	2,2,3,3,6,8,8	13824	13	59
7	7	2,2,3,3,7,7,7	12348	13	66	7	8	2,2,3,3,7,7,8	14112	13	58	7	8	2,2,3,3,7,8,8	16128	13	51
7	8	2,2,3,3,8,8,8	18432	14	89	7	4	2,2,3,4,4,4,4	3072	11	67	7	5	2,2,3,4,4,4,5	3840	11	53
7	6	2,2,3,4,4,4,6	4608	12	89	7	7	2,2,3,4,4,4,7	5376	12	76	7	8	2,2,3,4,4,4,8	6144	12	67
7	5	2,2,3,4,4,5,5	4800	12	85	7	6	2,2,3,4,4,5,6	5760	12	71	7	7	2,2,3,4,4,5,7	6720	12	61
7	8	2,2,3,4,4,5,8	7680	12	53	7	6	2,2,3,4,4,6,6	6912	12	59	7	7	2,2,3,4,4,6,7	8064	12	51
7	8	2,2,3,4,4,6,8	9216	13	89	7	7	2,2,3,4,4,7,7	9408	13	87	7	8	2,2,3,4,4,7,8	10752	13	76
7	8	2,2,3,4,4,8,8	12288	13	67	7	5	2,2,3,4,5,5,5	6000	12	68	7	6	2,2,3,4,5,5,6	7200	12	57
7	7	2,2,3,4,5,5,7	8400	13	98	7	8	2,2,3,4,5,5,8	9600	13	85	7	6	2,2,3,4,5,6,6	8640	13	95
7	7	2,2,3,4,5,6,7	10080	13	81	7	8	2,2,3,4,5,6,8	11520	13	71	7	7	2,2,3,4,5,7,7	11760	13	70
7	8	2,2,3,4,5,7,8	13440	13	61	7	8	2,2,3,4,5,8,8	15360	13	53	7	6	2,2,3,4,6,6,6	10368	13	79
7	7	2,2,3,4,6,6,7	12096	13	68	7	8	2,2,3,4,6,6,8	13824	13	59	7	7	2,2,3,4,6,7,7	14112	13	58
7	8	2,2,3,4,6,7,8	16128	13	51	7	8	2,2,3,4,6,8,8	18432	14	89	7	7	2,2,3,4,7,7,7	16464	14	100
7	8	2,2,3,4,7,7,8	18816	14	87	7	8	2,2,3,4,7,8,8	21504	14	76	7	8	2,2,3,4,8,8,8	24576	14	67
7	5	2,2,3,5,5,5,5	7500	12	55	7	6	2,2,3,5,5,5,6	9000	13	91	7	7	2,2,3,5,5,5,7	10500	13	78
7	8	2,2,3,5,5,5,8	12000	13	68	7	6	2,2,3,5,5,6,6	10800	13	76	7	7	2,2,3,5,5,6,7	12600	13	65
7	8	2,2,3,5,5,6,8	14400	13	57	7	7	2,2,3,5,5,7,7	14700	13	56	7	8	2,2,3,5,5,7,8	16800	14	98
7	8	2,2,3,5,5,8,8	19200	14	85	7	6	2,2,3,5,6,6,6	12960	13	63	7	7	2,2,3,5,6,6,7	15120	13	54
7	8	2,2,3,5,6,6,8	17280	14	95	7	7	2,2,3,5,6,7,7	17640	14	93	7	8	2,2,3,5,6,7,8	20160	14	81
7	8	2,2,3,5,6,8,8	23040	14	71	7	7	2,2,3,5,7,7,7	20580	14	80	7	8	2,2,3,5,7,7,8	23520	14	70
7	8	2,2,3,5,7,8,8	26880	14	61	7	8	2,2,3,5,8,8,8	30720	14	53	7	6	2,2,3,6,6,6,6	15552	13	53
7	7	2,2,3,6,6,6,7	18144	14	90	7	8	2,2,3,6,6,6,8	20736	14	79	7	7	2,2,3,6,6,7,7	21168	14	77
7	8	2,2,3,6,6,7,8	24192	14	68	7	8	2,2,3,6,6,8,8	27648	14	59	7	7	2,2,3,6,7,7,7	24696	14	66
7	8	2,2,3,6,7,7,8	28224	14	58	7	8	2,2,3,6,7,8,8	32256	14	51	7	8	2,2,3,6,8,8,8	36864	15	89
7	7	2,2,3,7,7,7,7	28812	14	57	7	8	2,2,3,7,7,7,8	32928	15	100	7	8	2,2,3,7,7,8,8	37632	15	87
7	8	2,2,3,7,8,8,8	43008	15	76	7	8	2,2,3,8,8,8,8	49152	15	67	7	4	2,2,4,4,4,4,4	4096	12	100
7	5	2,2,4,4,4,4,5	5120	12	80	7	6	2,2,4,4,4,4,6	6144	12	67	7	7	2,2,4,4,4,4,7	7168	12	57
7	8	2,2,4,4,4,4,8	8192	13	100	7	5	2,2,4,4,4,5,5	6400	12	64	7	6	2,2,4,4,4,5,6	7680	12	53
7	7	2,2,4,4,4,5,7	8960	13	91	7	8	2,2,4,4,4,5,8	10240	13	80	7	6	2,2,4,4,4,6,6	9216	13	89
7	7	2,2,4,4,4,6,7	10752	13	76	7	8	2,2,4,4,4,6,8	12288	13	67	7	7	2,2,4,4,4,7,7	12544	13	65
7	8	2,2,4,4,4,7,8	14336	13	57	7	8	2,2,4,4,4,8,8	16384	14	100	7	5	2,2,4,4,5,5,5	8000	12	51
7	6	2,2,4,4,5,5,6	9600	13	85	7	7	2,2,4,4,5,5,7	11200	13	73	7	8	2,2,4,4,5,5,8	12800	13	64

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	6	2,2,4,4,5,6,6	11520	13	71	7	7	2,2,4,4,5,6,7	13440	13	61	7	8	2,2,4,4,5,6,8	15360	13	53
7	7	2,2,4,4,5,7,7	15680	13	52	7	8	2,2,4,4,5,7,8	17920	14	91	7	8	2,2,4,4,5,8,8	20480	14	80
7	6	2,2,4,4,6,6,6	13824	13	59	7	7	2,2,4,4,6,6,7	16128	13	51	7	8	2,2,4,4,6,6,8	18432	14	89
7	7	2,2,4,4,6,7,7	18816	14	87	7	8	2,2,4,4,6,7,8	21504	14	76	7	8	2,2,4,4,6,8,8	24576	14	67
7	7	2,2,4,4,7,7,7	21952	14	75	7	8	2,2,4,4,7,7,8	25088	14	65	7	8	2,2,4,4,7,8,8	28672	14	57
7	8	2,2,4,4,8,8,8	32768	15	100	7	5	2,2,4,5,5,5,5	10000	13	82	7	6	2,2,4,5,5,5,6	12000	13	68
7	7	2,2,4,5,5,5,7	14000	13	59	7	8	2,2,4,5,5,5,8	16000	13	51	7	6	2,2,4,5,5,6,6	14400	13	57
7	7	2,2,4,5,5,6,7	16800	14	98	7	8	2,2,4,5,5,6,8	19200	14	85	7	7	2,2,4,5,5,7,7	19600	14	84
7	8	2,2,4,5,5,7,8	22400	14	73	7	8	2,2,4,5,5,8,8	25600	14	64	7	6	2,2,4,5,6,6,6	17280	14	95
7	7	2,2,4,5,6,6,7	20160	14	81	7	8	2,2,4,5,6,6,8	23040	14	71	7	7	2,2,4,5,6,7,7	23520	14	70
7	8	2,2,4,5,6,7,8	26880	14	61	7	8	2,2,4,5,6,8,8	30720	14	53	7	7	2,2,4,5,7,7,7	27440	14	60
7	8	2,2,4,5,7,7,8	31360	14	52	7	8	2,2,4,5,7,8,8	35840	15	91	7	8	2,2,4,5,8,8,8	40960	15	80
7	6	2,2,4,6,6,6,6	20736	14	79	7	7	2,2,4,6,6,6,7	24192	14	68	7	8	2,2,4,6,6,6,8	27648	14	59
7	7	2,2,4,6,6,7,7	28224	14	58	7	8	2,2,4,6,6,7,8	32256	14	51	7	8	2,2,4,6,6,8,8	36864	15	89
7	7	2,2,4,6,7,7,7	32928	15	100	7	8	2,2,4,6,7,7,8	37632	15	87	7	8	2,2,4,6,7,8,8	43008	15	76
7	8	2,2,4,6,8,8,8	49152	15	67	7	7	2,2,4,7,7,7,7	38416	15	85	7	8	2,2,4,7,7,7,8	43904	15	75
7	8	2,2,4,7,7,8,8	50176	15	65	7	8	2,2,4,7,8,8,8	57344	15	57	7	8	2,2,4,8,8,8,8	65536	16	100
7	5	2,2,5,5,5,5,5	12500	13	66	7	6	2,2,5,5,5,5,6	15000	13	55	7	7	2,2,5,5,5,5,7	17500	14	94
7	8	2,2,5,5,5,5,8	20000	14	82	7	6	2,2,5,5,5,6,6	18000	14	91	7	7	2,2,5,5,5,6,7	21000	14	78
7	8	2,2,5,5,5,6,8	24000	14	68	7	7	2,2,5,5,5,7,7	24500	14	67	7	8	2,2,5,5,5,7,8	28000	14	59
7	8	2,2,5,5,5,8,8	32000	14	51	7	6	2,2,5,5,6,6,6	21600	14	76	7	7	2,2,5,5,6,6,7	25200	14	65
7	8	2,2,5,5,6,6,8	28800	14	57	7	7	2,2,5,5,6,7,7	29400	14	56	7	8	2,2,5,5,6,7,8	33600	15	98
7	8	2,2,5,5,6,8,8	38400	15	85	7	7	2,2,5,5,7,7,7	34300	15	96	7	8	2,2,5,5,7,7,8	39200	15	84
7	8	2,2,5,5,7,8,8	44800	15	73	7	8	2,2,5,5,8,8,8	51200	15	64	7	6	2,2,5,6,6,6,6	25920	14	63
7	7	2,2,5,6,6,6,7	30240	14	54	7	8	2,2,5,6,6,6,8	34560	15	95	7	7	2,2,5,6,6,7,7	35280	15	93
7	8	2,2,5,6,6,7,8	40320	15	81	7	8	2,2,5,6,6,8,8	46080	15	71	7	7	2,2,5,6,7,7,7	41160	15	80
7	8	2,2,5,6,7,7,8	47040	15	70	7	8	2,2,5,6,7,8,8	53760	15	61	7	8	2,2,5,6,8,8,8	61440	15	53
7	7	2,2,5,7,7,7,7	48020	15	68	7	8	2,2,5,7,7,7,8	54880	15	60	7	8	2,2,5,7,7,8,8	62720	15	52
7	8	2,2,5,7,8,8,8	71680	16	91	7	8	2,2,5,8,8,8,8	81920	16	80	7	6	2,2,6,6,6,6,6	31104	14	53
7	7	2,2,6,6,6,6,7	36288	15	90	7	8	2,2,6,6,6,6,8	41472	15	79	7	7	2,2,6,6,6,7,7	42336	15	77
7	8	2,2,6,6,6,7,8	48384	15	68	7	8	2,2,6,6,6,8,8	55296	15	59	7	7	2,2,6,6,7,7,7	49392	15	66
7	8	2,2,6,6,7,7,8	56448	15	58	7	8	2,2,6,6,7,8,8	64512	15	51	7	8	2,2,6,6,8,8,8	73728	16	89

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	7	2,2,6,7,7,7,7	57624	15	57	7	8	2,2,6,7,7,7,8	65856	16	100	7	8	2,2,6,7,7,8,8	75264	16	87
7	8	2,2,6,7,8,8,8	86016	16	76	7	8	2,2,6,8,8,8,8	98304	16	67	7	7	2,2,7,7,7,7,7	67228	16	97
7	8	2,2,7,7,7,7,8	76832	16	85	7	8	2,2,7,7,7,8,8	87808	16	75	7	8	2,2,7,7,8,8,8	100352	16	65
7	8	2,2,7,8,8,8,8	114688	16	57	7	8	2,2,8,8,8,8,8	131072	17	100	7	3	2,3,3,3,3,3,3	1458	10	70
7	4	2,3,3,3,3,3,4	1944	10	53	7	5	2,3,3,3,3,3,5	2430	11	84	7	6	2,3,3,3,3,3,6	2916	11	70
7	7	2,3,3,3,3,3,7	3402	11	60	7	8	2,3,3,3,3,3,8	3888	11	53	7	4	2,3,3,3,3,4,4	2592	11	79
7	5	2,3,3,3,3,4,5	3240	11	63	7	6	2,3,3,3,3,4,6	3888	11	53	7	7	2,3,3,3,3,4,7	4536	12	90
7	8	2,3,3,3,3,4,8	5184	12	79	7	5	2,3,3,3,3,5,5	4050	11	51	7	6	2,3,3,3,3,5,6	4860	12	84
7	7	2,3,3,3,3,5,7	5670	12	72	7	8	2,3,3,3,3,5,8	6480	12	63	7	6	2,3,3,3,3,6,6	5832	12	70
7	7	2,3,3,3,3,6,7	6804	12	60	7	8	2,3,3,3,3,6,8	7776	12	53	7	7	2,3,3,3,3,7,7	7938	12	52
7	8	2,3,3,3,3,7,8	9072	13	90	7	8	2,3,3,3,3,8,8	10368	13	79	7	4	2,3,3,3,4,4,4	3456	11	59
7	5	2,3,3,3,4,4,5	4320	12	95	7	6	2,3,3,3,4,4,6	5184	12	79	7	7	2,3,3,3,4,4,7	6048	12	68
7	8	2,3,3,3,4,4,8	6912	12	59	7	5	2,3,3,3,4,5,5	5400	12	76	7	6	2,3,3,3,4,5,6	6480	12	63
7	7	2,3,3,3,4,5,7	7560	12	54	7	8	2,3,3,3,4,5,8	8640	13	95	7	6	2,3,3,3,4,6,6	7776	12	53
7	7	2,3,3,3,4,6,7	9072	13	90	7	8	2,3,3,3,4,6,8	10368	13	79	7	7	2,3,3,3,4,7,7	10584	13	77
7	8	2,3,3,3,4,7,8	12096	13	68	7	8	2,3,3,3,4,8,8	13824	13	59	7	5	2,3,3,3,5,5,5	6750	12	61
7	6	2,3,3,3,5,5,6	8100	12	51	7	7	2,3,3,3,5,5,7	9450	13	87	7	8	2,3,3,3,5,5,8	10800	13	76
7	6	2,3,3,3,5,6,6	9720	13	84	7	7	2,3,3,3,5,6,7	11340	13	72	7	8	2,3,3,3,5,6,8	12960	13	63
7	7	2,3,3,3,5,7,7	13230	13	62	7	8	2,3,3,3,5,7,8	15120	13	54	7	8	2,3,3,3,5,8,8	17280	14	95
7	6	2,3,3,3,6,6,6	11664	13	70	7	7	2,3,3,3,6,6,7	13608	13	60	7	8	2,3,3,3,6,6,8	15552	13	53
7	7	2,3,3,3,6,7,7	15876	13	52	7	8	2,3,3,3,6,7,8	18144	14	90	7	8	2,3,3,3,6,8,8	20736	14	79
7	7	2,3,3,3,7,7,7	18522	14	88	7	8	2,3,3,3,7,7,8	21168	14	77	7	8	2,3,3,3,7,8,8	24192	14	68
7	8	2,3,3,3,8,8,8	27648	14	59	7	4	2,3,3,4,4,4,4	4608	12	89	7	5	2,3,3,4,4,4,5	5760	12	71
7	6	2,3,3,4,4,4,6	6912	12	59	7	7	2,3,3,4,4,4,7	8064	12	51	7	8	2,3,3,4,4,4,8	9216	13	89
7	5	2,3,3,4,4,5,5	7200	12	57	7	6	2,3,3,4,4,5,6	8640	13	95	7	7	2,3,3,4,4,5,7	10080	13	81
7	8	2,3,3,4,4,5,8	11520	13	71	7	6	2,3,3,4,4,6,6	10368	13	79	7	7	2,3,3,4,4,6,7	12096	13	68
7	8	2,3,3,4,4,6,8	13824	13	59	7	7	2,3,3,4,4,7,7	14112	13	58	7	8	2,3,3,4,4,7,8	16128	13	51
7	8	2,3,3,4,4,8,8	18432	14	89	7	5	2,3,3,4,5,5,5	9000	13	91	7	6	2,3,3,4,5,5,6	10800	13	76
7	7	2,3,3,4,5,5,7	12600	13	65	7	8	2,3,3,4,5,5,8	14400	13	57	7	6	2,3,3,4,5,6,6	12960	13	63
7	7	2,3,3,4,5,6,7	15120	13	54	7	8	2,3,3,4,5,6,8	17280	14	95	7	7	2,3,3,4,5,7,7	17640	14	93
7	8	2,3,3,4,5,7,8	20160	14	81	7	8	2,3,3,4,5,8,8	23040	14	71	7	6	2,3,3,4,6,6,6	15552	13	53
7	7	2,3,3,4,6,6,7	18144	14	90	7	8	2,3,3,4,6,6,8	20736	14	79	7	7	2,3,3,4,6,7,7	21168	14	77

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	2,3,3,4,6,7,8	24192	14	68	7	8	2,3,3,4,6,8,8	27648	14	59	7	7	2,3,3,4,7,7,7	24696	14	66
7	8	2,3,3,4,7,7,8	28224	14	58	7	8	2,3,3,4,7,8,8	32256	14	51	7	8	2,3,3,4,8,8,8	36864	15	89
7	5	2,3,3,5,5,5,5	11250	13	73	7	6	2,3,3,5,5,5,6	13500	13	61	7	7	2,3,3,5,5,5,7	15750	13	52
7	8	2,3,3,5,5,5,8	18000	14	91	7	6	2,3,3,5,5,6,6	16200	13	51	7	7	2,3,3,5,5,6,7	18900	14	87
7	8	2,3,3,5,5,6,8	21600	14	76	7	7	2,3,3,5,5,7,7	22050	14	74	7	8	2,3,3,5,5,7,8	25200	14	65
7	8	2,3,3,5,5,8,8	28800	14	57	7	6	2,3,3,5,6,6,6	19440	14	84	7	7	2,3,3,5,6,6,7	22680	14	72
7	8	2,3,3,5,6,6,8	25920	14	63	7	7	2,3,3,5,6,7,7	26460	14	62	7	8	2,3,3,5,6,7,8	30240	14	54
7	8	2,3,3,5,6,8,8	34560	15	95	7	7	2,3,3,5,7,7,7	30870	14	53	7	8	2,3,3,5,7,7,8	35280	15	93
7	8	2,3,3,5,7,8,8	40320	15	81	7	8	2,3,3,5,8,8,8	46080	15	71	7	6	2,3,3,6,6,6,6	23328	14	70
7	7	2,3,3,6,6,6,7	27216	14	60	7	8	2,3,3,6,6,6,8	31104	14	53	7	7	2,3,3,6,6,7,7	31752	14	52
7	8	2,3,3,6,6,7,8	36288	15	90	7	8	2,3,3,6,6,8,8	41472	15	79	7	7	2,3,3,6,7,7,7	37044	15	88
7	8	2,3,3,6,7,7,8	42336	15	77	7	8	2,3,3,6,7,8,8	48384	15	68	7	8	2,3,3,6,8,8,8	55296	15	59
7	7	2,3,3,7,7,7,7	43218	15	76	7	8	2,3,3,7,7,7,8	49392	15	66	7	8	2,3,3,7,7,8,8	56448	15	58
7	8	2,3,3,7,8,8,8	64512	15	51	7	8	2,3,3,8,8,8,8	73728	16	89	7	4	2,3,4,4,4,4,4	6144	12	67
7	5	2,3,4,4,4,4,5	7680	12	53	7	6	2,3,4,4,4,4,6	9216	13	89	7	7	2,3,4,4,4,4,7	10752	13	76
7	8	2,3,4,4,4,4,8	12288	13	67	7	5	2,3,4,4,4,5,5	9600	13	85	7	6	2,3,4,4,4,5,6	11520	13	71
7	7	2,3,4,4,4,5,7	13440	13	61	7	8	2,3,4,4,4,5,8	15360	13	53	7	6	2,3,4,4,4,6,6	13824	13	59
7	7	2,3,4,4,4,6,7	16128	13	51	7	8	2,3,4,4,4,6,8	18432	14	89	7	7	2,3,4,4,4,7,7	18816	14	87
7	8	2,3,4,4,4,7,8	21504	14	76	7	8	2,3,4,4,4,8,8	24576	14	67	7	5	2,3,4,4,5,5,5	12000	13	68
7	6	2,3,4,4,5,5,6	14400	13	57	7	7	2,3,4,4,5,5,7	16800	14	98	7	8	2,3,4,4,5,5,8	19200	14	85
7	6	2,3,4,4,5,6,6	17280	14	95	7	7	2,3,4,4,5,6,7	20160	14	81	7	8	2,3,4,4,5,6,8	23040	14	71
7	7	2,3,4,4,5,7,7	23520	14	70	7	8	2,3,4,4,5,7,8	26880	14	61	7	8	2,3,4,4,5,8,8	30720	14	53
7	6	2,3,4,4,6,6,6	20736	14	79	7	7	2,3,4,4,6,6,7	24192	14	68	7	8	2,3,4,4,6,6,8	27648	14	59
7	7	2,3,4,4,6,7,7	28224	14	58	7	8	2,3,4,4,6,7,8	32256	14	51	7	8	2,3,4,4,6,8,8	36864	15	89
7	7	2,3,4,4,7,7,7	32928	15	100	7	8	2,3,4,4,7,7,8	37632	15	87	7	8	2,3,4,4,7,8,8	43008	15	76
7	8	2,3,4,4,8,8,8	49152	15	67	7	5	2,3,4,5,5,5,5	15000	13	55	7	6	2,3,4,5,5,5,6	18000	14	91
7	7	2,3,4,5,5,5,7	21000	14	78	7	8	2,3,4,5,5,5,8	24000	14	68	7	6	2,3,4,5,5,6,6	21600	14	76
7	7	2,3,4,5,5,6,7	25200	14	65	7	8	2,3,4,5,5,6,8	28800	14	57	7	7	2,3,4,5,5,7,7	29400	14	56
7	8	2,3,4,5,5,7,8	33600	15	98	7	8	2,3,4,5,5,8,8	38400	15	85	7	6	2,3,4,5,6,6,6	25920	14	63
7	7	2,3,4,5,6,6,7	30240	14	54	7	8	2,3,4,5,6,6,8	34560	15	95	7	7	2,3,4,5,6,7,7	35280	15	93
7	8	2,3,4,5,6,7,8	40320	15	81	7	8	2,3,4,5,6,8,8	46080	15	71	7	7	2,3,4,5,7,7,7	41160	15	80
7	8	2,3,4,5,7,7,8	47040	15	70	7	8	2,3,4,5,7,8,8	53760	15	61	7	8	2,3,4,5,8,8,8	61440	15	53

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	6	2,3,4,6,6,6,6	31104	14	53	7	7	2,3,4,6,6,6,7	36288	15	90	7	8	2,3,4,6,6,6,8	41472	15	79
7	7	2,3,4,6,6,7,7	42336	15	77	7	8	2,3,4,6,6,7,8	48384	15	68	7	8	2,3,4,6,6,8,8	55296	15	59
7	7	2,3,4,6,7,7,7	49392	15	66	7	8	2,3,4,6,7,7,8	56448	15	58	7	8	2,3,4,6,7,8,8	64512	15	51
7	8	2,3,4,6,8,8,8	73728	16	89	7	7	2,3,4,7,7,7,7	57624	15	57	7	8	2,3,4,7,7,7,8	65856	16	100
7	8	2,3,4,7,7,8,8	75264	16	87	7	8	2,3,4,7,8,8,8	86016	16	76	7	8	2,3,4,8,8,8,8	98304	16	67
7	5	2,3,5,5,5,5,5	18750	14	87	7	6	2,3,5,5,5,5,6	22500	14	73	7	7	2,3,5,5,5,5,7	26250	14	62
7	8	2,3,5,5,5,5,8	30000	14	55	7	6	2,3,5,5,5,6,6	27000	14	61	7	7	2,3,5,5,5,6,7	31500	14	52
7	8	2,3,5,5,5,6,8	36000	15	91	7	7	2,3,5,5,5,7,7	36750	15	89	7	8	2,3,5,5,5,7,8	42000	15	78
7	8	2,3,5,5,5,8,8	48000	15	68	7	6	2,3,5,5,6,6,6	32400	14	51	7	7	2,3,5,5,6,6,7	37800	15	87
7	8	2,3,5,5,6,6,8	43200	15	76	7	7	2,3,5,5,6,7,7	44100	15	74	7	8	2,3,5,5,6,7,8	50400	15	65
7	8	2,3,5,5,6,8,8	57600	15	57	7	7	2,3,5,5,7,7,7	51450	15	64	7	8	2,3,5,5,7,7,8	58800	15	56
7	8	2,3,5,5,7,8,8	67200	16	98	7	8	2,3,5,5,8,8,8	76800	16	85	7	6	2,3,5,6,6,6,6	38880	15	84
7	7	2,3,5,6,6,6,7	45360	15	72	7	8	2,3,5,6,6,6,8	51840	15	63	7	7	2,3,5,6,6,7,7	52920	15	62
7	8	2,3,5,6,6,7,8	60480	15	54	7	8	2,3,5,6,6,8,8	69120	16	95	7	7	2,3,5,6,7,7,7	61740	15	53
7	8	2,3,5,6,7,7,8	70560	16	93	7	8	2,3,5,6,7,8,8	80640	16	81	7	8	2,3,5,6,8,8,8	92160	16	71
7	7	2,3,5,7,7,7,7	72030	16	91	7	8	2,3,5,7,7,7,8	82320	16	80	7	8	2,3,5,7,7,8,8	94080	16	70
7	8	2,3,5,7,8,8,8	107520	16	61	7	8	2,3,5,8,8,8,8	122880	16	53	7	6	2,3,6,6,6,6,6	46656	15	70
7	7	2,3,6,6,6,6,7	54432	15	60	7	8	2,3,6,6,6,6,8	62208	15	53	7	7	2,3,6,6,6,7,7	63504	15	52
7	8	2,3,6,6,6,7,8	72576	16	90	7	8	2,3,6,6,6,8,8	82944	16	79	7	7	2,3,6,6,7,7,7	74088	16	88
7	8	2,3,6,6,7,7,8	84672	16	77	7	8	2,3,6,6,7,8,8	96768	16	68	7	8	2,3,6,6,8,8,8	110592	16	59
7	7	2,3,6,7,7,7,7	86436	16	76	7	8	2,3,6,7,7,7,8	98784	16	66	7	8	2,3,6,7,7,8,8	112896	16	58
7	8	2,3,6,7,8,8,8	129024	16	51	7	8	2,3,6,8,8,8,8	147456	17	89	7	7	2,3,7,7,7,7,7	100842	16	65
7	8	2,3,7,7,7,7,8	115248	16	57	7	8	2,3,7,7,7,8,8	131712	17	100	7	8	2,3,7,7,8,8,8	150528	17	87
7	8	2,3,7,8,8,8,8	172032	17	76	7	8	2,3,8,8,8,8,8	196608	17	67	7	4	2,4,4,4,4,4,4	8192	13	100
7	5	2,4,4,4,4,4,5	10240	13	80	7	6	2,4,4,4,4,4,6	12288	13	67	7	7	2,4,4,4,4,4,7	14336	13	57
7	8	2,4,4,4,4,4,8	16384	14	100	7	5	2,4,4,4,4,5,5	12800	13	64	7	6	2,4,4,4,4,5,6	15360	13	53
7	7	2,4,4,4,4,5,7	17920	14	91	7	8	2,4,4,4,4,5,8	20480	14	80	7	6	2,4,4,4,4,6,6	18432	14	89
7	7	2,4,4,4,4,6,7	21504	14	76	7	8	2,4,4,4,4,6,8	24576	14	67	7	7	2,4,4,4,4,7,7	25088	14	65
7	8	2,4,4,4,4,7,8	28672	14	57	7	8	2,4,4,4,4,8,8	32768	15	100	7	5	2,4,4,4,5,5,5	16000	13	51
7	6	2,4,4,4,5,5,6	19200	14	85	7	7	2,4,4,4,5,5,7	22400	14	73	7	8	2,4,4,4,5,5,8	25600	14	64
7	6	2,4,4,4,5,6,6	23040	14	71	7	7	2,4,4,4,5,6,7	26880	14	61	7	8	2,4,4,4,5,6,8	30720	14	53
7	7	2,4,4,4,5,7,7	31360	14	52	7	8	2,4,4,4,5,7,8	35840	15	91	7	8	2,4,4,4,5,8,8	40960	15	80

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	6	2,4,4,4,6,6,6	27648	14	59	7	7	2,4,4,4,6,6,7	32256	14	51	7	8	2,4,4,4,6,6,8	36864	15	89
7	7	2,4,4,4,6,7,7	37632	15	87	7	8	2,4,4,4,6,7,8	43008	15	76	7	8	2,4,4,4,6,8,8	49152	15	67
7	7	2,4,4,4,7,7,7	43904	15	75	7	8	2,4,4,4,7,7,8	50176	15	65	7	8	2,4,4,4,7,8,8	57344	15	57
7	8	2,4,4,4,8,8,8	65536	16	100	7	5	2,4,4,5,5,5,5	20000	14	82	7	6	2,4,4,5,5,5,6	24000	14	68
7	7	2,4,4,5,5,5,7	28000	14	59	7	8	2,4,4,5,5,5,8	32000	14	51	7	6	2,4,4,5,5,6,6	28800	14	57
7	7	2,4,4,5,5,6,7	33600	15	98	7	8	2,4,4,5,5,6,8	38400	15	85	7	7	2,4,4,5,5,7,7	39200	15	84
7	8	2,4,4,5,5,7,8	44800	15	73	7	8	2,4,4,5,5,8,8	51200	15	64	7	6	2,4,4,5,6,6,6	34560	15	95
7	7	2,4,4,5,6,6,7	40320	15	81	7	8	2,4,4,5,6,6,8	46080	15	71	7	7	2,4,4,5,6,7,7	47040	15	70
7	8	2,4,4,5,6,7,8	53760	15	61	7	8	2,4,4,5,6,8,8	61440	15	53	7	7	2,4,4,5,7,7,7	54880	15	60
7	8	2,4,4,5,7,7,8	62720	15	52	7	8	2,4,4,5,7,8,8	71680	16	91	7	8	2,4,4,5,8,8,8	81920	16	80
7	6	2,4,4,6,6,6,6	41472	15	79	7	7	2,4,4,6,6,6,7	48384	15	68	7	8	2,4,4,6,6,6,8	55296	15	59
7	7	2,4,4,6,6,7,7	56448	15	58	7	8	2,4,4,6,6,7,8	64512	15	51	7	8	2,4,4,6,6,8,8	73728	16	89
7	7	2,4,4,6,7,7,7	65856	16	100	7	8	2,4,4,6,7,7,8	75264	16	87	7	8	2,4,4,6,7,8,8	86016	16	76
7	8	2,4,4,6,8,8,8	98304	16	67	7	7	2,4,4,7,7,7,7	76832	16	85	7	8	2,4,4,7,7,7,8	87808	16	75
7	8	2,4,4,7,7,8,8	100352	16	65	7	8	2,4,4,7,8,8,8	114688	16	57	7	8	2,4,4,8,8,8,8	131072	17	100
7	5	2,4,5,5,5,5,5	25000	14	66	7	6	2,4,5,5,5,5,6	30000	14	55	7	7	2,4,5,5,5,5,7	35000	15	94
7	8	2,4,5,5,5,5,8	40000	15	82	7	6	2,4,5,5,5,6,6	36000	15	91	7	7	2,4,5,5,5,6,7	42000	15	78
7	8	2,4,5,5,5,6,8	48000	15	68	7	7	2,4,5,5,5,7,7	49000	15	67	7	8	2,4,5,5,5,7,8	56000	15	59
7	8	2,4,5,5,5,8,8	64000	15	51	7	6	2,4,5,5,6,6,6	43200	15	76	7	7	2,4,5,5,6,6,7	50400	15	65
7	8	2,4,5,5,6,6,8	57600	15	57	7	7	2,4,5,5,6,7,7	58800	15	56	7	8	2,4,5,5,6,7,8	67200	16	98
7	8	2,4,5,5,6,8,8	76800	16	85	7	7	2,4,5,5,7,7,7	68600	16	96	7	8	2,4,5,5,7,7,8	78400	16	84
7	8	2,4,5,5,7,8,8	89600	16	73	7	8	2,4,5,5,8,8,8	102400	16	64	7	6	2,4,5,6,6,6,6	51840	15	63
7	7	2,4,5,6,6,6,7	60480	15	54	7	8	2,4,5,6,6,6,8	69120	16	95	7	7	2,4,5,6,6,7,7	70560	16	93
7	8	2,4,5,6,6,7,8	80640	16	81	7	8	2,4,5,6,6,8,8	92160	16	71	7	7	2,4,5,6,7,7,7	82320	16	80
7	8	2,4,5,6,7,7,8	94080	16	70	7	8	2,4,5,6,7,8,8	107520	16	61	7	8	2,4,5,6,8,8,8	122880	16	53
7	7	2,4,5,7,7,7,7	96040	16	68	7	8	2,4,5,7,7,7,8	109760	16	60	7	8	2,4,5,7,7,8,8	125440	16	52
7	8	2,4,5,7,8,8,8	143360	17	91	7	8	2,4,5,8,8,8,8	163840	17	80	7	6	2,4,6,6,6,6,6	62208	15	53
7	7	2,4,6,6,6,6,7	72576	16	90	7	8	2,4,6,6,6,6,8	82944	16	79	7	7	2,4,6,6,6,7,7	84672	16	77
7	8	2,4,6,6,6,7,8	96768	16	68	7	8	2,4,6,6,6,8,8	110592	16	59	7	7	2,4,6,6,7,7,7	98784	16	66
7	8	2,4,6,6,7,7,8	112896	16	58	7	8	2,4,6,6,7,8,8	129024	16	51	7	8	2,4,6,6,8,8,8	147456	17	89
7	7	2,4,6,7,7,7,7	115248	16	57	7	8	2,4,6,7,7,7,8	131712	17	100	7	8	2,4,6,7,7,8,8	150528	17	87
7	8	2,4,6,7,8,8,8	172032	17	76	7	8	2,4,6,8,8,8,8	196608	17	67	7	7	2,4,7,7,7,7,7	134456	17	97

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	2,4,7,7,7,7,8	153664	17	85	7	8	2,4,7,7,7,8,8	175616	17	75	7	8	2,4,7,7,8,8,8	200704	17	65
7	8	2,4,7,8,8,8,8	229376	17	57	7	8	2,4,8,8,8,8,8	262144	18	100	7	5	2,5,5,5,5,5,5	31250	14	52
7	6	2,5,5,5,5,5,6	37500	15	87	7	7	2,5,5,5,5,5,7	43750	15	75	7	8	2,5,5,5,5,5,8	50000	15	66
7	6	2,5,5,5,5,6,6	45000	15	73	7	7	2,5,5,5,5,6,7	52500	15	62	7	8	2,5,5,5,5,6,8	60000	15	55
7	7	2,5,5,5,5,7,7	61250	15	53	7	8	2,5,5,5,5,7,8	70000	16	94	7	8	2,5,5,5,5,8,8	80000	16	82
7	6	2,5,5,5,6,6,6	54000	15	61	7	7	2,5,5,5,6,6,7	63000	15	52	7	8	2,5,5,5,6,6,8	72000	16	91
7	7	2,5,5,5,6,7,7	73500	16	89	7	8	2,5,5,5,6,7,8	84000	16	78	7	8	2,5,5,5,6,8,8	96000	16	68
7	7	2,5,5,5,7,7,7	85750	16	76	7	8	2,5,5,5,7,7,8	98000	16	67	7	8	2,5,5,5,7,8,8	112000	16	59
7	8	2,5,5,5,8,8,8	128000	16	51	7	6	2,5,5,6,6,6,6	64800	15	51	7	7	2,5,5,6,6,6,7	75600	16	87
7	8	2,5,5,6,6,6,8	86400	16	76	7	7	2,5,5,6,6,7,7	88200	16	74	7	8	2,5,5,6,6,7,8	100800	16	65
7	8	2,5,5,6,6,8,8	115200	16	57	7	7	2,5,5,6,7,7,7	102900	16	64	7	8	2,5,5,6,7,7,8	117600	16	56
7	8	2,5,5,6,7,8,8	134400	17	98	7	8	2,5,5,6,8,8,8	153600	17	85	7	7	2,5,5,7,7,7,7	120050	16	55
7	8	2,5,5,7,7,7,8	137200	17	96	7	8	2,5,5,7,7,8,8	156800	17	84	7	8	2,5,5,7,8,8,8	179200	17	73
7	8	2,5,5,8,8,8,8	204800	17	64	7	6	2,5,6,6,6,6,6	77760	16	84	7	7	2,5,6,6,6,6,7	90720	16	72
7	8	2,5,6,6,6,6,8	103680	16	63	7	7	2,5,6,6,6,7,7	105840	16	62	7	8	2,5,6,6,6,7,8	120960	16	54
7	8	2,5,6,6,6,8,8	138240	17	95	7	7	2,5,6,6,7,7,7	123480	16	53	7	8	2,5,6,6,7,7,8	141120	17	93
7	8	2,5,6,6,7,8,8	161280	17	81	7	8	2,5,6,6,8,8,8	184320	17	71	7	7	2,5,6,7,7,7,7	144060	17	91
7	8	2,5,6,7,7,7,8	164640	17	80	7	8	2,5,6,7,7,8,8	188160	17	70	7	8	2,5,6,7,8,8,8	215040	17	61
7	8	2,5,6,8,8,8,8	245760	17	53	7	7	2,5,7,7,7,7,7	168070	17	78	7	8	2,5,7,7,7,7,8	192080	17	68
7	8	2,5,7,7,7,8,8	219520	17	60	7	8	2,5,7,7,8,8,8	250880	17	52	7	8	2,5,7,8,8,8,8	286720	18	91
7	8	2,5,8,8,8,8,8	327680	18	80	7	6	2,6,6,6,6,6,6	93312	16	70	7	7	2,6,6,6,6,6,7	108864	16	60
7	8	2,6,6,6,6,6,8	124416	16	53	7	7	2,6,6,6,6,7,7	127008	16	52	7	8	2,6,6,6,6,7,8	145152	17	90
7	8	2,6,6,6,6,8,8	165888	17	79	7	7	2,6,6,6,7,7,7	148176	17	88	7	8	2,6,6,6,7,7,8	169344	17	77
7	8	2,6,6,6,7,8,8	193536	17	68	7	8	2,6,6,6,8,8,8	221184	17	59	7	7	2,6,6,7,7,7,7	172872	17	76
7	8	2,6,6,7,7,7,8	197568	17	66	7	8	2,6,6,7,7,8,8	225792	17	58	7	8	2,6,6,7,8,8,8	258048	17	51
7	8	2,6,6,8,8,8,8	294912	18	89	7	7	2,6,7,7,7,7,7	201684	17	65	7	8	2,6,7,7,7,7,8	230496	17	57
7	8	2,6,7,7,7,8,8	263424	18	100	7	8	2,6,7,7,8,8,8	301056	18	87	7	8	2,6,7,8,8,8,8	344064	18	76
7	8	2,6,8,8,8,8,8	393216	18	67	7	7	2,7,7,7,7,7,7	235298	17	56	7	8	2,7,7,7,7,7,8	268912	18	97
7	8	2,7,7,7,7,8,8	307328	18	85	7	8	2,7,7,7,8,8,8	351232	18	75	7	8	2,7,7,8,8,8,8	401408	18	65
7	8	2,7,8,8,8,8,8	458752	18	57	7	8	2,8,8,8,8,8,8	524288	19	100	7	3	3,3,3,3,3,3,3	2187	11	94
7	4	3,3,3,3,3,3,4	2916	11	70	7	5	3,3,3,3,3,3,5	3645	11	56	7	6	3,3,3,3,3,3,6	4374	12	94
7	7	3,3,3,3,3,3,7	5103	12	80	7	8	3,3,3,3,3,3,8	5832	12	70	7	4	3,3,3,3,3,4,4	3888	11	53

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	5	3,3,3,3,3,4,5	4860	12	84	7	6	3,3,3,3,3,4,6	5832	12	70	7	7	3,3,3,3,3,4,7	6804	12	60
7	8	3,3,3,3,3,4,8	7776	12	53	7	5	3,3,3,3,3,5,5	6075	12	67	7	6	3,3,3,3,3,5,6	7290	12	56
7	7	3,3,3,3,3,5,7	8505	13	96	7	8	3,3,3,3,3,5,8	9720	13	84	7	6	3,3,3,3,3,6,6	8748	13	94
7	7	3,3,3,3,3,6,7	10206	13	80	7	8	3,3,3,3,3,6,8	11664	13	70	7	7	3,3,3,3,3,7,7	11907	13	69
7	8	3,3,3,3,3,7,8	13608	13	60	7	8	3,3,3,3,3,8,8	15552	13	53	7	4	3,3,3,3,4,4,4	5184	12	79
7	5	3,3,3,3,4,4,5	6480	12	63	7	6	3,3,3,3,4,4,6	7776	12	53	7	7	3,3,3,3,4,4,7	9072	13	90
7	8	3,3,3,3,4,4,8	10368	13	79	7	5	3,3,3,3,4,5,5	8100	12	51	7	6	3,3,3,3,4,5,6	9720	13	84
7	7	3,3,3,3,4,5,7	11340	13	72	7	8	3,3,3,3,4,5,8	12960	13	63	7	6	3,3,3,3,4,6,6	11664	13	70
7	7	3,3,3,3,4,6,7	13608	13	60	7	8	3,3,3,3,4,6,8	15552	13	53	7	7	3,3,3,3,4,7,7	15876	13	52
7	8	3,3,3,3,4,7,8	18144	14	90	7	8	3,3,3,3,4,8,8	20736	14	79	7	5	3,3,3,3,5,5,5	10125	13	81
7	6	3,3,3,3,5,5,6	12150	13	67	7	7	3,3,3,3,5,5,7	14175	13	58	7	8	3,3,3,3,5,5,8	16200	13	51
7	6	3,3,3,3,5,6,6	14580	13	56	7	7	3,3,3,3,5,6,7	17010	14	96	7	8	3,3,3,3,5,6,8	19440	14	84
7	7	3,3,3,3,5,7,7	19845	14	83	7	8	3,3,3,3,5,7,8	22680	14	72	7	8	3,3,3,3,5,8,8	25920	14	63
7	6	3,3,3,3,6,6,6	17496	14	94	7	7	3,3,3,3,6,6,7	20412	14	80	7	8	3,3,3,3,6,6,8	23328	14	70
7	7	3,3,3,3,6,7,7	23814	14	69	7	8	3,3,3,3,6,7,8	27216	14	60	7	8	3,3,3,3,6,8,8	31104	14	53
7	7	3,3,3,3,7,7,7	27783	14	59	7	8	3,3,3,3,7,7,8	31752	14	52	7	8	3,3,3,3,7,8,8	36288	15	90
7	8	3,3,3,3,8,8,8	41472	15	79	7	4	3,3,3,4,4,4,4	6912	12	59	7	5	3,3,3,4,4,4,5	8640	13	95
7	6	3,3,3,4,4,4,6	10368	13	79	7	7	3,3,3,4,4,4,7	12096	13	68	7	8	3,3,3,4,4,4,8	13824	13	59
7	5	3,3,3,4,4,5,5	10800	13	76	7	6	3,3,3,4,4,5,6	12960	13	63	7	7	3,3,3,4,4,5,7	15120	13	54
7	8	3,3,3,4,4,5,8	17280	14	95	7	6	3,3,3,4,4,6,6	15552	13	53	7	7	3,3,3,4,4,6,7	18144	14	90
7	8	3,3,3,4,4,6,8	20736	14	79	7	7	3,3,3,4,4,7,7	21168	14	77	7	8	3,3,3,4,4,7,8	24192	14	68
7	8	3,3,3,4,4,8,8	27648	14	59	7	5	3,3,3,4,5,5,5	13500	13	61	7	6	3,3,3,4,5,5,6	16200	13	51
7	7	3,3,3,4,5,5,7	18900	14	87	7	8	3,3,3,4,5,5,8	21600	14	76	7	6	3,3,3,4,5,6,6	19440	14	84
7	7	3,3,3,4,5,6,7	22680	14	72	7	8	3,3,3,4,5,6,8	25920	14	63	7	7	3,3,3,4,5,7,7	26460	14	62
7	8	3,3,3,4,5,7,8	30240	14	54	7	8	3,3,3,4,5,8,8	34560	15	95	7	6	3,3,3,4,6,6,6	23328	14	70
7	7	3,3,3,4,6,6,7	27216	14	60	7	8	3,3,3,4,6,6,8	31104	14	53	7	7	3,3,3,4,6,7,7	31752	14	52
7	8	3,3,3,4,6,7,8	36288	15	90	7	8	3,3,3,4,6,8,8	41472	15	79	7	7	3,3,3,4,7,7,7	37044	15	88
7	8	3,3,3,4,7,7,8	42336	15	77	7	8	3,3,3,4,7,8,8	48384	15	68	7	8	3,3,3,4,8,8,8	55296	15	59
7	5	3,3,3,5,5,5,5	16875	14	97	7	6	3,3,3,5,5,5,6	20250	14	81	7	7	3,3,3,5,5,5,7	23625	14	69
7	8	3,3,3,5,5,5,8	27000	14	61	7	6	3,3,3,5,5,6,6	24300	14	67	7	7	3,3,3,5,5,6,7	28350	14	58
7	8	3,3,3,5,5,6,8	32400	14	51	7	7	3,3,3,5,5,7,7	33075	15	99	7	8	3,3,3,5,5,7,8	37800	15	87
7	8	3,3,3,5,5,8,8	43200	15	76	7	6	3,3,3,5,6,6,6	29160	14	56	7	7	3,3,3,5,6,6,7	34020	15	96

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	3,3,3,5,6,6,8	38880	15	84	7	7	3,3,3,5,6,7,7	39690	15	83	7	8	3,3,3,5,6,7,8	45360	15	72
7	8	3,3,3,5,6,8,8	51840	15	63	7	7	3,3,3,5,7,7,7	46305	15	71	7	8	3,3,3,5,7,7,8	52920	15	62
7	8	3,3,3,5,7,8,8	60480	15	54	7	8	3,3,3,5,8,8,8	69120	16	95	7	6	3,3,3,6,6,6,6	34992	15	94
7	7	3,3,3,6,6,6,7	40824	15	80	7	8	3,3,3,6,6,6,8	46656	15	70	7	7	3,3,3,6,6,7,7	47628	15	69
7	8	3,3,3,6,6,7,8	54432	15	60	7	8	3,3,3,6,6,8,8	62208	15	53	7	7	3,3,3,6,7,7,7	55566	15	59
7	8	3,3,3,6,7,7,8	63504	15	52	7	8	3,3,3,6,7,8,8	72576	16	90	7	8	3,3,3,6,8,8,8	82944	16	79
7	7	3,3,3,7,7,7,7	64827	15	51	7	8	3,3,3,7,7,7,8	74088	16	88	7	8	3,3,3,7,7,8,8	84672	16	77
7	8	3,3,3,7,8,8,8	96768	16	68	7	8	3,3,3,8,8,8,8	110592	16	59	7	4	3,3,4,4,4,4,4	9216	13	89
7	5	3,3,4,4,4,4,5	11520	13	71	7	6	3,3,4,4,4,4,6	13824	13	59	7	7	3,3,4,4,4,4,7	16128	13	51
7	8	3,3,4,4,4,4,8	18432	14	89	7	5	3,3,4,4,4,5,5	14400	13	57	7	6	3,3,4,4,4,5,6	17280	14	95
7	7	3,3,4,4,4,5,7	20160	14	81	7	8	3,3,4,4,4,5,8	23040	14	71	7	6	3,3,4,4,4,6,6	20736	14	79
7	7	3,3,4,4,4,6,7	24192	14	68	7	8	3,3,4,4,4,6,8	27648	14	59	7	7	3,3,4,4,4,7,7	28224	14	58
7	8	3,3,4,4,4,7,8	32256	14	51	7	8	3,3,4,4,4,8,8	36864	15	89	7	5	3,3,4,4,5,5,5	18000	14	91
7	6	3,3,4,4,5,5,6	21600	14	76	7	7	3,3,4,4,5,5,7	25200	14	65	7	8	3,3,4,4,5,5,8	28800	14	57
7	6	3,3,4,4,5,6,6	25920	14	63	7	7	3,3,4,4,5,6,7	30240	14	54	7	8	3,3,4,4,5,6,8	34560	15	95
7	7	3,3,4,4,5,7,7	35280	15	93	7	8	3,3,4,4,5,7,8	40320	15	81	7	8	3,3,4,4,5,8,8	46080	15	71
7	6	3,3,4,4,6,6,6	31104	14	53	7	7	3,3,4,4,6,6,7	36288	15	90	7	8	3,3,4,4,6,6,8	41472	15	79
7	7	3,3,4,4,6,7,7	42336	15	77	7	8	3,3,4,4,6,7,8	48384	15	68	7	8	3,3,4,4,6,8,8	55296	15	59
7	7	3,3,4,4,7,7,7	49392	15	66	7	8	3,3,4,4,7,7,8	56448	15	58	7	8	3,3,4,4,7,8,8	64512	15	51
7	8	3,3,4,4,8,8,8	73728	16	89	7	5	3,3,4,5,5,5,5	22500	14	73	7	6	3,3,4,5,5,5,6	27000	14	61
7	7	3,3,4,5,5,5,7	31500	14	52	7	8	3,3,4,5,5,5,8	36000	15	91	7	6	3,3,4,5,5,6,6	32400	14	51
7	7	3,3,4,5,5,6,7	37800	15	87	7	8	3,3,4,5,5,6,8	43200	15	76	7	7	3,3,4,5,5,7,7	44100	15	74
7	8	3,3,4,5,5,7,8	50400	15	65	7	8	3,3,4,5,5,8,8	57600	15	57	7	6	3,3,4,5,6,6,6	38880	15	84
7	7	3,3,4,5,6,6,7	45360	15	72	7	8	3,3,4,5,6,6,8	51840	15	63	7	7	3,3,4,5,6,7,7	52920	15	62
7	8	3,3,4,5,6,7,8	60480	15	54	7	8	3,3,4,5,6,8,8	69120	16	95	7	7	3,3,4,5,7,7,7	61740	15	53
7	8	3,3,4,5,7,7,8	70560	16	93	7	8	3,3,4,5,7,8,8	80640	16	81	7	8	3,3,4,5,8,8,8	92160	16	71
7	6	3,3,4,6,6,6,6	46656	15	70	7	7	3,3,4,6,6,6,7	54432	15	60	7	8	3,3,4,6,6,6,8	62208	15	53
7	7	3,3,4,6,6,7,7	63504	15	52	7	8	3,3,4,6,6,7,8	72576	16	90	7	8	3,3,4,6,6,8,8	82944	16	79
7	7	3,3,4,6,7,7,7	74088	16	88	7	8	3,3,4,6,7,7,8	84672	16	77	7	8	3,3,4,6,7,8,8	96768	16	68
7	8	3,3,4,6,8,8,8	110592	16	59	7	7	3,3,4,7,7,7,7	86436	16	76	7	8	3,3,4,7,7,7,8	98784	16	66
7	8	3,3,4,7,7,8,8	112896	16	58	7	8	3,3,4,7,8,8,8	129024	16	51	7	8	3,3,4,8,8,8,8	147456	17	89
7	5	3,3,5,5,5,5,5	28125	14	58	7	6	3,3,5,5,5,5,6	33750	15	97	7	7	3,3,5,5,5,5,7	39375	15	83

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	3,3,5,5,5,5,8	45000	15	73	7	6	3,3,5,5,5,6,6	40500	15	81	7	7	3,3,5,5,5,6,7	47250	15	69
7	8	3,3,5,5,5,6,8	54000	15	61	7	7	3,3,5,5,5,7,7	55125	15	59	7	8	3,3,5,5,5,7,8	63000	15	52
7	8	3,3,5,5,5,8,8	72000	16	91	7	6	3,3,5,5,6,6,6	48600	15	67	7	7	3,3,5,5,6,6,7	56700	15	58
7	8	3,3,5,5,6,6,8	64800	15	51	7	7	3,3,5,5,6,7,7	66150	16	99	7	8	3,3,5,5,6,7,8	75600	16	87
7	8	3,3,5,5,6,8,8	86400	16	76	7	7	3,3,5,5,7,7,7	77175	16	85	7	8	3,3,5,5,7,7,8	88200	16	74
7	8	3,3,5,5,7,8,8	100800	16	65	7	8	3,3,5,5,8,8,8	115200	16	57	7	6	3,3,5,6,6,6,6	58320	15	56
7	7	3,3,5,6,6,6,7	68040	16	96	7	8	3,3,5,6,6,6,8	77760	16	84	7	7	3,3,5,6,6,7,7	79380	16	83
7	8	3,3,5,6,6,7,8	90720	16	72	7	8	3,3,5,6,6,8,8	103680	16	63	7	7	3,3,5,6,7,7,7	92610	16	71
7	8	3,3,5,6,7,7,8	105840	16	62	7	8	3,3,5,6,7,8,8	120960	16	54	7	8	3,3,5,6,8,8,8	138240	17	95
7	7	3,3,5,7,7,7,7	108045	16	61	7	8	3,3,5,7,7,7,8	123480	16	53	7	8	3,3,5,7,7,8,8	141120	17	93
7	8	3,3,5,7,8,8,8	161280	17	81	7	8	3,3,5,8,8,8,8	184320	17	71	7	6	3,3,6,6,6,6,6	69984	16	94
7	7	3,3,6,6,6,6,7	81648	16	80	7	8	3,3,6,6,6,6,8	93312	16	70	7	7	3,3,6,6,6,7,7	95256	16	69
7	8	3,3,6,6,6,7,8	108864	16	60	7	8	3,3,6,6,6,8,8	124416	16	53	7	7	3,3,6,6,7,7,7	111132	16	59
7	8	3,3,6,6,7,7,8	127008	16	52	7	8	3,3,6,6,7,8,8	145152	17	90	7	8	3,3,6,6,8,8,8	165888	17	79
7	7	3,3,6,7,7,7,7	129654	16	51	7	8	3,3,6,7,7,7,8	148176	17	88	7	8	3,3,6,7,7,8,8	169344	17	77
7	8	3,3,6,7,8,8,8	193536	17	68	7	8	3,3,6,8,8,8,8	221184	17	59	7	7	3,3,7,7,7,7,7	151263	17	87
7	8	3,3,7,7,7,7,8	172872	17	76	7	8	3,3,7,7,7,8,8	197568	17	66	7	8	3,3,7,7,8,8,8	225792	17	58
7	8	3,3,7,8,8,8,8	258048	17	51	7	8	3,3,8,8,8,8,8	294912	18	89	7	4	3,4,4,4,4,4,4	12288	13	67
7	5	3,4,4,4,4,4,5	15360	13	53	7	6	3,4,4,4,4,4,6	18432	14	89	7	7	3,4,4,4,4,4,7	21504	14	76
7	8	3,4,4,4,4,4,8	24576	14	67	7	5	3,4,4,4,4,5,5	19200	14	85	7	6	3,4,4,4,4,5,6	23040	14	71
7	7	3,4,4,4,4,5,7	26880	14	61	7	8	3,4,4,4,4,5,8	30720	14	53	7	6	3,4,4,4,4,6,6	27648	14	59
7	7	3,4,4,4,4,6,7	32256	14	51	7	8	3,4,4,4,4,6,8	36864	15	89	7	7	3,4,4,4,4,7,7	37632	15	87
7	8	3,4,4,4,4,7,8	43008	15	76	7	8	3,4,4,4,4,8,8	49152	15	67	7	5	3,4,4,4,5,5,5	24000	14	68
7	6	3,4,4,4,5,5,6	28800	14	57	7	7	3,4,4,4,5,5,7	33600	15	98	7	8	3,4,4,4,5,5,8	38400	15	85
7	6	3,4,4,4,5,6,6	34560	15	95	7	7	3,4,4,4,5,6,7	40320	15	81	7	8	3,4,4,4,5,6,8	46080	15	71
7	7	3,4,4,4,5,7,7	47040	15	70	7	8	3,4,4,4,5,7,8	53760	15	61	7	8	3,4,4,4,5,8,8	61440	15	53
7	6	3,4,4,4,6,6,6	41472	15	79	7	7	3,4,4,4,6,6,7	48384	15	68	7	8	3,4,4,4,6,6,8	55296	15	59
7	7	3,4,4,4,6,7,7	56448	15	58	7	8	3,4,4,4,6,7,8	64512	15	51	7	8	3,4,4,4,6,8,8	73728	16	89
7	7	3,4,4,4,7,7,7	65856	16	100	7	8	3,4,4,4,7,7,8	75264	16	87	7	8	3,4,4,4,7,8,8	86016	16	76
7	8	3,4,4,4,8,8,8	98304	16	67	7	5	3,4,4,5,5,5,5	30000	14	55	7	6	3,4,4,5,5,5,6	36000	15	91
7	7	3,4,4,5,5,5,7	42000	15	78	7	8	3,4,4,5,5,5,8	48000	15	68	7	6	3,4,4,5,5,6,6	43200	15	76
7	7	3,4,4,5,5,6,7	50400	15	65	7	8	3,4,4,5,5,6,8	57600	15	57	7	7	3,4,4,5,5,7,7	58800	15	56

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	3,4,4,5,5,7,8	67200	16	98	7	8	3,4,4,5,5,8,8	76800	16	85	7	6	3,4,4,5,6,6,6	51840	15	63
7	7	3,4,4,5,6,6,7	60480	15	54	7	8	3,4,4,5,6,6,8	69120	16	95	7	7	3,4,4,5,6,7,7	70560	16	93
7	8	3,4,4,5,6,7,8	80640	16	81	7	8	3,4,4,5,6,8,8	92160	16	71	7	7	3,4,4,5,7,7,7	82320	16	80
7	8	3,4,4,5,7,7,8	94080	16	70	7	8	3,4,4,5,7,8,8	107520	16	61	7	8	3,4,4,5,8,8,8	122880	16	53
7	6	3,4,4,6,6,6,6	62208	15	53	7	7	3,4,4,6,6,6,7	72576	16	90	7	8	3,4,4,6,6,6,8	82944	16	79
7	7	3,4,4,6,6,7,7	84672	16	77	7	8	3,4,4,6,6,7,8	96768	16	68	7	8	3,4,4,6,6,8,8	110592	16	59
7	7	3,4,4,6,7,7,7	98784	16	66	7	8	3,4,4,6,7,7,8	112896	16	58	7	8	3,4,4,6,7,8,8	129024	16	51
7	8	3,4,4,6,8,8,8	147456	17	89	7	7	3,4,4,7,7,7,7	115248	16	57	7	8	3,4,4,7,7,7,8	131712	17	100
7	8	3,4,4,7,7,8,8	150528	17	87	7	8	3,4,4,7,8,8,8	172032	17	76	7	8	3,4,4,8,8,8,8	196608	17	67
7	5	3,4,5,5,5,5,5	37500	15	87	7	6	3,4,5,5,5,5,6	45000	15	73	7	7	3,4,5,5,5,5,7	52500	15	62
7	8	3,4,5,5,5,5,8	60000	15	55	7	6	3,4,5,5,5,6,6	54000	15	61	7	7	3,4,5,5,5,6,7	63000	15	52
7	8	3,4,5,5,5,6,8	72000	16	91	7	7	3,4,5,5,5,7,7	73500	16	89	7	8	3,4,5,5,5,7,8	84000	16	78
7	8	3,4,5,5,5,8,8	96000	16	68	7	6	3,4,5,5,6,6,6	64800	15	51	7	7	3,4,5,5,6,6,7	75600	16	87
7	8	3,4,5,5,6,6,8	86400	16	76	7	7	3,4,5,5,6,7,7	88200	16	74	7	8	3,4,5,5,6,7,8	100800	16	65
7	8	3,4,5,5,6,8,8	115200	16	57	7	7	3,4,5,5,7,7,7	102900	16	64	7	8	3,4,5,5,7,7,8	117600	16	56
7	8	3,4,5,5,7,8,8	134400	17	98	7	8	3,4,5,5,8,8,8	153600	17	85	7	6	3,4,5,6,6,6,6	77760	16	84
7	7	3,4,5,6,6,6,7	90720	16	72	7	8	3,4,5,6,6,6,8	103680	16	63	7	7	3,4,5,6,6,7,7	105840	16	62
7	8	3,4,5,6,6,7,8	120960	16	54	7	8	3,4,5,6,6,8,8	138240	17	95	7	7	3,4,5,6,7,7,7	123480	16	53
7	8	3,4,5,6,7,7,8	141120	17	93	7	8	3,4,5,6,7,8,8	161280	17	81	7	8	3,4,5,6,8,8,8	184320	17	71
7	7	3,4,5,7,7,7,7	144060	17	91	7	8	3,4,5,7,7,7,8	164640	17	80	7	8	3,4,5,7,7,8,8	188160	17	70
7	8	3,4,5,7,8,8,8	215040	17	61	7	8	3,4,5,8,8,8,8	245760	17	53	7	6	3,4,6,6,6,6,6	93312	16	70
7	7	3,4,6,6,6,6,7	108864	16	60	7	8	3,4,6,6,6,6,8	124416	16	53	7	7	3,4,6,6,6,7,7	127008	16	52
7	8	3,4,6,6,6,7,8	145152	17	90	7	8	3,4,6,6,6,8,8	165888	17	79	7	7	3,4,6,6,7,7,7	148176	17	88
7	8	3,4,6,6,7,7,8	169344	17	77	7	8	3,4,6,6,7,8,8	193536	17	68	7	8	3,4,6,6,8,8,8	221184	17	59
7	7	3,4,6,7,7,7,7	172872	17	76	7	8	3,4,6,7,7,7,8	197568	17	66	7	8	3,4,6,7,7,8,8	225792	17	58
7	8	3,4,6,7,8,8,8	258048	17	51	7	8	3,4,6,8,8,8,8	294912	18	89	7	7	3,4,7,7,7,7,7	201684	17	65
7	8	3,4,7,7,7,7,8	230496	17	57	7	8	3,4,7,7,7,8,8	263424	18	100	7	8	3,4,7,7,8,8,8	301056	18	87
7	8	3,4,7,8,8,8,8	344064	18	76	7	8	3,4,8,8,8,8,8	393216	18	67	7	5	3,5,5,5,5,5,5	46875	15	70
7	6	3,5,5,5,5,5,6	56250	15	58	7	7	3,5,5,5,5,5,7	65625	16	100	7	8	3,5,5,5,5,5,8	75000	16	87
7	6	3,5,5,5,5,6,6	67500	16	97	7	7	3,5,5,5,5,6,7	78750	16	83	7	8	3,5,5,5,5,6,8	90000	16	73
7	7	3,5,5,5,5,7,7	91875	16	71	7	8	3,5,5,5,5,7,8	105000	16	62	7	8	3,5,5,5,5,8,8	120000	16	55
7	6	3,5,5,5,6,6,6	81000	16	81	7	7	3,5,5,5,6,6,7	94500	16	69	7	8	3,5,5,5,6,6,8	108000	16	61

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	7	3,5,5,5,6,7,7	110250	16	59	7	8	3,5,5,5,6,7,8	126000	16	52	7	8	3,5,5,5,6,8,8	144000	17	91
7	7	3,5,5,5,7,7,7	128625	16	51	7	8	3,5,5,5,7,7,8	147000	17	89	7	8	3,5,5,5,7,8,8	168000	17	78
7	8	3,5,5,5,8,8,8	192000	17	68	7	6	3,5,5,6,6,6,6	97200	16	67	7	7	3,5,5,6,6,6,7	113400	16	58
7	8	3,5,5,6,6,6,8	129600	16	51	7	7	3,5,5,6,6,7,7	132300	17	99	7	8	3,5,5,6,6,7,8	151200	17	87
7	8	3,5,5,6,6,8,8	172800	17	76	7	7	3,5,5,6,7,7,7	154350	17	85	7	8	3,5,5,6,7,7,8	176400	17	74
7	8	3,5,5,6,7,8,8	201600	17	65	7	8	3,5,5,6,8,8,8	230400	17	57	7	7	3,5,5,7,7,7,7	180075	17	73
7	8	3,5,5,7,7,7,8	205800	17	64	7	8	3,5,5,7,7,8,8	235200	17	56	7	8	3,5,5,7,8,8,8	268800	18	98
7	8	3,5,5,8,8,8,8	307200	18	85	7	6	3,5,6,6,6,6,6	116640	16	56	7	7	3,5,6,6,6,6,7	136080	17	96
7	8	3,5,6,6,6,6,8	155520	17	84	7	7	3,5,6,6,6,7,7	158760	17	83	7	8	3,5,6,6,6,7,8	181440	17	72
7	8	3,5,6,6,6,8,8	207360	17	63	7	7	3,5,6,6,7,7,7	185220	17	71	7	8	3,5,6,6,7,7,8	211680	17	62
7	8	3,5,6,6,7,8,8	241920	17	54	7	8	3,5,6,6,8,8,8	276480	18	95	7	7	3,5,6,7,7,7,7	216090	17	61
7	8	3,5,6,7,7,7,8	246960	17	53	7	8	3,5,6,7,7,8,8	282240	18	93	7	8	3,5,6,7,8,8,8	322560	18	81
7	8	3,5,6,8,8,8,8	368640	18	71	7	7	3,5,7,7,7,7,7	252105	17	52	7	8	3,5,7,7,7,7,8	288120	18	91
7	8	3,5,7,7,7,8,8	329280	18	80	7	8	3,5,7,7,8,8,8	376320	18	70	7	8	3,5,7,8,8,8,8	430080	18	61
7	8	3,5,8,8,8,8,8	491520	18	53	7	6	3,6,6,6,6,6,6	139968	17	94	7	7	3,6,6,6,6,6,7	163296	17	80
7	8	3,6,6,6,6,6,8	186624	17	70	7	7	3,6,6,6,6,7,7	190512	17	69	7	8	3,6,6,6,6,7,8	217728	17	60
7	8	3,6,6,6,6,8,8	248832	17	53	7	7	3,6,6,6,7,7,7	222264	17	59	7	8	3,6,6,6,7,7,8	254016	17	52
7	8	3,6,6,6,7,8,8	290304	18	90	7	8	3,6,6,6,8,8,8	331776	18	79	7	7	3,6,6,7,7,7,7	259308	17	51
7	8	3,6,6,7,7,7,8	296352	18	88	7	8	3,6,6,7,7,8,8	338688	18	77	7	8	3,6,6,7,8,8,8	387072	18	68
7	8	3,6,6,8,8,8,8	442368	18	59	7	7	3,6,7,7,7,7,7	302526	18	87	7	8	3,6,7,7,7,7,8	345744	18	76
7	8	3,6,7,7,7,8,8	395136	18	66	7	8	3,6,7,7,8,8,8	451584	18	58	7	8	3,6,7,8,8,8,8	516096	18	51
7	8	3,6,8,8,8,8,8	589824	19	89	7	7	3,7,7,7,7,7,7	352947	18	74	7	8	3,7,7,7,7,7,8	403368	18	65
7	8	3,7,7,7,7,8,8	460992	18	57	7	8	3,7,7,7,8,8,8	526848	19	100	7	8	3,7,7,8,8,8,8	602112	19	87
7	8	3,7,8,8,8,8,8	688128	19	76	7	8	3,8,8,8,8,8,8	786432	19	67	7	4	4,4,4,4,4,4,4	16384	14	100
7	5	4,4,4,4,4,4,5	20480	14	80	7	6	4,4,4,4,4,4,6	24576	14	67	7	7	4,4,4,4,4,4,7	28672	14	57
7	8	4,4,4,4,4,4,8	32768	15	100	7	5	4,4,4,4,4,5,5	25600	14	64	7	6	4,4,4,4,4,5,6	30720	14	53
7	7	4,4,4,4,4,5,7	35840	15	91	7	8	4,4,4,4,4,5,8	40960	15	80	7	6	4,4,4,4,4,6,6	36864	15	89
7	7	4,4,4,4,4,6,7	43008	15	76	7	8	4,4,4,4,4,6,8	49152	15	67	7	7	4,4,4,4,4,7,7	50176	15	65
7	8	4,4,4,4,4,7,8	57344	15	57	7	8	4,4,4,4,4,8,8	65536	16	100	7	5	4,4,4,4,5,5,5	32000	14	51
7	6	4,4,4,4,5,5,6	38400	15	85	7	7	4,4,4,4,5,5,7	44800	15	73	7	8	4,4,4,4,5,5,8	51200	15	64
7	6	4,4,4,4,5,6,6	46080	15	71	7	7	4,4,4,4,5,6,7	53760	15	61	7	8	4,4,4,4,5,6,8	61440	15	53
7	7	4,4,4,4,5,7,7	62720	15	52	7	8	4,4,4,4,5,7,8	71680	16	91	7	8	4,4,4,4,5,8,8	81920	16	80

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	6	4,4,4,4,6,6,6	55296	15	59	7	7	4,4,4,4,6,6,7	64512	15	51	7	8	4,4,4,4,6,6,8	73728	16	89
7	7	4,4,4,4,6,7,7	75264	16	87	7	8	4,4,4,4,6,7,8	86016	16	76	7	8	4,4,4,4,6,8,8	98304	16	67
7	7	4,4,4,4,7,7,7	87808	16	75	7	8	4,4,4,4,7,7,8	100352	16	65	7	8	4,4,4,4,7,8,8	114688	16	57
7	8	4,4,4,4,8,8,8	131072	17	100	7	5	4,4,4,5,5,5,5	40000	15	82	7	6	4,4,4,5,5,5,6	48000	15	68
7	7	4,4,4,5,5,5,7	56000	15	59	7	8	4,4,4,5,5,5,8	64000	15	51	7	6	4,4,4,5,5,6,6	57600	15	57
7	7	4,4,4,5,5,6,7	67200	16	98	7	8	4,4,4,5,5,6,8	76800	16	85	7	7	4,4,4,5,5,7,7	78400	16	84
7	8	4,4,4,5,5,7,8	89600	16	73	7	8	4,4,4,5,5,8,8	102400	16	64	7	6	4,4,4,5,6,6,6	69120	16	95
7	7	4,4,4,5,6,6,7	80640	16	81	7	8	4,4,4,5,6,6,8	92160	16	71	7	7	4,4,4,5,6,7,7	94080	16	70
7	8	4,4,4,5,6,7,8	107520	16	61	7	8	4,4,4,5,6,8,8	122880	16	53	7	7	4,4,4,5,7,7,7	109760	16	60
7	8	4,4,4,5,7,7,8	125440	16	52	7	8	4,4,4,5,7,8,8	143360	17	91	7	8	4,4,4,5,8,8,8	163840	17	80
7	6	4,4,4,6,6,6,6	82944	16	79	7	7	4,4,4,6,6,6,7	96768	16	68	7	8	4,4,4,6,6,6,8	110592	16	59
7	7	4,4,4,6,6,7,7	112896	16	58	7	8	4,4,4,6,6,7,8	129024	16	51	7	8	4,4,4,6,6,8,8	147456	17	89
7	7	4,4,4,6,7,7,7	131712	17	100	7	8	4,4,4,6,7,7,8	150528	17	87	7	8	4,4,4,6,7,8,8	172032	17	76
7	8	4,4,4,6,8,8,8	196608	17	67	7	7	4,4,4,7,7,7,7	153664	17	85	7	8	4,4,4,7,7,7,8	175616	17	75
7	8	4,4,4,7,7,8,8	200704	17	65	7	8	4,4,4,7,8,8,8	229376	17	57	7	8	4,4,4,8,8,8,8	262144	18	100
7	5	4,4,5,5,5,5,5	50000	15	66	7	6	4,4,5,5,5,5,6	60000	15	55	7	7	4,4,5,5,5,5,7	70000	16	94
7	8	4,4,5,5,5,5,8	80000	16	82	7	6	4,4,5,5,5,6,6	72000	16	91	7	7	4,4,5,5,5,6,7	84000	16	78
7	8	4,4,5,5,5,6,8	96000	16	68	7	7	4,4,5,5,5,7,7	98000	16	67	7	8	4,4,5,5,5,7,8	112000	16	59
7	8	4,4,5,5,5,8,8	128000	16	51	7	6	4,4,5,5,6,6,6	86400	16	76	7	7	4,4,5,5,6,6,7	100800	16	65
7	8	4,4,5,5,6,6,8	115200	16	57	7	7	4,4,5,5,6,7,7	117600	16	56	7	8	4,4,5,5,6,7,8	134400	17	98
7	8	4,4,5,5,6,8,8	153600	17	85	7	7	4,4,5,5,7,7,7	137200	17	96	7	8	4,4,5,5,7,7,8	156800	17	84
7	8	4,4,5,5,7,8,8	179200	17	73	7	8	4,4,5,5,8,8,8	204800	17	64	7	6	4,4,5,6,6,6,6	103680	16	63
7	7	4,4,5,6,6,6,7	120960	16	54	7	8	4,4,5,6,6,6,8	138240	17	95	7	7	4,4,5,6,6,7,7	141120	17	93
7	8	4,4,5,6,6,7,8	161280	17	81	7	8	4,4,5,6,6,8,8	184320	17	71	7	7	4,4,5,6,7,7,7	164640	17	80
7	8	4,4,5,6,7,7,8	188160	17	70	7	8	4,4,5,6,7,8,8	215040	17	61	7	8	4,4,5,6,8,8,8	245760	17	53
7	7	4,4,5,7,7,7,7	192080	17	68	7	8	4,4,5,7,7,7,8	219520	17	60	7	8	4,4,5,7,7,8,8	250880	17	52
7	8	4,4,5,7,8,8,8	286720	18	91	7	8	4,4,5,8,8,8,8	327680	18	80	7	6	4,4,6,6,6,6,6	124416	16	53
7	7	4,4,6,6,6,6,7	145152	17	90	7	8	4,4,6,6,6,6,8	165888	17	79	7	7	4,4,6,6,6,7,7	169344	17	77
7	8	4,4,6,6,6,7,8	193536	17	68	7	8	4,4,6,6,6,8,8	221184	17	59	7	7	4,4,6,6,7,7,7	197568	17	66
7	8	4,4,6,6,7,7,8	225792	17	58	7	8	4,4,6,6,7,8,8	258048	17	51	7	8	4,4,6,6,8,8,8	294912	18	89
7	7	4,4,6,7,7,7,7	230496	17	57	7	8	4,4,6,7,7,7,8	263424	18	100	7	8	4,4,6,7,7,8,8	301056	18	87
7	8	4,4,6,7,8,8,8	344064	18	76	7	8	4,4,6,8,8,8,8	393216	18	67	7	7	4,4,7,7,7,7,7	268912	18	97

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	4,4,7,7,7,7,8	307328	18	85	7	8	4,4,7,7,7,8,8	351232	18	75	7	8	4,4,7,7,8,8,8	401408	18	65
7	8	4,4,7,8,8,8,8	458752	18	57	7	8	4,4,8,8,8,8,8	524288	19	100	7	5	4,5,5,5,5,5,5	62500	15	52
7	6	4,5,5,5,5,5,6	75000	16	87	7	7	4,5,5,5,5,5,7	87500	16	75	7	8	4,5,5,5,5,5,8	100000	16	66
7	6	4,5,5,5,5,6,6	90000	16	73	7	7	4,5,5,5,5,6,7	105000	16	62	7	8	4,5,5,5,5,6,8	120000	16	55
7	7	4,5,5,5,5,7,7	122500	16	53	7	8	4,5,5,5,5,7,8	140000	17	94	7	8	4,5,5,5,5,8,8	160000	17	82
7	6	4,5,5,5,6,6,6	108000	16	61	7	7	4,5,5,5,6,6,7	126000	16	52	7	8	4,5,5,5,6,6,8	144000	17	91
7	7	4,5,5,5,6,7,7	147000	17	89	7	8	4,5,5,5,6,7,8	168000	17	78	7	8	4,5,5,5,6,8,8	192000	17	68
7	7	4,5,5,5,7,7,7	171500	17	76	7	8	4,5,5,5,7,7,8	196000	17	67	7	8	4,5,5,5,7,8,8	224000	17	59
7	8	4,5,5,5,8,8,8	256000	17	51	7	6	4,5,5,6,6,6,6	129600	16	51	7	7	4,5,5,6,6,6,7	151200	17	87
7	8	4,5,5,6,6,6,8	172800	17	76	7	7	4,5,5,6,6,7,7	176400	17	74	7	8	4,5,5,6,6,7,8	201600	17	65
7	8	4,5,5,6,6,8,8	230400	17	57	7	7	4,5,5,6,7,7,7	205800	17	64	7	8	4,5,5,6,7,7,8	235200	17	56
7	8	4,5,5,6,7,8,8	268800	18	98	7	8	4,5,5,6,8,8,8	307200	18	85	7	7	4,5,5,7,7,7,7	240100	17	55
7	8	4,5,5,7,7,7,8	274400	18	96	7	8	4,5,5,7,7,8,8	313600	18	84	7	8	4,5,5,7,8,8,8	358400	18	73
7	8	4,5,5,8,8,8,8	409600	18	64	7	6	4,5,6,6,6,6,6	155520	17	84	7	7	4,5,6,6,6,6,7	181440	17	72
7	8	4,5,6,6,6,6,8	207360	17	63	7	7	4,5,6,6,6,7,7	211680	17	62	7	8	4,5,6,6,6,7,8	241920	17	54
7	8	4,5,6,6,6,8,8	276480	18	95	7	7	4,5,6,6,7,7,7	246960	17	53	7	8	4,5,6,6,7,7,8	282240	18	93
7	8	4,5,6,6,7,8,8	322560	18	81	7	8	4,5,6,6,8,8,8	368640	18	71	7	7	4,5,6,7,7,7,7	288120	18	91
7	8	4,5,6,7,7,7,8	329280	18	80	7	8	4,5,6,7,7,8,8	376320	18	70	7	8	4,5,6,7,8,8,8	430080	18	61
7	8	4,5,6,8,8,8,8	491520	18	53	7	7	4,5,7,7,7,7,7	336140	18	78	7	8	4,5,7,7,7,7,8	384160	18	68
7	8	4,5,7,7,7,8,8	439040	18	60	7	8	4,5,7,7,8,8,8	501760	18	52	7	8	4,5,7,8,8,8,8	573440	19	91
7	8	4,5,8,8,8,8,8	655360	19	80	7	6	4,6,6,6,6,6,6	186624	17	70	7	7	4,6,6,6,6,6,7	217728	17	60
7	8	4,6,6,6,6,6,8	248832	17	53	7	7	4,6,6,6,6,7,7	254016	17	52	7	8	4,6,6,6,6,7,8	290304	18	90
7	8	4,6,6,6,6,8,8	331776	18	79	7	7	4,6,6,6,7,7,7	296352	18	88	7	8	4,6,6,6,7,7,8	338688	18	77
7	8	4,6,6,6,7,8,8	387072	18	68	7	8	4,6,6,6,8,8,8	442368	18	59	7	7	4,6,6,7,7,7,7	345744	18	76
7	8	4,6,6,7,7,7,8	395136	18	66	7	8	4,6,6,7,7,8,8	451584	18	58	7	8	4,6,6,7,8,8,8	516096	18	51
7	8	4,6,6,8,8,8,8	589824	19	89	7	7	4,6,7,7,7,7,7	403368	18	65	7	8	4,6,7,7,7,7,8	460992	18	57
7	8	4,6,7,7,7,8,8	526848	19	100	7	8	4,6,7,7,8,8,8	602112	19	87	7	8	4,6,7,8,8,8,8	688128	19	76
7	8	4,6,8,8,8,8,8	786432	19	67	7	7	4,7,7,7,7,7,7	470596	18	56	7	8	4,7,7,7,7,7,8	537824	19	97
7	8	4,7,7,7,7,8,8	614656	19	85	7	8	4,7,7,7,8,8,8	702464	19	75	7	8	4,7,7,8,8,8,8	802816	19	65
7	8	4,7,8,8,8,8,8	917504	19	57	7	8	4,8,8,8,8,8,8	1048576	20	100	7	5	5,5,5,5,5,5,5	78125	16	84
7	6	5,5,5,5,5,5,6	93750	16	70	7	7	5,5,5,5,5,5,7	109375	16	60	7	8	5,5,5,5,5,5,8	125000	16	52
7	6	5,5,5,5,5,6,6	112500	16	58	7	7	5,5,5,5,5,6,7	131250	17	100	7	8	5,5,5,5,5,6,8	150000	17	87

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	7	5,5,5,5,5,7,7	153125	17	86	7	8	5,5,5,5,5,7,8	175000	17	75	7	8	5,5,5,5,5,8,8	200000	17	66
7	6	5,5,5,5,6,6,6	135000	17	97	7	7	5,5,5,5,6,6,7	157500	17	83	7	8	5,5,5,5,6,6,8	180000	17	73
7	7	5,5,5,5,6,7,7	183750	17	71	7	8	5,5,5,5,6,7,8	210000	17	62	7	8	5,5,5,5,6,8,8	240000	17	55
7	7	5,5,5,5,7,7,7	214375	17	61	7	8	5,5,5,5,7,7,8	245000	17	53	7	8	5,5,5,5,7,8,8	280000	18	94
7	8	5,5,5,5,8,8,8	320000	18	82	7	6	5,5,5,6,6,6,6	162000	17	81	7	7	5,5,5,6,6,6,7	189000	17	69
7	8	5,5,5,6,6,6,8	216000	17	61	7	7	5,5,5,6,6,7,7	220500	17	59	7	8	5,5,5,6,6,7,8	252000	17	52
7	8	5,5,5,6,6,8,8	288000	18	91	7	7	5,5,5,6,7,7,7	257250	17	51	7	8	5,5,5,6,7,7,8	294000	18	89
7	8	5,5,5,6,7,8,8	336000	18	78	7	8	5,5,5,6,8,8,8	384000	18	68	7	7	5,5,5,7,7,7,7	300125	18	87
7	8	5,5,5,7,7,7,8	343000	18	76	7	8	5,5,5,7,7,8,8	392000	18	67	7	8	5,5,5,7,8,8,8	448000	18	59
7	8	5,5,5,8,8,8,8	512000	18	51	7	6	5,5,6,6,6,6,6	194400	17	67	7	7	5,5,6,6,6,6,7	226800	17	58
7	8	5,5,6,6,6,6,8	259200	17	51	7	7	5,5,6,6,6,7,7	264600	18	99	7	8	5,5,6,6,6,7,8	302400	18	87
7	8	5,5,6,6,6,8,8	345600	18	76	7	7	5,5,6,6,7,7,7	308700	18	85	7	8	5,5,6,6,7,7,8	352800	18	74
7	8	5,5,6,6,7,8,8	403200	18	65	7	8	5,5,6,6,8,8,8	460800	18	57	7	7	5,5,6,7,7,7,7	360150	18	73
7	8	5,5,6,7,7,7,8	411600	18	64	7	8	5,5,6,7,7,8,8	470400	18	56	7	8	5,5,6,7,8,8,8	537600	19	98
7	8	5,5,6,8,8,8,8	614400	19	85	7	7	5,5,7,7,7,7,7	420175	18	62	7	8	5,5,7,7,7,7,8	480200	18	55
7	8	5,5,7,7,7,8,8	548800	19	96	7	8	5,5,7,7,8,8,8	627200	19	84	7	8	5,5,7,8,8,8,8	716800	19	73
7	8	5,5,8,8,8,8,8	819200	19	64	7	6	5,6,6,6,6,6,6	233280	17	56	7	7	5,6,6,6,6,6,7	272160	18	96
7	8	5,6,6,6,6,6,8	311040	18	84	7	7	5,6,6,6,6,7,7	317520	18	83	7	8	5,6,6,6,6,7,8	362880	18	72
7	8	5,6,6,6,6,8,8	414720	18	63	7	7	5,6,6,6,7,7,7	370440	18	71	7	8	5,6,6,6,7,7,8	423360	18	62
7	8	5,6,6,6,7,8,8	483840	18	54	7	8	5,6,6,6,8,8,8	552960	19	95	7	7	5,6,6,7,7,7,7	432180	18	61
7	8	5,6,6,7,7,7,8	493920	18	53	7	8	5,6,6,7,7,8,8	564480	19	93	7	8	5,6,6,7,8,8,8	645120	19	81
7	8	5,6,6,8,8,8,8	737280	19	71	7	7	5,6,7,7,7,7,7	504210	18	52	7	8	5,6,7,7,7,7,8	576240	19	91
7	8	5,6,7,7,7,8,8	658560	19	80	7	8	5,6,7,7,8,8,8	752640	19	70	7	8	5,6,7,8,8,8,8	860160	19	61
7	8	5,6,8,8,8,8,8	983040	19	53	7	7	5,7,7,7,7,7,7	588245	19	89	7	8	5,7,7,7,7,7,8	672280	19	78
7	8	5,7,7,7,7,8,8	768320	19	68	7	8	5,7,7,7,8,8,8	878080	19	60	7	8	5,7,7,8,8,8,8	1003520	19	52
7	8	5,7,8,8,8,8,8	1146880	20	91	7	8	5,8,8,8,8,8,8	1310720	20	80	7	6	6,6,6,6,6,6,6	279936	18	94
7	7	6,6,6,6,6,6,7	326592	18	80	7	8	6,6,6,6,6,6,8	373248	18	70	7	7	6,6,6,6,6,7,7	381024	18	69
7	8	6,6,6,6,6,7,8	435456	18	60	7	8	6,6,6,6,6,8,8	497664	18	53	7	7	6,6,6,6,7,7,7	444528	18	59
7	8	6,6,6,6,7,7,8	508032	18	52	7	8	6,6,6,6,7,8,8	580608	19	90	7	8	6,6,6,6,8,8,8	663552	19	79
7	7	6,6,6,7,7,7,7	518616	18	51	7	8	6,6,6,7,7,7,8	592704	19	88	7	8	6,6,6,7,7,8,8	677376	19	77
7	8	6,6,6,7,8,8,8	774144	19	68	7	8	6,6,6,8,8,8,8	884736	19	59	7	7	6,6,7,7,7,7,7	605052	19	87
7	8	6,6,7,7,7,7,8	691488	19	76	7	8	6,6,7,7,7,8,8	790272	19	66	7	8	6,6,7,7,8,8,8	903168	19	58

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	6,6,7,8,8,8,8	1032192	19	51	7	8	6,6,8,8,8,8,8	1179648	20	89	7	7	6,7,7,7,7,7,7	705894	19	74
7	8	6,7,7,7,7,7,8	806736	19	65	7	8	6,7,7,7,7,8,8	921984	19	57	7	8	6,7,7,7,8,8,8	1053696	20	100
7	8	6,7,7,8,8,8,8	1204224	20	87	7	8	6,7,8,8,8,8,8	1376256	20	76	7	8	6,8,8,8,8,8,8	1572864	20	67
7	7	7,7,7,7,7,7,7	823543	19	64	7	8	7,7,7,7,7,7,8	941192	19	56	7	8	7,7,7,7,7,8,8	1075648	20	97
7	8	7,7,7,7,8,8,8	1229312	20	85	7	8	7,7,7,8,8,8,8	1404928	20	75	7	8	7,7,8,8,8,8,8	1605632	20	65
7	8	7,8,8,8,8,8,8	1835008	20	57	7	8	8,8,8,8,8,8,8	2097152	21	100	-	-	-	-	-	-

Appendix B: Difference-of-2 Sequence Listing

Note: This is a computer generated table. Within this table, efficiency, η, is rounded to the nearest whole number. As a result, the efficiency shown may not be accurate but, if the true efficiency is needed, a division of the number of mapped waveforms, 2^b, over the total number of waveforms, W, can be performed for a specific sequence. See 3.1.2 for a usage example.

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
1	2	2	2	1	100	1	3	3	3	1	67	1	4	4	4	2	100
1	5	5	5	2	80	1	6	6	6	2	67	1	7	7	7	2	57
1	8	8	8	3	100	2	2	2,2	4	2	100	2	3	2,3	6	2	67
2	4	2,4	8	3	100	2	3	3,3	9	3	89	2	4	3,4	12	3	67
2	5	3,5	15	3	53	2	4	4,4	16	4	100	2	5	4,5	20	4	80
2	6	4,6	24	4	67	2	5	5,5	25	4	64	2	6	5,6	30	4	53
2	7	5,7	35	5	91	2	6	6,6	36	5	89	2	7	6,7	42	5	76
2	8	6,8	48	5	67	2	7	7,7	49	5	65	2	8	7,8	56	5	57
2	8	8,8	64	6	100	3	2	2,2,2	8	3	100	3	3	2,2,3	12	3	67
3	4	2,2,4	16	4	100	3	3	2,3,3	18	4	89	3	4	2,3,4	24	4	67
3	4	2,4,4	32	5	100	3	3	3,3,3	27	4	59	3	4	3,3,4	36	5	89
3	5	3,3,5	45	5	71	3	4	3,4,4	48	5	67	3	5	3,4,5	60	5	53
3	5	3,5,5	75	6	85	3	4	4,4,4	64	6	100	3	5	4,4,5	80	6	80
3	6	4,4,6	96	6	67	3	5	4,5,5	100	6	64	3	6	4,5,6	120	6	53
3	6	4,6,6	144	7	89	3	5	5,5,5	125	6	51	3	6	5,5,6	150	7	85
3	7	5,5,7	175	7	73	3	6	5,6,6	180	7	71	3	7	5,6,7	210	7	61

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
3	7	5,7,7	245	7	52	3	6	6,6,6	216	7	59	3	7	6,6,7	252	7	51
3	8	6,6,8	288	8	89	3	7	6,7,7	294	8	87	3	8	6,7,8	336	8	76
3	8	6,8,8	384	8	67	3	7	7,7,7	343	8	75	3	8	7,7,8	392	8	65
3	8	7,8,8	448	8	57	3	8	8,8,8	512	9	100	4	2	2,2,2,2	16	4	100
4	3	2,2,2,3	24	4	67	4	4	2,2,2,4	32	5	100	4	3	2,2,3,3	36	5	89
4	4	2,2,3,4	48	5	67	4	4	2,2,4,4	64	6	100	4	3	2,3,3,3	54	5	59
4	4	2,3,3,4	72	6	89	4	4	2,3,4,4	96	6	67	4	4	2,4,4,4	128	7	100
4	3	3,3,3,3	81	6	79	4	4	3,3,3,4	108	6	59	4	5	3,3,3,5	135	7	95
4	4	3,3,4,4	144	7	89	4	5	3,3,4,5	180	7	71	4	5	3,3,5,5	225	7	57
4	4	3,4,4,4	192	7	67	4	5	3,4,4,5	240	7	53	4	5	3,4,5,5	300	8	85
4	5	3,5,5,5	375	8	68	4	4	4,4,4,4	256	8	100	4	5	4,4,4,5	320	8	80
4	6	4,4,4,6	384	8	67	4	5	4,4,5,5	400	8	64	4	6	4,4,5,6	480	8	53
4	6	4,4,6,6	576	9	89	4	5	4,5,5,5	500	8	51	4	6	4,5,5,6	600	9	85
4	6	4,5,6,6	720	9	71	4	6	4,6,6,6	864	9	59	4	5	5,5,5,5	625	9	82
4	6	5,5,5,6	750	9	68	4	7	5,5,5,7	875	9	59	4	6	5,5,6,6	900	9	57
4	7	5,5,6,7	1050	10	98	4	7	5,5,7,7	1225	10	84	4	6	5,6,6,6	1080	10	95
4	7	5,6,6,7	1260	10	81	4	7	5,6,7,7	1470	10	70	4	7	5,7,7,7	1715	10	60
4	6	6,6,6,6	1296	10	79	4	7	6,6,6,7	1512	10	68	4	8	6,6,6,8	1728	10	59
4	7	6,6,7,7	1764	10	58	4	8	6,6,7,8	2016	10	51	4	8	6,6,8,8	2304	11	89
4	7	6,7,7,7	2058	11	100	4	8	6,7,7,8	2352	11	87	4	8	6,7,8,8	2688	11	76
4	8	6,8,8,8	3072	11	67	4	7	7,7,7,7	2401	11	85	4	8	7,7,7,8	2744	11	75
4	8	7,7,8,8	3136	11	65	4	8	7,8,8,8	3584	11	57	4	8	8,8,8,8	4096	12	100
5	2	2,2,2,2,2	32	5	100	5	3	2,2,2,2,3	48	5	67	5	4	2,2,2,2,4	64	6	100
5	3	2,2,2,3,3	72	6	89	5	4	2,2,2,3,4	96	6	67	5	4	2,2,2,4,4	128	7	100
5	3	2,2,3,3,3	108	6	59	5	4	2,2,3,3,4	144	7	89	5	4	2,2,3,4,4	192	7	67
5	4	2,2,4,4,4	256	8	100	5	3	2,3,3,3,3	162	7	79	5	4	2,3,3,3,4	216	7	59
5	4	2,3,3,4,4	288	8	89	5	4	2,3,4,4,4	384	8	67	5	4	2,4,4,4,4	512	9	100
5	3	3,3,3,3,3	243	7	53	5	4	3,3,3,3,4	324	8	79	5	5	3,3,3,3,5	405	8	63
5	4	3,3,3,4,4	432	8	59	5	5	3,3,3,4,5	540	9	95	5	5	3,3,3,5,5	675	9	76
5	4	3,3,4,4,4	576	9	89	5	5	3,3,4,4,5	720	9	71	5	5	3,3,4,5,5	900	9	57
5	5	3,3,5,5,5	1125	10	91	5	4	3,4,4,4,4	768	9	67	5	5	3,4,4,4,5	960	9	53
5	5	3,4,4,5,5	1200	10	85	5	5	3,4,5,5,5	1500	10	68	5	5	3,5,5,5,5	1875	10	55

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
5	4	4,4,4,4,4	1024	10	100	5	5	4,4,4,4,5	1280	10	80	5	6	4,4,4,4,6	1536	10	67
5	5	4,4,4,5,5	1600	10	64	5	6	4,4,4,5,6	1920	10	53	5	6	4,4,4,6,6	2304	11	89
5	5	4,4,5,5,5	2000	10	51	5	6	4,4,5,5,6	2400	11	85	5	6	4,4,5,6,6	2880	11	71
5	6	4,4,6,6,6	3456	11	59	5	5	4,5,5,5,5	2500	11	82	5	6	4,5,5,5,6	3000	11	68
5	6	4,5,5,6,6	3600	11	57	5	6	4,5,6,6,6	4320	12	95	5	6	4,6,6,6,6	5184	12	79
5	5	5,5,5,5,5	3125	11	66	5	6	5,5,5,5,6	3750	11	55	5	7	5,5,5,5,7	4375	12	94
5	6	5,5,5,6,6	4500	12	91	5	7	5,5,5,6,7	5250	12	78	5	7	5,5,5,7,7	6125	12	67
5	6	5,5,6,6,6	5400	12	76	5	7	5,5,6,6,7	6300	12	65	5	7	5,5,6,7,7	7350	12	56
5	7	5,5,7,7,7	8575	13	96	5	6	5,6,6,6,6	6480	12	63	5	7	5,6,6,6,7	7560	12	54
5	7	5,6,6,7,7	8820	13	93	5	7	5,6,7,7,7	10290	13	80	5	7	5,7,7,7,7	12005	13	68
5	6	6,6,6,6,6	7776	12	53	5	7	6,6,6,6,7	9072	13	90	5	8	6,6,6,6,8	10368	13	79
5	7	6,6,6,7,7	10584	13	77	5	8	6,6,6,7,8	12096	13	68	5	8	6,6,6,8,8	13824	13	59
5	7	6,6,7,7,7	12348	13	66	5	8	6,6,7,7,8	14112	13	58	5	8	6,6,7,8,8	16128	13	51
5	8	6,6,8,8,8	18432	14	89	5	7	6,7,7,7,7	14406	13	57	5	8	6,7,7,7,8	16464	14	100
5	8	6,7,7,8,8	18816	14	87	5	8	6,7,8,8,8	21504	14	76	5	8	6,8,8,8,8	24576	14	67
5	7	7,7,7,7,7	16807	14	97	5	8	7,7,7,7,8	19208	14	85	5	8	7,7,7,8,8	21952	14	75
5	8	7,7,8,8,8	25088	14	65	5	8	7,8,8,8,8	28672	14	57	5	8	8,8,8,8,8	32768	15	100
6	2	2,2,2,2,2,2	64	6	100	6	3	2,2,2,2,2,3	96	6	67	6	4	2,2,2,2,2,4	128	7	100
6	3	2,2,2,2,3,3	144	7	89	6	4	2,2,2,2,3,4	192	7	67	6	4	2,2,2,2,4,4	256	8	100
6	3	2,2,2,3,3,3	216	7	59	6	4	2,2,2,3,3,4	288	8	89	6	4	2,2,2,3,4,4	384	8	67
6	4	2,2,2,4,4,4	512	9	100	6	3	2,2,3,3,3,3	324	8	79	6	4	2,2,3,3,3,4	432	8	59
6	4	2,2,3,3,4,4	576	9	89	6	4	2,2,3,4,4,4	768	9	67	6	4	2,2,4,4,4,4	1024	10	100
6	3	2,3,3,3,3,3	486	8	53	6	4	2,3,3,3,3,4	648	9	79	6	4	2,3,3,3,4,4	864	9	59
6	4	2,3,3,4,4,4	1152	10	89	6	4	2,3,4,4,4,4	1536	10	67	6	4	2,4,4,4,4,4	2048	11	100
6	3	3,3,3,3,3,3	729	9	70	6	4	3,3,3,3,3,4	972	9	53	6	5	3,3,3,3,3,5	1215	10	84
6	4	3,3,3,3,4,4	1296	10	79	6	5	3,3,3,3,4,5	1620	10	63	6	5	3,3,3,3,5,5	2025	10	51
6	4	3,3,3,4,4,4	1728	10	59	6	5	3,3,3,4,4,5	2160	11	95	6	5	3,3,3,4,5,5	2700	11	76
6	5	3,3,3,5,5,5	3375	11	61	6	4	3,3,4,4,4,4	2304	11	89	6	5	3,3,4,4,4,5	2880	11	71
6	5	3,3,4,4,5,5	3600	11	57	6	5	3,3,4,5,5,5	4500	12	91	6	5	3,3,5,5,5,5	5625	12	73
6	4	3,4,4,4,4,4	3072	11	67	6	5	3,4,4,4,4,5	3840	11	53	6	5	3,4,4,4,5,5	4800	12	85
6	5	3,4,4,5,5,5	6000	12	68	6	5	3,4,5,5,5,5	7500	12	55	6	5	3,5,5,5,5,5	9375	13	87
6	4	4,4,4,4,4,4	4096	12	100	6	5	4,4,4,4,4,5	5120	12	80	6	6	4,4,4,4,4,6	6144	12	67

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
6	5	4,4,4,4,5,5	6400	12	64	6	6	4,4,4,4,5,6	7680	12	53	6	6	4,4,4,4,6,6	9216	13	89
6	5	4,4,4,5,5,5	8000	12	51	6	6	4,4,4,5,5,6	9600	13	85	6	6	4,4,4,5,6,6	11520	13	71
6	6	4,4,4,6,6,6	13824	13	59	6	5	4,4,5,5,5,5	10000	13	82	6	6	4,4,5,5,5,6	12000	13	68
6	6	4,4,5,5,6,6	14400	13	57	6	6	4,4,5,6,6,6	17280	14	95	6	6	4,4,6,6,6,6	20736	14	79
6	5	4,5,5,5,5,5	12500	13	66	6	6	4,5,5,5,5,6	15000	13	55	6	6	4,5,5,5,6,6	18000	14	91
6	6	4,5,5,6,6,6	21600	14	76	6	6	4,5,6,6,6,6	25920	14	63	6	6	4,6,6,6,6,6	31104	14	53
6	5	5,5,5,5,5,5	15625	13	52	6	6	5,5,5,5,5,6	18750	14	87	6	7	5,5,5,5,5,7	21875	14	75
6	6	5,5,5,5,6,6	22500	14	73	6	7	5,5,5,5,6,7	26250	14	62	6	7	5,5,5,5,7,7	30625	14	53
6	6	5,5,5,6,6,6	27000	14	61	6	7	5,5,5,6,6,7	31500	14	52	6	7	5,5,5,6,7,7	36750	15	89
6	7	5,5,5,7,7,7	42875	15	76	6	6	5,5,6,6,6,6	32400	14	51	6	7	5,5,6,6,6,7	37800	15	87
6	7	5,5,6,6,7,7	44100	15	74	6	7	5,5,6,7,7,7	51450	15	64	6	7	5,5,7,7,7,7	60025	15	55
6	6	5,6,6,6,6,6	38880	15	84	6	7	5,6,6,6,6,7	45360	15	72	6	7	5,6,6,6,7,7	52920	15	62
6	7	5,6,6,7,7,7	61740	15	53	6	7	5,6,7,7,7,7	72030	16	91	6	7	5,7,7,7,7,7	84035	16	78
6	6	6,6,6,6,6,6	46656	15	70	6	7	6,6,6,6,6,7	54432	15	60	6	8	6,6,6,6,6,8	62208	15	53
6	7	6,6,6,6,7,7	63504	15	52	6	8	6,6,6,6,7,8	72576	16	90	6	8	6,6,6,6,8,8	82944	16	79
6	7	6,6,6,7,7,7	74088	16	88	6	8	6,6,6,7,7,8	84672	16	77	6	8	6,6,6,7,8,8	96768	16	68
6	8	6,6,6,8,8,8	110592	16	59	6	7	6,6,7,7,7,7	86436	16	76	6	8	6,6,7,7,7,8	98784	16	66
6	8	6,6,7,7,8,8	112896	16	58	6	8	6,6,7,8,8,8	129024	16	51	6	8	6,6,8,8,8,8	147456	17	89
6	7	6,7,7,7,7,7	100842	16	65	6	8	6,7,7,7,7,8	115248	16	57	6	8	6,7,7,7,8,8	131712	17	100
6	8	6,7,7,8,8,8	150528	17	87	6	8	6,7,8,8,8,8	172032	17	76	6	8	6,8,8,8,8,8	196608	17	67
6	7	7,7,7,7,7,7	117649	16	56	6	8	7,7,7,7,7,8	134456	17	97	6	8	7,7,7,7,8,8	153664	17	85
6	8	7,7,7,8,8,8	175616	17	75	6	8	7,7,8,8,8,8	200704	17	65	6	8	7,8,8,8,8,8	229376	17	57
6	8	8,8,8,8,8,8	262144	18	100	7	2	2,2,2,2,2,2,2	128	7	100	7	3	2,2,2,2,2,2,3	192	7	67
7	4	2,2,2,2,2,2,4	256	8	100	7	3	2,2,2,2,2,3,3	288	8	89	7	4	2,2,2,2,2,3,4	384	8	67
7	4	2,2,2,2,2,4,4	512	9	100	7	3	2,2,2,2,3,3,3	432	8	59	7	4	2,2,2,2,3,3,4	576	9	89
7	4	2,2,2,2,3,4,4	768	9	67	7	4	2,2,2,2,4,4,4	1024	10	100	7	3	2,2,2,3,3,3,3	648	9	79
7	4	2,2,2,3,3,3,4	864	9	59	7	4	2,2,2,3,3,4,4	1152	10	89	7	4	2,2,2,3,4,4,4	1536	10	67
7	4	2,2,2,4,4,4,4	2048	11	100	7	3	2,2,3,3,3,3,3	972	9	53	7	4	2,2,3,3,3,3,4	1296	10	79
7	4	2,2,3,3,3,4,4	1728	10	59	7	4	2,2,3,3,4,4,4	2304	11	89	7	4	2,2,3,4,4,4,4	3072	11	67
7	4	2,2,4,4,4,4,4	4096	12	100	7	3	2,3,3,3,3,3,3	1458	10	70	7	4	2,3,3,3,3,3,4	1944	10	53
7	4	2,3,3,3,3,4,4	2592	11	79	7	4	2,3,3,3,4,4,4	3456	11	59	7	4	2,3,3,4,4,4,4	4608	12	89
7	4	2,3,4,4,4,4,4	6144	12	67	7	4	2,4,4,4,4,4,4	8192	13	100	7	3	3,3,3,3,3,3,3	2187	11	94

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	4	3,3,3,3,3,3,4	2916	11	70	7	5	3,3,3,3,3,3,5	3645	11	56	7	4	3,3,3,3,3,4,4	3888	11	53
7	5	3,3,3,3,3,4,5	4860	12	84	7	5	3,3,3,3,3,5,5	6075	12	67	7	4	3,3,3,3,4,4,4	5184	12	79
7	5	3,3,3,3,4,4,5	6480	12	63	7	5	3,3,3,3,4,5,5	8100	12	51	7	5	3,3,3,3,5,5,5	10125	13	81
7	4	3,3,3,4,4,4,4	6912	12	59	7	5	3,3,3,4,4,4,5	8640	13	95	7	5	3,3,3,4,4,5,5	10800	13	76
7	5	3,3,3,4,5,5,5	13500	13	61	7	5	3,3,3,5,5,5,5	16875	14	97	7	4	3,3,4,4,4,4,4	9216	13	89
7	5	3,3,4,4,4,4,5	11520	13	71	7	5	3,3,4,4,4,5,5	14400	13	57	7	5	3,3,4,4,5,5,5	18000	14	91
7	5	3,3,4,5,5,5,5	22500	14	73	7	5	3,3,5,5,5,5,5	28125	14	58	7	4	3,4,4,4,4,4,4	12288	13	67
7	5	3,4,4,4,4,4,5	15360	13	53	7	5	3,4,4,4,4,5,5	19200	14	85	7	5	3,4,4,4,5,5,5	24000	14	68
7	5	3,4,4,5,5,5,5	30000	14	55	7	5	3,4,5,5,5,5,5	37500	15	87	7	5	3,5,5,5,5,5,5	46875	15	70
7	4	4,4,4,4,4,4,4	16384	14	100	7	5	4,4,4,4,4,4,5	20480	14	80	7	6	4,4,4,4,4,4,6	24576	14	67
7	5	4,4,4,4,4,5,5	25600	14	64	7	6	4,4,4,4,4,5,6	30720	14	53	7	6	4,4,4,4,4,6,6	36864	15	89
7	5	4,4,4,4,5,5,5	32000	14	51	7	6	4,4,4,4,5,5,6	38400	15	85	7	6	4,4,4,4,5,6,6	46080	15	71
7	6	4,4,4,4,6,6,6	55296	15	59	7	5	4,4,4,5,5,5,5	40000	15	82	7	6	4,4,4,5,5,5,6	48000	15	68
7	6	4,4,4,5,5,6,6	57600	15	57	7	6	4,4,4,5,6,6,6	69120	16	95	7	6	4,4,4,6,6,6,6	82944	16	79
7	5	4,4,5,5,5,5,5	50000	15	66	7	6	4,4,5,5,5,5,6	60000	15	55	7	6	4,4,5,5,5,6,6	72000	16	91
7	6	4,4,5,5,6,6,6	86400	16	76	7	6	4,4,5,6,6,6,6	103680	16	63	7	6	4,4,6,6,6,6,6	124416	16	53
7	5	4,5,5,5,5,5,5	62500	15	52	7	6	4,5,5,5,5,5,6	75000	16	87	7	6	4,5,5,5,5,6,6	90000	16	73
7	6	4,5,5,5,6,6,6	108000	16	61	7	6	4,5,5,6,6,6,6	129600	16	51	7	6	4,5,6,6,6,6,6	155520	17	84
7	6	4,6,6,6,6,6,6	186624	17	70	7	5	5,5,5,5,5,5,5	78125	16	84	7	6	5,5,5,5,5,5,6	93750	16	70
7	7	5,5,5,5,5,5,7	109375	16	60	7	6	5,5,5,5,5,6,6	112500	16	58	7	7	5,5,5,5,5,6,7	131250	17	100
7	7	5,5,5,5,5,7,7	153125	17	86	7	6	5,5,5,5,6,6,6	135000	17	97	7	7	5,5,5,5,6,6,7	157500	17	83
7	7	5,5,5,5,6,7,7	183750	17	71	7	7	5,5,5,5,7,7,7	214375	17	61	7	6	5,5,5,6,6,6,6	162000	17	81
7	7	5,5,5,6,6,6,7	189000	17	69	7	7	5,5,5,6,6,7,7	220500	17	59	7	7	5,5,5,6,7,7,7	257250	17	51
7	7	5,5,5,7,7,7,7	300125	18	87	7	6	5,5,6,6,6,6,6	194400	17	67	7	7	5,5,6,6,6,6,7	226800	17	58
7	7	5,5,6,6,6,7,7	264600	18	99	7	7	5,5,6,6,7,7,7	308700	18	85	7	7	5,5,6,7,7,7,7	360150	18	73
7	7	5,5,7,7,7,7,7	420175	18	62	7	6	5,6,6,6,6,6,6	233280	17	56	7	7	5,6,6,6,6,6,7	272160	18	96
7	7	5,6,6,6,6,7,7	317520	18	83	7	7	5,6,6,6,7,7,7	370440	18	71	7	7	5,6,6,7,7,7,7	432180	18	61
7	7	5,6,7,7,7,7,7	504210	18	52	7	7	5,7,7,7,7,7,7	588245	19	89	7	6	6,6,6,6,6,6,6	279936	18	94
7	7	6,6,6,6,6,6,7	326592	18	80	7	8	6,6,6,6,6,6,8	373248	18	70	7	7	6,6,6,6,6,7,7	381024	18	69
7	8	6,6,6,6,6,7,8	435456	18	60	7	8	6,6,6,6,6,8,8	497664	18	53	7	7	6,6,6,6,7,7,7	444528	18	59
7	8	6,6,6,6,7,7,8	508032	18	52	7	8	6,6,6,6,7,8,8	580608	19	90	7	8	6,6,6,6,8,8,8	663552	19	79
7	7	6,6,6,7,7,7,7	518616	18	51	7	8	6,6,6,7,7,7,8	592704	19	88	7	8	6,6,6,7,7,8,8	677376	19	77

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	8	6,6,6,7,8,8,8	774144	19	68	7	8	6,6,6,8,8,8,8	884736	19	59	7	7	6,6,7,7,7,7,7	605052	19	87
7	8	6,6,7,7,7,7,8	691488	19	76	7	8	6,6,7,7,7,8,8	790272	19	66	7	8	6,6,7,7,8,8,8	903168	19	58
7	8	6,6,7,8,8,8,8	1032192	19	51	7	8	6,6,8,8,8,8,8	1179648	20	89	7	7	6,7,7,7,7,7,7	705894	19	74
7	8	6,7,7,7,7,7,8	806736	19	65	7	8	6,7,7,7,7,8,8	921984	19	57	7	8	6,7,7,7,8,8,8	1053696	20	100
7	8	6,7,7,8,8,8,8	1204224	20	87	7	8	6,7,8,8,8,8,8	1376256	20	76	7	8	6,8,8,8,8,8,8	1572864	20	67
7	7	7,7,7,7,7,7,7	823543	19	64	7	8	7,7,7,7,7,7,8	941192	19	56	7	8	7,7,7,7,7,8,8	1075648	20	97
7	8	7,7,7,7,8,8,8	1229312	20	85	7	8	7,7,7,8,8,8,8	1404928	20	75	7	8	7,7,8,8,8,8,8	1605632	20	65
7	8	7,8,8,8,8,8,8	1835008	20	57	7	8	8,8,8,8,8,8,8	2097152	21	100	-	-	-	-	-	-

Appendix C: Difference-of-1 Sequence Listing

Note: This is a computer generated table. Within this table, efficiency, η, is rounded to the nearest whole number. As a result, the efficiency shown may not be accurate but, if the true efficiency is needed, a division of the number of mapped waveforms, 2^b, over the total number of waveforms, W, can be performed for a specific sequence. See 3.1.2 for a usage example.

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
1	2	2	2	1	100	1	3	3	3	1	67	1	4	4	4	2	100
1	5	5	5	2	80	1	6	6	6	2	67	1	7	7	7	2	57
1	8	8	8	3	100	2	2	2,2	4	2	100	2	3	2,3	6	2	67
2	3	3,3	9	3	89	2	4	3,4	12	3	67	2	4	4,4	16	4	100
2	5	4,5	20	4	80	2	5	5,5	25	4	64	2	6	5,6	30	4	53
2	6	6,6	36	5	89	2	7	6,7	42	5	76	2	7	7,7	49	5	65
2	8	7,8	56	5	57	2	8	8,8	64	6	100	3	2	2,2,2	8	3	100
3	3	2,2,3	12	3	67	3	3	2,3,3	18	4	89	3	3	3,3,3	27	4	59
3	4	3,3,4	36	5	89	3	4	3,4,4	48	5	67	3	4	4,4,4	64	6	100
3	5	4,4,5	80	6	80	3	5	4,5,5	100	6	64	3	5	5,5,5	125	6	51
3	6	5,5,6	150	7	85	3	6	5,6,6	180	7	71	3	6	6,6,6	216	7	59
3	7	6,6,7	252	7	51	3	7	6,7,7	294	8	87	3	7	7,7,7	343	8	75
3	8	7,7,8	392	8	65	3	8	7,8,8	448	8	57	3	8	8,8,8	512	9	100
4	2	2,2,2,2	16	4	100	4	3	2,2,2,3	24	4	67	4	3	2,2,3,3	36	5	89
4	3	2,3,3,3	54	5	59	4	3	3,3,3,3	81	6	79	4	4	3,3,3,4	108	6	59
4	4	3,3,4,4	144	7	89	4	4	3,4,4,4	192	7	67	4	4	4,4,4,4	256	8	100

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
4	5	4,4,4,5	320	8	80	4	5	4,4,5,5	400	8	64	4	5	4,5,5,5	500	8	51
4	5	5,5,5,5	625	9	82	4	6	5,5,5,6	750	9	68	4	6	5,5,6,6	900	9	57
4	6	5,6,6,6	1080	10	95	4	6	6,6,6,6	1296	10	79	4	7	6,6,6,7	1512	10	68
4	7	6,6,7,7	1764	10	58	4	7	6,7,7,7	2058	11	100	4	7	7,7,7,7	2401	11	85
4	8	7,7,7,8	2744	11	75	4	8	7,7,8,8	3136	11	65	4	8	7,8,8,8	3584	11	57
4	8	8,8,8,8	4096	12	100	5	2	2,2,2,2,2	32	5	100	5	3	2,2,2,2,3	48	5	67
5	3	2,2,2,3,3	72	6	89	5	3	2,2,3,3,3	108	6	59	5	3	2,3,3,3,3	162	7	79
5	3	3,3,3,3,3	243	7	53	5	4	3,3,3,3,4	324	8	79	5	4	3,3,3,4,4	432	8	59
5	4	3,3,4,4,4	576	9	89	5	4	3,4,4,4,4	768	9	67	5	4	4,4,4,4,4	1024	10	100
5	5	4,4,4,4,5	1280	10	80	5	5	4,4,4,5,5	1600	10	64	5	5	4,4,5,5,5	2000	10	51
5	5	4,5,5,5,5	2500	11	82	5	5	5,5,5,5,5	3125	11	66	5	6	5,5,5,5,6	3750	11	55
5	6	5,5,5,6,6	4500	12	91	5	6	5,5,6,6,6	5400	12	76	5	6	5,6,6,6,6	6480	12	63
5	6	6,6,6,6,6	7776	12	53	5	7	6,6,6,6,7	9072	13	90	5	7	6,6,6,7,7	10584	13	77
5	7	6,6,7,7,7	12348	13	66	5	7	6,7,7,7,7	14406	13	57	5	7	7,7,7,7,7	16807	14	97
5	8	7,7,7,7,8	19208	14	85	5	8	7,7,7,8,8	21952	14	75	5	8	7,7,8,8,8	25088	14	65
5	8	7,8,8,8,8	28672	14	57	5	8	8,8,8,8,8	32768	15	100	6	2	2,2,2,2,2,2	64	6	100
6	3	2,2,2,2,2,3	96	6	67	6	3	2,2,2,2,3,3	144	7	89	6	3	2,2,2,3,3,3	216	7	59
6	3	2,2,3,3,3,3	324	8	79	6	3	2,3,3,3,3,3	486	8	53	6	3	3,3,3,3,3,3	729	9	70
6	4	3,3,3,3,3,4	972	9	53	6	4	3,3,3,3,4,4	1296	10	79	6	4	3,3,3,4,4,4	1728	10	59
6	4	3,3,4,4,4,4	2304	11	89	6	4	3,4,4,4,4,4	3072	11	67	6	4	4,4,4,4,4,4	4096	12	100
6	5	4,4,4,4,4,5	5120	12	80	6	5	4,4,4,4,5,5	6400	12	64	6	5	4,4,4,5,5,5	8000	12	51
6	5	4,4,5,5,5,5	10000	13	82	6	5	4,5,5,5,5,5	12500	13	66	6	5	5,5,5,5,5,5	15625	13	52
6	6	5,5,5,5,5,6	18750	14	87	6	6	5,5,5,5,6,6	22500	14	73	6	6	5,5,5,6,6,6	27000	14	61
6	6	5,5,6,6,6,6	32400	14	51	6	6	5,6,6,6,6,6	38880	15	84	6	6	6,6,6,6,6,6	46656	15	70
6	7	6,6,6,6,6,7	54432	15	60	6	7	6,6,6,6,7,7	63504	15	52	6	7	6,6,6,7,7,7	74088	16	88
6	7	6,6,7,7,7,7	86436	16	76	6	7	6,7,7,7,7,7	100842	16	65	6	7	7,7,7,7,7,7	117649	16	56
6	8	7,7,7,7,7,8	134456	17	97	6	8	7,7,7,7,8,8	153664	17	85	6	8	7,7,7,8,8,8	175616	17	75
6	8	7,7,8,8,8,8	200704	17	65	6	8	7,8,8,8,8,8	229376	17	57	6	8	8,8,8,8,8,8	262144	18	100
7	2	2,2,2,2,2,2,2	128	7	100	7	3	2,2,2,2,2,2,3	192	7	67	7	3	2,2,2,2,2,3,3	288	8	89
7	3	2,2,2,2,3,3,3	432	8	59	7	3	2,2,2,3,3,3,3	648	9	79	7	3	2,2,3,3,3,3,3	972	9	53
7	3	2,3,3,3,3,3,3	1458	10	70	7	3	3,3,3,3,3,3,3	2187	11	94	7	4	3,3,3,3,3,3,4	2916	11	70
7	4	3,3,3,3,3,4,4	3888	11	53	7	4	3,3,3,3,4,4,4	5184	12	79	7	4	3,3,3,4,4,4,4	6912	12	59

N	m	S	W	b	η	N	m	S	W	b	η	N	m	S	W	b	η
7	4	3,3,4,4,4,4,4	9216	13	89	7	4	3,4,4,4,4,4,4	12288	13	67	7	4	4,4,4,4,4,4,4	16384	14	100
7	5	4,4,4,4,4,4,5	20480	14	80	7	5	4,4,4,4,4,5,5	25600	14	64	7	5	4,4,4,4,5,5,5	32000	14	51
7	5	4,4,4,5,5,5,5	40000	15	82	7	5	4,4,5,5,5,5,5	50000	15	66	7	5	4,5,5,5,5,5,5	62500	15	52
7	5	5,5,5,5,5,5,5	78125	16	84	7	6	5,5,5,5,5,5,6	93750	16	70	7	6	5,5,5,5,5,6,6	112500	16	58
7	6	5,5,5,5,6,6,6	135000	17	97	7	6	5,5,5,6,6,6,6	162000	17	81	7	6	5,5,6,6,6,6,6	194400	17	67
7	6	5,6,6,6,6,6,6	233280	17	56	7	6	6,6,6,6,6,6,6	279936	18	94	7	7	6,6,6,6,6,6,7	326592	18	80
7	7	6,6,6,6,6,7,7	381024	18	69	7	7	6,6,6,6,7,7,7	444528	18	59	7	7	6,6,6,7,7,7,7	518616	18	51
7	7	6,6,7,7,7,7,7	605052	19	87	7	7	6,7,7,7,7,7,7	705894	19	74	7	7	7,7,7,7,7,7,7	823543	19	64
7	8	7,7,7,7,7,7,8	941192	19	56	7	8	7,7,7,7,7,8,8	1075648	20	97	7	8	7,7,7,7,8,8,8	1229312	20	85
7	8	7,7,7,8,8,8,8	1404928	20	75	7	8	7,7,8,8,8,8,8	1605632	20	65	7	8	7,8,8,8,8,8,8	1835008	20	57
7	8	8,8,8,8,8,8,8	2097152	21	100	-	-	-	-	-	-	-	-	-	-	-	-

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Conference Presentations:
B. J. Wedemire and B. R. Petersen, "Fractional-Bits per Symbol Using Non-Powers-of-2-Point Constellations," in *ICTRS 2021: 10th International Conference on Telecommunications and Remote Sensing*, vol. 1, Virtual Conference, Sofia, Bulgaria, ACM, Nov. 15-16, 2021., https://doi.org/10.1145/3495535.3495540.

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Alex DiTommaso, Alex Voisine, Ben Wedemire, Sarah Siddiqua, Marco Mendonça, Shiva V. I. David, Koffi V. C. Kevin de Souza, Tobias Nießen, Alexander G. B. Colpitts, Nicholas Kozma, Troy T. Lavigne, William E. Ward, Richard B. Langley, Tiger Jeans, Yassine Bouslimani, Saadmaan Rahman, Brent R. Petersen,"VIOLET, a Student CubeSat for Space Weather", COSPAR 2021, *43rd COSPAR Scientific Assembly*, Virtual Presentation, Sydney, Australia, Jan. 28 - Feb. 4, 2021.